CHAPTER II

UNSTEADY HYDRO - MAGNETIC FLOW OF AN INCOMPRESSIBLE VISCOS FLUID IN A ROTATING PARALLEL PLATE CHANNEL WITH A POROUS LINING - BRINKMAN MODEL.
2.1. INTRODUCTION:

The MHD fluid flow in a rotating channel is an interesting area in the study of fluid mechanics because of its relevance to various engineering applications. It is a challenging approach to atmospheric science that exerts its influence of rotation to help in understanding the behaviour of oceanic circulation and formation of galaxies. The effect of Coriolis force in the atmosphere is exposed to oceanic circulation and the formation of galaxies in taking into account the flow of electron is continuously liberated from the sun what is called "Solar wind". The MHD flow in the rotating environment leads to a startup process implying thereby a viscous layer at the boundary is suddenly set into motion and the rate of rotation becomes important in the application of various branches of geophysics, astrophysics and fluid engineering. Currently, MHD effects are widely exploited in different industrial processes ranging from metallurgy to the production pure crystals (18). A field in which MHD will play an essential role is nuclear fusion, where it is involved in at least two different problems: the confinement and dynamics of plasma, and the behaviour of the liquid metal alloys employed in some of the currently considered designs of tritium breeding blankets. In recently years the hydromagnetic flow in a rotating channel in the presence of an applied uniform magnetic field as well as constant pressure gradient has been considered by a number of research workers, taking into account the various aspects of the problem.

The motion of rotating fluids enclosed with in a body or vice versa, was given by Green Span (31) who discussed these problems relating to the boundary layers
and their interaction in rotating flows and gave so many examples relating to such interaction. The flow of a fluid under a constant pressure gradient between two rotating parallel plates has been given by Vidyanidhi (30). The rotating viscous flow equations yield a layer known as Eckman boundary layer after the Swedish oceanographer V-W Eckman who discovered it. Attempts to observe the structure of the Eckman layer in the surface layers of the sea have been successful. Eckman layers are easy to produce and observe in the laboratory. Such boundary layers or similar ones, are required to connect principally geostrophic flow in the interior of the fluid to the horizontal boundaries where conditions like a prescribed horizontal stress or no slip on a solid bottom are given. In a similar way other kinds of various boundaries have been studied so as to connect geostrophic flow to vertical boundaries (for example a vertical well along which the depth varies) on which boundary conditions consistent with geostrophic flow are given.

G.S.Seth and Ghosh (25) was investigated to the unsteady hydro magnetic flow of a viscous incompressible electrically conducting fluid in rotating channel under the influence of a periodic pressure gradient and of uniform magnetic field, which is inclined with the axes of rotation. The problem of steady laminar micro polar fluid flow through porous walls of different permeability had been discussed by R.S. Agarwal and C. Dhanapal (1). Steady and unsteady hydromagnetic flow of viscous incompressible electrically conducting fluid under the influence of constant and periodic pressure gradient in the presence of include magnetic field had been investigated by S.K Ghosh (8) to study the effect of slowly rotating systems with low frequency of oscillation when the conductivity of the fluid is low and the applied magnetic field is weak. El-Mistikawy
et al. (5 & 6) were discussed the rotating disk flow in the presence of strong magnetic field and weak magnetic field. Later Hazem Ali Allia (10) developed the MHD flow of incompressible, viscous and electrically conducting fluid above an infinite rotating porous disk was extended to flow starting impulsively from rest. The fluid was subjected to an external uniform magnetic field perpendicular to the plane of the disk. The effects of uniform suction or injection through the disk on the unsteady MHD flow were also considered.

Laszlo Fuchs and Yongnian Yang (16) studied a numerical study of the incompressible viscous flow in rotating channel with rectangular cross section. K.N. Mehta and Shobha sood (17) investigated the effect of temperature dependent viscosity on the heat transfer rate for a transient free convection flow along a non-isothermal vertical surface. They observed that a decrease in viscosity increases the heat transfer rate and reduces the time to reach steady state. Hazem A. Attia (11) studied the unsteady flow and heat transfer of a dusty conducting fluid between two parallel plates with variable viscosity and electric conductivity. The effect of the variation in the viscosity and electric conductivity of the fluid and the uniform magnetic field on the velocity and temperature fields for both the fluid and dust particles was discussed. D.A Nield et al. (19) investigated the thermal development of forced convection in a parallel plate channel filled by a saturated porous medium, with wall held at uniform temperature and with the effects of axial conduction and viscous dissipation included. A numerical solution of the transient free convection MHD flow of an incompressible viscous fluid past a semi-
infinite inclined plate with variable surface heat and mass flux was studied by
P.Ganeshan et al.(7).

The channel flow problems where the flow is maintained by torsional or
non-torsional oscillations of one or both the boundaries, threw some light in finding out
the growth and development of boundary layers associated with the flows occurring in
geothermal phenomena. D.V.Krishna et al (15) studied the hydromagnetic conversion
flow of a viscous electrically conducting fluid through a porous medium in a rotating
parallel plate channel. O.Jumbul et al. (13) studied laminar heat transfer in parallel plates
and circular ducts subject to uniform wall temperature by taking into account both
viscous dissipation and fluid axial heat conductions in an infinite region. And extension
of this work was done by O.Jambal et al.(14). Later M. Guria et al (9) studied the
unsteady couette flow of an viscous incompressible fluid confined between parallel
plates, rotating with an uniform angular velocity about an axis normal to the plates, here
the flow was induced by the motion of the upper plate and the fluid and plates rotate in
unison with the same angular velocity. Claire Jacobs (3) studied the transient effects
considering the small amplitude torsional oscillations of disks. This problem had been
extended to the hydro-magnetic case by Murthy (32), who discussed torsional oscillations
of the disks maintained at different temperatures. Debnath(4) considered an unsteady
hydrodynamic and hydro magnetic boundary flow in a rotating viscous fluid due to
oscillations of plates including the effects of uniform pressure gradients and uniform
suction. The structure of the velocity field and the associated Stokes, Ekman and
Rayleigh boundary layers on the plates are determined for the resonant and non-resonant
cases. Rao.D.R.V. Krishna.D.V. & Debanath, Rao et al (21) have made an initial value investigation of the combined free and forced convection effects in an unsteady hydro magnetic viscous incompressible rotating fluid between two disks under a uniform transverse magnetic field. This analysis has been extended to porous boundaries by Sarojamma and Krishna(23), and later by Siva Prasad (26) to include the Hall current effects.

In this chapter, we make an initial value investigation of the unsteady flow of an incompressible viscous fluid in a rotating parallel plate channel bounded on one side by a porous bed under the influence of a uniform transverse magnetic field. The perturbations are created by a constant pressure gradient along the plates in addition to the non-torsional oscillations of the lower plate. The flow in the clean fluid region is governed by Navier-Stoke's equations while in the porous bed the equations are based on Brinkman's model. The exact solutions of the velocity in the clean fluid and the porous medium consist of steady state and transient state. The time required for the transient state to decay is evaluated in detail and ultimate quasi-steady state solution has been derived analytically, its behaviour is computationally discussed with reference to the various governing parameters. The shear stresses on the boundaries and the mass flux are also obtained analytically and their behaviour is computationally discussed.
2.2. Formulation and Solution of the problem:

We consider the unsteady flow of an incompressible viscous fluid in a rotating parallel plate channel bounded on one side by a porous bed subjected to a uniform transverse magnetic field normal to the channel. In the initial undisturbed state both the plates and the fluid rotate with the same angular velocity $\Omega$. At $t > 0$ the fluid is driven by a constant pressure gradient parallel to the channel walls and in addition the lower plate perform non-torsional oscillations in its own plane.

We choose a Cartesian system $0(x, y, z)$ such that the boundary walls are at $z=0$ and $z=1$, $z$-axis being the axis of rotation of the plates. The fluid medium consists of two zones namely zone 1 and zone 2. Zone 1 consists of clean fluid governed by Navier-Stokes equations and zone 2 corresponds to the flow through porous bed governed by Brinkman's equations. At the interface the fluid satisfies the continuity condition of velocity and shear stress. The unsteady hydro magnetic equations governing the incompressible viscous fluid in zone 1 under the influence of transverse magnetic field with reference to a frame rotating with a constant angular velocity $\Omega$ are

\[
\frac{\partial u}{\partial t} - 2\Omega v = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \nu \frac{\partial^2 u}{\partial z^2} - \frac{\sigma H_o^2}{\rho} u - \frac{v}{k} u_p \quad (2.2.1)
\]

\[
\frac{\partial v}{\partial t} + 2\Omega u = -\frac{1}{\rho} \frac{\partial P}{\partial y} + \nu \frac{\partial^2 v}{\partial z^2} - \frac{\sigma H_o^2}{\rho} v - \frac{v}{k} v_p \quad (2.2.2)
\]

The Brinkman-equations governing the flow through porous medium with respect to the rotating frame zone 2.

\[
\frac{\partial u_p}{\partial t} - 2\Omega v_p = -\frac{1}{\rho} \frac{\partial P}{\partial x} + v_{eff} \frac{\partial^2 u_p}{\partial z^2} - \frac{\sigma H_o^2}{\rho} u - \frac{v}{k} u_p \quad (2.2.3)
\]

\[
\frac{\partial v_p}{\partial t} + 2\Omega u_p = -\frac{1}{\rho} \frac{\partial P}{\partial y} + v_{eff} \frac{\partial^2 v_p}{\partial z^2} - \frac{\sigma H_o^2}{\rho} v_p - \frac{v}{k} v_p \quad (2.2.4)
\]
zone 1 Clean fluid region

zone 2 porous bed

$z = h$

$z = 0$

$z = l$

$z = h$

$z = 0$
Where \((u, v)\) and \((u_p, v_p)\) are velocity components along \(O(x, y)\) directions respectively, \(\rho\) the density of the fluid, \(\sigma\) the conductivity of the medium, \(\mu_e\) the magnetic permeability, \(\nu\) the coefficient of kinematic viscosity, \(\nu_{\text{eff}}\) the coefficient of effective kinematic viscosity, \(k\) the permeability of the medium, \(H_0\) is the applied magnetic field. Since the plates extends to infinity along \(x\) and \(y\) directions, all the physical quantities except the pressure depend on \(z\) and \(t\) alone. Hence \(u, v\) and \(u_p, v_p\) are function of \(z\) and \(t\) alone and hence the respective equations of continuity are trivially satisfied.

Let
\[ q = u + iv, \quad \xi = x - iy, \]
\[ q_p = u_p + iv_p. \]

Now combining equations (2.2.1) and (2.2.2), we obtain
\[ \frac{\partial q}{\partial t} + 2i\Omega q = -\frac{1}{\rho} \frac{\partial p}{\partial \xi} + \nu \frac{\partial^2 q}{\partial z^2} - \frac{\sigma \mu_e^2 H_0^2}{\rho} q \tag{2.2.5} \]

and combining equations (2.2.3) and (2.2.4), we obtain
\[ \frac{\partial q}{\partial t} + 2i\Omega q_p = -\frac{1}{\rho} \frac{\partial p}{\partial \xi} + \nu \frac{\partial^2 q_p}{\partial z^2} - \frac{\sigma \mu_e^2 H_0^2}{\rho} q_p - \frac{\nu}{k} q_p \tag{2.2.6} \]

The boundary and initial conditions are
\[ q_p = a e^{iat} + b e^{-iat}, \quad z = 0 \tag{2.2.7} \]
\[ q = 0, \quad t \neq 0, \quad z = l \tag{2.2.8} \]
\[ q = 0, \quad q_p = 0, \quad t \leq 0, \quad \text{for all } z \tag{2.2.9} \]
The interfacial conditions are

\[ q = q_p, \quad v \frac{dq}{dz} = v_{eff} \frac{dq_p}{dz} \]

at \( z = h \) \hspace{1cm} (2.2.10)

we introduce the following non dimensional variables are

\[ z^* = \frac{z}{l}, q^* = q l, q_{p}^* = \frac{q_p l}{\nu}, t^* = \frac{t}{l^2} \]

\[ \omega^* = \frac{\omega l^2}{\nu}, \xi^* = \frac{\xi}{\xi l}, p^* = \frac{p l^2}{\rho \nu^2}, h^* = \frac{h}{l} \]

The governing non-dimensional equations are (dropping asterisks in all forms)

\[ \frac{\partial q}{\partial t} + 2iE^{-1} q = -\frac{\partial P}{\partial \xi} + \frac{\partial^2 q}{\partial \xi^2} - M^2 q \] \hspace{1cm} (2.2.11)

\[ \frac{\partial q_p}{\partial t} + 2iE^{-1} q_p = -\frac{\partial P}{\partial \xi} + \frac{\partial^2 q_p}{\partial \xi^2} - M^2 q_p - D^{-1} q_p \] \hspace{1cm} (2.2.12)

where,

\[ M^2 = \frac{\sigma H^2 H_0^2 l^2}{\rho \nu} \] \hspace{1cm} is the Hartmann number

\[ D^{-1} = \frac{l^2}{k} \] \hspace{1cm} is the Inverse Darcy Parameter

\[ E = \frac{\nu}{\Omega l^2} \] \hspace{1cm} is the Eckmann number
Whose corresponding initial and boundary conditions are

\[ q_p = a e^{i\omega t} + b e^{-i\omega t}, \quad t > 0 \quad z = 0 \quad (2.2.13) \]
\[ q = 0, \quad t > 0 \quad z = l \quad (2.2.14) \]
\[ q = q_p = 0, \quad at \quad t \leq 0, \quad for \ all \ z \quad (2.2.15) \]

The interfacial conditions are

\[
\begin{aligned}
q &= q_p, \\
\frac{dq}{dz} &= \beta \frac{dq_p}{dz}, \\
\end{aligned} \quad \begin{cases} \quad z = h \end{cases} \quad (2.2.16)
\]

Taking Laplace transforms of equations (2.2.11) and (2.2.12) using initial condition (2.2.15) the governing equations in terms of the transformed variable in zone 1 reduces to

\[
\frac{d^2 \tilde{q}}{dz^2} - \left( M^3 + 2iE^{-1} + s \right) \tilde{q}_p = -\frac{P}{s} \quad (2.2.17)
\]

The relevant transformed boundary condition is

\[
\tilde{q} = 0, \quad z = l, \quad (2.2.18)
\]

Likewise the governing equation in zone 2 is

\[
\frac{d^2 \tilde{q}_p}{dz^2} - \left( M^2 + 2iE^{-1} + D^{-1} + s \right) \tilde{q}_p = -\frac{P}{s} \quad (2.2.19)
\]
the corresponding transformed condition is

\[ q_p = \frac{a}{s - i\omega} + \frac{b}{s + i\omega}, \quad z = 0 \]  

(2.2.20)

The transformed interfacial conditions are

\[ \bar{q} = \bar{q}_p, \quad z = h \]  

(1.2.21)

\[ \frac{d\bar{q}}{dz} = \beta \frac{d\bar{q}_p}{dz}, \quad z = h \]  

(1.2.22)

Solving equation (2.2.17) subjected to the condition (2.2.18), we get

\[ ACosh\lambda_1 + B Sinh\lambda_1 + \frac{P}{\lambda_1^2 s} = 0 \]  

(1.2.23)

Solving (2.2.19) subjected to the conditions (2.2.20)

\[ C + \frac{P}{\lambda_2^2 s} = \frac{a}{s - i\omega} + \frac{a}{s + i\omega} \]  

(1.2.24)
Making use of the interfacial conditions (2.2.21) and (2.2.22)

\[ \begin{align*}
ACosh\lambda_1 h + B Sinh\lambda_1 h &+ \frac{P}{\lambda_1^2 s} \\
&= C Cosh\lambda_1 h + D Sinh\lambda_1 h + \frac{P}{\lambda_2^2 s} \\
&= \beta\left[C\lambda_2 Sinh\lambda_2 h + D\lambda_2 Cosh\lambda_2 h\right] \\
\end{align*} \]  

(2.2.25)

(2.2.26)

Solving equations (2.2.23), (2.2.24), (2.2.25) and (2.2.26) and we obtain

\[ \bar{q} = \left\{ \begin{array}{c}
\left\{ \frac{a}{s-i\omega} + \frac{b}{s+i\omega} - \frac{P}{\lambda_2^2 s} \right\} Cosh\lambda_2 h + \frac{P}{\lambda_1^2 s} Sinh\lambda_1 h + \frac{p}{s} \left( \frac{\lambda_1^2 - \lambda_2^2}{\lambda_1^2} \right) \lambda_1^{\text{eff}} Cosh\lambda_1 h - \\
- \left\{ \frac{\lambda_2^{\text{eff}}}{\lambda_1} \left( \frac{a}{s-i\omega} + \frac{b}{s+i\omega} - \frac{P}{\lambda_2^2 s} \right) Sinh\lambda_2 h + \frac{Cosh\lambda_1 h}{Sinh\lambda_1} \frac{p}{\lambda_1^2 s} \right\} Sinh\lambda_2 h \\
+ \frac{\lambda_2^{\text{eff}}}{\lambda_1} \left[ Cosh\lambda_2 h - \frac{Cosh\lambda_1}{Sinh\lambda_1} Sinh\lambda_1 h \right] + Sinh\lambda_2 h \left[ Sinh\lambda_2 h - \frac{Cosh\lambda_1}{Sinh\lambda_1} Sinh\lambda_1 h \right] \\
+ \frac{Sinh\lambda_2 z}{\lambda_1} \{ - \frac{P}{\lambda_1^2 s} \}
\end{array} \right. \]  

\[ \bar{q} = \left\{ \begin{array}{c}
\left\{ \frac{a}{s-i\omega} + \frac{b}{s+i\omega} - \frac{P}{\lambda_2^2 s} \right\} Cosh\lambda_2 h + \frac{P}{\lambda_1^2 s} Sinh\lambda_1 h + \frac{p}{s} \left( \frac{\lambda_1^2 - \lambda_2^2}{\lambda_1^2} \right) \lambda_1^{\text{eff}} Cosh\lambda_1 h - \\
- \left\{ \frac{\lambda_2^{\text{eff}}}{\lambda_1} \left( \frac{a}{s-i\omega} + \frac{b}{s+i\omega} - \frac{P}{\lambda_2^2 s} \right) Sinh\lambda_2 h + \frac{Cosh\lambda_1 h}{Sinh\lambda_1} \frac{p}{\lambda_1^2 s} \right\} Sinh\lambda_2 h \\
+ \frac{\lambda_2^{\text{eff}}}{\lambda_1} \left[ Cosh\lambda_2 h - \frac{Cosh\lambda_1}{Sinh\lambda_1} Sinh\lambda_1 h \right] + Sinh\lambda_2 h \left[ Sinh\lambda_2 h - \frac{Cosh\lambda_1}{Sinh\lambda_1} Sinh\lambda_1 h \right] \\
+ \frac{Sinh\lambda_2 z}{\lambda_1} \{ - \frac{P}{\lambda_1^2 s} \}
\end{array} \right. \]  

(2.2.27)
Taking inverse Laplace transforms to the equations (2.2.27) and (2.2.28) on both sides.

We obtain

\[ q = \left\{ - \frac{p}{\sqrt{d_z f_i}} + \frac{p \cos(\sqrt{d_z} h)}{\sqrt{d_z f_i}} \cdot \frac{\sinh(\sqrt{d_z} h) \cdot \cos(\sqrt{d_z} h)}{\sqrt{d_z f_i} \cdot \sinh(\sqrt{d_z} h)} + \frac{p \sqrt{d_z} \cosh(\sqrt{d_z} h) \sinh(\sqrt{d_z} h)}{d_z f_i} \right\} - \left\{ \frac{p \beta}{d_z f_i} \cosh(\sqrt{d_z} h) \sinh(\sqrt{d_z} h) \right\} . \]
\[
+ a \left\{ \beta d_z \sinh \left( d_z \left( 1 - z \right) \right) \right\} e^{izt} + b \left\{ \beta d_z \sinh \left( d_z \left( 1 - z \right) \right) \right\} e^{-izt} +
\]
\[
+ \left\{ \frac{p \beta D^{-1/2} \left( 1 - h \right) \cosh \left( D^{-1/2} h \right)}{d_1 f_{d}^4} + \frac{p \sinh \left( D^{-1/2} h \right)}{d_1 f_{d}^4} - \frac{p}{d_1} \right\} \left( 1 - z \right) e^{-dz}
\]

and
\[
q_p = \frac{pc\cosh \left( \sqrt{d_2} z \right) - p \sqrt{d_2} \cosh^2 \left( \sqrt{d_2} h \right) \cosh \left( \sqrt{d_1} h \right) \sinh \left( \sqrt{d_1} z \right)}{f_1 d_1 \sinh \left( \sqrt{d_2} h \right)} +
\]
\[
+ \frac{p \beta \sqrt{d_2} \cdot \cosh \left( \sqrt{d_2} h \right) \sinh \left( \sqrt{d_1} h \right) \cosh \left( \sqrt{d_1} h \right) \sinh \left( \sqrt{d_2} z \right)}{f_1 d_1 \sinh \left( \sqrt{d_2} h \right)} -
\]
\[
- \frac{p \beta \sqrt{d_2} \cdot \cosh \left( \sqrt{d_2} h \right) \sinh \left( \sqrt{d_1} h \right) \cosh \left( \sqrt{d_1} h \right) \sinh \left( \sqrt{d_2} z \right)}{f_1 d_1 \sinh \left( \sqrt{d_2} h \right)} +
\]
\[
+ \frac{p \cosh \left( \sqrt{d_2} h \right) \cdot \cosh \left( \sqrt{d_1} h \right) \sinh \left( \sqrt{d_1} h \right) \cdot \sinh \left( \sqrt{d_2} z \right)}{f_1 d_1 \sinh \left( \sqrt{d_2} h \right)} +
\]
\[
+ \frac{p \sinh \left( \sqrt{d_2} h \right) \cdot \cosh \left( \sqrt{d_1} h \right) \sinh \left( \sqrt{d_1} h \right) \cdot \sinh \left( \sqrt{d_2} z \right)}{\sqrt{d_2} \cdot f_1} -
\]
\[
- \frac{p \cosh^2 \left( \sqrt{d_2} h \right) \sinh \left( \sqrt{d_2} z \right)}{\sqrt{d_1} \cdot f_1} +
\]
\[
+ \frac{p \cosh^2 \left( \sqrt{d_2} h \right) \cdot \cosh \left( \sqrt{d_1} h \right) \sinh \left( \sqrt{d_1} h \right) \cdot \sinh \left( \sqrt{d_2} z \right)}{f_1 \sqrt{d_1} \cdot \sinh \left( \sqrt{d_2} h \right)} -
\]
\[
- \frac{p \beta \sqrt{d_2} \cdot \cosh \left( \sqrt{d_2} h \right) \cdot \sinh^2 \left( \sqrt{d_1} h \right) \cosh \left( \sqrt{d_1} h \right) \cdot \sinh \left( \sqrt{d_2} z \right)}{f_1 d_1 \sinh \left( \sqrt{d_1} \right) \sinh \left( \sqrt{d_2} h \right)} +
\]
\[
\]
\[ p \beta \cdot \sqrt{d_2} \cdot \cosh(\sqrt{d_2} h) \cdot \cosh(\sqrt{d_1} h) \cdot \sinh(\sqrt{d_2} z) - f_1, d_1, \sinh(\sqrt{d_2} h) \cdot \cosh(\sqrt{d_1} h) \cdot \sinh(\sqrt{d_2} z) - f_1, d_2, \sinh(\sqrt{d_2} h) \cdot \cosh(\sqrt{d_1} h) \cdot \sinh(\sqrt{d_2} z) \]

\[ p \sinh(\sqrt{d_2} h) \cdot \cosh(\sqrt{d_1} h) \cdot \sinh(\sqrt{d_1} h) \cdot \sinh(\sqrt{d_2} z) - \sqrt{d_2} \cdot f_1 \]

\[ p \cdot \cosh(\sqrt{d_1} h) \cdot \cosh(\sqrt{d_2} h) \cdot \sinh(\sqrt{d_1} h) \cdot \sinh(\sqrt{d_2} z) - f_1, \sqrt{d_1}, \sinh(\sqrt{d_2} h) \cdot \cosh(\sqrt{d_1} h) \cdot \sinh(\sqrt{d_2} z) \]

\[ p \cdot \sinh(\sqrt{d_2} h) \cdot \cosh(\sqrt{d_1} h) \cdot \sinh(\sqrt{d_1} h) \cdot \sinh(\sqrt{d_2} z) - \beta \, d_2 \cdot \sinh(\sqrt{d_2} h) \]

\[ \frac{p \sinh(\sqrt{d_2} z)}{d_1 \cdot \sinh(\sqrt{d_2} h)} - \frac{p \sinh(\sqrt{d_2} z)}{d_2 \beta \sinh(\sqrt{d_1} h)} + \frac{p}{d_2 \beta} \]

\[ + a \left\{ \cosh(d_2 z) + \frac{\beta \, d_2 \cosh^2(d_2 h) \cdot \cosh(d_2 h) \cdot \sinh(d_2 z)}{f_2 \sinh(d_2 h)} \right\} \]

\[ - \frac{\beta \, d_2 \sinh(d_2 h) \cdot \cosh(d_2 h) \cdot \sinh(d_2 z)}{f_2} \]

\[ - \frac{\beta \, d_2 \cosh^2(d_2 h) \cdot \cosh(d_2 h) \cdot \sinh(d_2 z)}{f_2 \sinh(d_2 h)} \]
\[
\beta \cdot \frac{d_z \sinh(d_z h) \cdot \cosh(d_z) \cdot \sinh(d_z h) \cdot \sinh(d_z z)}{f_2} + \frac{\cosh(d_z h) \cdot \sinh(d_z z)}{\sinh(d_z h)} e^{\omega r} +
\]
\[
+ b \left\{ \cosh(d_z z) + \beta \frac{d_z \cosh'(d_z h) \cdot \cosh(d_z) \cdot \sinh(d_z) \cdot \sinh(d_z z)}{f_3 \sinh(d_z h)} \right\}
\]
\[
- \frac{\beta \cdot d_z \sinh(d_z h) \cdot \cosh(d_z h) \cdot \sinh(d_z) \cdot \sinh(d_z z)}{f_3} +
\]
\[
+ \frac{\beta \cdot d_z \sinh(d_z h) \cdot \cosh(d_z h) \cdot \sinh(d_z) \cdot \sinh(d_z z)}{\sinh(d_z h) \cdot f_3} +
\]
\[
+ \frac{\beta \cdot d_z \sinh(d_z h) \cdot \cosh(d_z h) \cdot \sinh(d_z) \cdot \sinh(d_z z)}{f_3} -
\]
\[
- \frac{\cosh(d_z h) \cdot \sinh(d_z z)}{\sinh(d_z h)} \} e^{\omega r} +
\]
\[
+ \left\{ \frac{-p \beta D^{-1/2} \cosh(D^{-1/2} h) \sinh(D^{-1/2} z)}{d_i \cdot \sinh(D^{-1/2} h) \cdot f_4} \right\}
\]
\[
+ \frac{p \beta D^{-1/2} \cdot \cosh(D^{-1/2} h) \cdot \sinh(D^{-1/2} z)}{d_i \cdot \sinh(D^{-1/2} h) \cdot f_4} + \frac{p \cdot \sinh(D^{-1/2} z)}{d_i \cdot f_4} +
\]
\[
+ \frac{p \beta D^{-1/2} \cdot h^2 \cosh(D^{-1/2} h) \cdot \sinh(D^{-1/2} z)}{d_i \cdot f_4 \sinh(D^{-1/2} h)} -
\]

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The shear stresses on the upper plate and lower plate are given by

\[
\tau_{uv} = \left( \frac{dq}{dz} \right)_{z=1} = \frac{p \beta D^{-1/2} \cdot h \cosh(D^{-1/2}h) \cdot \sinh(D^{-1/2}z)}{d_1 \cdot f_1 \sinh(D^{-1/2}h)} +
\]

\[
+ \frac{ph \cdot \sinh(D^{-1/2}z)}{(d_1) \sinh(D^{-1/2}h)} \left\{ \frac{p \sinh(D^{-1/2}z)}{d_1 \cdot \sinh(D^{-1/2}h)} \right\} e^{-d_1t} +
\]

\[
+ \left\{ \frac{p z \cosh(D^{-1/2}h)}{h \cdot d_2} + \frac{pz}{h \cdot d_2} \right\} e^{-d_2t}
\]

(2.2.30)

\[
\tau_{uv} = \left( \frac{dq}{dz} \right)_{z=1} = \frac{p \beta D^{-1/2} \cdot h \cosh(D^{-1/2}h) \cdot \sinh(D^{-1/2}z)}{d_1 \cdot f_1 \sinh(D^{-1/2}h)} +
\]

\[
+ \frac{ph \cdot \sinh(D^{-1/2}z)}{(d_1) \sinh(D^{-1/2}h)} \left\{ \frac{p \sinh(D^{-1/2}z)}{d_1 \cdot \sinh(D^{-1/2}h)} \right\} e^{-d_1t} +
\]

\[
+ \left\{ \frac{p z \cosh(D^{-1/2}h)}{h \cdot d_2} + \frac{pz}{h \cdot d_2} \right\} e^{-d_2t}
\]

The shear stresses on the upper plate and lower plate are given by

\[
\tau_{uv} = \left( \frac{dq}{dz} \right)_{z=1} = \frac{p \beta D^{-1/2} \cdot h \cosh(D^{-1/2}h) \cdot \sinh(D^{-1/2}z)}{d_1 \cdot f_1 \sinh(D^{-1/2}h)} +
\]

\[
+ \frac{ph \cdot \sinh(D^{-1/2}z)}{(d_1) \sinh(D^{-1/2}h)} \left\{ \frac{p \sinh(D^{-1/2}z)}{d_1 \cdot \sinh(D^{-1/2}h)} \right\} e^{-d_1t} +
\]

\[
+ \left\{ \frac{p z \cosh(D^{-1/2}h)}{h \cdot d_2} + \frac{pz}{h \cdot d_2} \right\} e^{-d_2t}
\]

(2.2.30)
We also determine the mass flux by the formula,

\[
Q_x + i Q_y = \int_{h} q dz
\]

Mass flux \( = Q = \sqrt{Q_x^2 + Q_y^2} \)

\[
Q_x + i Q_y = \left\{ -\frac{P}{\sqrt{d_z f_i}} + \frac{p \text{Cosh} (\sqrt{d_z} h)}{\sqrt{d_z f_i}} - \frac{p \text{Sinh} (\sqrt{d_z} h) \cdot \text{Cosh} (\sqrt{d_z} h)}{\sqrt{d_z f_i} \text{Sinh} (\sqrt{d_z} h)} + \right. \\
\left. + \frac{p \beta \cdot \text{Cosh} (\sqrt{d_z} h) \cdot \text{Sinh} (\sqrt{d_z} h)}{d_z f_i \text{Sinh} (\sqrt{d_z} h)} \right\}
\]

\[
\frac{p \beta \cdot \sqrt{d_z} \cdot \text{Cosh} (\sqrt{d_z} h)}{d_z f_i} - \frac{p}{\sqrt{d_z} \text{Sinh} (\sqrt{d_z} h)} - \\
-a \left\{ \frac{\beta d_z d_i \text{Cosh} (d_i)}{f_z} \right\} e^{i \omega t} - b \left\{ \frac{\beta d_z d_i \text{Cosh} (d_i)}{f_z} \right\} e^{-i \omega t} - \\
- \left\{ \frac{p \beta D^{-1/2} (1 - h) \text{Cosh} (D^{-1/2} h) + p \text{Sinh} (D^{-1/2} h)}{d_z f_i} - \frac{p}{d_z} \right\} e^{-i \omega t}
\]
\[
+ \frac{\beta}{d_1} (1-h) + \alpha \left\{ \frac{\beta d_2 \cosh(d_2(1-h))}{d_2 f_2} \right\} e^{iw} + b \left\{ \frac{\beta d_6 \cosh(d_6(1-h))}{d_6 f_6} \right\} e^{-iw} + \\
+ \left\{ \frac{p \beta D^{-1/2} (1-h) \cosh(D^{-1/2}h)}{d_4 f_4} + \frac{p \sinh(D^{-1/2}h)}{d_4 f_4} \right\} \frac{1}{d_4} \left( \frac{1}{2} - h + \frac{h^2}{2} \right) e^{-d_4}.
\]

where,

\[\beta = \frac{v_{\text{eff}}}{v}\]
\[d_1 = M^2 + 2iE^{-1}\]
\[d_2 = M^2 + 2iE^{-1} + D^{-1}\]
\[d_3 = \sqrt{i\omega + M^2 + 2iE^{-1}}\]
\[d_4 = \sqrt{i\omega + M^2 + 2iE^{-1} + D^{-1}}\]
\[d_5 = \sqrt{-i\omega + M^2 + 2iE^{-1}}\]
\[d_6 = \sqrt{-i\omega + M^2 + 2iE^{-1} + D^{-1}}\]

\[f_1 = \sqrt{d_2} \beta \cosh(\sqrt{d_2} h) \cdot \sinh(\sqrt{d_2} (1-h)) + \sqrt{d_2} \cdot \sinh(\sqrt{d_2} h) \cdot \cosh(\sqrt{d_2} (1-h))\]
\[f_2 = d_4 \beta \cosh(d_4 h) \cdot \sinh(d_4 (1-h)) + d_4 \cdot \sinh(d_4 h) \cdot \cosh(d_4 (1-h))\]
\[f_3 = d_6 \beta \cosh(d_6 h) \cdot \sinh(d_6 (1-h)) + d_6 \cdot \sinh(d_6 h) \cdot \cosh(d_6 (1-h))\]
\[f_4 = D^{-1/2} \beta (1-h) \cdot \cosh(D^{-1/2}h) + \sinh(D^{-1/2}h)\]
1.3. DISCUSSION:

The flow is governed by the non-dimensional parameters $E$, Eckmann number, $D^{-1}$ the inverse Darcy parameter and $M$, the Hartmann number. The velocity field in each zone of composite medium is evaluated analytically its behaviour with reference to variations in the governing parameters has been computationally analyzed. We notice that the thickness of the porous bed significantly affects the flow in the clean fluid region. Two different cases namely viz, the thickness of porous bed is relatively small compare to the width of the rotating channel and the other thickness of the porous bed is large are discussed in detail. The profiles for $u$ and $v$ have been plotted in the entire flow field with an understanding that $u$ is $u_p$ and $v$ is $v_p$ in the porous bed. This has been carried out in all other chapters.

The solutions for the combined velocity $q$ and $q_p$ consists of three kinds of terms 1) The steady state 2) The quasi steady state terms associated with non-torsional oscillations in the boundary 3) The transient terms involving exponentially varying time dependence. From the expressions (1.2.29) and (1.2.30), it follows that the transient components in the velocity in the clean fluid region decays in dimensionless time

$$t > \frac{1}{|d|} = \frac{1}{\{M^4 + 4E^{-2}\}^{1/2}}$$

while in the porous bed these transient terms the time

$$t > \text{Max} \left\{ \frac{1}{\{M^4 + 4E^{-2}\}^{1/2}} \cdot \frac{1}{\{M^4 + D^{-1}\}^{1/2}} \cdot \frac{1}{\{M^4 + 4E^{-2}\}^{1/2}} \right\}$$
Thus in a composite medium in the presence of clean fluid the time of decay of the transient velocity continues to be higher although the time of decay involving the porous parameter is comparatively less.

When the transient terms decay the steady oscillatory solutions in the clean fluid and the porous bed are given by

\[
(q)_{\text{steady}} = \frac{p}{\sqrt{d_z f_1}} + \frac{p \cosh(\sqrt{d_z h})}{\sqrt{d_z f_1}} - \frac{p \sinh(\sqrt{d_z h}) \cosh(\sqrt{d_z h})}{\sqrt{d_z f_1} \sinh(\sqrt{d_z h})} + \\
+ \frac{p \beta \cosh(\sqrt{d_z h}) \sinh(\sqrt{d_z h})}{d_z f_1} - \frac{p \beta \cosh(\sqrt{d_z h}) \sinh(\sqrt{d_z h})}{d_z f_1} \sinh(\sqrt{d_z h}) + \frac{p}{d_z},
\]

\[
(q)_{\text{oscillatory}} = a \left( \frac{\beta d_z \sinh(d_z (1-z))}{f_2} \right) e^{i\omega t} + b \left[ \frac{\beta d_z \sinh(d_z (1-z))}{f_3} \right] e^{-i\omega t}
\]

and

\[
(q_p)_{\text{steady}} = - \frac{p \cosh(\sqrt{d_z h})}{d_z \beta} - \frac{p \sqrt{d_z h} \cosh(\sqrt{d_z h}) \cosh(\sqrt{d_z h}) \sinh(\sqrt{d_z h})}{f_1 d_z \sinh(\sqrt{d_z h})} \\
+ \frac{p \beta \sqrt{d_z} \cosh(\sqrt{d_z h}) \sinh(\sqrt{d_z h}) \cosh(\sqrt{d_z h}) \sinh(\sqrt{d_z h})}{f_1 d_z \sinh(\sqrt{d_z h})} \\
+ \frac{p \beta \sqrt{d_z} \cosh(\sqrt{d_z h}) \sinh(\sqrt{d_z h}) \cosh(\sqrt{d_z h}) \sinh(\sqrt{d_z h})}{f_1 d_z \sinh(\sqrt{d_z h})} +
\]
\[ p \cosh(\sqrt{d_2} h) \cdot \cosh(\sqrt{d_1} h) \cdot \sinh(\sqrt{d_1} z) \cdot \sinh(\sqrt{d_2} z) \]

\[ + \frac{p \cosh(\sqrt{d_2} h) \cdot \cosh(\sqrt{d_1} h) \cdot \sinh(\sqrt{d_1} z) \cdot \sinh(\sqrt{d_2} z)}{f_1 \sqrt{d_2} \cdot \sinh(\sqrt{d_2} h)} \]

\[ + \frac{p \sinh(\sqrt{d_2} h) \cdot \cosh(\sqrt{d_1} h) \cdot \sinh(\sqrt{d_1} z) \cdot \sinh(\sqrt{d_2} z)}{\sqrt{d_2} \cdot f_1} \]

\[ - \frac{p \cosh^2(\sqrt{d_2} h) \cdot \sinh(\sqrt{d_2} z)}{\sqrt{d_1} \cdot f_1} + \]

\[ + \frac{p \cosh^2(\sqrt{d_2} h) \cdot \cosh(\sqrt{d_1} h) \cdot \sinh^2(\sqrt{d_1} z) \cdot \sinh(\sqrt{d_2} z) \cdot \sinh(\sqrt{d_1} z)}{f_1 \sqrt{d_2} \cdot \sinh(\sqrt{d_2} h)} \]

\[ - \frac{p \beta \cdot \sqrt{d_2} \cdot \cosh(\sqrt{d_2} h) \cdot \sinh^2(\sqrt{d_1} h) \cdot \cosh(\sqrt{d_1} h) \cdot \sinh(\sqrt{d_2} z) \cdot \sinh(\sqrt{d_1} z)}{f_1 d_1 \cdot \sinh(\sqrt{d_1} h) \cdot \sinh(\sqrt{d_2} h) \cdot \sinh(\sqrt{d_2} z)} + \]

\[ + \frac{p \beta \cdot \sqrt{d_2} \cdot \cosh(\sqrt{d_2} h) \cdot \cosh(\sqrt{d_1} h) \cdot \sinh(\sqrt{d_1} h) \cdot \sinh(\sqrt{d_2} h) \cdot \sinh(\sqrt{d_2} h)}{f_1 d_1 \cdot \sinh(\sqrt{d_2} h)} \]

\[ - \frac{p \cosh(\sqrt{d_2} h) \cdot \cosh(\sqrt{d_1} h) \cdot \sinh(\sqrt{d_1} h) \cdot \sinh(\sqrt{d_2} h) \cdot \sinh(\sqrt{d_2} h)}{f_1 \sqrt{d_2} \cdot \sinh(\sqrt{d_2} h)} \]

\[ - \frac{p \sinh(\sqrt{d_2} h) \cdot \cosh(\sqrt{d_1} h) \cdot \sinh(\sqrt{d_1} h) \cdot \sinh(\sqrt{d_2} h) \cdot \sinh(\sqrt{d_2} h)}{\sqrt{d_2} \cdot f_1} \]

\[ - \frac{p \cdot \cosh(\sqrt{d_1} h) \cdot \cosh(\sqrt{d_1} h) \cdot \sinh(\sqrt{d_1} h) \cdot \sinh(\sqrt{d_2} h) \cdot \sinh(\sqrt{d_2} h)}{f_1 \sqrt{d_1} \cdot \sinh(\sqrt{d_1} h)} + \]

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We find that from the behaviour of steady and quasi steady state solutions no boundary layers are formed near the boundary plates, although the presence of porous bed influences the clean fluid region and vice versa. This is in contrast to the case of the hydro magnetic flow through a porous medium in a rotating channel under similar conditions (11). We now discuss the quasi steady solutions for the velocity for different sets of governing parameters namely E,M and D' thickness of the porous bed h, P the non dimensional pressure gradient, the frequency oscillation ω, a and b the constants related to non-torsional oscillations of the boundary. For computational purpose we fix the axial pressure gradient as well as ‘a and b’.

The flow takes place in between the rotating plates in planes parallel to the boundaries. The velocity components relative to the rotating frame are obtained solving Navier-Stokes’s equation (boundary layer type) in the clean fluid region, and the Brinkman’s equation in the porous bed. The velocity in the clean and porous region is matched using the interfacial conditions.

Figures (1-6) corresponding to the velocity component u along the imposed pressure gradient for different sets of governing parameters when the lower boundary plate executes non-torsional oscillations. Figures (7-12) corresponds to the velocity component for v. Similarly figures (13-18) corresponds to the velocity...
component for $u$ when the both the boundary plates are at rest. Figures (19-24) represent the velocity components for $v$.

The velocity profiles for $u$ in the case of porous bed of small thickness are plotted (1-3). We notice that the velocity $u$ in the clean fluid region enhances with increase in the Eckmann number $E$ while $u$ reduces in the porous bed (fig 1). Likewise $u$ increases with $M$ in the clean fluid region where as $u$ reduces in porous bed with increase in $M$ (fig 2). From figure 3 we observe that $u$ reduces in the lower part of the clean fluid region $(0.6 \leq z \leq 1)$ while it experiences enhancement upper half $(0.3 \leq z \leq 0.5)$, while in the porous region $u$ increases with $D^{-1}$. When the thickness of the porous bed is sufficiently large, an increase in $E$ enhances $u$ in the clean fluid region while reduces in the porous bed (fig 4). The similar variation is noticed with increase in $M$ (fig 5). However the variation with the Darcy parameter $D^{-1}$ shows that the lower permeability higher the magnitude of $u$ in the both clean and porous regions (Fig 6).

Figures (7-9) correspond to the velocity profiles for $v$ in case of porous beds of small thickness while figures (10-12) represent their profiles in case of the larger thickness of the porous bed. From (fig 7), we observe that the behaviour of $v$ with reference to the variation in $E$ is similar to that of $u$ with its magnitude increasing with increase in $E$ except in the layer adjacent to the oscillating boundary plate where its magnitude reduces with $E$ we also notice that, the magnitude of $v$ reduces in the entire fluid region for an increase in $M$, the other parameters being fixed (fig 8). Likewise its magnitude lessens with increase in inverse Darcy parameter $D^{-1}$ (Fig 9).

From (fig 10) $|v|$ enhances in the clean fluid region while reduces in the porous bed with increase in $E$, where as the reversal behaviour is observed with
reference to variation in $M$ with $v$ increasing the porous bed reducing in the clean fluid region (fig 11). Lesser the permeability lower the magnitude of $v$ in the entire flow region (fig 12).

In all above mentioned cases the resultant velocity in the clean fluid region is directed away from the central axis of the channel with phase difference greater than $\frac{7\pi}{4}$ from the x-axis the directions of the imposed pressure gradient. In the porous region the resultant velocity is again directed from the central axis with phase difference less than $\frac{7\pi}{4}$ from the x-axis. The magnitude of the resultant increases with the Eckmann number $E$ but reduces with $M$ and $D^{-1}$. Thus in general, higher the Eckmann number greater the resultant fluid speed. In other words the distance created due to the non-torsional oscillations of the boundary in a rotating channel is more pronouncing for lower angular speeds of the channel. Also the transverse magnetic field applied on the rotating channel acts as a retarded force is evident from the fact that higher the magnetic parameter lesser the fluid speed. The other important factor to be noted that, lower the permeability of the porous bed lesser the fluid speed in both the clean fluid region and the porous region. This evidently exhibits the influence of the presence of the porous bed on the clean fluid flow. It is interesting to note the larger thickness of the porous bed higher the magnitude of the resultant velocity for $M \leq 5 (or) D^{-1} \leq 2 \times 10^3$. However for higher values of $M (or) D^{-1} \geq 3 \times 10^3$ these magnitudes are larger in the case of smaller thickness of porous bed.
Similar observations have been made regarding the behaviour of $u$ fig (13-18) and $v$ fig (19-24) in both the cases of smaller and larger thickness of porous bed when both the boundary plates are at rest.

The shear stresses $\tau_x$ and $\tau_y$ on the upper and lower plates have been calculated for the variations in the governing parameters and tabulated in the tables (1-4). We notice from table (1) that on the upper plate both the shear stresses enhances for an increase in $E \geq 0.2$. However $\tau_x$ increases while $\tau_y$ reduces with increase in $M$. In contrast $\tau_x$ reduces and $\tau_y$ increases for an increase in $D^{-1}$ also we notice that both the stresses on the upper plate enhances with increase in the thickness of the porous bed. This once again conform the influence of the thickness of the porous bed on the stresses experienced by the upper plate bounding the clean fluid (table 2). On the lower plate bounding the porous medium, similar observations can be made with variations in governing parameters. We also notice that the magnitudes of the shear stresses on the lower plate are very small compare to its values of the upper plate (tables 3 & 4).

Table (5) corresponds to the behavior of mass flux for variations in the governing parameters. We find that the mass flux increases with $E$ and reduces with $M$ and $D$. 

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The velocity profiles when the lower plate execute non-torsional oscillations

I. Profiles of $u$ when the thickness of the porous bed ($h$) is small.

Fig 1: The velocity profile for $u$ with $E$.
$a=b=1, \beta=1.2, \omega=\frac{\pi}{4}, h=0.2, M=5, D^{-1}=1000$.

Fig 2: The velocity profile for $u$ with $M$.
$a=b=1, \beta=1.2, \omega=\frac{\pi}{4}, h=0.2, E=0.01, D^{-1}=1000$. 
II. Profiles of $u$ when the thickness of the porous bed ($h$) is large.

Fig 3: The velocity profile for $u$ with $D^{-1}$.

$a=b=1, \beta=1.2, \omega=\frac{\pi}{4}, h=0.2, E=0.01, M=5$.

Fig 4: The velocity profile for $u$ with $E$.

$a=b=1, \beta=1.2, \omega=\frac{\pi}{4}, h=0.5, M=5, D^{-1}=1000$.

$a=b=1, \beta=1.2, \omega=\frac{\pi}{4}, h=0.2, E=0.01, M=5$.
Fig 5: The velocity profile for $u$ with $M$.

$a=b=1, \beta=1.2, \omega=\frac{\pi}{4}, h=0.5, E=0.01, D^{-1}=1000.$

Fig 6: The velocity profile for $u$ with $D^{-1}$.

$a=b=1, \beta=1.2, \omega=\frac{\pi}{4}, h=0.5, E=0.01, M=5.$
III. Profiles of $v$ when the thickness of the porous bed ($h$) is small.

Fig 7: The velocity profile for $v$ with $E$.

$a=b=1$, $\beta=1.2$, $\omega=\frac{\pi}{4}$, $h=0.2$, $M=5$, $D^{-1}=1000$.

Fig 8: The velocity profile for $v$ with $M$.

$a=b=1$, $\beta=1.2$, $\omega=\frac{\pi}{4}$, $h=0.2$, $E=0.01$, $D^{-1}=1000$. 
Fig 9: The velocity profile for $v$ with $D^{-1}$.

$a=b=1$, $\beta=1.2$, $\omega=\frac{\pi}{4}$, $h=0.2$, $E=0.01$, $M=5$.

IV. Profiles of $v$ when the thickness of the porous bed ($h$) is large.

Fig 10: The velocity profile for $v$ with $E$.

$a=b=1$, $\beta=1.2$, $\omega=\frac{\pi}{4}$, $h=0.5$, $M=5$, $D^{-1}=1000$. 
Fig 11: The velocity profile for $v$ with $M$.  
$a=b=1, \beta=1.2, \omega=\frac{\pi}{4}, h=0.5, E=0.01, D^{-1}=1000$.

Fig 12: The velocity profile for $v$ with $D^{-1}$.  
$a=b=1, \beta=1.2, \omega=\frac{\pi}{4}, h=0.5, E=0.01, M=5$. 
The velocity profiles when the both the plates are at rest

I. Profiles of $u$ when the thickness of the porous bed ($h$) is small.

![Graph of velocity profile for $u$ with $E$.](image1)

$a=b=1$, $\beta=1.2$, $\omega=\frac{\pi}{2}$, $h=0.3$, $M=5$, $D^{-1}=1000$.

![Graph of velocity profile for $u$ with $M$.](image2)

$a=b=1$, $\beta=1.2$, $\omega=\frac{\pi}{2}$, $h=0.3$, $E=0.01$, $D^{-1}=1000$. 
II. Profiles of $u$ when the thickness of the porous bed ($h$) is large.

Fig 15: The velocity profile for $u$ with $D^{-1}$.
$a=b=1$, $\beta=1.2$, $\omega=\frac{\pi}{2}$, $h=0.3$, $E=0.01$, $M=5$.

Fig 16: The velocity profile for $u$ with $E$.
$a=b=1$, $\beta=1.2$, $\omega=\frac{\pi}{2}$, $h=0.8$, $M=5$, $D^{-1}=1000$. 
Fig 17: The velocity profile for $u$ with $M$.

$a=b=1$, $\beta=1.2$, $\omega=\frac{\pi}{2}$, $h=0.8$, $E=0.01$, $D^{-1}=1000$.

Fig 18: The velocity profile for $u$ with $D^{-1}$.

$a=b=1$, $\beta=1.2$, $\omega=\frac{\pi}{2}$, $h=0.8$, $E=0.01$, $M=5$. 
III. Profiles of $v$ when the thickness of the porous bed ($h$) is small.

Fig 19: The velocity profile for $v$ with $E$.
$a=b=1$, $\beta=1.2$, $\omega=\frac{\pi}{2}$, $h=0.3$, $M=5$, $D^{-1}=1000$.

Fig 20: The velocity profile for $v$ with $M$.
$a=b=1$, $\beta=1.2$, $\omega=\frac{\pi}{2}$, $h=0.3$, $E=0.01$, $D^{-1}=1000$. 
Fig 21: The velocity profile for \( v \) with \( D^{-1} \).

\[ a=b=1, \quad \beta=1.2, \quad \omega=\frac{\pi}{2}, \quad h=0.3, \quad E=0.01, \quad M=5. \]

IV. Profiles of \( v \) when the thickness of the porous bed (\( h \)) is large.

Fig 22: The velocity profile for \( v \) with \( E \).

\[ a=b=1, \quad \beta=1.2, \quad \omega=\frac{\pi}{2}, \quad h=0.8, \quad M=5, \quad D^{-1}=1000. \]
Fig 23: The velocity profile for $v$ with $M$.

$a=b=1$, $\beta=1.2$, $\omega=\frac{\pi}{2}$, $h=0.8$, $E=0.01$, $D^{-1}=1000$.

Fig 24: The velocity profile for $v$ with $D^{-1}$.

$a=b=1$, $\beta=1.2$, $\omega=\frac{\pi}{2}$, $h=0.8$, $E=0.01$, $M=5$. 
## Table 1

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The shear stress ($\tau_x$) on the upper plate.

$$a=b=1, \ \beta=1.2, \ \omega=\frac{\pi}{4}.$$
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The shear stress ($\tau_y$) on the lower plate.

\[ a=b=1, \beta=1.2, \omega=\frac{\pi}{4}. \]

Table 4

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<td>1000</td>
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The shear stress ($\tau_y$) on the lower plate.

\[ a=b=1, \beta=1.2, \omega=\frac{\pi}{4}. \]
Table 5

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Mass flux (Q)

\[
a = b = 1, \quad \beta = 1.2, \quad \omega = \frac{\pi}{4}.
\]
REFERENCES:


   G. Davaa and S. Momoki
   "Effects of viscous dissipation and fluid
   axial heat conduction on heat transfer for
   non-Newtonian fluids inducts with
   uniform wall temperature part II: Annular
   ducts", International communications in

    G. Davaa and S. Momoki
    "Effects of viscous dissipation and fluid
    axial heat conduction on heat transfer for
    non-Newtonian fluids inducts with
    uniform wall temperature part I: parallel
    plates and circularducts, International
    Communications in Heat and Mass
    transfer, vol 32, pp1165-1173.

    Rao and A.S. Rama
    chandra murthy.
    "Hydromagnetic convection flow through
    a porous medium in a rotating channel",
    Journal of engg. physics and thermo

16. Lasozal Fuchs and
    Yongnian Yang 1993
    "Numerical study of Viscous flow in
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<td>26</td>
<td>Siva Prasad R.</td>
<td>1985</td>
<td>Ph.D., thesis, S.K. University, Anantapur</td>
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<td>27</td>
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<td>1993</td>
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<td></td>
<td>Tanveer, Itagi SheebaRani</td>
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