ABSTRACT

This thesis entitled “Accelerated Life Test Designs” is submitted to the Aligarh Muslim University, Aligarh, India, to supplicate the degree of Doctor of Philosophy in Statistics. It embodies the research work carried out by me in the Department of Statistics and Operations Research, Aligarh Muslim University, Aligarh.

Models and methods of accelerated life testing (ALTs) are useful when technical systems under test tend to have very long lifetimes. Under normal operating conditions, as systems usually last long, the corresponding life tests become very time-consuming and expensive. In these cases, accelerated life tests (ALTs) can be applied to reduce the experimental time and hence the cost associated with experiment. In Accelerated life test (ALT) the failure times of component are accelerated by applying higher stress. This implies that the failure time is a function of stress factor. Higher stress may bring quicker failure but hopefully do not change the failure mechanisms of the component. By this process failures which under normal conditions would occur only after a long testing can be observed quicker and the size of data can be increased without a large cost and long time. Most of the ALT models have two assumptions, first a life distribution for a unit’s behavior at a constant stress level, and second is an assumed relationship between the life characteristics and the stress factor which expresses the effect of changing factors like temperature, voltage, humidity on a product’s failure time. Based on these assumptions, the life distribution under normal stress levels can be estimated.

This thesis consists of different designs of ALTs such as Step Stress Accelerated Life Test (SSALT), Partially Accelerated Life Test (PALT) and
Constant Stress Accelerated Life Test (CSALT) using Geometric Process (GP).
The method of maximum likelihood (ML) estimation is used to obtain the
estimates of model parameters because it provides estimators that have both a
reasonable intuitive basis and many desirable statistical properties. In order to
obtain consistent estimators, the minimum variance principle of estimator is
used. In order to get the asymptotic variance (AV) of the ML estimator, the
Fisher information matrix is constructed. In particular, this thesis focused on
the derivation of different ALT plans and designs for the lifetimes of units that
are assumed to follow different life distributions at use stress with different
types of censoring schemes.

This thesis consists of six chapters. Chapter 1 is the usual introductory
chapter. It provides a detailed introduction to ALT designs and their analysis.
An overview of the available literature is also given in this chapter.

In Chapter 2, a simple SSALT is applied to the Pareto distribution, with a
scale parameter which is a log-linear function of the stress and a cumulative
exposure (CE) model is assumed. ML estimates of parameters are obtained.
The AV of desired ML estimates is then obtained by using the Fisher
information matrix, and then by using this AV the asymptotic confidence
intervals (CIs) of parameters are derived. An optimal plan for the stress change
times which minimizes the AV of the ML estimate of the $P^{th}$ percentile of the
lifetime distribution at normal stress condition $t_p(x_0)$ is also discussed. A
simulation study is also presented to demonstrate the performances of the
estimates.

In ALTg analysis, there are situations where life stress relationship is not
known or cannot be assumed. In such situations, partially accelerated life tests
(PALTs) are used. In PALT, test units are run at both use and accelerated
Absract

conditions. Constant stress partially accelerated life test (CSPALT) and step
stress partially accelerated life test (SSPALT) are two commonly used methods
in PALT analysis. In CSPALT products are tested at either normal use or
accelerated condition only until the test is terminated. In SSPALT, a sample of
test items is run at use condition and, if it does not fail for a specified time, then
it is run at accelerated condition.

Chapter 3 deals with a simple CSPALT plan for type-I censored data
when the lifetimes of test item are assumed to follow inverted Weibull
distribution. The ML estimates are obtained for the distribution parameter and
acceleration factor (AF). Asymptotic variance and covariance matrix of the
estimators is then obtained by using the Fisher information matrix. CIs for
parameters and AF are also obtained. A simulation study is performed to
illustrate the statistical properties of the parameters and the confidence bounds.

ALTg, generally, deals with the log linear function between life and stress
to obtain the estimates of original parameters of the life distribution. The log
linear relationship between life and stress is just a simple re-parameterization
of the original parameter and hard to use in mathematical calculations.
Therefore, it is preferable from statistical point of view that we work with the
original parameters instead of developing inferences for the parameters of the
log-linear link function. In this situation, the use of GP may be a good
alternative in ALTg to obtain the estimates of original parameters of life
distribution directly.

In Chapter 4, an attempt is made to obtain the ML estimates of the
parameters directly. By assuming that the lifetimes under increasing stress
levels form a GP, the ML estimates of the parameters of Pareto distribution in
ALTg with complete, type-I and type-II censored data are obtained. The CIs for
parameters are also constructed by using the asymptotic properties of normal distribution. The statistical properties of estimates and CIs are examined through a simulation study.

Chapter 5 extends the use of GP in CSALTg for the case of Weibull distribution with type-I and type-II censored data. Here, we also assume that the lifetimes under increasing stress levels form a GP. The estimates of the parameters are obtained by using the ML method. In addition, asymptotic interval estimates of the parameters of the distribution using Fisher information matrix are also constructed. A Simulation study is used to illustrate the statistical properties of the parameters and the CIs.

Finally, Chapter 6 is aimed at critically analyzing and evaluating the overall thesis briefly. In the light of the results obtained in the thesis prominent and invaluable conclusions are presented. Some relevant and pertinent suggestions are also given for the future research which is tantamount to provide a pathway for future researchers in the field of ALT.

A comprehensive list of references is provided at the end of the thesis.
LIST OF AUTHOR'S PUBLICATIONS USED IN THIS THESIS

Chapter 2 based on


Chapter 3 based on


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