CHAPTER 6

PERFORMANCE ANALYSIS OF MULTICARRIER SYSTEM
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6.1 Introduction

An OFDM system is modeled using Matlab to allow various parameters of the system to be varied and tested. The aim of doing these simulations is to measure the performance of OFDM under different channel conditions, and to allow for different OFDM configurations to be tested. The inter carrier interference caused by frequency drift is eliminated by equalizing the complex weighted coefficients of interference. Four main criteria are used to assess the performance of the OFDM system, which were its tolerance to multipath delay spread, channel noise, different modulation schemes and time synchronization errors.

6.2 OFDM Model Used

The OFDM system was modeled using Matlab and is shown in Figure 6.1. A brief description of the model is provided below.

Fig. 6.1 OFDM Model used for simulations
6.3 Details of OFDM Model Chosen for Simulation

6.3.1 Delay Spread and inter symbol interference

The received radio signal from a transmitter consists of typically a direct signal, plus reflections off objects such as buildings, mountings, and other structures. The reflected signals arrive at a later time than the direct signal because of the extra path length, giving rise to a slightly different arrival times, spreading the received energy in time. Delay spread is the interval for which a symbol remains inside the multipath channel as shown in figure 6.2. The delay spread can lead to inter-symbol interference (ISI). The spread spectrum under ideal conditions are as shown in figure 6.3.

Fig. 6.2 Multi path signals

Fig. 6.3 Spread spectrum wave form for OFDM
6.4 Qualitative Description of OFDM

OFDM stands for Orthogonal Frequency Division multiplexing. OFDM is based on a parallel data transmission scheme that reduces the effect of multipath fading and makes the use of complex equalizers unnecessary. OFDM derived from the fact the digital data is sent using many carriers, each of a different frequency and these carriers are orthogonal to each other, hence orthogonal frequency division multiplexing. The frequency carriers is chosen in such a way that the modulated carriers are orthogonal and do not interfere with one other. is a modulation technique for transmission based upon the idea of frequency-division multiplexing (FDM), where each frequency channel carries a separate stream of data. in OFDM the frequencies are chosen so that the modulated, which greatly simplifies the design of both the transmitter and the receiver, and also allows high spectral efficiency. although the principles and some of the benefits have been known since the 1960s, OFDM is made popular today by the lower cost and availability of digital signal processing and VLSI based components

6.5 OFDM Simulation

Simulation is done for a multi path (frequency selective fading) channel for a given number of Multi Paths. Channel. Simulation of Modulator and Demodulator Structures for the Single and Multi Carrier PSK Transmission System along with “Symbol Generation” system for Frequency Selective Fading in the Wireless Channel is done. Equal degree of FEC or forward error correction such as Rate Punctured Convolution Encoder/Decoder (RCPC) and compared the BER performance of both the Single and Multi Carrier Transmission systems against symbol period and SNR.
6.5.1 Source Encoder / Source decoder

The data is usually image, wave or text and it is converted into binary data bits. In source coding, the encoder maps the digital signal generated at the source output into another signal to digital form. The mapping is one to one and the objective is to eliminate or reduce the redundancy so as to provide an efficient representation of the source output. Since, the source encoder mapping is one to one, the source decoder simply performs the inverse mapping and thereby delivers to the user destination a reproduction of the original digital source output. The primary benefit gained from the application of source coding is a reduced band width requirement. [Simon Haykin “digital communication” Wiley pub.].

6.5.2 Channel Encoder / Channel Decoder

In channel coding, the objective is for the encoder to map the incoming digital signal into a channel input and for the decoder to map the channel output into an output digital signal in such a way that the effect of channel noise is minimized. The combined role of encoder and decoder is to provide reliable communication over a noisy channel. Because of its high performance, the Viterbi algorithm is commonly used for decoding the convolution coders and is widely used in different communication standards and communication environments.

6.5.3 Serial to Parallel Conversion

The input serial data stream is formatted into the word size required for transmission, e.g. 2 bits/word for QPSK, and shifted into a parallel format. The
data is then transmitted in parallel by assigning each data word to one carrier in the transmission.

6.5.4 Modulation of Data

The data to be transmitted on each carrier is then differential encoded with previous symbols, then mapped into a Phase Shift Keying (PSK) format. Since differential encoding requires an initial phase reference an extra symbol is added at the start for this purpose. The data on each symbol is then mapped to a phase angle based on the modulation method. For example, for QPSK the phase angles used are 0, 90, 180, and 270 degrees. The use of phase shift keying produces a constant amplitude signal and was chosen for its simplicity and to reduce problems with amplitude fluctuations due to fading. The modulation scheme is implemented for BPSK, QPSK, 8PSK, 16PSK, 8QAM, 16QAM, 32QAM AND 64QAM to study the BER performance.

6.5.5 Inverse Fourier Transform

After the required spectrum is worked out, an inverse Fourier transform is used to find the corresponding time waveform. The guard period is then added to the start of each symbol.

6.5.6 Guard Period

The level of multipath robustness can be further increased by the addition of a guard period between transmitted symbols. The guard period allows time for multipath signals from the previous symbol to die away before the information from the current symbol is gathered. The most effective guard period to use is a cyclic prefix of the symbol. If a mirror in time, of the end of the symbol waveform is put at the start of the symbol as the guard period, this
effectively extends the length of the symbol, while maintaining the orthogonality of the waveform. Using this cyclic prefix symbol the samples required for performing the FFT (to decode the symbol), can be taken anywhere over the length of the symbol. This provides multipath immunity as well as symbol time synchronization tolerance. The guard period is as shown in figure 6.4.

![Diagram of cyclic prefixing with symbols and extension](image)

**Fig. 6.4 Guard Period (Cyclic Prefixing)**

The guard interval then begins to cause inter-symbol interference. However, provided the echoes are sufficiently small they do not cause significant problems. This is true most of the time as multipath echoes delayed longer than the guard period will have been reflected of very distant objects. The effect of cyclic prefix is shown in figure 6.5

![Diagram of cyclic prefixing process](image)

Addition of Cyclic prefix to the OFDM signal further improves its ability to deal with fading and interference.

**Fig. 6.5**
6.5.7 Channel

A channel model is then applied to the transmitted signal. The model allows for the signal to noise ratio, multipath, and peak power clipping to be controlled. The signal to noise ratio is set by adding a known amount of white noise to the transmitted signal. Multipath delay spread then added by simulating the delay spread using an FIR filter. The length of the FIR filter represents the maximum delay spread, while the coefficient amplitude represents the reflected signal magnitude.

6.5.8 Transmitter

Locate the carrier positions in the frequency domain using the spectrum of a particular symbol. Convert the stream of symbols to parallel from serial and place them at the positions of the carriers along with pilot signals next to them. Get OFDM time signal /block with the help of IFFT for each block and concatenate all the blocks to get the total OFDM time signal. Insert the guard time(cyclic prefix) to prevent inter block interference. Filter this signal through an FIR filter and add noise to obtain an received signal.

6.5.9 Receiver

The receiver basically does the reverse operation to the transmitter. The guard period is removed and divide the time signal into OFDM block. The FFT of each block is then taken to find the original transmitted spectrum and also pilot symbols as per carrier location. The phase angle of each transmission carrier is then evaluated and converted back to the data word by demodulating the received phase. The data words are then combined back to the same word size as the original data.
6.6 Simulation Results (Single Carrier Verses Multi Carrier)

In order to examine the performance of the proposed scheme, simulations of the proposed scheme, with single carrier system and normal OFDM system are presented in this section. To compare the performance of various schemes symbol period, user SNR and BER are computed. The OFDM system with 64 sub carrier with BPSK, QPSK, 8PSK, 16PSK, 8QAM, 16QAM, 32QAM AND 64 QAM modulations is considered for the study. Figure 6.6 gives the symbol period verses Bit Error Period. From the graph it observed that Bit Error rate(BER) of multicarrier PSK communication system is less than $10^{-2}$ and decreases further to $10^{-4}$ as symbol period increase further and BER is of multicarrier is very much lower compare to single carrier.

Proposed multicarrier system scheme provides significant improvement in terms of BER compare to single carrier. BER versus user SNR graph is shown in Figure 6.7 From the graph we can observe that BER is less than $10^{-2}$ and decreases further as SNR increases and multicarrier performance is very high when it compare with single carrier. Number paths verses BER graph is shown in Figure 6.8. In case of multicarrier system the BER is zero when number paths are less than 3 and increase little by increasing the number of multi paths beyond 3. However, the BER is less than $10^{-2}$ for all multi paths. In case of single carrier the BER is very high for all multi path compare to multi carriers Similar performance is observed with various modulation schemes when it is compared with single carrier as shown in Figure 6.9.
Fig. 6.6 Symbol period verses Bit Error Rate for PSK system

Fig. 6.7 User SNR verses Bit Error Rate

Fig. 6.8 Number paths verses Bit Error Rate
6.7 OFDM Systems by Interference Coefficients Equalization

6.7.1 System Model

The present scheme selects an appropriate filter to neutralize the effect of complex weighting coefficients reducing the ICI in the system. Simulation is performed for different frequency offset and signal to noise ratio (SNR) using 4 QAM and 16-QAM modulations. The performance of various schemes considered in this study is evaluated on the basis of carrier to interference ration (CIR) and bit error rate (BER).

In OFDM, the wide transmission spectrum is divided into narrower bands and data is transmitted in parallel on these narrow bands and data is transmitted in parallel on these narrow bands. Therefore, OFDM symbol period is increased by the number of subcarriers. This multicarrier transmitter partitions the data stream into a block of N data symbols that are transmitted in parallel by modulating the N carriers. Thus if \( t_s \) is the input data symbol duration, the OFDM symbol duration \( T \) becomes \( Nt_s \). These \( N \) data symbols \( X_0, X_1, \ldots, X_{N-1} \) are then converted into parallel form using a serial to parallel
converter. The $N$ data symbols are used to modulate $N$ carriers resulting in frequency division multiplexing. The OFDM modulation process can be easily accomplished by the use of inverse discrete Fourier transform (IDFT) on the $N$ parallel data symbols.

The signal at the output of OFDM transmitter resulting from the $i^{th}$ transmitted symbol can be given by

$$x(t) = e^{j2\pi f_c t} \sum_{k=0}^{N-1} x_k, i \ p(t-kT/N) \quad \ldots \quad (6.1)$$

Where $x_k, i$ is the IDFT of $X_k, k=0, 1, \ldots, N-1$, in the $i^{th}$ symbol, $f_c$ is the carrier frequency and $p(t)$ is the impulse response of the low pass filter at the transmitter.

At the receiver, the received noisy signal is mixed with a local oscillator signal having frequency $\Delta f$ above the correct frequency $f_c$ ignoring the effect of noise as considered in [J. Armstrong, 1999], the demodulated signal is given by

$$y(t) = e^{j2\pi \Delta f t + 0} \sum_{k=0}^{N-1} x_k, i q(t-kT/N) \quad \ldots \quad (6.2)$$

Where $q(t)$ is combined impulse response of channel, transmitter and receiver filters. $\theta_0$ is the difference between the phase of the receiver local oscillator and the carrier phase at the start of the received symbol. We assume that the $q(t)$ satisfies the Nyquist criterion for samples taken at intervals $T/N$ and that $y(t)$ is sampled at the optimum instant, then the samples, of $y(t)$ are given by

$$y_k, i = e^{j\theta_0} x_k, i e^{j2\pi k\Delta f T} \quad \ldots \quad (6.3)$$

Where $\varepsilon = \Delta f T$ is the normalized frequency offset.
These samples are converted into parallel form and then used for computing DFT. The output of the DFT is given by

\[ Y_{m,i} = \sum_{k=0}^{N-1} y_{k,i} e^{(-j2\pi km / N)} \]  

Then from (6.3) and (6.4) it can be shown that

\[ Y_{m,i} = \frac{1}{N} e^{(j\theta_0)} \sum_{k=0}^{N-1} X_{1,i} \sum_{l=0}^{N-1} \exp (j2\pi k(l-m+\varepsilon)/N) \]  

After some simplification this can also be expressed as

\[ Y_{m,i} = e^{(j\theta_0)} \sum_{l=0}^{N-1} c_{1-m} X_{1,i} \]  

Where

\[ c_{1-m} = \frac{1}{N} \sum_{k=0}^{N-1} \exp(j2\pi k(l-m+\varepsilon)/N) \]  

From (6.6), it can be seen that, if \( \varepsilon = 0 \) then \( Y_{m,i} = e^{(j\theta_0)} X_{m,i} \) and each decode value is simply the phase rotated version of the transmitted value. If \( \varepsilon \neq 0 \), the ICI will occur and each output decoded value will depend on all the input values. The decoded output \( Y_{m,i} \) therefore consists of a wanted component which is due to \( X_{m,i} \) but is subject to a change in phase and amplitude given by

\[ c_0 = \frac{1}{N} \sum_{k=0}^{N-1} \exp(j2\pi \varepsilon/N) \]  

therefore, the ICI depends on the N complex weighting coefficients \( c_0, c_1, \ldots, c_{N-1} \). The carrier to interference ratio (CIR) can be expressed as

\[ \text{CIR} = \frac{|c_0|}{\sum_{i=0, i \neq m}^{N-1} |c_{i-m}|^2} \]
6.7.2 Proposed Scheme

In order to design an equalizer for equalizing the weighting coefficients $c_0, c_1, \ldots, c_{N-1}$ that cause ICI, the matrix of the coefficients is obtained as:

$$
C = \begin{pmatrix}
  c_0 & c_1 & \cdots & c_{N-1} \\
  c_{-1} & c_0 & \cdots & c_{N-2} \\
  c_{-2} & c_{-1} & c_0 & \cdots & c_{N-3} \\
  \vdots & \vdots & \ddots & \ddots & \vdots \\
  c_{N+1} & c_{N+2} & \cdots & c_0 \\
\end{pmatrix}
$$

(6.10)

The received signal at the receiver in the matrix form can be written as

$$
Y = WAW^{-1}X \text{ OR } Y = CX 
$$

(6.11)

Where $X = [x_0, x_1, x_2, \ldots, x_{N-1}]^T$ is $N \times 1$ vector, $W$ is $N \times N$ DFT matrix and $W^{-1}$ is $N \times N$ IDFT matrix. When an equalizer is used the output is given by

$$
Z = DY 
$$

(6.12)

Where $D = C^{-1}$

(6.13)

is a circulant matrix which denotes the equalizer filter matrix.

Commonly, the equalizer is designed by using training symbols. In the present scheme, however, an appropriate filter is chosen from a small set of predefined filters. This is based on the observation that an equalizer designed for a given value of $\varepsilon$ is effective for a variation $\pm \Delta \varepsilon$ around $\varepsilon$.

If the maximum expected value of normalized frequency offset is $\varepsilon_{\text{max}}$, then the given range of $\varepsilon$ can be divided into $p$ segments, where $p = \frac{\varepsilon_{\text{max}}}{2\Delta \varepsilon}$. Let the midpoint of these segment be denoted by $\varepsilon^{(i)} = 1, 2, \ldots, p$ can be obtained from (6.13).
The appropriate equalizer from this set of predefined equalizers \( D^{(i)} \), \( i = 1, 2, \ldots, p \), is selected according to the following criterion.

Minimize \( \{ J(D^{(i)}) = E[|Z^{(i)} - Z_{k}^{(i)}|^2] \} \) \hspace{1cm} \ldots \hspace{1cm} (6.14)

with \( Z^{(i)} = D^{(i)} Y \) where \( Z_{k}^{(i)} \) denotes the decision on \( Z^{(i)} \) and \( E\{\} \) denotes the expectation.

Alternatively, in place of the cost function defined in (6.14), another cost function such as CMA cost function [C. Richard Johnson et al, 1998] can also be used. In the proposed scheme the cost function is merely used for the selection of the appropriate equalizer filter and is not minimized iteratively. The cost function is required to be differentiable when it is used for the designing of the equalizer using some higher order statistics (HOS) based blind equalization scheme such as CMA. These schemes usually suffer from slow convergence and are not suitable for the present case.

### 6.7.3 Simulation and Results

In order to examine the performance of the proposed scheme, simulations of the proposed scheme, ICI self-cancellation scheme of [J. Armstrong, 1999] and normal OFDM system are presented in this section. To compare the performance of various schemes CIR and BER are computed. The OFDM system with 64 subcarrier with 4QAM and 16QAM modulations is considered for the study. The appropriate equalizer is selected from a set of three filters designed at \( \varepsilon^{(1)} = 0.15, \varepsilon^{(2)} = 0.2, \) and \( \varepsilon^{(3)} = 0.3 \). For better accuracy a larger set of filters may be considered.
Fig. 6.10 CIR as a Function Of Normalized Frequency Offset

Fig. 6.11 BER Comparison for normal OFDM, ICI self cancellation with proposed scheme at $\epsilon = 0.05$

Fig. 6.12 BER Comparison for normal OFDM, ICI self cancellation with proposed scheme at $\epsilon = 0.15$
Figure 6.13 gives the CIR for the above three schemes at different values of normalized frequency offset $\varepsilon$. Simulation results show that the proposed scheme provides significant improvement in terms of CIR. In present scheme the CIR for ideal equalizer is maximum and is always higher than that of ICI self-cancellation scheme of [J.Armstrong, 1999] at other value of $\varepsilon$. The CIR for the proposed scheme can be computed by using the elements of any row of the product matrix $DCIN (6.13)$. 
BER versus Eb/No for the various schemes at different normalized frequency offset $\epsilon$ is shown in Figure 6.11 – Figure 6.13. The results in Figure 6.11 represent the BER versus Eb/No for the standard OFDM and ICI self-cancellation scheme. It can be observed from Figure 6.11 that BER is decreased as the SNR increases when normalized frequency offset $\epsilon$ is small (<0.1). However, when $\epsilon$ is large (>0.15) the decrease in BER is not very significant as can be seen from Figure 6.13 it indicates that it may not be possible to improve the system performance only by increasing transmitter power when normalized frequency offset is very large.

BER versus Eb/No plots for the proposed scheme at $\epsilon=0.15$, and $\epsilon=0.2$ are shown in Figure 6.12, Figure 6.13 respectively, Simulation results, in fig.6.13, show that the BER performance of the proposed scheme is better than that of ICI self-cancellation scheme, and at $\epsilon=0.15$, the proposed scheme is about 10 dB better than the ICI self cancellation scheme when BER is $10^{-4}$. Also, the problem of reduction is bandwidth efficiency of ICI self-cancellation scheme is effectively overcome in the present scheme. The variation in BER of the proposed scheme at different values of frequency offset is always less than the ICI self-cancellation scheme as observed from Figure 6.12 and Figure 6.13

6.8 Conclusions

A) From the above simulation results the following conclusions are made for multi carrier communication system (OFDM) when it is compared with single carrier:

1. The Bit Error rate (BER) is exponentially decrease from $10^{-2}$ as symbol period increases and it is very low.
2. BER is very low for different modulation schemes. The performance is very high for the modulation schemes like QPSK, 8PSK and 8QAM compare to other modulation schemes. Performance at 64QAM is poor compare to other schemes.

3. BER is zero with respect to multi paths, when number of multi paths less than 4. But little increase in BER as number of multi paths are more 4. However is less than $10^{-3}$.

4. BER is decrease as user SNR increases and still at low SNR the BER is less than $10^{-2}$.

B) The following conclusions are made for equalized OFDM when compared with normal and self ICI cancellation method.

OFDM is an effective technology for high data rate transmissions. The frequency offset in mobile radio channels distorts the orthogonality between subcarriers resulting in ICI, which seriously degrades the performance of systems. Based on ICI coefficients analysis, this work has presented an inter carrier interference cancellation method using a selection scheme to choose an appropriate filter for equalization of these ICI coefficients. As the scheme does not require any training symbols, the high spectral efficiency of OFDM system is maintained. The present scheme gives better performance than the ICI self-cancellation scheme in terms of CIR and BER. However, these benefits are derived at the cost of slight increase in the system complexity.

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