Chapter 7

Magnetic field sensors using exponentially tapered quarter annular Josephson junctions

A novel fluxon based magnetic field sensor is proposed using an exponentially tapered quarter annular Josephson junction geometry. Theoretical studies shows that quarter annular geometry provides asymmetric boundary conditions in a parallel magnetic field and exponentially tapered width of the dielectric barrier provides a geometrical driving force for the fluxons facilitating unidirectional flow of fluxons from one end to the other end in the junction when a magnetic field parallel to the dielectric barrier is applied to the junction. The proposed device acts as a field to voltage transducer and does not require electric power for its operation.

7.1 Introduction

Josephson junctions are best transducers which can convert magnetic energy into electrical energy. They are widely used in SQUID magnetometers[121], SIS mixers[122] and in voltage standard applications[123]. SQUIDs are used for an extremely sensitive measurement of the magnetic fields. They can detect even the magnetic fields of the biological cells. In many applications, the extraordinary
sensitivity of the SQUID based sensors are not required. For such applications, we can make use of some simple LJJ devices to fabricate sensors with less sensitivity than SQUID sensors but exhibiting better performance compared to other commercially available magnetic field sensors.

Detection of both static and \( rf \) magnetic field signals for nondestructive testing and evaluation needs a wide range of different sensor types depending on spatial resolution and field sensitivity. In this chapter, an exponentially tapered quarter annular LJJ terminated with a load resistor at one end is studied and demonstrates that the device can be effectively used as a magnetic field to voltage transducer. Quarter annular geometry provides non-uniform boundary conditions to a parallel magnetic field. Exponential tapering in LJJ was introduced in Ref.[105, 106] as a means to produce coherent unidirectional flow of fluxons in a flux-flow oscillator. Exponential tapering provides a geometrical force for the fluxons and avoids the presence of trapped fluxons and gives perfect impedance matching to an external load. The load resistor connected at one end of the junction can be used to terminate the fluxon chain moving in the junction. This is a unique device in which fluxons enter the junction from one end and move unidirectionally to the other end of the junction under the influence of the geometrical driving force and also due to the fluxon-fluxon repulsive interaction.

### 7.2 Theoretical model

A LJJ with a quarter annular geometry is considered with an external magnetic field applied parallel to the dielectric barrier of uniform thickness. A sketch of the quarter annular geometry is shown in Fig.7.1a. The width of the junction is exponentially tapered (i.e., \( w(x) = w_0 e^{-\lambda x} \)), decreasing towards the load as represented schematically in Fig.7.1(b). The external field is applied in such a way that it is directed radially at the left end \( (x = 0) \) of the junction. The magnetic flux linked with the junction can be expressed as \( d\varphi(x) = \varepsilon (\vec{H} \cdot \vec{n}) = \frac{1}{2} \nabla_{\perp} \cdot \vec{E} \).
The effects of an applied magnetic field is to induce currents in closed form across the junction. The induced current in the junction due to the applied field is \( \frac{d\varphi(x)}{dx} = \varepsilon H \cos(kx) \). This current term gives a net positive value over the length of the junction and therefore cannot circulate in closed form across the junction. This means that the external field cannot have any influence in the interior part of the junction. Thus an exponentially tapered quarter annular LJJ under a static magnetic field is modelled with the general perturbed sG partial differential equation\[105, 12\]

\[
\varphi_{tt} - \varphi_{xx} + \sin \varphi = -\alpha \varphi_t - \lambda \varphi_x
\]  

(7.1)

Where \( \lambda \) is the tapering factor. The boundary conditions of the junction in an external field with a passive load of impedance \( z \) (representing a connection to a microwave circuit) connected at \( x = l \) (right-end of the junction) are

\[
\varphi_x(0, t) = \varepsilon H = b; \quad \varphi_x(l, t) = -\frac{\varphi_t}{z}
\]  

(7.2)

These boundary conditions are consistent with the fact that the effective field linked with the junction has asymmetric boundary conditions. Due to this boundary conditions, flux penetration is possible only from the left-end of the junction. The penetrated fluxons are pushed towards right-end due to the geometrical driving force. The fluxon-fluxon repulsive interaction maintain constant distance between the fluxons. The transit of the fluxons from the left-end to the right-end in the junction produce periodic voltage pulses in the load which can be averaged over a time. The passive load \( (z) \) connected at the right-end absorbs the fluxon chain in the junction.

In nonuniform junctions, a static field can produce a preferential direction for the flux-flow even in the absence of an external \( dc \) bias. This effect gives rise to ZCFFS in the current voltage characteristics of the junction \[87, 49\]. ZCFFS is a manifestation of flux-flow in the absence of a \( dc \) current. Thus, in nonuniform
junctions or junctions with asymmetric boundary conditions, it is possible to extract work from a constant magnetic field.

Eq. (7.1) with boundary conditions, Eq. (7.2), represents an exponentially tapered quarter annular LJJ in a static magnetic field. Eqs. (7.1) and (7.2) are mathematically equivalent to Eqs. (11), (12) and (13) in Ref.[105] and therefore all the results obtained in that work is applicable to the present model. In Ref.[105], a coherent unidirectional flow of fluxons was achieved by feeding a dc bias from one end of a rectangular junction. In the present work, the same phenomenon is achieved by considering an exponentially tapered quarter annular junction and applying a magnetic field parallel to the dielectric barrier.

In the case of exponentially tapered junctions, impedance can be exactly matched. From Eq. (7.1), we can see that any travelling wave \( \phi(x,t) = f(x-ut) \) has a solution with the velocity \( u = \lambda/\alpha \). The condition for impedance matching can be obtained by equating this limiting velocity to \( -\varphi_t/\varphi_z \). Therefore the impedance matching load can be calculated as \( z = \lambda/\alpha \).

In the case, \( 0 < \lambda \leq 1 \), the maximum value of \( b \), for a static solution to exist is \( b_e = 2 - 2\lambda \) (cf. Sec. 6.2.1). This expression shows that exponential tapering decreases critical magnetic field value.

To determine the dynamical properties of the device, we introduce simple analytical models describing a smooth phase flow of the fluxons through the junction which is referred as laminar flow[106]. We approximate the laminar flow by taking the variational approach and consider the high voltage limit in which small changes in the instantaneous voltage due to the changes in the parameters \( x \) and \( t \) are neglected to a first approximation. The variational analysis is made on the basis of conservation of energy by the sG Hamiltonian. The energy of the unperturbed sG system is given by Eq. (1.37). Perturbational parameters modulate the velocity of the solitons and may cause to dissipate energy. The rate of dissipation is calculated by computing
\[ \frac{d}{dt}(H) = [\varphi_x \varphi_t]_0^l - \int_0^l [\alpha \varphi_t^2 + \lambda \varphi_x \varphi_t] \, dx \]  

(7.3)

where the first term on the right side account for the boundary conditions. From Eq. (1.38), we get \( \varphi_t = -u \varphi_x \). Inserting Eq. (1.38) in Eq. (7.4) and following perturbative analysis[105, 106], we get

\[ \frac{du}{dt} = (1 - u^2)(\lambda - \alpha u) \]  

(7.4)

This equation shows that for \( \lambda > \alpha \), fluxon will always be accelerated towards the limiting value \( u = 1 \). When \( \lambda < \alpha \), the fixed point \( u = \lambda / \alpha \) is linearly stable. Assuming a linear flow of the fluxons, we get the average voltage across the load as[105]

\[ V = -\frac{(2 + \lambda l)zb}{2 + 2\alpha l z - \lambda l} \]  

(7.5)

and the travelling wave speed of the fluxon as

\[ u = \frac{(1 + \lambda l / 2)z}{(\lambda - \alpha z) + 1 + \alpha l z - \lambda l / 2} \]  

(7.6)

### 7.3 Static field detection properties

The influences of an external static magnetic field on the dynamical properties of an exponentially tapered junction is studied and seen that at low magnetic fields, the static solution in the junction has only a half-fluxon content. This static solution exists up to a critical value of the magnetic field \( b_c \), above this value, static solution becomes unstable and gives rise to a train of fluxons moving in the junction. In Fig.7.2, static distribution of the flux profile \( (\varphi_x) \) in the junction at \( b = 1.9 \) (solid line) and dynamic distribution of the fluxons at \( b = 2.0 \) (circles) are presented. This figure illustrates the process of fluxon penetration into the junction at higher magnetic fields. The data were obtained by numerical solution of Eqs. (7.1) and (7.2) which automatically takes into account the fluxon interaction with the edges and with each other. The unidirectional flow of fluxons
produces an average voltage across the junction. The magnetic field \(b\) versus average voltage \(<u>\) is presented in Fig.7.3. For generality, different tapering factors are considered. It is found that as the tapering factor increases, the critical field required for flux penetration decreases. Average voltage is zero below the critical value \(b_c\) and linearly increases with the applied field above the critical value. These graphs demonstrates that in exponentially tapered quarter annular junctions, due to the geometrical driving force and due to the asymmetrically linked magnetic field, unidirectional flux-flow takes place even in the absence of a \(dc\) bias. This peculiar property of the exponentially tapered quarter annular junctions make them superior in the design of the magnetic field sensors.

### 7.4 \(rf\) field detection

To determine the \(rf\) field detection capabilities of the device, we have considered a harmonically varying \(rf\) field parallel to the dielectric barrier of the junction. Theoretical model suggests that the corresponding boundary conditions of the junction become:

\[
\varphi_x(0, t) = \varepsilon H \sin(\omega t) = b \sin(\omega t) ; \quad \varphi_x(l, t) = -\frac{\varphi_t}{z}
\]  

These boundary conditions show that the flux linked with the junction at every alternate half cycles of the field changes in sign. Thus in the first half cycle of the field, fluxons enter the junction while in the second half cycle antifluxons enter the junction. Thus this device support fluxon and antifluxon propagation in the same direction one after another. In Fig.7.4, we plot the spatial profiles \(\varphi_x\) of the fluxons and antifluxons moving in the same direction along the junction. Fluxons on reaching the load produce positive voltage pulses while antifluxons produce negative voltage pulses. Thus the \(rf\) field produces an alternating voltage across the load. The amplitude of the induced \(ac\) voltage will be proportional to the \(rf\) field intensity.
7.5 Conclusions

The proposed device is very simple to fabricate and can be operated as a static device as this device does not require electric power for its operation. Absence of an electric bias current minimizes the heating effect and decreases the degradation of the device and therefore make them suitable in space applications. The device gives output voltage which is linearly proportional to the applied field. Both static and time varying magnetic fields can be detected using this device. One limitation of the device is that it can detect only fields of strength higher than the first critical field of the LJJ and that is parallel to the dielectric barrier. Junctions of large $\lambda_J$ will give lower critical field and therefore can be preferred in making the device. As it can be seen from Eq. (7.6), low dissipative junctions give higher voltages and are suitable for making the sensor. The transit time of the fluxons can be reduced using shorter junctions and therefore the delay in the detection can be minimized. A static magnetic field produces a proportional $dc$ voltage across the load and a $rf$ field produces a proportional alternating voltage across the load. Using vertically stacked junctions output voltage can be increased. Instead of the exponential tapering, a properly chosen $dc$ bias can be used to drive the fluxons. This device is extremely useful in detecting comparatively higher fields with less precision. Experimental realization of the device will create potential market in the superconducting electronic industry.
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Fig. 7.1a A sketch of the quarter annular LJJ geometry with the applied field parallel to the dielectric barrier.

Fig. 7.1b Schematic representation of the top view of the exponentially tapered width of the junction.

Fig. 7.2 Fluxon penetration into an exponentially tapered quarter annular junction at a constant magnetic field. Static solution (solid circles) and dynamic solution (open circles) in a junction of length $l=20$ with parameters $z=1.0$, $\alpha=0.1$ and $\lambda=0.02$. 
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Fig. 7.3  Applied static magnetic field $b$ versus the average normalized velocity $<u>$ computed at different tapering factors. The parameters of the junctions are $l=20$, $\alpha=1.0$ and $z=0.05$.

Fig. 7.4  The spatial profiles along the junction showing fluxons and antifluxons moving in the same direction towards the load when an rf field is applied to the junction. The parameters are $l=30$, $z=1.0$, $\alpha=0.05$, $b=2.0$, $\omega=0.5$ and $\lambda=0.05$. 

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