CHAPTER 3

MULTI CARRIER MULTI FREQUENCY MODULATION SCHEMES
3.1 Introduction

Orthogonal frequency division multiplexing can be implemented through different modulation schemes such as

1. Single carrier modulation
   a. Parallel data transfer.
   b. Serial data transfer.
3. Multi carrier multi frequency modulation.

The above mentioned modulation schemes are briefly described in the next section.

3.2 Single Carrier Approach

In figure 1 the general structure of a single carrier transmission system is depicted. The transmitted symbols are pulse formed by a transmitter filter. After passing the multi-path channel in the receiver a filter matched to the channel is used to maximize signal to noise ratio and used to extract the data.

![Fig. 3.1 Basic structure of a single carrier system](image)

The present scenario is characterized by the following conditions:

\[ R = \frac{1}{T} = 7.4 \frac{M \text{ sym}}{s} \]

Transmission Rate: ... (3.1)
Maximum channel delay: \( \tau_{\text{max}} = 224 \mu s \)

For the single carrier system this results in an ISI of \( \frac{T_{\text{max}}}{T} = 1600 \)

3.3 Multi Carrier Approach

3.3.1 Multi carrier transmitter for serial data

![Diagram of multi carrier transmitter for serial data]

**Fig. 3.2** Multi Carrier Transmitter for serial data transfer

In this scheme data of multi user can be transmitted serially but not simultaneously. Each data bit of individual user is sent serially one after the other. At first the data bit of each user is coded with different codes using the code book as shown in figure 3.2 to get \( b(t)/U_k \). Now by using M-ary PSK modulator each bit of individual user is modulated with same frequency but different phase.

Let for example, an 8-ary PSK modulation scheme be used, where \( s(t) = b(t) \cos(2\pi fct + (2i-1)\pi/8) \) with \( i = 1 \) to \( 8 \).
Table 3.1 The different values of $s(t)$ is shown in

<table>
<thead>
<tr>
<th>Value of $i$</th>
<th>$s(t) = b(t) \cos(2\pi f_c t + (2i-1)\pi/8)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\cos(2\pi f_c t + \pi/8)$</td>
</tr>
<tr>
<td>2</td>
<td>$\cos(2\pi f_c t + 3\pi/8)$</td>
</tr>
<tr>
<td>3</td>
<td>$\cos(2\pi f_c t + 5\pi/8)$</td>
</tr>
<tr>
<td>4</td>
<td>$\cos(2\pi f_c t + 7\pi/8)$</td>
</tr>
<tr>
<td>5</td>
<td>$\cos(2\pi f_c t + 9\pi/8)$</td>
</tr>
<tr>
<td>6</td>
<td>$\cos(2\pi f_c t + 11\pi/8)$</td>
</tr>
<tr>
<td>7</td>
<td>$\cos(2\pi f_c t + 13\pi/8)$</td>
</tr>
<tr>
<td>8</td>
<td>$\cos(2\pi f_c t + 15\pi/8)$</td>
</tr>
</tbody>
</table>

Fig. 3.3 Signal space diagram of the 8-ary PSK
The plotted values of s(t) in a graph by taking $\cos 2\pi f t$ in the X axis and $\sin 2\pi f t$ in the y axis the following graph which is known as constellation diagram or signal space diagram is obtained and is shown in figure 3.3. In the above figure the vector distance between any two points of s(t) is $d$ and the safe metric is $d/2$. If we can maintain the distance $d/2$ then there will not be any error in data recovery.

3.3.2 Multicarrier Transmitter for Parallel Data

Figure 3.4 shows block diagram of a multi-carrier system for parallel data.

![Fig. 3.4 Basic structure of a multicarrier system]

The original data stream of rate $R$ is multiplexed into $N$ parallel data streams of rate $R_{mc} = 1/T_{mc} = R/N$ and each of the data streams is modulated with a different frequency and the resulting signals are transmitted together in the same band. Correspondingly the receiver consists of $N$ parallel receiver paths. Due to the prolonged distance in between the transmitted symbols the ISI for each sub system reduces to

$$\frac{T_{mc}}{T_{mc}} - \frac{T_{mc}}{N \cdot T}$$

... (3.2)
With $N=8192$ the resultant ISI is

$$\frac{T_{\text{max}}}{T_{\text{mc}}} = 0.2 \quad \text{... (3.3)}$$

This little ISI can often be tolerated and no extra counter measure such as an equalizer is needed. Also as far as the complexity of a receiver is concerned a system with 8192 parallel paths still isn't feasible. This asks for a slight modification of the approach which leads us to the concept of OFDM. Though the speed of data transfer is high, the circuit complexity is more and hence the cost is also high. The following figure 3.5 shows a practical parallel multi carrier transmitter with 4-PSK

![Fig. 3.5 Practical Parallel Multi Carrier Transmitter](image)

In the transmitter shown in figure 3.5, four users are using the same frequency but with different phase and the data are transmitted in parallel.

The values of $S_i(t)$ generated for $i=1$ to $4$ is as follows:

$$s_1(t) = \cos(2\pi fct + \pi/4) = \cos(2\pi fct \cos \pi/4 - \sin 2\pi fct \sin \pi/4)$$

$$s_2(t) = \cos(2\pi fct + 3\pi/4) = \cos(2\pi fct \cos 3\pi/4 - \sin 2\pi fct \sin 3\pi/4)$$
s3(t) = \cos(2\pi f_c t + 5\pi/4) = \cos2\pi f_c t \cos5\pi/4 - \sin2\pi f_c t \sin5\pi/4
s4(t) = \cos(2\pi f_c t + 7\pi/4) = \cos2\pi f_c t \cos7\pi/4 - \sin2\pi f_c t \sin7\pi/4 \quad \cdots \quad (3.4)

Now, plotting the values of s1, s2, s3 and s4 in the graph taking \cos2\pi f_c t in the X axis and \sin2\pi f_c t in the Y axis we get the constellation diagram shown in figure 3.6 is obtained.

![Constellation Diagram For 4-PSK](image)

**Fig. 3.6 Constellation Diagram For 4-PSK**

### 3.4 OFDM Transmitter

![Block diagram of an OFDM transmitter](image)

**Fig. 3.7 Block diagram of an OFDM transmitter**

In figure 3.7 S[N] is a serial stream of binary digits. These are first de-multiplexed into N parallel streams, and each one mapped to a (possibly complex) symbol stream using some modulation constellation (QAM, PSK, etc.). Note that the constellations may be different, so some streams may carry a higher bit-rate than others.
3.4.1 Transmission System

The transmission system model used for software radios is depicted in Fig.3.8(a) for real waveform, and in Fig. 3.8(b) for complex waveform. A sequence of M-ary symbols $s_n$, is expressed by the information vector $s = [s_1, s_2, \ldots, s_N]$, with $s_n = \pm d, \pm 3d, \ldots, \pm (M-1)d$. The initial $N$-dimensional orthonormal vectors are given by $e_1 = [100\ldots000]$, $e_2 = [010\ldots000], \ldots, e_N = [000\ldots001]$, where $e_n$ may be regarded as the $n$th symbol on a time axis. They are then transformed into orthonormal row rectors[Ikuo Oka and Marc. P.C Fossorier,2006].

$h_1 = [h_{11}h_{12} \ldots h_{1N}]$, $h_2 = [h_{21}h_{22} \ldots h_{2N}]$,

$\ldots$, $h_N = [h_{N1}h_{N2}\ldots h_{NN}]$
By a transfer matrix $H$, so that
\[ H = HE \]  \hspace{2cm} (3.5)

Where $H = [h_1, h_2, \ldots, h_N]^T$, and $E = [e_1, e_2, \ldots, e_N]^T$ is the identity matrix of the initial orthonormal bases. The orthonormal transform $H$ will be given by consecutive multidimensional rotations. The weights $s_n$ and orthonormal row vectors $h_n$ define the modulation.

Next the sequence is transformed into a real or a complex waveform.

**Real waveform:** The row vector $c = [c_1, c_2, \ldots, c_N]$ is defined by
\[ C = sH. \]  \hspace{2cm} (3.6)

After passing through the pulse generator, the signal is expressed by a time function.
\[ X(t) = \sum_{n=1}^{N} c_n \delta(t-n \tau) \]  \hspace{2cm} (3.7)

Where $\tau$ is the symbol duration and equals the symbol block length $T$ divided by $N$. The impulse signal $x(t)$ is fed into the root Nyquist filter for pulse shaping without ISI, and transformed into
\[ Y(t) = \sum_{n=1}^{N} c_n p(t-n \tau) \]  \hspace{2cm} (3.8)

Where $p(t)$ is the impulse response of the filter. The Amplitude Modulation (AM) is obtained without orthonormal transform on $e_1, e_2, \ldots, e_N$.

**Complex waveform:** The $N/2$ complex symbols $u_1, u_2, \ldots, u_{N/2}$ produced by a mapper define a row vector $u$ of $N$ elements
\[ U = [\Re\{u_1\}, \Re\{u_{N/2}\}, \Im\{u_1\}, \Im\{u_2\}, \ldots, \Im\{u_{N/2}\}] \]

The vector $u$ is expressed by
\[ U = sH \]  \hspace{2cm} (3.9)
And is used as input to the pulse generator.

First, modulation in time is considered, where the inverse discrete Fourier transform (IDFT) and DFT are not employed in Fig. 1(b). The pulse generator produces

\[ x(t) = \sum_{n=1}^{N/2} u_n \delta(t-2n\tau) \quad \ldots \quad (3.10) \]

where the symbol duration of \( 2\tau \) indicates a half-band-width reduction, compared with that of the real waveform case. The root Nyquist filter yields

\[ y(t) = \sum_{n=1}^{N/2} u_n p(t-2n\tau) \quad \ldots \quad (3.11) \]

The frequency-modulation design requires the IDFT and DFT in Fig. 3.8(b). The identity matrix of \( H \) corresponds to OFDM. The DFT can be included in the orthonormal transform \( H \) with the multidimensional rotations. Consequently, the modulation designs in time and frequency have no basic difference.

### 3.4.2 Widely Used Modulations

An N-dimensional rotation is expressed by the product of the rotation matrix \( R \) and the transfer matrix to be rotated. The Rotation angle is defined as \( \theta_{ij} \) in the \( i \)th and \( j \)th dimensions, where \( i < j \) is assumed without loss of generality. For \( \theta_{ij} = \alpha \), the rotation matrix \( R \) is written by [B.Jacob,1995], [G.STrang1988]

\[ R = [\theta_{ij} = \alpha] = [\tau_{ij}] \quad \ldots \quad (3.12) \]

Where \( \tau_{ij} = \cos \alpha \), \( \tau_{ij} = \sin \alpha \), \( \tau_{ik} = \cos \alpha \), \( \tau_{kk} = 1 \quad (k \neq i, j) \), and \( \tau_{kj} = 0 \quad (k \neq j) \). Let us denote \( L \) consecutive rotations by the matrices \( R_1, R_2, \ldots, R_L \). Then the new transfer matrix \( H \) is expressed by
To produce OFDM, the orthonormal vectors \( e_1, e_2, \ldots, e_N \) are applied to the multiplication block with identity matrix \( H (=E) \), where IDFT and DFT are employed in Fig. 3.8(b). On the other hand, without IDFT or DFT, the orthonormal transform \( H \) is the Fourier Transform.

A four-point DFT for \( N = 8 \) is expressed by

\[
H = \frac{1}{2} \begin{pmatrix}
1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & -1 & 0 & 0 & 1 & 0 & -1 \\
1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\
1 & 0 & -1 & 0 & 0 & -1 & -1 & -1 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
0 & 1 & 0 & -1 & 1 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\
0 & 1 & 0 & -1 & -1 & 0 & 1 & 0
\end{pmatrix}
\]

The intuitive calculations gives

\[
H = \begin{pmatrix}
\theta_{68} = \frac{\pi}{2}; \theta_{48} = -\frac{\pi}{2} \\
\theta_{46} = \frac{\pi}{4}; \theta_{28} = \frac{\pi}{4} \\
\theta_{57} = -\frac{\pi}{4}; \theta_{68} = -\frac{\pi}{4}; \theta_{58} = -\frac{3\pi}{4}; \theta_{67} = -\frac{3\pi}{4}; \theta_{78} = -\frac{3\pi}{2}; \theta_{56} = -\pi; \theta_{13} = -\frac{\pi}{4}; \theta_{24} = -\frac{\pi}{2}; \theta_{14} = -\frac{3\pi}{4}; \theta_{23} = -\frac{3\pi}{2}; \theta_{34} = -\frac{3\pi}{2}; \theta_{12} = -\pi
\end{pmatrix}
\]

where the rotation \([\theta_{68} = \pi/2; \theta_{48} = -\pi/2]\) is introduced to interchange rows, and may be omitted if the symbols are interchanged at the mapping in Fig.,
3.8(b). The result by the Gaussian elimination method is also given by 14 rotations

\[ H = \begin{bmatrix} 
\theta_{78} &= -0.500 \pi; 
\theta_{67} &= 0.250 \pi; 
\theta_{58} &= 0.250 \pi; \\
\theta_{57} &= 0.250 \pi; 
\theta_{48} &= 0.195 \pi; 
\theta_{45} &= 0.500 \pi; \\
\theta_{45} &= -0.000 \pi; 
\theta_{34} &= -0.196 \pi; 
\theta_{25} &= -0.167 \pi; \\
\theta_{26} &= -0.196 \pi; 
\theta_{23} &= -0.333 \pi; 
\theta_{14} &= -0.250 \pi; \\
\theta_{13} &= -0.196 \pi; 
\theta_{12} &= -0.250 \pi 
\end{bmatrix} \]

An inverse FFT is computed on each set of symbols, giving a set of complex time-domain samples. These samples are then quadrature-mixed to pass band in the standard way. The real and imaginary components are first converted to the analogue domain using digital-to-analog converters (DACs); the analogue signals are then used to modulate cosine and sine waves at the carrier frequency, \( F_c \), respectively. These signals are then summed to give the transmission signal, \( S(T) \).

3.5 OFDM RECEIVER

![Block diagram of the OFDM receiver](image)

The receiver of figure 3.9 picks up the signal \( R(T) \), which is then quadrature-mixed down to base band using cosine and sine waves at the carrier frequency. This also creates signals centered on \( 2F_c \), and low-pass filters are used to reject these. The base band signals are then sampled and digitized...
using analog to digital converters (ADCs), and a forward FFT is used to convert it back to the frequency domain. This returns $N$ parallel streams, each of which is converted to a binary stream using an appropriate symbol detectors. These streams are then re-combined into a serial stream, $\hat{s}[n]$, which is an estimate of the original binary stream at the transmitter.

### 3.6 MODELLING MULTICARRIER SYSTEM

The aim was to develop a mathematical model of the performance (BER) of OFDM verses the channel noise. This was so that the simulated results could be verified, and to get a more in depth understanding of the transmission mechanism.

The model developed is based on the transmission modulation technique being phase shift keying, and that the channel noise is Gaussian noise (i.e. white noise)[Eric Lawrey, 1997]

#### 3.6.1 Mathematical description

The low-pass equivalent of OFDM signal is expressed as

$$\nu(t) = \sum_{k=0}^{N-1} I_k e^{i2\pi k t / T}, \quad 0 \leq t < T;$$

... (3.17)

where $\{I_k\}$ are the data symbols, $N$ is the number of subcarriers, and $T$ is the OFDM block time. The subcarriers spacing of $1/T$ Hz makes the subcarriers orthogonal; this property can be expressed as

$$\frac{1}{T} \int_0^T (e^{i2\pi k_1 t / T})^* (e^{i2\pi k_2 t / T}) dt = \frac{1}{T} \int_0^T e^{i2\pi (k_2 - k_1) t / T} dt = \begin{cases} 1, & k_1 = k_2, \\ 0, & k_1 \neq k_2; \end{cases}$$

where $(\cdot)^*$ denotes the complex conjugate operator. ... (3.18)
To avoid intersymbol interference in multipath fading channels, a guard interval $-T_g \leq t < 0$, where $T_g$ is the guard period, is inserted prior to the OFDM block. During this interval, a cyclic prefix is transmitted. The cyclic prefix is equal to the last $T_g$ of the OFDM block. The OFDM signal with cyclic prefix is thus:

$$
\nu(t) = \sum_{k=0}^{N-1} I_k e^{i2\pi kt/T}, \quad -T_g \leq t < T.
$$

The low-pass signal above can be either real or complex-valued. Real-valued low-pass equivalent signals are typically transmitted at baseband—wireline applications. For wireless applications, the low-pass signal is typically complex-valued; in which case, the transmitted signal is up-converted to a carrier frequency $f_c$. In general, the transmitted signal can be represented as

$$
s(t) = \frac{1}{2} \Re \left\{ \nu(t)e^{i2\pi f_c t} \right\}.
$$

(3.19)

3.6.2 RMS Demodulated Phase Error

If we assume that the transmission modulation method used is phase shift keyed then any noise added to the transmitted signal will result in a phase error. If we look at the IQ diagram of the transmitted signal then the transmitted signal will be a phasor of fixed magnitude, and of phase corresponding to the data to be transmitted. The noise can then be considered as the random vector added to the transmitted signal. The magnitude of the phase error depends on two things, the relative phase angle of the noise vector, and the magnitude of the noise vector.
The received vector will be the vector sum of the transmitted signal and the noise. If we assume that the noise is a constant magnitude vector equal to its RMS magnitude, and that it has a random phase angle then the problem of working out the received angle would be as follows

3.6.3 BER verses Channel Noise

Figure 3.7 show the effect of noise on the received phase angle. If we let the amplitude of the transmitted signal be 1, and the length of the noise vector be $A$ with angle $\phi$, then the received phase error is $\theta_{err}$.

![Diagram of received vector and noise](image)

Fig. 3.10 Received Pharos, showing effect of noise on the received phase angle.
Using trigonometry,

\[ x = l + a \cos \varphi \]
\[ y = a \sin \varphi \]

Since,

\[ \beta_{\text{err}} = \tan^{-1} \left( \frac{y}{x} \right) \]

Therefore,

\[ \beta_{\text{err}} = \tan^{-1} \left( \frac{a \sin \varphi}{1 + a \cos \varphi} \right) \]

... (3.20)

The signal to ratio determines the relative amplitude of the received signal and the noise level. Since the signal is scaled to an amplitude of 1, the amplitude of the noise is:

\[ a = \frac{1}{S_{NR}} \]

Note: The SNR is base on the amplitudes of the signals thus must be scaled correctly when converting it to dB

If we substitute this in we get,

\[ \beta_{\text{err}} = \tan^{-1} \left( \frac{1}{S_{NR}} \sin \varphi \right) \]

\[ \beta_{\text{err}} = \tan^{-1} \left( \frac{\sin \varphi}{S_{NR} + \cos \varphi} \right) \]

... (3.21)

\[ \beta_{\text{err}} = \tan^{-1} \left( \frac{\sin \varphi}{S_{NR} + \cos \varphi} \right) \]

... (3.22)

The noise signal can be of any phase angle. What we need is to find is the RMS phase error, so if we find the average phase error (assuming the noise phase angle is always positive) then this can be scaled to find the RMS
error. The average phase angle can be found by integrating $\theta_{err}$ over a half circle, i.e \( \phi \) varies from 0 to \( \pi \)

\[
A\theta_{av} = \frac{1}{\pi} \int_0^\pi \tan^{-1} \left( \frac{\sin \phi}{\text{SNR} + \cos \phi} \right) d\phi
\]

... (3.23)

The RMS phase error will be greater by root 2, thus

\[
\text{RMS}_{err} = \sqrt{2} \int_0^\pi \tan^{-1} \left( \frac{\sin \phi}{\text{SNR} + \cos \phi} \right) d\phi
\]

... (3.24)

### 3.6.4 Maximum Allowable Phase Angle

$\theta_{max}$ is the maximum phase error allowed on the received symbol, before an error will occur on the received word

\[\text{Fig. 3.11 IQ diagram for QPSK, showing the phase locations for data (crosses) and that } \theta_{max} \text{ is 45 degrees}\]

<table>
<thead>
<tr>
<th>Modulation Technique</th>
<th>Maximum Phase Error Allowed $\theta_{max}$ in degrees</th>
</tr>
</thead>
<tbody>
<tr>
<td>BPSK</td>
<td>90</td>
</tr>
<tr>
<td>QPSK</td>
<td>45</td>
</tr>
<tr>
<td>16PSK</td>
<td>11.25</td>
</tr>
<tr>
<td>256PSK</td>
<td>0.70313</td>
</tr>
</tbody>
</table>
Once $\theta_{\text{max}}$ and $\theta_{\text{error(rms)}}$ have been established, $Z$ can be calculated, and the BER

$$Z = \frac{\theta_{\text{max}}}{\theta_{\text{error(rms)}}}$$

**Sample**

For a QPSK transmission if the signal to noise ratio (SNR) of the channel is 10dB, find the BER:

$\theta_{\text{error(rms)}}=16.5$ degrees,

For QPSK, $\theta_{\text{max}}=45$ degrees.

Therefore,

$$Z = \frac{45}{16.5} = 2.727$$

the BER is between 0.0053 and 0.0091, with a result of 0.0077 if the results are interpolated

### 3.6.5 Effect of AWGN and fading channels on constellation points

In AWGN channel, the received band pass signals in the $k$-th signaling interval may be written as

$$r(t, k) = s_m(t, k) + n(t, k)$$

$$kT_s \leq (k + 1)T_s$$

where $T_s$: symbol duration, $s_m(t)$ is the message waveform corresponding to the M-PSK symbol $s_m$, $m=1, 2, 3, ....M$. Assuming perfect carrier synchronization and timing recovery as in and employing I-Q demodulation we get

$$r(k) = [r_i(k), r_Q(k)]$$

$$= [s_m + n_i(k), s_mQ + n_Q(k)]$$

$$kT_s \leq (k + 1)T_s$$

(3.25)

(3.26)
Thus in the signal space the received signal points wander around signal points in a completely random fashion, in the sense it may lie anywhere inside a Gaussian distributed noise cloud centered on the message point. The effect of AWGN on signal points for MPSK signals at the receiver is shown in the fig.3.12(B). For wireless communication scenarios, in addition to AWGN, there will be the effect of multipath fading. Multipath fading channel can be modeled by a Tapped Delay Line (TDL) [Jhon g. Proakis, 2001]: the test signal is convolved with the impulse response of the TDL to account for the effect of fading induced by the channel.

![Figure 3.12 Effect of Noise and Fading on MPSK constellation at SNR =15dB](image1)

The TDL parameters are chosen corresponding to power delay profile of physical channels [S.Chen and E.S.Chang, 2004]. We have considered a frequency selective channel in performing simulations. Fig 3.12 (C) Fig. 3.12(D) respectively show the faded received signal constellation and equalized signal constellation after CM equalization [J.C.Lin, 2002, Simon Haykin, 1994].
3.6.6 Method For Automatic Blind Modulation Recognition

In this section, we briefly discuss the Automatic Blind Modulation Recognition (ABMR) algorithm developed [M.Vastram Nayk et al,2005], for estimating modulation schemes based Mean Square Error (MSE) criterion.

A sequence of N received signal sample \( \{r(k)\}, k = 1,2,\ldots,N \) are collected at demodulator output. Using this sequence, we check how closely the received signal samples "match" with each of the prototype constellations available at the receiver library. The degree of "closeness" or "match" is measured in terms of a Mean Square Error power defined as

\[
MSE(M) = \frac{1}{N} \sum_{k=1}^{N} D_{k, M}^2, \quad M=2^q, \quad q=1,2,\ldots \quad (3.27)
\]

Where

\[
D_{k, M} = \min_{m} \| r(k) - s_m \|, \quad m=1,2,\ldots,M \quad (3.28)
\]

\[
= \min_{m} \| d_{k-m} \| \quad (3.29)
\]

The computation of \( D_{k, M} \) can be simplified by confining the search to that quadrant in which \( r(k) \) lies. For example, as shown in Fig 3.12, as \( r(k) \) lies in the first quadrant (Q₁), we need to compute only the distances \( d_{k,1}, d_{k,2} \) and \( d_{k,3} \) to find \( d_{k,3} \).
We make the following observations:

Lower-order PSK constellations are sub-sets of the higher order PSK schemes; therefore, when lower-order PSK symbols are transmitted, the received signals sequence \{r(k)\} will find a "match" not only with the corresponding prototype constellation, it will also "match with the higher-order constellation (with more or less the same degree of accuracy).

Case 1 BPSK is transmitted

In this case, the received signal points will be scattered around the symbols \(s_2\) and \(s_6\) shown in Fig.3.13

(a) Majority of the points will be confined in the first and third quadrants \((Q_1\) and \(Q_3\)) especially at high SNR. The contribution of theses points towards MSE power will be the same in both BPSK and QPSK, i.e.

\[
\text{MSE}(2) = \text{MSE}(4),
\]

\[
\forall r(k) \in Q_1 \cup Q_3 \quad \ldots \quad (3.30)
\]
However, this same set of points will result in the slightly lower MSE when matched to 8-PSK as some of these points will have closer match to 8-PSK symbols \( s_1 \) or \( s_3 \) and \( s_5 \) or \( s_7 \) shown in Fig 3.13. Thus,

\[
\text{MSE}(8) < \text{MSE}(2), \text{MSE}(4),
\]

\[
\forall \mathbf{r}(k) \in Q_1 \cup Q_3
\]  

(b) For a small fraction of the received points which lie in the \( Q_2 \) and \( Q_4 \), their 'match' with the BPSK prototype will be proper 9 the nearest symbols being \( s_2 \) and \( s_6 \) as compared to QPSK prototype (nearest symbols \( s_4 \) and \( s_8 \)) and 8-PSK (nearest symbols \( s_3, s_4, s_5 \) and \( s_7, s_8, s_1 \)) thus,

\[
\text{MSE}(8) < \text{MSE}(4) < \text{MSE}(2),
\]

\[
\forall \mathbf{r}(k) \in Q_2 \cup Q_4
\]  

Conclusion:

(i) when BPSK is transmitted, at any SNR, we shall find \( \text{MSE}(8) < \text{MSE}(4) < \text{MSE}(2) \)

(ii) at high SNR, the differences in MSE are negligibly small; only at low SNR, the differences are distinguishable

Case 2. QPSK is transmitted

Now \( \{\mathbf{r}(k)\}'s \) are scattered around the four symbols \( s_2, s_4, s_6, s_8 \). It follows that \( \{\mathbf{r}(k)\} \) will match well with QPSK and 8-PSK prototypes while there will be large mismatch with BPSK prototype. Thus,

\[
\text{MSE}(2) > \text{MSE}(4), \text{MSE}(8), \text{at all SN}
\]

\[
\text{MSE}(8) \approx \text{MSE}(4) \text{ at high SNR}
\]

\[
\text{MSE}(8) < \text{MSE}(4) \text{ at low SNR}
\]
Case 3. I-PSK is transmitted

Following similar reasoning we conclude

\[ \text{MSE}(2) \gg \text{MES}(4) \gg \text{MSE}(8) \] at all SNR.

Recognition of QPSK Variants

In the previous section we have described how to blindly identify the modulation type of a received M-PSK signal. In this section, we present a novel approach to identify QPSK signal variants, such as QPSK, OQPSK and \( \pi/4 - \) DQPSK. OQPSK signaling is similar to a QPSK signaling, except the time alignment of the odd and even bit stream by half-symbol period [Simon Haykin, 1994, T.S. Rappaport, 2004]. This implies that the maximum phase shift of the transmitted signal at any given time instant is limited to ±90°. The constellation diagram of QPSK and OQPSK is shown in Fig.3.14(A) and Fig.3.14(B) respectively. It is observed that, for APSK signal there is a bidirectional phase transition from signals point \( s_1 \), to \( s_3 \) or \( s_2 \) to \( s_4 \) and vice versa. Hence the relative Euclidian distance denoted by \( D' \) between the adjacent received signal symbol in a frame for QPSK is greater than OQPSK signal.

\[ \text{Fig. 3.14} \quad \text{Constellation diagrams of a QPSK and OPQSK} \]
In a $\pi/4$-DQPSK modulator, constellation is formed by superimposing two QPSK signal constellations offset by $\pi/4$ relative to each other, resulting in eight signals phases. During each symbol period a phase angle from only one of two QPSK constellation is transmitted. This results in maximum phase transient of $135^\circ$. The above mentioned constellation based ABMR algorithm does not classify $\pi/4$-DQPSK and 8-PSK signals, because both schemes have eight constellation points.

An alternate approach to classify $\pi/4$-DQPSK and 8-PSK signal depends on relative phase of received symbols in a frame. Fig.3.15(A) and Fig.3.15(B) depict the relative phase distribution of 8-PSK and $\pi/4$-DQPSK signal respectively.

The Bit Error Rate (BER) of an OFDM link can be predicted based on the channel signal to noise ratio (SNR) and phase modulation used (e.g. BPSK, QPSK, etc). This is done by finding out what the expected RMS phase error ($\theta_{\text{error(rms)}}$) there will be on the signal (due to the channel noise). The bit
error rate can then be found by comparing the magnitude of the RMS phase error to that of the maximum phase allowed for the particular phase modulation used ($\theta_{\text{max}}$).

3.7 MULTIPLE ACCESS TECHNIQUES

Multiple access schemes are used to allow many simultaneous users to use the same fixed bandwidth radio spectrum. In any radio system, the bandwidth that is allocated to it is always limited. For mobile phone systems the total bandwidth is typically 50 MHz, which is split in half to provide the forward and reverse links of the system. Sharing of the spectrum is required in order increase the user capacity of any wireless network. FDMA, TDMA and CDMA are the three major methods of sharing the available bandwidth to multiple users in wireless system. There are many extensions, and hybrid techniques for these methods, such as OFDM, and hybrid TDMA and FDMA systems. However, an understanding of the three major methods is required for understanding of any extensions to these methods [Eric Lawrey, 1997].

3.7.1 Frequency Division Multiple Access

For systems using Frequency Division Multiple Access (FDMA), the available bandwidth is subdivided into a number of narrower band channels. Figure 3.16 and Figure 3.17 shows the allocation of the available bandwidth into several channels.
Fig. 3.16 FDMA showing that the each narrow band channel is allocated to a single user

Fig. 3.17 FDMA spectrum, where the available bandwidth is subdivided into narrower band channels

3.7.2 Time Division Multiple Access

Time Division Multiple Access (TDMA) divides the available spectrum into multiple time slots, by giving each user a time slot in which they can transmit or receive. Figure 3.18 shows how the time slots are provided to users in a round robin fashion, with each user being allotted one time slot per frame.

Fig. 3.18 TDMA scheme where each user is allocated a small time slot

TDMA is normally used in spread on the transmission. Figure 3.19 shows the use of TDMA with FDMA.
Fig. 3.19  TDMA / FDMA hybrid, showing that the bandwidth is split into frequency channels and time slots

3.7.3 Code Division Multiple Access

Code Division Multiple Access (CDMA) is a spread spectrum technique that uses neither frequency channels nor time slots. With CDMA, the narrow band message (typically digitized voice data) is multiplied by a large bandwidth signal that is a pseudo random noise code (PN code). All users in a CDMA system use the same frequency band and transmit simultaneously. The transmitted signal is recovered by correlating the received signal with the PN code used by the transmitter. Figure 13.20 shows the general use of the spectrum using CDMA.

Fig. 3.20 Code division multiple access (CDMA)
3.8 FREQUENCY HOPPING MULTI CARRIER SYSTEM (FH/MC)

Broadband multiple access candidate scheme meeting the user requirements is constituted by frequency-hopping (FH)-based multi-carrier direct sequence (DS) FH/MC [L. L. Yan et al., sep., 2000, Jul, 2001]. The transmitter schematic of FH/MC arrangement is depicted in Fig. 3.21. Each sub-carrier of a user is assigned a pseudo-noise (PN) spreading sequence. These PN sequences can be simultaneously assigned to a number of users, provided that no more than one user activates the same PN sequence on a given sub-carrier.

![Transmitter diagram of the FH/MC system using adaptive transmission.](image)

**Fig. 3.21** Transmitter diagram of the FH/MC system using adaptive transmission.

As shown in Fig. 3.21, the original bit stream having a bit duration of $T_b$ is first serial-to-parallel (S-P) converted. Then these parallel bit streams are grouped and mapped to the potentially time-variant modulation constellations of the $U$ active sub-carriers. The system is designed for achieving a high processing gain and mitigating the intersymbol interference (ISI) in a constant-rate transmission scheme, the symbol duration can be extended to a multiple of the bit duration can be extended to a multiple of the bit duration (i.e., $Ts = mT_b$). In contrast, the design aims to support multiple transmission rates or channel-quality matched variable information rates, a constant symbol
duration of $T_0$ may be employed. Both multi-rate and variable-rate transmissions can be implemented by employing a different number of sub-carriers associated with different modulation constellations as well as different spreading gains. As seen in Fig. 3.21, after the constellation mapping stage, each branch is DS spread using the assigned PN sequence, and then this spread signal is carrier modulated using one of the active sub-carrier frequencies derived from the constant-weight code $C(Q, U)$. Finally, all $U$ active branch signals are multiplexed in order to form the transmitted signal.

In the FH/MC receiver of Fig.3.22 the received signal associated with each active sub-carrier is detected using, for example, a combiner. Alternatively, multi-user detection (MUD) can be invoked in order to approach the single-user bound. In contrast to the transmitter side, where only $U$ out of $Q$ sub-carriers are transmitted by a user, at the receiver different detector structures might be implemented based on the availability [L.-L. Yang, et al Jul, 2001] or lack [L.-L. Yangk] of the FH pattern information. The transmitted information can be detected and the FH patterns acquired simultaneously by using blind joint detection algorithms exploiting the characteristics of the constant-weight codes. However, the receiver has explicit knowledge of the FH patterns, only $U$ sub-carriers have to be demodulated. However, if fast Fourier transform (FFT) techniques are employed for demodulation, as is often the case in MC or OFDM systems, all $Q$ sub-carriers might be demodulated, where the inactive sub-carriers only output noise. In the decision unit of Fig. 3.18, these noise-output-only branches can be eliminated by exploiting knowledge of the FH patterns. Hence, the decision unit only outputs the information transmitted by the active sub-carriers. Finally, the
decision unit's output information parallel-to-serial converted to form the output data.

![A receiver block diagram of the FH/MC system using a conventional receive](image)

**Fig. 3.22** A receiver block diagram of the FH/MC system using a conventional receive

3.8.1 Characteristics Of The FH/MC DS-CDMA Systems

3.8.1.1 Broadband Wireless Mobile System

To elaborate a little further, our advocated FH/MC system is a broadband wireless mobile system constituted by multiple narrowband subsystems. Again, FH techniques are employed for each user, in order to evenly occupy the whole system bandwidth and efficiently utilize the available frequency resource.

3.8.1.2 Compatibility

The broadband FH/MC system can be rolled out over the bands of 2G and 3G mobile wireless systems and/or in the band licensed for future broadband wireless communication systems.

3.8.1.3 FH Strategy
In FH/MC systems both slow and fast FH techniques can be invoked, depending on the system's design and the state of the art.

**Fig. 3.23 Spectrum of a FH/MC signal using a subchannel bandwidth of 1.25MHz and/or 5Mhz**

### 3.8.1.4 Implementation of MC Modulation

The MC modulation and MC demodulation blocks in Fig. 3.22 can be implemented using FFT techniques, provided that each of the sub-channels occupies the same bandwidth.

### 3.8.1.5 New User Access and Transmission

### 3.8.1.6 Multirate And Variable Services

In FH/MC systems multi-rate and variable-rate services can be implemented using a variety of approaches.

### 3.8.1.7 Diversity

The FH/MC system includes FH, MC modulation, as well as DS spreading; hence, a variety of diversity schemes and their combinations can be implemented. The possible diversity schemes include the following arrangements:

- If the chip duration of the spreading sequences is lower than the maximum delay spread of the fading channels, frequency diversity can be achieved on each of the sub-carrier signals.
• Frequency diversity can also be achieved by transmitting the same signal using a number of different sub-carriers.
• Time diversity can be achieved by using slow FH in conjunction with error control coding as well as interleaving.
• Time-frequency diversity can be achieved by using fast FH techniques, where the same symbol is transmitted using several time slots assigned to different frequencies.
• Spatial diversity can be achieved by using multiple transmit antennas, multiple receiver antennas and polarization.

3.8.1.8 Interference Resistance

The FH/MC system concerned can mitigate the effects of intersymbol interference encountered during high-speed transmissions, and readily supports partial-band and multi-tone interference suppression.

3.8.1.9 Advanced Technologies

The FH/MC system can efficiently amalgamate the techniques of FH, OFDM, and DS-CDMA.

3.8.1.10 Flexibility

The future-generation broadband mobile wireless systems will aim to support a wide range of services and bit rates.

3.9 QUADRATURE AMPLITUDE MODULATION

QAM developments focused on the benign AWGN telephone line and point-to-point radio applications [G.Forney, 1984], which led to the definition of
the International Telecommunication Union's (ITU) telephone circuit modem standards based on various QAM constellations ranging from un-coded 16QAM to trellis coded (TC) 256QAM [L.Hanzo et al, 2000]. In recent years, QAM research for hostile fading mobile channels has been motivated by the ever-increasing bandwidth efficiency demand for mobile telephony.

3.9.1 Modem Schematic

Multilevel full-response modulation schemes have been considered in depth in reference [L.Hanzo et al, 2000]. The fundamental modem schematic is shown in figure 3.24. If an analog source signal must be transmitted, the signal is first low-pass filtered and analog-to-digital converted (ADC) using a sampling frequency satisfying they Nyquist criterion. The general digital bit-stream is then mapped to complex modulation symbols by the MAP block, as seen in Figure 3.25 in case of mapping 4 bits/symbol to a 16QAM constellation.

![Fig. 3.24 Simplified QAM Modem schematic](image)

When designing a constellation, consideration must be given to:
1. The minimum Euclidean distance among phasors, which is characteristic of the noise immunity of the scheme.
2. The minimum phase rotation among constellation points, determining the phase jitter immunity and hence the scheme's resilience against clock recovery imperfections and channel phase rotations.
3. The ratio of the peak-to-average phasor power, which is a measure of robustness against nonlinear distortions introduced by the power amplifier.

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Fig. 3.25 16 QAM square constellation