Chapter 1

Preliminary Concepts and Summary

1.1 Introduction

The analysis of experimental data observed at different points in time leads to new and unique problems in statistical modeling and inference. The impact of time series analysis on scientific applications can be partially documented by producing an abbreviated listing of the diverse fields in which important time series problems may arise. For example, many familiar time series occur in the field of economics, where we are continually exposed to daily stock market quotations, monthly unemployment figures etc. Social scientists follow population series, such as birthrates or school enrollments. An epidemiologist might be interested in the number of influenza cases observed over some time period. In medicine, blood pressure measurements traced over time could be useful for evaluating drugs used in treating hypertension. Functional magnetic resonance imaging of brain-wave time series patterns might be used to study how the brain reacts to certain stimuli under various experimental conditions.

Many of the most intensive and sophisticated applications of time series methods have been applied to problems in the physical and environmental sciences. One of the earliest recorded time series is the monthly sunspot numbers studied by Schuster (1906). More modern investigations may center on whether a warming trend is present in
global temperature measurements or whether levels of pollution may influence daily mortality in a particular region. The modeling of speech series is an important problem related to the efficient transmission of voice recordings. Geophysical time series such as those produced by early depositions of various kinds can provide long-range proxies for temperature and rainfall. Seismic recordings can aid in mapping fault lines or in distinguishing between earthquakes and nuclear explosions.

There are two approaches to time series analysis- the time domain approach and the frequency domain approach. The time domain approach focuses on modeling some future value of a time series as a parametric function of the current and past values. In this scenario, we begin with linear regressions of the present value of a time series on its own past values and on the past values of other series. This modeling leads one to use the results of the time domain approach as a forecasting tool, which is particularly popular with economists for this reason.

One approach, advocated in the landmark work of Box and Jenkins (1976), develops a systematic class of models called Auto-Regressive Integrated Moving Average (ARIMA) models to handle time – correlated modeling and forecasting.

The frequency domain approach assumes primary interest in time series analysis related to periodic or systematic sinusoidal variations found naturally in most data. These periodic variations are often caused by biological, physical or environmental phenomena of interest. A series of periodic shocks may influence certain areas of the brain; wind may affect vibrations on an airplane wing; sea surface
temperatures caused by El Niño oscillations may affect the number of fish in the ocean etc. The study of periodicity extends to economics and social sciences, where one may be interested in yearly periodicities in such series as monthly unemployment or monthly birth rates.

1.2 Preliminaries

A time series can be defined as a collection of random variables indexed according to the order in which they are obtained in time.

Definition 1.2.1

A time series \( \{X_t\} \), is a family of real-valued random variables indexed by \( t \in \mathbb{Z} \), where \( \mathbb{Z} \) denotes the set of integers.

A time series is said to be continuous parameter when observations are made continuously in time. A time series is said to be discrete parameter when observations are taken only at specific times, usually equally spaced.

1.2.1 Stationary time series

A sort of regularity may exist over time in the behaviour of a time series. Shumway and Stoffer (2000), introduce the notion of regularity using the concept called stationarity. Let the value of the time series at some time point \( t \) be denoted by \( X_t \). A \textit{strictly stationary time series} is one for which the probabilistic behaviour of

\[
(X_{t-1}, X_t, \ldots, X_{t+h})
\]

is identical to that of the shifted set

\[
(X_{t-1+h}, X_{t+h}, \ldots, X_{t+h+h})
\]
for any collection of time points \( t_1, t_2, \ldots, t_k \), for any number \( k = 1, 2, \ldots, \) and for any shift \( h = 0, \pm 1, \pm 2, \ldots \). This means that all of the multivariate distribution functions for subsets of variables must agree with their counter parts in the shifted set for all values of the shift parameter \( h \). For the distribution functions, we would have

\[
P ( X_{t_1} \leq c_1, X_{t_2} \leq c_2, \ldots, X_{t_k} \leq c_k ) = P ( X_{t_1+h} \leq c_1, X_{t_2+h} \leq c_2, \ldots, X_{t_k+h} \leq c_k ),
\]

where \( c_1, c_2, \ldots, c_k \) are real numbers.

The version of stationarity in (1.2.1) is too strong for most applications, and we will use a milder version and define a weakly stationary time series as one which imposes conditions only on the first two moments of a time series. A series which is weakly stationary is usually refered to as \textit{wide sense stationary or covariance stationary} if its mean is a constant and its autocovariance function depends only on the lag, so that

\[
E(X_t) = \mu
\]

and

\[
Cov (X_t, X_{t+h}) = \gamma (h),
\]

where \( h \) is the lag.

The time series \( \{ X_t, \ t \in \mathbb{Z} \} \) is \textit{marginally stationary} or \textit{first order stationary} if \( X_t \)'s are identically distributed for all \( t \in \mathbb{Z} \).

The time series \( \{ X_t ; t \in \mathbb{Z} \} \) is \textit{stationary of order} \( k \geq 1 \) if the joint distribution of \( (X_t, X_{t+1}, \ldots, X_{t+k}) \) and \( (X_{t+k}, \ldots, X_{t+sk}) \) are same for all \( t \) and given \( k \).

\( \{ X_t \} \) is called a \textit{Gaussian process} if, for all \( t_1, t_2, \ldots, t_n \) the set of random variables \( (X_{t_1}, X_{t_2}, \ldots, X_{t_n}) \) has a multivariate normal
distribution. Since it is completely specified by its mean and variance, it follows that for a Gaussian process weak stationarity implies complete stationarity. But for non-Gaussian processes, this does not hold.

1.2.2 Autoregressive models

Autoregressive models are developed with the idea that the present value of the series, $X_t$, can be explained as a function of past values, $X_{t-1}, X_{t-2}, \ldots, X_{t-p}$, where $p$ determines the number of steps into the past needed to forecast the current value.

The standard form of an autoregressive model of order $p$, denoted by AR ($p$), is given by

$$X_t = a_1 X_{t-1} + a_2 X_{t-2} + \ldots + a_p X_{t-p} + \epsilon_t,$$

where $\{\epsilon_t\}$ are independent and identically distributed random variables called innovations and $a_1, a_2, \ldots, a_p$ are fixed parameters, with $a_p \neq 0$. Also $\epsilon_t$ is independent of $X_{t-1}, X_{t-2} \ldots$

1.2.3 Non-Gaussian time series models

A time series model which is not Gaussian is called a non-Gaussian time series model. The need for developing non-Gaussian time series models have been long felt from the fact that most of the observed time series data have non-Gaussian marginal distributions. For example, in economics, Nelson and Granger (1979) considered a set of 21 time series data of which only six were found to be Gaussian. Lawrance and Kottegoda (1977) explain the need for using time series models having non-Gaussian marginal distributions for modeling river flow and other hydrological time series data. Brown et al. (1984)
describe the need for constructing time series models with Weibull marginal distribution for the modeling of wind velocity data. Gibson (1986) described the use of autoregressive processes with Laplace marginal distribution for image source modeling. Anderson and Arnold (1993) describe the use of Linnik marginal distribution in modeling stock price returns and other financial data.

A direct approach towards the construction of non-Gaussian time series models is to develop time series driven by innovations with a prespecified common distribution. Damsleth and El-Shaarawi (1989) constructed a first order autoregressive model with double exponential innovations and applied it to a series of weekly measurements of sulphate concentration. They obtained a significantly better fit when compared with the Gaussian model.

Another approach is to abandon linearity and develop non-linear time series models. Priestly (1980) describes a general class of non-linear models called state dependent models, which includes the Bilinear, Threshold and Ozaki’s exponential models as special cases and allows more flexibility in the character of the non-linear structure.

Yet another approach, which is more realistic, is the construction of models which have a predesignated marginal distribution. This thesis is concerned with this approach. The pioneer work in this field of non-Gaussian autoregressive time series modeling with a specified stationary marginal distribution was by Gaver and Lewis (1980). A number of models for stationary time series of continuous variables with negative exponential and gamma marginal distributions were developed in a series of papers by Gaver and Lewis (1980) and Lawrance and Lewis (1977, 1980, 1981, 1982, 1985). The
original formulations of these models were as first order autoregressive processes.


The present study aims at exploring some classes of distributions for which stationary solutions to minification autoregressive models exist.

1.3 Review of results and concepts

1.3.1 Adding a parameter to a family of distributions

Exponential distributions play a central role in the analysis of lifetime or survival data in part because of their convenient statistical theory. In circumstances where the one-parameter family of
exponential distributions is not sufficiently broad a number of wider families are in common use. By various methods new parameters can be introduced to expand families of distributions for added flexibility or to construct covariate models. Introduction of a scale parameter leads to the accelerated life model and taking powers of the survival function introduces a parameter that leads to the proportional hazards model.

A new way of introducing a parameter to expand a family of distributions is developed by Marshall and Olkin (1997) and applied to yield a new two-parameter extension of the exponential distribution which may serve as a competitor to such commonly used two parameter families of life distributions. A general method is applied to yield a new parameter into a family of distributions. Starting with a survival function $\overline{F}$, we develop the one-parameter family of survival functions

$$
\overline{G}(x; \alpha) = \frac{\alpha \overline{F}(x)}{1 - (1 - \alpha) \overline{F}(x)} ; \ -\infty < x < \infty, \ 0 < \alpha < \infty. \quad (1.3.1)
$$

Whenever $F$ has a density, the family of survival functions $\overline{G}$ given by (1.3.1) have easily computed densities. In particular, if $F$ has a density $f$ and hazard rate $\gamma_F$, then $G$ has the density $g$ given by

$$
g(x; \alpha) = \frac{\alpha f(x)}{\left(1 - (1 - \alpha) \overline{F}(x)\right)^2} ; \ -\infty < x < \infty, \ 0 < \alpha < \infty
$$

and hazard rate

$$
\gamma_G(x; \alpha) = \frac{\gamma_F(x)}{\left(1 - (1 - \alpha) \overline{F}(x)\right)} ; \ -\infty < x < \infty, \ 0 < \alpha < \infty.
$$
1.3.2 Geometric infinite divisibility and geometric extreme stability

Klebanov, Maniya and Melamed (1984) introduced the concept of geometric infinite divisibility (g.i.d.). Pillai and Sandhya (1990) showed that the class of distributions with complete monotone derivative is a proper subclass of geometrically infinitely divisible distributions.

**Definition 1.3.2.1** A random variable $X$ is said to be geometric infinite divisible (g.i.d.) if there exists an independent identically distributed (i.i.d.) sequence of random variables $X^{(p)}_j; j = 1, 2, \ldots, N_p$ such that for any $p \in (0, 1)$

$$X \overset{d}{=} \sum_{j=1}^{N_p} X^{(p)}_j$$

(1.3.2.1)

Where $P(N_p = k) = p (1 - p)^{k-1}, k = 1, 2, \ldots$ and $X$, $N_p$ and $X^{(p)}_j$ are independent ($\overset{d}{=}$ stands for equality of distributions).

In terms of characteristic functions (1.3.2.1) can be expressed as

$$\phi_X(t) = \frac{p\phi_p(t)}{1 - (1 - p)\phi_p(t)}$$

for any $p \in (0, 1)$ where $\phi_X(t)$ and $\phi_p(t)$ are the characteristic functions of $X$ and $X^{(p)}_j$ respectively.

**Definition 1.3.2.2**

Let $X_1, X_2, \ldots$ be a sequence of i.i.d. random variables with distribution in the family (1.3.1) and suppose $N$ is independent of the $X_i$’s with a geometric $(p)$ distribution such that

$$P(N = n) = p (1 - p)^{n-1}; n = 1, 2 \ldots$$

Let $U_N = \min (X_1, X_2, \ldots, X_N)$ and $V_N = \max (X_1, X_2, \ldots, X_N)$. 
If \( F \in \) implies that the distribution of \( U (V) \) is in \( \), then \( F \) is said to be geometric minimum stable (geometric maximum stable). If \( F \) is both geometric minimum and geometric maximum stable, then \( F \) is said to be geometric extreme stable.

### 1.3.3 Semi-Weibull distributions

We say that a random variable \( X \) with positive support has a semi-Weibull distribution and write \( X = SW (\beta, \rho) \) if its survival function is given by

\[
\bar{F}_X(x) = P(X > x) = \exp(-\psi(x))
\]  
\( \tag{1.3.3.1} \)

where \( \psi(x) \) satisfies the functional equation

\[
\rho \psi(x) = \psi(\rho^{1/\beta} x); \; \beta > 0, \; 0 < \rho < 1.
\]  
\( \tag{1.3.3.2} \)

Equation (1.3.3.2) will give on iteration

\[
\rho^n \psi(x) = \psi(\rho^{n/\beta} x).
\]

On solving (1.3.3.2) we obtain \( \psi(x) = x^\beta h(x) \) where \( h(x) \) is periodic in \( \ln x \) with period \( \left( \frac{-2\pi \beta}{\ln x} \right) \). For details see Jose (1994). Also by selecting \( \rho_1 \) and \( \rho_2 \) satisfying (1.3.3.2) such that \( \frac{\ln \rho_1}{\ln \rho_2} \) is irrational we get \( \psi(x) = c x^\beta \) where \( c \) is a constant, since a continuous periodic function with periods in irrational ratio is a constant. (For proof see Kagan et al. (1973) p.163).

As an example of \( h(x) \) we have \( h_1(x) = 1 - a \cos (k \log x) \) where \( k = \frac{-2\pi}{\ln b}, \; 0 < b < 1 \) and \( 0 < a < 1 \). Then \( h_1(x) \) satisfies (1.3.3.2) with \( b = \rho^{1/\beta} \) and \( \psi(x) \) is monotone increasing.
Similarly \( h_2(x) = \exp(r \cos(\beta \ln x)) \) satisfies (1.3.3.2) with 
\[ \rho = \exp(-2\pi) \]
and \( \psi(x) \) is monotone increasing with \( 0 < r < 1 \).

It can be seen that semi-Weibull distribution is a more general class of distributions which includes the Weibull distribution in the sense that for \( h(x) = 1 \), we get
\[ \bar{F}(x) = \exp(-x^\beta). \]
When \( h(x) = 1 \) and \( \beta = 1 \), it reduces to the exponential distribution with unit mean.

### 1.3.4 Semi–Pareto distributions

A random variable \( X \) with positive support is said to follow semi-Pareto family of distributions denoted by \( SP(\beta, \rho) \) if the survival function is of the form
\[ \bar{F}(x) = P(X > x) = \frac{1}{1 + \psi(x)}, \]
(1.3.4.1)
where \( \psi(x) \) satisfies the functional equation (1.3.3.2).

### 1.3.5 Semi–Logistic distributions

Pillai (1991) introduced the semi-Pareto distribution. In a similar manner we introduce the semi-logistic distribution as one having the survival function
\[ \bar{F}(x) = P(X > x) = \frac{1}{1 + \exp(\psi(x))}, \]
(1.3.5.1)
where \( \psi(x) \) satisfies the functional equation (1.3.3.2).

### 1.3.6 Extreme value distributions

Extreme value distributions are generally considered to comprise of the three following families given below. For details see Johnson et al. (1983).
Type I Gumbel Family

\[ P(X \leq x) = \exp\left(-\exp\left(-\frac{x - \delta}{\lambda}\right)\right); \ -\infty < x < \infty. \]  

(1.3.6.1)

Type II Frechet Family

\[ P(X \leq x) = \exp\left(-\left[\left(\frac{x - \delta}{\lambda}\right)\right]^{\beta}\right); \ x \geq \lambda \]

\[ = 0; \ x < \lambda. \]  

(1.3.6.2)

Under the transformation \( Z = \log(X - \lambda) \), this distribution reduces to Type I Gumbel family.

Type III Weibull Family

\[ P(X \leq x) = \exp\left(-\left(\frac{\lambda - x}{\delta}\right)^{\beta}\right); \ x \leq \lambda \]

\[ = 1; \ x > \lambda \]  

(1.3.6.3)

where \( \lambda, \delta > 0 \) and \( \beta > 0 \) are parameters. The corresponding distributions of \((-X)\) are also called extreme value distributions.

Here also under the transformation \( Z = -\log(\lambda - x) \), this distribution reduces to Type I Gumbel family. Type III distribution of \((-X)\) is a Weibull distribution.

1.3.7 Domain of attraction

Let \( L_n(x) = P[W_n \leq x] = 1 - [F(x)]^n \) be the distribution function of \( W_n = \min(X_1, X_2, \ldots, X_n) \). Then the distribution function \( F(x) \) is said to belong to the domain of attraction for minimum of a given cumulative distribution function \( L(x) \); if there exists sequences \( \{a_n\} \) and \( \{b_n > 0\} \) where \( a_n \) and \( b_n \) are constants depending on \( n \) such that the limit distribution

\[ \lim_{n \to \infty} L_n(a_n + b_n x) = \lim_{n \to \infty} [F(a_n + b_n x)]^n = L(x) \]

for all \( x \) become non-degenerated. For detail see Castillo, (1988).
1.3.8 Some Reliability concepts

Most of the difficulties in reliability modeling can be substantially reduced by appealing to certain concepts associated with the failure process that permit different distributions to be distinguished. In this section we discuss these concepts.

1.3.8.1 Univariate case

Let \( X \) be a non-negative random variable on a probability space \((\Omega, \theta, P)\) with distribution function \( F(x) = P(X \leq x) \) where \( F \in \). In the reliability context, \( X \) generally represents the length of life of a device measured in units of time and the function,

\[
R(x) = 1 - F(x) = P(X > x),
\]

is called the survival or reliability function because it gives the probability that the device will operate without failure for a mission time \( x \).

Failure rate

Defining the right extremity \( L \) of \( F(x) \) by

\[
L = \inf \{x : F(x) = 1\},
\]

The failure rate \( h(x) \) of \( X \), when \( F(x) \) is absolutely continuous with respect to Lebesgue measure with probability density \( f(x) \), is defined for \( x < L \) by

\[
h(x) = \lim_{u \to 0^+} \frac{P(x < X \leq x + u / X > x)}{u} = \frac{f(x)}{R(x)},
\]

\[
= \frac{d}{dx} (-\log R(x)). \tag{1.3.8.1}
\]
This function has a useful probabilistic interpretation, namely $h(x)\,dx$ represents the probability that an object of age $x$ will fail in the interval $(x, x + dx)$. It is known by a variety of names. In actuaries it is called "force of mortality"; its reciprocal for the normal distribution is known as "Mills ratio" in statistics: in extreme value theory it is called the "intensity function", and in reliability theory it has been called "hazard rate".

1.3.8.2 Bivariate case

The main problem in generalizing the univariate concepts introduced in the previous section into higher dimensions is that it cannot be accomplished uniquely. The definitions in the bivariate set up largely depend on how one visualizes the physical situation in a manner comparable to the univariate case. As we shall see, most natural extensions of the univariate case fail to provide us a meaningful definition that takes care of the joint variation or dependency structure underlying the component variables. In the following, we shall assume that $X = (X_1, X_2)$ is a non-negative random vector admitting absolutely continuous distribution function $F(x_1, x_2)$ with respect to Lebesgue measure. The survival function of $X$ denoted by

$$ R(x_1, x_2) = P(X_1 > x_1, X_2 > x_2) $$

is related to $F$ as

$$ R(x_1, x_2) = 1 - F_1(x_1) - F_2(x_2) + F(x_1, x_2) $$

where $F_i(x_i)$ is the distribution function of $X_i$.

The density of $X$ is given by

$$ f(x_1, x_2) = \frac{\partial^2 R}{\partial x_1 \partial x_2}. $$
1.3.8.3 Bivariate failure rate

Assuming that \((X_1, X_2)\) represents the lives of the components in a two component system, the bivariate failure rate at \((x_1, x_2)\) is defined as

\[
a(x_1, x_2) = \frac{f((x_1, x_2))}{R(x_1, x_2)}
\]  

(1.3.8.2)

for \(x_i > 0, i = 1, 2\).

This definition is due to Basu (1971). It is important to note that \(a(x_1, x_2)\) is a scalar quantity and that when \(X_1\) and \(X_2\) are independent

\[a(x_1, x_2) = h_1(x_1) h_2(x_2),\]

where \(h_i(x_i)\) is the univariate failure rate of \(X_i\) as defined in (1.3.8.1).

Basu (1971) shows in his paper that \(a(x_1, x_2)\) is a constant, independent of \(X_1\) and \(X_2\) if and only if \(X_1\) and \(X_2\) are independent and exponentially distributed. Galambos and Kotz (1978, p.125) claims that under suitable assumptions on the survival function \(a(x_1, x_2)\) defined by (1.3.8.2) can be expressed as

\[
a(x_1, x_2) = \frac{\partial H}{\partial x_1} + \frac{\partial H}{\partial x_2} + \frac{\partial^2 H}{\partial x_1 \partial x_2}
\]  

(1.3.8.3)

where

\[H = H(x_1, x_2) = -\log R(x_1, x_2)\]  

(1.3.8.4)

However, we note that from (1.3.8.4)

\[\frac{\partial H}{\partial x_1} = -\frac{1}{R} \frac{\partial R}{\partial x_1},\]

and

\[\frac{\partial^2 H}{\partial x_1 \partial x_2} = \frac{1}{R^2} \frac{\partial R}{\partial x_1} \frac{\partial R}{\partial x_2} - \frac{1}{R} \frac{\partial^2 R}{\partial x_1 \partial x_2}\]
which gives
\[ a(x_1, x_2) = \frac{\partial H}{\partial x_1} \frac{\partial H}{\partial x_2} - \frac{\partial^2 H}{\partial x_1 \partial x_2} \]  \tag{1.3.8.5}
as the differential equation connecting the failure rate and the survival function. It is easy to observe from (1.3.8.5) that being a second order partial differential equation, \( H \) and \( R \) need not be determined uniquely from \( a(x_1, x_2) \). A simple proof of Basu’s result that \( a(x_1, x_2) = c, \) a constant implies
\[ R(x_1, x_2) = \exp(-a_1 x_1 - a_2 x_2), \]
can be derived from (1.3.8.5).

A second approach to defining bivariate failure rate is provided by Johnson and Kotz (1975) who take it as the vector valued function
\[
h(x_1, x_2) = \left( \frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2} \right) H(x_1, x_2),
\]
\[
= \begin{pmatrix} \frac{\partial H}{\partial x_1} & \frac{\partial H}{\partial x_2} \end{pmatrix},
\]
\[
= (h_1(x_1, x_2), h_2(x_1, x_2)) \tag{1.3.8.6}
\]
Observe that \( h_i(x_1, x_2) = \frac{-\partial R / \partial x_i}{R} \).

Each of the components in \( h(x_1, x_2) \) depends, in general, on \( x_1 \) and \( x_2 \) and to reduce this redundancy of variables in the structure of the failure rate, Shanbhag and Kotz (1987) have proposed some modifications. In the case of non-negative continuous random vector \( X \) the modification in (1.3.8.6) is to take the vector
\[ h^+(x_1, x_2) = (h_1(x_1, x_2), h_2(x_2)), \]
where
\[ h_t(x_1 / x_2) = -\frac{\partial}{\partial x_1} \log P(X_1 > x_1 / X_2 > x_2) \]  \hspace{1cm} (1.3.8.7) 

and

\[ h_2(x_2) = -\frac{\partial}{\partial x_2} \log P(X_2 > x_2) \]  \hspace{1cm} (1.3.8.8) 

From (1.3.8.7) and (1.3.8.8)

\[ P(X_1 > x_1 / X_2 > x_2) = \exp(-\int_{0}^{x_2} h_1(t / x_2)dt) \]

and

\[ P(X_2 > x_2) = \exp(-\int_{0}^{x_2} h_2(t)dt), \]

so that

\[ R(x_1, x_2) = \exp(-\int_{0}^{x_2} h_1(t / x_2)dt - \int_{0}^{x_2} h_2(t)dt). \]  \hspace{1cm} (1.3.8.9) 

\subsection*{1.4 Summary of the Present Study}

This thesis is mainly concerned with some theoretical as well as practical aspects of non-Gaussian autoregressive time series modeling. Here we consider a class of autoregressive models with minification structure in detail. In our study we obtain the class of marginal distributions under which the models of consideration are strictly Markovian and stationary. In this study we introduce a new class of distributions and discuss the applications in autoregressive time series modeling. Bivariate and multivariate extensions are done. Also some reliability problems are addressed in the univariate and bivariate contexts.
The present work is organized into eight chapters. After the present introductory chapter where we have pointed out the relevance and scope of the study, along with a review of the basic concepts used in time series modeling and reliability analysis, the remaining chapters are addressed to some new results except where due references are given.

In the second chapter we consider a new family of distributions namely two-parameter Marshall-Olkin generalized exponential distribution refered to as MO-GE. Some characterizations are investigated. A first order time series model with exponential innovations is constructed and it is shown to be marginally distributed as MO-GE. The sample path behaviour of MO-GEAR (1) process seems to be distinctive and is adjustable through the parameters. This makes the model very rich. An autoregressive model of order k with marginal distribution MO-GE is also constructed.

In the third chapter, we generalize the semi-Weibull distributions to a wider class called Marshall-Olkin semi-Weibull distributions(MO-SW) by adding a parameter to it. Its properties are studied. We investigate some characterizations of Marshall-Olkin semi-Weibull (MO-SW) distributions. Then we construct a minification AR (1) model and AR (k) model with MO-SW marginal distribution. Generalizing the semi-Weibull to Weibull, the new distribution MO-GW is explored. The distributional properties and some characterizations are investigated. Also autoregressive models with MO-GW marginals are developed and sample path behaviour of
the process is explored. In section 4, we developed the bivariate concept and constructed AR (1) and AR (k) models with marginal bivariate semi-Weibull and Weibull distributions. Some characterizations are also investigated.

Chapter 4 is devoted to the study of Marshall–Olkin semi-Pareto survival functions. The definition and properties of semi-Pareto and Pareto distributions are presented. Minification models of AR (1) and AR (k) with marginal semi-Pareto and Pareto distributions are constructed. The sample path behaviour of the Marshall-Olkin generalized Pareto (MO-GP) process is studied. Marshall-Olkin bivariate semi-Pareto (MO-BSP) and Pareto distributions (MO-BP) are introduced and characterized using geometric minimization. Autoregressive minification models for bivariate random vectors with MO-BP and MO-BSP distributions are also discussed. The problem of estimating the parameters is also considered.

In chapter 5, a new class of distributions, namely Marshall-Olkin generalized logistic (MO-GL) distribution is introduced and its properties are studied. First order and kth order autoregressive processes are developed with MO-GL as marginal distributions. The sample path behaviour of the process is considered. We introduce the Marshall- Olkin bivariate semi-logistic distributions and study the properties using geometric minimization. The AR (1) and AR (k) models for random vectors with the above distributions as marginals are discussed.
Marshall – Olkin extreme value distributions such as Gumbel maximum, Gumbel minimum, Weibull maximum, Weibull minimum, Frechet maximum and Frechet minimum are introduced and their properties are studied in chapter 6. Minification processes of order 1 and k are developed in each case. Some characterizations are also investigated.

In chapter 7, multivariate models having new Marshall-Olkin multivariate stationary marginal distributions are developed. Characterizations are investigated. AR (1) and AR (k) models are developed.

In chapter 8, some reliability applications and time series applications are discussed. Reliability characteristics of MO-BP and MO-GE are discussed and some characterizations are investigated. Application to analysis of incomes are discussed. Case study about time series modeling with MO-GW as the marginal distribution of MO-GWAR (1) process is discussed. For this we consider the total daily discharge of Neyyar river in Kerala at the location Amaravilla during 1993. The model developed is shown to be adequate in this case.