CHAPTER-I

INTRODUCTION
1.1. INTRODUCTION:

All fluid phenomena on earth involve rotation to a greater or lesser extent. Because of the basic rotation of the earth most of the large scale motions in atmosphere and seas/oceans fall under the category in which rotation is an absolutely essential factor. The atmosphere and the ocean are not homogeneous incompressible fluids, but in many cases the essentials physical features of atmospheric or oceanic flows are not dependent on this factor and satisfactory theories can be based on mathematical models assuming the fluid as incompressible. The atmosphere and oceans are located on the earth which is a spheroid (almost a sphere) and there scale of motion is relatively small compared to the earth radius. The fact that the geophysical fluids are really thin spherical shells is felt in such models in two different ways: a) There are purely geometrical effects associated with any planar map of the sphere and b) The angular velocity of the rotating sphere makes different angles with the vertical at different latitudes. The geometrical effects do not seen to be of scale comparable to the radius, but the effects of the rotating sphere in some respects create qualitative differences already on an intermediate scale. It turns out essentially because of the fact that the geophysical flows normally have a very large horizontal scale large compared to the vertical scale and hence the velocity vectors are nearly horizontal. In such cases where a small range of latitude is involved, the vertical component may be regarded as constant, a satisfactory model is obtained by considering a plane horizontal layer of fluids rotating about a vertical axis with angular velocity equal to the vertical component $\Omega \sin \lambda$ ($\lambda$ being the latitude). This is commonly called the “$f$-plane model” because the parameter measuring the rotation which occurs in the equation is denoted by $f$. For motions at greater latitudinal scale a reasonably satisfactory
model is obtained by using the same basic equation as in the $f$-plane model, except that $f$ is no longer taken as constant, but its variation with latitude is included. A simplified version of this is called the "$\beta$-plane model" ($\beta$ is the measure of the rate of change of $f$ with latitude). It does not consider the geometrical effects of the spherical surface, but takes partial account of the variation with latitude of the angle between the axis of rotation and the vertical at the same time minimizing the mathematical complexities. There is another interesting aspect of the problem of rotating viscous incompressible fluid. Here viscous boundary layers are formed on the solid bodies and there exists a strong interaction between the boundary layers and the outer flow.

Several problems of interest in Industry involve flow of fluids through composite media consisting partially of porous bed. Examples include use of filtration to purify water and treat Sewage, movement of fertilizers in the soil in Agriculture. Transitions zone between salt water and fresh water in coastal aquifers. Such problems involving the flow past a porous bed is also of immense use in Bio-medical problems involving transport process in lungs and kidneys excetra. This prompted several researchers investigate the flow through in composite system with porous lining abutting the boundaries. Sasthry (43) discuss the flow of a rotating parallel plate channel with porous lining on one or both the sides of the channel, making use of Beavers-Joseph slip condition. He observes that the wall permeability has an effect on the flow with the flux increasing with the permeable parameter.

The problem of homogeneous rotating fluid bounded by a permeable bed was studied by Kroll and Veronis (28) who observed that the effect the permeable
medium is to speed up the spin-up process and viscous decay of Ekman layer and residual inertial oscillations. The permeable medium acts like a centrifugal fan, sucking the fluid from the interior down into the Ekman layer and forcing the spin-up process.

- This lead Bretherton and Spiegel (7) to propose a rotating fluid bounded by permeable bed as a model for the study of turbulent process in Geophysical and Astrophysical applications. Nanda and Mohanty (33) have analyzed the steady flow in a parallel plate channel rotating with an angular velocity and bounded below by a permeable bed. Arun kumar varma (2) has discussed the Eckman flow induced by a naturally permeable bed employing a set of generalized equation for the Darcy's law within the porous medium.

Couple stresses and the concept of internal spin are two main physical concepts that go into building theories of fluids with microstructure. Couple stresses are a consequence of assuming that the mechanical action of one part of a body on another across a surface is equivalent to a force and a moment distribution. In classical non-polar mechanics, moment distributions are not considered, and the mechanical action is assumed to be equivalent to a force distribution only. The laws of motion can then be used for defining the stress tensor which, necessarily, turns out to be symmetric. Thus, in non-polar mechanics, the state of stress at a point is defined by a symmetric second order tensor which is a point function that has six independent components. However, in polar mechanics the mechanical action is assumed to be equivalent to both a force and a moment distribution. The state of stress is then measured by a stress tensor and a couple stress tensor. In general neither of these second order tensors is symmetric, so the state of stress at a point is measured by eighteen independent components. Thus, the concept of
couple stresses results from a study of the mechanical interaction taking place across a surface and conceptually is not related to the kinematics of motion.

Experimentally, it is observed that an addition of a small amount of long-chain polymer to the lubricant oils not only reduces their sensitivity to changes in the shear rate but also enhances the load-bearing capacity of these fluids to a great extent and thereby increases the life of the bearings (9). The lubricant, containing polymer additives, has to be represented by proper constitutive equations, in any theoretical model. The presence of long-chain molecules in a viscous liquid suggests that the couple stress fluid model of Stokes (47) can be used to represent these solutions. Other fluids that may be represented by the model are the infinitely dilute solutions of high polymers in theta solvents and the bio-fluids in synovial joints (8).

Stokes (47) proposed the constitutive equations for force and couple stress in an effort to generalize the classical theory which would allow polar effects. Many of the flow problems discussed by Stokes indicate that the effects of couple stresses on the velocity field are quite large for small values of a non-dimensional parameter $n = h \left( \frac{\mu}{\eta} \right)^{\frac{1}{2}}$, where $h$ is a typical dimension of the flow geometry and $\left( \frac{\mu}{\eta} \right)^{\frac{1}{2}}$ is the characteristic microstructural length of the material. $\left( \frac{\mu}{\eta} \right)^{\frac{1}{2}}$, being a function of molecular dimension, is responsible for the size-dependent effect in polar fluids. Thus $n$ characterizes the microstructure of the fluid. The smaller $n$, the coarser the microstructure. The solutions
of the boundary value problems, discussed by Stokes, show that the shear stress for a 
couple stress fluid is quantitatively greater than that for the Newtonian fluids under 
similar conditions. This suggests that the polar fluids can have important applications in 
lubrication theory.

Stokes (47) gave the equation of motion and constitutive equations for a 
couples stress fluid on the basis of couple stress in elastic materials. The equation of 
motion for the flow of an incompressible fluid with couple stresses are

\[ T_{,j,j} + \rho \dot{f}_j = \rho \frac{DV_j}{Dt} \]  

(1.1)

\[ E_{ijkl} T_{jk}^A + M_{j,j} + \rho C_j = 0 \]  

(1.2)

where \( f_j \) is body force per unit mass, \( C_j \) is body moment per unit mass, 
\( V_j \) is velocity vector, \( T_{jk}^S \) and \( T_{jk}^A \) are the symmetric and anti-symmetric parts of the 
stress tensor \( T_{jk} \) respectively, \( \rho \) is the density of the fluid, \( M_j \) is the couple stress 
tensor and the other terms have their usual meaning of tensor analysis (19).

The constitutive equations for an isotropic incompressible fluid with couple 
stress are (47, 19).

\[ T_{jk}^S = -pS_{jk} + 2 \mu d_{jk} \]  

(1.3)

\[ \mu_j = 4 \eta W_{j,j} + 4 \eta' W_{i,j} \]  

(1.4)

where \( \mu \) is the shear viscosity which is different from the solvent viscosity, 
\( \mu, \eta, \eta' \) are the constants associated with couples stress, \( 'p' \) is pressure, \( \mu_j \) is
deviatoric part of $M_y$, $d_y$ is symmetric part of velocity gradient and $W$, is the vorticity vector.

In case of incompressible fluids, when the body forces and body moments are absent, the momentum equation in the vector notation become

$$\rho \left[ \frac{\partial \mathbf{q}}{\partial t} + (\mathbf{q} \cdot \nabla) \mathbf{q} \right] = -\nabla p + \rho g + \mu \nabla^2 \mathbf{q} - \eta \nabla^4 \mathbf{q} \quad (1.5)$$

The momentum equation for couple stress fluid through sparsely packed porous medium is given by (8)

$$\rho \left[ \frac{1}{\delta} \frac{\partial \mathbf{q}}{\partial t} + \frac{1}{\delta^2} (\mathbf{q} \cdot \nabla) \mathbf{q} \right] = -\nabla p + \rho g + \frac{\mu_s}{k} \mathbf{q} + \mu_c (\nabla^2 \mathbf{q}) - \eta \nabla^4 \mathbf{q} \quad (1.6)$$

In recent years, there has been a considerable interest in rotating hydro magnetic fluid flows due to possible applications to geophysical and astrophysical problems. The magnitude analysis shows that in the basic field equations, the Coriolis force is very significant as compared to the inertial force. Furthermore, it reveals that the Coriolis and magneto hydro dynamic forces are of comparable magnitude. It is generally admitted that a number of astronomical bodies (e.g. the Sun, Earth, Jupiter, Magnetic Stars and pulsars) possess fluid interiors and (at least surface) magnetic fields. Changes in the rotation rate of such objects suggest the possible importance of hydro magnetic spin-up. This problem of spin-up in magneto hydro dynamic rotating fluids has been
examined under varied conditions by many researchers notably Gilman and Benton (22), Benton and Loper (6), Chawala (11), Debnath (13,15), and Singh (46). Rao et.al (37) made an initial value investigation of the combined free and forced convection effects in an unsteady hydro magnetic viscous incompressible rotating fluid between two disks under a uniform transverse magnetic field. This analysis was extended to porous boundaries by Sarojamma and Krishna (41).

In all these analyses, the effects of the Hall current are not considered. Therefore, the results in these investigations cannot be applied to the flow of ionized gases. This is because in an ionized gas where the density is low and/or the applied magnetic field is strong, the effect of Hall currents may be significant. Debnath et al. (14) discussed its effects on unsteady flow in a rotating viscous fluid. The effects of the Hall current on unsteady hydro magnetic rotating non-Newtonian fluid flows. Such work seems to be important and useful partly for gaining a basic understanding of such flows, and partly for possible applications of these fluids in Chemical process industries, movement of Biological fluids, in Petroleum production and in Power engineering. Another important field of application is the electromagnetic propulsion. Basically, an electromagnetic propulsion system consists of a power source, such as a nuclear reactor, plasma and a tube through which the plasma is accelerated by electromagnetic forces. The study of such systems, which is closely associated with Magneto chemistry, requires a complete understanding of the equation of state and transport properties such as diffusion, shear stress-shear rate relationship, thermal conductivity, electrical conductivity, and radiation. Some of these properties will undoubtedly be influenced by
the presence of an external magnetic field which sets the plasma in hydrodynamic motion.

Keeping the above mentioned facts in view, in this thesis an attempt has been made to discuss the steady or the unsteady magneto-hydrodynamic flow of a Newtonian viscous incompressible fluid as well as viscous couple stress fluid through a composite medium in a rotating parallel plate channel. We calculate the velocity, shear stress and mass flux numerically by using Mathematica, computer software.
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