Chapter 2

Theory of Ultra Low Frequency Waves
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2.1 Introduction: -

It is generally agreed that ultra low frequency waves (magnetic pulsations) are caused by hydromagnetic waves that may be generated as a result of different types of plasma instabilities in the magnetosphere or on its boundary in a very complicated manner. In the current chapter, the generation of hydromagnetic waves, their sources within and external to the magnetosphere and their propagation and modification within the magnetosphere and ionosphere are briefly discussed. A very good summary of these topics, with references to the most important publications dealing with ULF waves, has elegantly been reported by McPherron (2005) and by Southwood and Hughes (1983) and also presented in the books, “Introduction to Space Physics”, edited by M.G. Kivelson and C.T. Russell (1995) and “Geomagnetic Micropulsations” by Jacobs (1970). The informations provided in this chapter are mainly reproduced from these publications and the references therein.

2.2 Generation of Ultra Low Frequency Waves: -

Any process that modifies the equilibrium of the plasma and the magnetic field can serve as an energy source for waves. Plasma is highly ionized low-density gas threaded by a magnetic field. H. Alfvén showed in 1942 that in a steady magnetic field, waves of low enough frequency could propagate in fluids of high electrical conductivity (plasma). The direct confirmation of the existence of the waves was difficult to obtain, as they decay rapidly in most laboratory situations. The situation, however, is very different in problems of cosmic physics because of the enormous dimensions involved and one can experience these waves, as the decay rate can be small at these large spatial scales. Alfvén showed that at sufficiently low collision frequency, the charged particles simply gyrate around the magnetic field and travel along it. Any force that moves the
charged particles also moves the magnetic field and vice versa. In this situation the field and plasma are "frozen together". At low frequencies electric field $\vec{E}$ and plasma flow velocity $\vec{V}$ are related by

$$\vec{E} = -\vec{V} \times \vec{B}$$

where $\vec{B}$ is magnetic field flux density.

So with the low frequency approximations, Faraday's law takes the form

$$\nabla \times (\vec{V} \times \vec{B}) = \frac{\partial \vec{B}}{\partial t}$$

and can be used to show that any two particles initially connected by a field line which move with velocity $\vec{V}$ (perpendicular to $\vec{B}$) remain connected by the field line.

Alfvén's model for wave generation is summarized in Fig. 2.1 (Alfvén and Falthammar, 1963). Consider an infinite volume of fully ionized hydrogen. As shown in Fig. 2.1(a), imagine that a force is applied to the plasma perpendicular to $\vec{B}$ displacing a rectangular section of plasma with velocity $\vec{V}$. As the charges begin to move they experience a Lorentz force $\vec{F} = q(\vec{V} \times \vec{B})$, where $q$ is the charge in Coulombs. The electrons move to the left side of the moving slab, and the protons to the right side as shown in Fig. 2.1(b). This polarization of the charges creates an electric field $\vec{E}$ orthogonal to both $\vec{V}$ and $\vec{B}$. If the slab were in a vacuum, the charges would be trapped at the edges of the column and the force of the electric field would eventually stop any further transfer of charge. At this point the electric force would be equal and opposite to the Lorentz force so that,

$$q\vec{E} = -q(\vec{V} \times \vec{B})$$

or

$$\vec{E} = (\vec{V} \times \vec{B})$$

However, in a plasma the charges can flow through the surrounding fluid in an attempt to neutralize the polarization. This charge motion creates an
electrical current density \( \mathbf{J} \) as shown in the Fig. 2.1(c). As this current flows across the magnetic field above and below the moving slab, it exerts a force on the gas, \( \mathbf{F} = (\mathbf{J} \times \mathbf{B}) \). The direction of the force is the same as the initial motion of the slab. As a result, the plasma above and below begins to move.

Similarly these two moving slabs above and below the initial slab become polarized, driving currents that cause slabs further above and below the initial slab to start moving as seen in the Fig. 2.1(d). Clearly, the initial disturbance is propagating in both directions along the magnetic field away from the initial disturbance. The displacement of the slab distorts the magnetic field that is frozen into the plasma, which is clear from the Fig. 2.1(e). A tension develops due to bending of field, which causes a restoring force that brings the slab to a stop and then returns it towards its initial...
location. As this happens the moving slabs above and below the initial slab distort the field line in the same way so that two pulses appear to propagate away from the origin as illustrated in Fig. 2.1(f). These pulses are called Alfvén waves.

MHD waves are found as solutions to the basic fluid equations, Maxwell’s equations and Ohm’s law. In a magnetized plasma the two electromagnetic waves are coupled to the sound wave by the “frozen in” magnetic field, and there are three solutions to the basic equations. A low frequency approximation to the basic equations is called magnetohydrodynamics (MHD). The wave solutions to these equations are called MHD waves. The fluids equations are

\[ \frac{\partial}{\partial t} \rho + \nabla \cdot (\rho \vec{v}) = 0 \] (Equation of continuity) (1)

\[ \frac{\partial}{\partial t} \vec{v} = -\nabla p + \vec{j} \times \vec{B} \] (Equation of motion) (2)

\[ \frac{p}{\rho^\gamma} = \text{constant} \] (Equation of state) (3)

Here, \( \rho \) is the plasma density, \( \vec{V} \) is the velocity, \( p \) is the pressure, \( \vec{J} \) is current density, \( \vec{B} \) is the magnetic flux density and \( \gamma \) is the ratio of specific heats. The Maxwell’s equations are:

\[ \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \] (Gauss law in electrostatics) (4)

\[ \nabla \cdot \vec{B} = 0 \] (Gauss law in magnetostatics) (5)

\[ \nabla \times \vec{E} = -\frac{\partial}{\partial t} \vec{B} \] (Faraday’s law) (6)

\[ \nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \vec{E} \] (Ampere’s law) (7)

In the problems of cosmic physics with low frequency approximations, displacement currents are negligible in comparison to conduction current.
So the second term proportional to the electric field can be dropped. Thus we get,

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$  \hspace{1cm} \text{(Ampere's law in MHD limits)} \hspace{1cm} (8)

Here, \( \varepsilon_0 \) and \( \mu_0 \) are the permittivity and permeability of free space, respectively.

Also, Ohm's law in the frame in which the plasma velocity is measured can be written as

$$\vec{J} = \sigma \left( \vec{E} + \vec{V} \times \vec{B} \right)$$  \hspace{1cm} (9)

where \( \sigma \) is the electrical conductivity.

Let us assume that the plasma is initially at rest, which means that there are neither flows nor electric fields, and also assume that no currents are flowing. The wave perturbations introduce finite but small \( \vec{E} \), \( \vec{u} \) and \( \vec{J} \). The magnetic field, mass density and pressure also changes. So the velocity of plasma and magnetic field can be written in terms of initial (background) values and small perturbations as

$$\vec{V} = 0 + \vec{u}, \hspace{1cm} \vec{B} = \vec{B}_0 + \vec{b},$$

Similarly the density and pressure can be written as

$$\rho = \rho_0 + \delta \rho, \hspace{1cm} \text{and} \hspace{1cm} p = p_0 + \delta p$$

All of the perturbed quantities, \( \vec{b}, \delta \rho, \delta p, \vec{u}, \vec{E} = -\vec{u} \times \vec{B} \) and \( \vec{J} = \nabla \times \vec{b} / \mu_0 \) are assumed to be small enough that only terms linear in any of them are retained and squares or high powers and cross products will be dropped. Putting these expressions into the preceding equations and after several substitutions and algebraic manipulation, we get a set of four equations.

$$\rho_0 \frac{\partial \vec{u}}{\partial t} = -\nabla p + \frac{1}{\mu_0} \left[ \left( \nabla \times \vec{b} \right) \times \vec{B}_0 \right]$$  \hspace{1cm} (10)
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\[ \nabla p = \nabla \rho = \gamma \frac{P_0}{\rho_0} \nabla \delta \rho \quad (11) \]

\[ \frac{\partial}{\partial t} \delta \rho + \rho_0 (\nabla \cdot \vec{u}) = 0 \quad (12) \]

\[ \frac{\partial}{\partial t} b = (B_0 \cdot \nabla) \vec{u} = B_0 (\nabla \cdot \vec{u}) \quad (13) \]

For a plane wave propagating with wavelength \( \lambda \) and frequency \( f \), all oscillating quantities can be taken as proportional to

\[ e^{i \vec{k} \cdot \vec{r}} e^{-i \omega t} = e^{i \vec{k} \cdot \vec{r} - i \omega t} \]

where \( k = 2\pi / \lambda \), is the wave number and \( \omega = 2\pi f \), is the angular frequency of the wave and \( \vec{r} \) is the position vector. Again by substituting and simplifying, we get a set of four algebraic equations. These four equations can be solved obtaining a single vector equation for the velocity perturbation of the plasma. This equation is

\[ \omega^2 \vec{u} - \gamma \frac{P_0}{\rho_0} \frac{k \cdot \vec{u}}{k} \frac{1}{\mu_0 \rho_0} \left( \frac{B_0^2 k \cdot \vec{u} - B_0 (k \cdot \vec{u}) (k \cdot B_0) - k (k \cdot B_0) (B_0 \cdot \vec{u})}{(k \cdot B_0)^2} \right) = 0 \quad (14) \]

For simplifying this equation we can choose a special coordinate system in which the magnetic field \( \vec{B}_0 \) lies along the z-axis, and the wave number \( k = k \vec{n} \) with \( \vec{n} \) the direction of propagation, lies in the y-z plane along the unit vector \( \vec{n} \) at an angle \( \theta \) to the z-axis and can also choose two characteristic velocities. First \( s^2 = \gamma (\rho_0 / \rho_0) \) is the square of the sound speed and the second is the square of the Alfvën velocity given as \( V_A^2 = B_0^2 / \mu_0 \rho \). By taking these choices, the equation (14) gives a vector equation corresponding to the three elements of the plasma velocity \( \vec{u} \) and by expressing it in matrix form, the equation appears much simpler as
This set of equations has three solutions corresponding to three different modes of wave propagation. The first wave is the Alfvén wave, also called shear Alfvén wave, transverse or guided mode. It has only an x-component of the perturbation velocity (i.e. $u_x \neq 0$, & $u_y = u_z = 0$) orthogonal to the plane containing the ambient field and the direction of propagation. In this condition the first equation requires that

$$\frac{\omega^2}{k^2} - V_A^2 \cos^2 \theta = 0$$

or

$$V_{ph}^2 = \frac{\omega^2}{k^2} = V_A^2 \cos^2 \theta$$

(16)

where $V_{ph} = V_A \cos \theta$ is phase velocity of shear Alfvén wave. The wave energy propagates along the direction of the Poynting flux vector:

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

This direction will be along the ambient field ($\pm \vec{B}_0$) in the shear Alfvén wave. Thus the shear Alfvén wave propagates with the phase velocity $V_A \cos \theta$ and sets the fluid into motion in the direction perpendicular to the plane containing the propagation vector $\vec{k}$ and the background field $\vec{B}_0$. As the wave magnetic perturbation is transverse to the ambient field so the magnetic pressure does not change in the wave and also the wave Poynting flux is always along $\vec{B}_0$ and thus the ambient magnetic field strictly guides the energy flow and information contents.
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The other two wave modes are coupled. One has a speed that is fast compared to the shear Alfv'\'en wave, and the other has a speed that is slow. The fast wave or compressional wave is a combination of pressure and magnetic field fluctuations. When the fast wave propagates perpendicular to the background field it is seen as alternating compressions and rarefactions of both the field and plasma density. The Poynting flux vector is in the direction of propagation (at an arbitrary angle relative to $\vec{B}_0$). Thus

\[
V_{ph}^2 = \frac{\omega^2}{k^2} = V_A^2
\]

(17)

The slow wave is closest to being a pure sound wave. Each of these waves has unique polarization properties with the electric, magnetic, and plasma velocity fluctuations being oriented in different directions relative
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to the direction of wave propagation and background field. Fig. 2.2 illustrates the differences in polarization of the fast and shear Alfvén modes in a uniform, cold plasma (Dungey, 1968), [Taken from Southwood and Hughes (1983)].

In general, the ULF waves seen at the ground originate in space as either fast mode, or shear Alfvén mode, or a combination of the two. They propagate along or across the magnetic field until they reach the ionosphere. At the ionosphere they drive electrical currents that radiate pure electromagnetic waves into the neutral atmosphere. Thus magnetic pulsations measured on the ground are MHD waves converted to purely electromagnetic waves in the ionosphere.

2.3 Sources of Ultra Low Frequency Waves:

Ultra low frequency waves measured on the ground originate outside the magnetosphere as well as in various regions of the magnetosphere by a variety of physical processes. As for most magnetospheric phenomena, the energy comes principally from the solar wind, but other energy sources in the ionosphere or internal to the magnetosphere can be important. A brief description of these sources is given in the following subsections.

2.3.1 External Sources of ULF Waves:

The ULF waves can be generated by different sources outside the magnetosphere. One of them is the solar wind, particularly for waves at low frequencies around 1 mHz (Belcher and Davis, 1971). Some of these waves originate at the Sun and are both carried by, and propagate through, the solar wind. There is variation in the dynamic pressure of the solar wind as a consequence of quasiperiodic variations in its density found many times. These pressure changes cause the magnetosphere to expand and contract creating global changes in the internal magnetic field. Kepko et al. (2002)
showed that the spectrum of the dynamic pressure has multiple peaks at frequencies including $f = 0.4, 0.7, 1.0, \text{ and } 1.3 \text{ mHz}$. The simultaneous spectrum of the total field at synchronous orbit is virtually identical. The authors reported that these waves may have originated at the Sun as pressure waves caused by granulation, and not in a magnetospheric waveguide as has sometimes been suggested (Samson et al., 1991).

The ion foreshock is another external source of ULF waves seen in the magnetosphere. Ion beams are found upstream of the quasi-parallel shock and interact with the coming solar wind plasma to produce large amplitude waves, called upstream waves as shown in Fig. 2.3 [after Russell and Hoppe (1983), taken from Russell (1995)].

![Image of low-frequency waves](image)

Fig. 2.3: A variety of low-frequency waves stimulated by back-streaming ions and electrons [From Russell and Hoppe (1983)].

As the ions move upstream they interact with the solar wind producing waves with frequencies that depend on the strength of the solar wind magnetic field (Russell and Hoppe, 1983). These waves propagate very
slowly relative to the speed of the solar wind so they are swept downstream towards the bow shock. It is not well understood that how the various ion distributions are produced but the properties of the waves depend on the nature of the ion distributions (Hoppe et al., 1982).

Another external source of ULF waves is the bow shock. Shocks are places where the plasma and field go through dramatic changes such as change in density, temperature, field strength and flow speed. These changes, combined with the collisionless nature of the space plasma and the wide variety of wave modes, produce different shock waves. The bow shock is a fast mode wave standing in the solar wind (Russell, 1985). As a sharp discontinuity in space, it is made up of waves with many different frequencies. The structure of bow shock was found to be sensitive to the direction of interplanetary magnetic field. When the interplanetary magnetic field is almost aligned with the direction of propagation of the shock, i.e., the shock normal, the shock is said to be parallel shock. When the magnetic field is perpendicular to the shock normal, it produces a perpendicular shock. The parallel shock is a source of ULF waves that propagate downstream and, under certain circumstances, can enter the magnetosphere (Greenstadt 1985). The entrance of these waves as well as upstream and solar wind waves depends on the values of interplanetary magnetic field (IMF) cone angle $\theta_{xB}$, that is, the angle between the IMF direction and the Earth-Sun line. When the cone angle of IMF is low ($< 45^\circ$) then the upstream waves and the parallel shock are located on the dawn side of the Earth. Waves from this region will be swept downstream through the shock and into the magnetosheath [Russell et al. (1983), Takahashi et al. (1981)].

In addition to these sources the magnetopause, in connection with the solar wind parameters, is also a source of several types of ULF waves. Increase or decrease in pressure of solar wind will strengthen or weaken the
magnetopause current and move this current closer or out to the earth. This will globally increase or reduce the magnetic field inside. The step like changes in the dynamic pressure also creates conditions, which are also responsible for the observed magnetic impulses on the ground.

2.3.2 Internal Sources of ULF Waves:

Other than the external sources, ULF waves can also be generated by a variety of internal plasma instabilities. The charged particle distributions that provide the energy tend to control the frequencies of these waves. In general there are three periodicities associated with charged particles trapped in the dipole geomagnetic field; gyro frequency, bounce frequency, and longitudinal drift frequency. These depend on the energy and equatorial pitch angle of the particles. Instabilities, corresponding to these frequencies, are the cyclotron instability, the bounce resonance instability, and the drift instability. Gyro resonance (or cyclotron resonance) applies to the situation where a circularly polarized wave and a charged particle rotate (gyrate) about the magnetic field at the same frequency. Because the electric field of the wave and the particle velocity maintain a constant angle with respect to each other, the field is able to exert a force on the charge for a long interval. This allows energy transfer to occur (Brice, 1964). The direction of the energy flow depends on the relative angle. Cyclotron resonance between a left circularly polarized wave and protons is the mechanism that creates Pc 1 magnetic pulsations (0.5–5 Hz) in the magnetosphere.

The drift-bounce resonance mechanism is also an important internal source of ULF waves. The waves are generated by particles energy as the particle distributions give up some of their energy through the generation of waves [Southwood and Kivelson (1982), Hughes et al. (1978)]. Other processes also produce waves in the magnetosphere like the earthward directed flows that occur in the plasma sheet during substorms. These flows
are transient with durations of a few minutes and localized in azimuth to a few earth radii. Alfvén waves are radiated by these flows that travel to the auroral ionosphere where they are reflected and travel back to interact with the flow. This process is thought to create Pi 2 waves. These ULF waves are always associated with the onset of substorms, and intensifications of an ongoing substorm.

2.4 Propagation of Ultra Low Frequency Waves:

The ULF wave energy generated outside the magnetosphere penetrates the magnetopause and passes through the deep magnetosphere and are registered at the ground. Theories have been developed to describe how these wave disturbances at the magnetopause boundary pump energy into the magnetosphere cavity. In fact the wave energy transformed through the magnetosphere undergoes several modifications before reaching the ground.

2.4.1 Field Line Resonance:

The ULF wave energy that enters the magnetosphere from outside is generally thought to be transformed by two processes. One is called field line resonance and the other cavity resonance. Southwood (1974) and Chen and Hasegawa (1974) independently developed the field line resonance (FLR) theory. With an extremely simplified magnetospheric model, made up of straight magnetic field lines with the same length along the z-axis and a plasma density varying along the x-axis only, Southwood analyzed the FLR effects. In this theory, it was assumed that the source of FLRs is a monochromatic surface wave generated at the equatorial magnetopause, probably through the Kelvin-Helmholtz instability, i.e., by velocity shear instability. It is thought that the streaming of the solar wind around the magnetopause excites this K-H instability (see left side of Figure 2.4).
polarization of surface waves excited via the K-H instability indicates left-hand (LH) oscillations at dawn and right-hand (RH) oscillations at dusk.

Such a surface wave propagates into the magnetosphere perpendicular to the ambient magnetic field and its amplitude would decay exponentially with distance from the magnetopause. As a result, a local resonant field line oscillation occurs at an L-shell where the period of the surface wave is matched to the field line eigenperiod (see right side of Figure 2.4).

In general, any process, either externally generated or internal to the magnetosphere, which displaces a field line can excite field line resonances. Field lines of the earth’s dipole behave like vibrating strings as illustrated in Fig. 2.5. The ends of the field lines are frozen in the conducting ionosphere and cannot move, yet they can bend. Although, there is no force orthogonal to the field lines in their equilibrium positions but a tension force develops when the field line is displaced by some process. This tension tries to restore it to its equilibrium shape but the field line picks up momentum due
to the loaded gyrating particles and cross the equilibrium. The field line thus oscillates until other processes cause damping.

There are two primary modes of oscillation of a dipole field line (Southwood and Hughes, 1983). The toroidal mode is a displacement in the azimuthal direction creating an azimuthal magnetic perturbation, as shown in the right panels of Fig. 2.5. The poloidal mode is a radial displacement with radial magnetic perturbations as shown in the middle panels. Either mode may oscillate with different harmonics. The fundamental harmonic depicted in the top row contains an odd number (one) of half wavelengths between the ends of a field line. The second harmonic in the bottom row contains an even number (two) of half wavelengths. Many different harmonics may be simultaneously excited when the field line is excited by a broadband source.

Fig. 2.5: The stretched field lines of earth's magnetic field like stretched string (left), radial (center) and azimuthal (right) oscillations in the dipole field lines [Sugiura and Wilson (1964), Taken from McPherron (2005)].
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Since the azimuthal perturbations do not change the field magnitude or cause plasma density changes, the toroidal mode of field line resonances is most commonly observed in space. Also, field lines azimuthally adjacent to the vibrating line can vibrate in phase with the initially disturbed line as they have nearly the same resonant frequency. In fact, these properties identify this mode as the Alfvén wave.

The poloidal mode is harder to excite because field lines are oscillating in a radial direction and radially adjacent field lines have different frequencies. So, the adjacent field lines will oscillate out of phase and there will be compressions and rarefactions of the field. This mode of field line resonance corresponds to the fast mode.

2.4.2 Cavity or Wave Guide Mode:

Usually, Kelvin-Helmholtz instability could generate unstable waves with a rather broad spectrum, so that the period of the excited FLRs should have a continuous spectrum. However, characteristics of most of pulsation events having only certain discrete period(s) lead to the development of new mode of wave transmission. Kivelson et al. [1984] introduced the concept of standing global compressional modes to explain the discrete (or monochromatic) pulsation spectrum. The existence of such global compressional hydromagnetic modes in the magnetosphere was first suggested by Dungey [1954b]. Later, Kivelson and Southwood [1985, 1986] developed the global cavity mode theory. It was thought that the boundaries within the magnetosphere form cavities that may also resonate in response to external excitation. If the magnetosphere is thought like a spherical cavity, it can be easily seen that the region between the magnetopause and the plasmapause forms a complex, doughnut-shaped cavity. This cavity will have normal modes with standing waves, structured between the magnetopause and the inner turning points (TP) as shown in

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Fig. 2.6, where the plasmapause, low-latitude boundary layer, magnetosheath and bow shock are indicated as PP, LLBL, MS and BS respectively. The oscillations produced will be both radial and azimuthal, as well as along the field lines. The fact that the magnetospheric cavity is open in the tailward direction led to the extension of the cavity mode model into a waveguide model [Samson et al. (1992)]. Waves can ‘stand’ radially, but will propagate azimuthally and be lost down the tail.

Fig. 2.6: Cavity modes throughout the dayside magnetosphere produced by a compression of the nose of the magnetosphere (a), [from Kivelson, 1995]. Conceptual model for the generation of standing modes in the equatorial magnetosphere (b) [Harrold and Samson, 1992].

Since the magnetospheric cavity as a whole oscillates at its own eigenperiods, it behaves like a filter for an original broadband energy source applied to the magnetosphere. The field lines whose eigenperiods match one of the cavity eigenperiods, couple to the cavity mode and resonate strongly, in just the same way as the coupling with the surface wave. Several studies demonstrated the cavity mode nature of magnetic pulsations using satellite and ground station data (Takahashi et al. 1995). Sutcliffe and Lühr (2003) reported good correlation between the
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H component recorded at ground station observatory at Sutherland and the compressional and poloidal components recorded by CHAMP satellite and discussed it as the indicative of a cavity mode resonance nature of low latitude Pi2 magnetic pulsations.

Although the concept of the waveguide/cavity modes has been widely applied as a driving mechanism to explain observed long-period geomagnetic pulsations, but there is a lack of strong observational evidence for the existence of global compressional waves quantized at certain discrete frequencies. The real magnetosphere is extremely complex because of the asymmetric magnetic field with strong gradients in the field and plasma in every direction. In addition, its dimensions are continually changing in response to the solar wind so the cavity properties are always changing. These facts have made it difficult to demonstrate conclusively the existence of cavity modes experimentally and made doubt about the validity of the waveguide/cavity model [Samson et al. (1991), Kivelson et al. (1997)].