CHAPTER 6

TRADE LIBERALISATION, TERMS OF TRADE AND SKILL FORMATION IN A NORTH – SOUTH MODEL
6.1. Introduction:

The literature on North–South models (Findlay (1980), Burgstaller and Saavedra-Rivano (1984), Quibria (1986), Sarkar (1989) and Gupta (1996)) mostly contains the idea that the labour market in both the North and South are composed of homogeneous units. But, in reality, though the Northern labour market consists mostly of skilled workers, the labour force in the South is split up into skilled and unskilled workers. The advent of globalisation and openness in the developing countries coupled with technological progress, which helps to introduce sophistication in the nature of the products of the manufacturing sector has raised the internal demand for skill. Increasing trade and investment possibilities require specialised skills that may be in short supply in a developing country. So, with expansion of trade and liberalisation of foreign investment, the process of skill formation gets a natural boost, which leads to an increase in the quantity of skilled labour in the South. In fact, deregulation, liberalisation and globalisation help in creating new opportunities to talented and skilled workers. Consequently, it is in their own interest that some unskilled workers get themselves educated and trained, which thus leads to skill formation rather than choosing the option of remaining employed as unskilled workers. As in Marjit & Acharyya (2003), here also we consider that the process of skill formation in the South involves both capital and unskilled labour.
In a North–South interaction model, terms of trade is endogenously determined and it plays an important role in influencing the factor-returns. In Findlay (1980), we see that the terms of trade has been regarded as a key index of the distribution of the benefits from the international division of labour and the development prospects of the South. Thus it may be interesting to examine how terms of trade for the South changes and how does it influence the supply of skilled labour of the South.

We develop a North-South model in which the North produces a single good and the South is a two-sector economy. The export sector of the South uses unskilled labour and the import sector uses skilled labour as specific factors. Skilled labour has been considered as an intermediate input and its supply depends on the skill formation process. This process requires capital and unskilled labour as inputs. We consider that there is free and perfect mobility of capital between the North and the South as is found in Burgstaller and Saavedra-Rivano (1984), Quibria (1986) and Gupta (1996). The South is thus a net receiver of Northern capital. The foreign capital that comes into the South is assumed to be a perfect substitute to the domestic capital and its inflow depends, among others, on the stock of Northern capital and the net rate of return to foreign capital used in the South. In this North-South trade model, terms of trade, defined as the ratio of export price to import price, has been determined by the trade-balance condition. Although our analysis follows Gupta’s (1996) approach in some respects, there exists wide difference in the characterisation of the Southern economy which is relevant for the purpose of our analysis. Gupta (1996) has considered a HT framework for the South whereas in our
model the South is a neoclassical full employment economy. In Gupta's (1996) model, there is no classification of the total labour force into skilled and unskilled sections, which has been incorporated in our model. On the other hand, we have not considered any dynamic analysis as done by Gupta (1996).

The present chapter analyses how a policy of liberalisation taken in the South can affect the supply of skilled personnel by influencing the terms of trade and also the internal demand for skill. Liberalisation of imports in the South would deteriorate its terms of trade. If Southern exportables are relatively less capital-intensive, return to its capital will increase. Consequently, larger inflow of foreign capital provides the much needed resource for skill formation process in the South. We also analyse the impact of a rise in the stock of capital upon the terms of trade and the supply of skilled labour. The growth in the stock of capital improves the terms of trade for the South, and it may also raise the supply of skilled labour.

We organise the chapter in the following manner. Section 6.2 analyses the model. Section 6.3 describes some comparative static results. Finally, concluding remarks are made in section 6.4.

6.2. The Model:

In this section, a model has been developed on the basis of the fact that there are two regions in the world: the North and the South. We consider that the two regions differ in their endowment of capital stocks and also in the characterisation of their labour
markets. The North is considered to be a Solow type neoclassical economy. It produces only one good with two factors of production: capital & labour. The product of the North, $Y_N$, is used as export to the South and as a capital good. A part of the Northern capital flows to the South. We assume $K_N$ as the total stock of Northern capital and $K_F$ as the amount of Northern capital that flows to the South. The Northern labour force, $L_N$, consisting of homogenous units, is fully employed with flexible wages.

The Northern production function is

$$Y_N = F_N(K_N - K_F, L_N)$$

(6.1)

In equation (5.1), $K_N - K_F$ is the net quantity of capital used in the North. It obeys all the standard neoclassical properties, where the marginal productivity of capital is given by

$$\frac{\partial Y_N}{\partial (K_N - K_F)} = F_N'$$

(6.2)

The free mobility of capital from the North to the South requires that the effective rate of return on the Northern capital used in the South, $r_{NS}$, should be equal to the marginal productivity of capital in the North. Thus

$$F_N' = r_{NS}$$

(6.3)

The South produces two traded goods. Sector x produces an exportable good, $X$, with the help of unskilled labour and capital; while sector y produces an importable good, $Y$, with capital and skilled labour. The skilled labour is considered as an intermediate input. Although it is used as an input in sector y, we should note that the skilled labour is an output of the skill-producing sector. The creation of this particular product, i.e., skilled

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^{48}$ $Y_N$ may be used for capital formation if we consider a dynamic version of the model.

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^{49}$ This type of Northern production function has been used by Gupta (1996).
labour, requires capital and unskilled labour as inputs. A section of the unskilled labour force takes training and education for skill formation without working as unskilled labourers. We assume that final product sectors use neoclassical technology while the intermediate good (i.e., skilled labour) producing sector uses fixed coefficient technology. The skill formation process converts one unskilled labour into one skilled labour.\footnote{This is also implicitly assumed in Marjit and Acharyya (2003). This is the basis for the use of fixed coefficient technology in the skill-producing sector.} We also assume that importables are relatively more capital intensive than the exportables and the latter is relatively more capital intensive than the skilled sector. There is full employment with free mobility and flexible prices of capital and unskilled labour. Since skilled labour is an intermediate input, it is also fully employed. It is further assumed, following Marjit & Acharyya (2003), that at any particular point of time the South is endowed with a stock of capital, consisting of domestic and foreign units,\footnote{We have thus assumed that, for the South, domestic capital and foreign capital are perfect substitutes. This assumption is borrowed from the literature. See Marjit and Acharyya (2003), Gupta (1996), etc.} and unskilled labour.

We use the following notations to describe the Southern economy: \( x \) and \( y \) represent two traded sectors and they produce outputs \( X \) and \( Y \) respectively.

\( P_i \) – price of the product of the \( i \)-th sector, with \( i = x \) & \( y \);

\( P \) – relative price of the exportables in terms of the importables or the terms of trade;

\( (i.e., P = \frac{P_x}{P_y}) \)

\( W_s \) – wage rate of the skilled labour;

\( W_u \) – wage rate of the unskilled labour;

\( r \) - rate of return on Southern capital;
$a_{ij}$ – quantity of $i$-th input required to produce one unit of the $j$-th good, with $i = K, U, S$
& $j = X, Y, S$;

$S$ – quantity of skilled labour;

$U$ – stock of unskilled labour;

$K_S$ – stock of domestic capital in the South;

$K_F$ – stock of foreign capital used in the South;

$M_S$ – imports of the South;

$E_S$ – exports of the South;

$\theta$ – proportion of the foreign capital income repatriated to the North;

$k$ – ratio of the endowment of total capital stock of the South to its endowment of unskilled labour;

$k_i$ – capital intensity of the $i$-th sector.

$Y_d$ – demand for importables in the South

From the competitive equilibrium conditions, we get the following price-unit cost equations:

$$P = a_{ux} W_U + a_{kx} r \quad (6.4)$$

$$\& l = a_{uy} W_S + a_{ky} r \quad (6.5)$$

Since we assume that the skill formation process requires capital and unskilled labour, the wage rate of the skilled labour is equivalent to the foregone wage of a typical unskilled labour and the cost of providing training to him.\(^{52}\) Thus we have

\(^{52}\) Free mobility of capital within the South ensures equality of the rate of return in the sector producing skilled labour.
\[ W_S = \bar{a}_{us} W_U + \bar{a}_{ks} r \]  \hspace{1cm} (6.6)

In equation (6.6) above, \( \bar{a}_{us} \) is equal to unity and the wage rate of the skilled labour is flexible since \( W_U \) and \( r \) are flexible.

Full employment condition prevails in the market for capital. This is shown in equation (6.7)

\[ a_{KX} X + a_{KY} Y + \bar{a}_{KS} S = K \]  \hspace{1cm} (6.7)

\[ \text{with } K = K_S + K_F \]  \hspace{1cm} (6.8)

When the Northern capital stock \( (K_N) \) is given, the inflow of foreign capital \( (K_F) \) depends on the net rate of return to foreign capital used in the South.\(^{53}\) So, we have

\[ K_F = K_F (r_{ns}) \]  \hspace{1cm} (6.9)

We should like to mention here that "0" portion (where, \( 0 < \theta < 1 \)) of the return to the foreign capital income is repatriated to the North. So the net rate of return per unit of foreign capital that the North receives from the South is^\(^{54}\)

\[ r_{ns} = \theta r \]  \hspace{1cm} (6.10)

Again, by the condition of full employment in the market for unskilled labour we have

\[ a_{UX} X + \bar{a}_{us} S = U \]  \hspace{1cm} (6.11)

The stock of skilled labour is demand determined. So, it is given by

\[ S = a_{SY} Y \]  \hspace{1cm} (6.12)

\(^{53}\) Let us recall that the marginal productivity of Northern capital is a decreasing function of \( (K_N - K_F) \).

\(^{54}\) As \( K_S \) and \( K_F \) are perfect substitutes the rates of return are same.
The right hand side of equation (6.12) gives the demand for skilled labour as intermediates. Thus in the model described above we have two final goods: X and Y and two primary factors of production, capital and unskilled labour.

In a North-South model, the relative commodity price or terms of trade can be determined by the trade balance condition of the South. In the South the export supply function has been given by

\[ E_s = E_s(P) \quad (6.13) \]

with \( \frac{\partial E_s}{\partial P} < 0 \), since Northern demand for South’s exports falls as the relative price rises.

Again, the demand function for importables is given by

\[ Y_d = Y_d(P) \quad (6.14) \]

Thus the import demand of the South becomes the difference between the demand for importables, \( Y_d \) and the output of importables, \( Y \),

i.e.,

\[ M_s = Y_d(P) - Y = M_s(P, Y) \quad (6.15) \]

with \( \frac{\partial M_s}{\partial P} > 0 \), since the South’s demand for imports rise as the relative price of the importables falls and \( \frac{\partial M_s}{\partial Y} < 0 \), since the South’s import demand falls when the production of importables rises.

So, the trade balance condition of the South becomes:

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55 I am indebted to an anonymous referee for specifying such an export supply function and the import demand function as described in equation (6.15). We assume here that there is equilibrium in the world market for exports. So, the supply function for exports of the South may be interpreted as the demand function for imports of the North.

56 The demand for importables is, in fact, function of both relative commodity price (P) and national income (I). But it could be shown that the income of the South, again, depends, under some reasonable assumptions, upon the relative commodity price or the international terms of trade (P). So, the demand for the importables is a function of P only. For more detail, see the appendix, 6.1.
The trade equilibrium is stable on the assumption that the Marshall-Lerner condition for trade equilibrium holds.\(^57\)

Equations (6.4) to (6.16) thus represent a system of 13 equations with 13 variables: \(r\), \(W_u\), \(W_s\), \(X\), \(Y\), \(S\), \(P\), \(K\), \(K_F\), \(r_{NS}\), \(Y_d\), \(E_s\) and \(M_s\). We can solve the system in the following way:

From equation (6.6) we can write \(W_s = W_s (W_u, r)\). So, from equation (6.5) we get \(W_u = W_u (r)\). Hence, substituting \(W_u = W_u (r)\) in equation (6.4), we find that \(r\) can be expressed as a function of \(P\) only, i.e., \(r = r (P)\). Thus the factor prices can be determined when the relative commodity price or the international terms of trade \((P)\) is determined. It is easy to see that \(r\) is negatively related to the terms of trade, \(P\). The result follows from the straightforward application of the Stolper-Samuelson theorem, when exportables are assumed to be relatively less capital intensive than the importables.\(^58\) Once the factor prices are determined, the factor intensities are also determined and they are functions of the relative commodity price. But we cannot determine the equilibrium value of the terms of trade from the trade balance condition alone, as described by equation (6.16), independently of the output of the tradeables. Thus the output levels of the tradeables, the level of skilled labour and the terms of trade are simultaneously determined. From equation (6.8), we see that total capital stock \((K)\) of the South depends on \(K_S\) and \(K_F\). But by equations (6.9) and (6.10), \(K_F\) depends on \(r\) when \(\theta\) is given. Since \(r\) is a function of \(P\)

\(^57\) The Marshall-Lerner condition states that the sum of the elasticity of demand for imports of the North (or, the elasticity of supply of exports of the South) and the elasticity of demand for imports in the South, both considered in absolute sense, should exceed unity.

\(^58\) For mathematical derivation see the appendix, 6.2.
then $K_F$ is also a function of $P$ only, i.e., $K_F = K_F(P)$, with $K'_F < 0$. Larger inflow of foreign capital takes place when net rate of return to foreign capital rises due to worsening of the terms of trade for the South. Hence we can determine the equilibrium values of $X$, $Y$, $S$ and $P$ from equations (6.7), (6.11), (6.12) and (6.16). We get positive values of $X$ and $Y$ on the assumption that the factor endowment ratio, $k$, is a weighted average of $(k_S + k_Y)$ and $k_X$ with $(k_S + k_Y) > k_X$.\(^\text{59}\) Again, we see that the relative commodity price, the factor prices, the factor intensities and the output levels depend on the exogenously given factor endowment levels of the unskilled labour and domestic capital stock ($K_S$) only, as inflow of foreign capital depends on the terms of trade\(^\text{60}\). When $P$ is determined, we can determine the rest of the variables of the system. This completes the working of the model.

### 6.3 The Comparative Static Exercises:

Recently, developing countries have been adopting massive liberalisation programmes either on the basis of the advice of the International Monetary Fund (IMF)/World Bank or out of their own compulsions. In terms of the model presented above, we thus study the impact of trade liberalisation measures followed in the South and the impact of rise in the stock of domestic capital upon terms of trade and the supply of skilled labour of the South.

In order to study the impact of trade liberalisation policies, we now introduce a parameter, $\beta$, representing import control measure, like tariffs and other measures, in the

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\(^{59}\) For detail, see the appendix, 6.3.

\(^{60}\) The decomposability property, as found in Gupta (1996), no longer holds here.
import demand function of the South. So, we rewrite equation (6.15) of the model as:

\[ M_S = M_S(P, Y, \beta) \quad (6.15') \]

with \( \frac{\partial M_S}{\partial \beta} < 0 \), since as \( \beta \) falls the imports of the South rise. We should note here that the introduction of \( \beta \) does not alter the basic results of the model.

Let us assume that the South liberalises its imports and reduces tariffs on imports from the North. To study the impact of a fall in \( \beta \) let us express equations (6.7) and (6.11) by using equation (6.12) in terms of equations (6.7') and (6.11') respectively. They are shown below as

\[ a_{KX}(P)X + a_{KY}(P)Y + \alpha_{KS}a_{SY}(P)Y = K = K_S + K_F(P) \quad (6.7') \]

\[ a_{UX}(P)X + \alpha_{US}a_{SY}(P)Y = U \quad (6.11') \]

By using equation (6.15'), the trade balance condition, as given by equation (6.16), can now be rewritten as

\[ P E_S(P) - M_S(P, Y, \beta) = 0 \quad (6.16') \]

By total differentiation of (6.7'), (6.11') and (6.16') we get the equations (6.17), (6.18) and (6.19) respectively as

\[ (a'_{KX}X + a'_{KY}Y + \alpha_{KS}a'_{SY}Y - K'_F) dP + a_{KX}dX + (a_{KY} + \alpha_{KS}a_{SY})dY = dK_S \quad (6.17) \]

\[ (a'_{UX}X + a'_{SY}Y) dP + a_{UX}dX + a_{SY}dY = dU \quad (6.18) \]

\[ (E_S + P'E_{SP} - M'_{SP})dP - M'_{SY}dY = M'_S \beta d\beta \quad (6.19) \]

Solving equations (6.17), (6.18) and (6.19) for \( dP/\beta \) and \( dY/\beta \), we get

\[ \frac{dP}{\beta} = \left( \frac{1}{\Delta} \right) M'_S \beta a_{UX} \alpha_{US}a_{SY} \{ k_x - (k_S + k_F) \} \quad (6.20) \]

and \( \frac{dY}{\beta} = (1/\Delta) M'_S \beta \{(a'_{KX}X + a'_{KY}Y + \alpha_{KS}a'_{SY}Y - K'_F)a_{UX} - a_{KX}(a'_{UX}X + a'_{SY}Y)\} \)

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\[ ^{61} \text{We know } a_{xx}(W_U(r(P)), r(P)) = a_{xx}(P). \text{ For more detail, see the appendix, 6.4.} \]
or, \( \frac{dY}{d\beta} = \left( \frac{1}{\Delta} \right) M'_{SP} \left\{ (a'_{kx} X + a'_{ky} Y - a_{kx} a'_{ux} X - K'_F) 
+ a'_{sy} Y a_{ux} \bar{a}_{us}(k_s - k_x) \right\} \)  

(6.21)

where \( \Delta = \left\{ - a'_{kx} X a_{ux} M'_{SY} - a'_{ky} Y a_{ux} M'_{SY} + K'_F a_{ux} M'_{SY} + a_{kx} a'_{ux} X M'_{SY} + a'_{sy} Y M'_{SY} a_{ux} \bar{a}_{us}(k_x - k_s) \right\} + E_S \left\{ 1 + \left( \frac{P}{E_S} \right) E'_{SP} - (M_S/P)M'_{SP} \right\} a_{ux} \bar{a}_{us} a_{sy} \left\{ k_x - (k_x + k_y) \right\} \)

Now, given \( a'_{kx} > 0, a'_{ky} > 0, a'_{ux} < 0, a'_{sy} < 0, M'_{SY} < 0, k_x < (k_x + k_y), k_x > k_s, \bar{a}_{us} = 1, \)
and \( 1 + \left( \frac{P}{E_S} \right) E'_{SP} - (M_S/P)M'_{SP} < 0 \) by the Marshall-Lerner condition, we get \( \Delta > 0. \)

So, for given \( M'_S \beta < 0, \frac{dP}{d\beta} > 0, \) terms of trade will deteriorate as tariffs are reduced. In fact, import liberalisation leads to an increase in the demand for imports and thereby worsens the terms of trade for the South. Again, given the value of \( \Delta, \) the sign restriction upon \( M'_S \beta \) and the assumption about the factor intensity ranking, we see that \( \frac{dY}{d\beta} < 0, \) i.e., the production of importables rises when the South liberalises its imports and the relative price of importables rises.

We can write equation (6.12) as: \( S = a_{sy}(r(P))Y \)  

(6.12')

Differentiating (6.12') with respect to \( \beta \) we get

\[
\frac{dS}{d\beta} = a'_{sy} Y \frac{dP}{d\beta} + a_{sy} \frac{dY}{d\beta}
\]

(6.22)

Now, given \( a'_{sy} < 0, \frac{dP}{d\beta} > 0 \) and \( \frac{dY}{d\beta} < 0, \) we have \( \frac{dS}{d\beta} < 0, \) i.e., the stock of skilled labour in the South will rise due to the rise in the internal demand for skill when import liberalisation policies are adopted. The internal demand for skilled labour rises as

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62 For detail derivation of \( \Delta, \) and for justification of the signs of \( a'_{kx}, a'_{ky}, a'_{ux}, \) etc., see the appendix, 6.5.
the per unit requirement of skilled labour \( (a_{sv}) \) and the output of skilled labour-intensive good \( (Y) \) rise when terms of trade \( (P) \) deteriorates.

We have seen from the model that inflow of foreign capital into the South is inversely related to the terms of trade faced by it. Thus when liberalisation of trade leads to the fall in terms of trade, then there will be a rise in the inflow of foreign capital from the North. Now we can summarise our results due to trade liberalisation in terms of proposition 6.1.

**Proposition 6.1:** *When the South liberalises its trade in the form of reducing tariffs, its terms of trade will deteriorate, but the inflow of foreign capital and the stock of skilled labour in the South will rise.*

Let us now assume that the stock of capital in the South rises due to domestic capital formation. Thus by solving equations (6.17), (6.18) and (6.19) for \( dP/dK_s \) and \( dY/dK_s \) we get

\[
dP / dK_s = - (1 / \Delta) a_{ux} M'_{sy}
\]

(6.23)

Since \( M'_{sy} < 0 \) and \( \Delta > 0 \), we get \( dP / dK_s > 0 \), i.e., rise in the stock of domestic capital improves the terms of trade for the South. Again, \( dY / dK_s = - (1 / \Delta) a_{ux} E_s \{1 + (P/E_s)E_{sp}' - (M_s/P)M'_{sp}\} \).

Now, given the Marshall-Lerner condition and the value of \( \Delta \), we get \( dY / dK_s > 0 \). When capital stock rises due to an increase in the stock of domestic capital, the production of
the relatively capital intensive product, importables, will rise by the application of the Rybczynski theorem. So, the Southern demand for imports falls and its terms of trade improves.

In order to get the impact of rise in the stock of capital upon the stock of skilled labour let us differentiate equation (6.12') with respect to $K_S$. Thus we get

$$\frac{dS}{dK_S} = a_{sy}' Y \frac{dP}{dK_S} + a_{sy} \frac{dY}{dK_S}$$

(6.24)

Given $a_{sy}' < 0$, and $\frac{dP}{dK_S} > 0$, the first term on the right hand side of the above relation becomes negative and when $\frac{dY}{dK_S} > 0$, the second term of the right hand side becomes positive. Thus when stock of domestic capital rises and the return to it falls due to the Stolper-Samuelson effect resulting from the improvement in terms of trade, the factor substitution leads to fall in the unit requirement of skilled labour. But the rise in the output of importables due to the rise in the stock of domestic capital raises the need for skilled labour. Hence, we cannot say whether the positive impact of rise in the output (the second term in equation (6.24)) is greater than or less than or equal to the negative impact resulting from a fall in the unit requirement of skilled labour due to an improvement in terms of trade (the first term in equation (6.24)). So, there is an ambiguity about the impact of the growth in the stock of domestic capital upon the stock of skilled labour. Proposition 6.2 below summarises this result.

**Proposition 6.2:** Any rise in the stock of domestic capital in the South improves its terms of trade but its impact upon the stock of skilled labour is ambiguous.
6.4. Concluding Remarks:

We have considered a North-South model with free and perfect mobility of capital from the North to the South. The North is a one-sector and the South is a two-sector economy. We have considered full employment of the factors of production in both regions. The two regions differ in their endowment of capital stocks and also in their characterisation of the labour markets. The North is endowed with larger stock of capital than the South. Again, while the labour force of the North is composed of homogeneous units the Southern labour force can be decomposed into skilled and unskilled workers. A section of the unskilled labour is converted into skilled labour through the provision of education and training. This process of skill formation requires capital and unskilled labour.

This chapter analyses the fact that terms of trade, although endogenously determined, can play a major role in influencing the stock of skilled labour in the South and also the inflow of foreign capital from the North into the South. So, the South can manipulate its trade policy to influence the terms of trade and the output of the importables to increase its stock of skilled labour and the inflow of foreign capital. We have seen that if the South liberalises its trade by reducing or lifting the import control measures, then terms of trade for the South would deteriorate but there would be an increase in its stock of skilled labour and a larger inflow of foreign capital from the North would take place. It is also seen that if the stock of domestic capital increases in the South, then the impact upon the stock of skilled labour would remain indeterminate but the terms of trade for the South would surely improve.
APPENDIX 6

6.1: Justification for equation (6.14) of the text

Let us consider that the demand for importables in the South is a function of the relative commodity price \( P \) and the national income of the South \( I \).

Thus \( Y_d = Y_d(P, I) \) \hspace{1cm} (A.6.1)

Where \( I = rK_S + (1-\theta) rK_F + W_U(U - S) + W_S S \)

\[= rK_S + (1-\theta) rK_F + W_U U + (W_S - W_U) S\]

From equation (6.6) we get \( W_S = W_S(W_U, r) \). Substituting for \( W_S \) in equation (6.5) we can express \( W_U = W_U(r) \). Finally, substituting for \( W_U = W_U(r) \) in equation (6.4) we can express \( r = r(P) \).

In terms of our model \( r = r(P), W_U = W_U(r(P)), W_S = W_S(r(P)), S = S(P, Y) \), where ‘\( Y \)’ represents the output of the importables. Since, for given factor endowments, \( S, P \) and \( Y \) are simultaneously determined, we can ignore the influence of \( P \) and \( Y \) upon \( S \). Thus, for given factor endowments, we can write ‘\( I \)’ as a function of \( r, W_U \) and \( W_S \).

Using the above relations, equation (A.6.1) can be written as

\[Y_d = Y_d(P, r(P), W_U(r(P)), W_S(r(P)))\]

So, \( Y_d = Y_d(P) \)

6.2: Proof of the relation between ‘\( r \)’ and ‘\( P \)’

Equation (6.5) of the text can be written as

\[a_{SY} W_S = 1 - a_{KY} r\]

or, \( W_S = 1/a_{SY} - a_{KY}/a_{SY} r \)

So, equation (6.6) of the text becomes
Solving equation (6.4) of the text and (A6.2) for ‘r’ we get

\[ r = \frac{(a_{Kv} / a_{SY} - P)}{\Delta} \]

so, \( \frac{dr}{dP} = -\frac{1}{\Delta} \)

where \( \Delta = a_{UX} (\bar{a}_{KS} + a_{Kv} / a_{SY}) - a_{KX} \)

\[ = a_{UX} \bar{a}_{US} \{(\bar{a}_{KS} / \bar{a}_{US} + a_{Kv} / a_{SY}) - a_{KX} / a_{UX}\} \]

\[ = a_{UX} \bar{a}_{US} \{(k_s + k_y) - k_x\} \]

Now, \( \Delta > 0 \) by the assumption that \( k_x < (k_s + k_y) \)

So, \( \frac{dr}{dP} < 0 \)

**6.3: Solution for the output levels**

Combining equations (6.7) and (6.12) we get

\[ a_{Kx} X + a_{Ky} Y + \bar{a}_{KS} a_{SY} Y = K \] \hspace{1cm} (A.6.3)

Again, combining equations (6.11) and (6.12) we get

\[ a_{UX} X + a_{SY} Y = U \] \hspace{1cm} (A.6.4)

Let us now recall equation (6.16) as given by

\[ P E_S (P) = M_S (P, Y) \] \hspace{1cm} (6.16)

Solving equations (A.6.3), (A.6.4) and (6.16) of the text we get

\[ X = \frac{1}{\Delta} E_S (P) \bar{a}_{US} a_{SY} U \{ k - (k_s + k_y)\} \]

Where \( \Delta = E_S (P) \bar{a}_{US} a_{SY} a_{UX} \{ k_x - (k_s + k_y)\} \)

\( \Delta < 0 \) on the assumption that \( k_x < (k_s + k_y) \). So, \( X > 0 \), when we assume that \( k < (k_s + k_y) \)

Again \( Y = \frac{1}{\Delta} E_S (P) a_{UX} U \{ k_x - k\} \). We get \( Y > 0 \), when \( \Delta < 0 \) and \( k_x < k \).
6.4: Justification for equations (6.7') and (6.11')

From the analysis of the model we see that \( W_S = W_S(W_U, r) \) and \( W_U = W_U(r) \).

Hence, \( W_S = W_S(r) \). Again, we know that \( r = r(P) \).

So, we get \( a_{kX} = a_{kX}(W_U(r(P)), r(P)) = a_{kX}(P) \). Similarly, \( a_{kY} = a_{kY}(W_S(r(P)), r(P)) = a_{kY}(P) \), \( a_{UX} = a_{UX}(W_U(r(P)), r(P)) = a_{UX}(P) \) and \( a_{SY} = a_{SY}(W_S(r(P)), r(P)) = a_{SY}(P) \).

6.5: Derivation of the value of \( \Delta \)

\[
\Delta = (a'_{kX} X + a'_{kY} Y + \bar{a}_{US} a'_{SY} Y - K_F') (-a_{UX} M'_{SY}) - a_{kX} [a'_{UX} X + a'_{SY} Y] (-M'_{SY})
\]

\[-Es \{1 + (P/Es)E_{SP}' - (M_S/P)M_{SP}' \} a_{SY}] + (a_{kY} + \bar{a}_{US} a_{SY}) \{ -Es \{1 + (P/Es)E_{SP}' - (M_S/P)M_{SP}' \} \}

(In the above relation we make use of the relation that \( Es = M_S/P \), when trade is balanced.)

So, \( \Delta = (-a'_{kX} X a_{UX} M'_{SY} - a'_{kY} Y a_{UX} M'_{SY} + a_{kX} a'_{UX} X M'_{SY} + a'_{SY} YM'_{SY} a_{UX} \bar{a}_{US} (k_x - k_s)) + Es \{1 + (P/Es)E_{SP}' - (M_S/P)M_{SP}' \} a_{UX} \bar{a}_{US} a_{SY} \{ k_x - (k_s + k_y) \} \}

When \( P \) falls and \( 'r' \) rises, the use of capital becomes costlier. So, factor substitution leads to fall in the use of capital and rise in the use of unskilled labour and skilled labour.

So, \( a'_{kX} > 0, a'_{kY} > 0, a'_{UX} < 0 \) and \( a'_{SY} < 0 \). Again, we know \( M'_{SY} < 0, k_x < (k_s + k_y), k_x > k_s, \bar{a}_{US} = 1 \) and \( \{1 + (P/Es)E_{SP}' - (M_S/P)M_{SP}' \} < 0 \) by the Marshall-Lerner condition.

Thus, \( \Delta > 0 \).