HALL EFFECTS ON STEADY HYDRO-MAGNETIC FLOW OF A COUPLE STRESS FLUID IN A ROTATING PARALLEL PLATE CHANNEL WITH POROUS BED ON THE LOWER HALF.
5.1. INTRODUCTION:

The flow between parallel plates is a classical problem that has important applications in magneto hydrodynamic (MHD) power generators and pumps, accelerators, aerodynamic heating, electrostatic precipitation polymer technology, petroleum industry, purification of crude oil and fluid droplets, sprays, designing cooling systems with liquid metal, centrifugal separation of matter from fluid and flow meters. Hartman and Lazarus (6) studied the influence of a transverse uniform magnetic field on the flow of a viscous incompressible electrically conducting fluid between two infinite parallel stationary and insulating plates. Then the problem was extended in numerous ways. Closed form solutions for the velocity fields were obtained in Ref (1, 3, 19 & 21) under the different physical effects. Some exact and numerical solutions for the heat transfer problem are found in Ref (2 & 8). In the above mentioned cases the Hall term was ignored in applying Ohm’s Law as it has no marked effect for small and moderate values of the magnetic field. However, the current trend for the application of magneto-hydrodynamics is to words a strong magnetic field, so that the influence of electromagnetic force is noticeable by Cramer et al(3). Under these conditions, the Hall current is important and it has a marked effect on the magnitude and direction of the current density and consequently on the magnetic force.

In a partially ionized gas there occurs a Hall current (Cowling (4)) when the strength of the impressed magnetic field is very strong. These Hall effects play a significant role in determining the flow features. Pop(9), Sato(15), Yamanishi(23), Sherman and Sutton(16) have discussed the Hall effects on the steady hydromagnetic
flow between two parallel plates. Tani (20) studied the Hall effect on the steady motion of electrically conducting and viscous fluids in channels. Soundalgekar and co-workers (17 & 18) studied the effect of Hall currents on the steady MHD couette flow with heat transfer. The temperatures of two plates were assumed either to be constant (17) or varying linearly along the plates in the direction of the flow (18). Linga Raju .T and Ramana Rao. V.V.(7) studied steady viscous incompressible fluid flow between two parallel walls in the presence of a uniform magnetic field applied transversely to the flow and when rotated at an angular velocity about an axis perpendicular to the walls, taking Hall current into account.

The unsteady hydromagnetic rotating viscous flow through a nonporous or porous medium has drawn attention in the recent years for possible applications in Geophysical and Cosmical fluid dynamics. The Hall effects in the unsteady case were discussed by Vatazhin(22), Pop(9) and Sakhonovskii (13). Debnath et.al (5) have studied the effects of Hall current on unsteady hydromagnetic flow past a porous plate in a rotating fluid system and the structure of the steady and unsteady flow fields is investigated. Rao and Krishna(10) studied Hall effects on the non-torsionally generated unsteady hydromagnetic flow in semi-infinite expansion of an electrically conducting viscous rotating fluid. Krishna and Rao (11 & 12) discussed the Stokes and Ekman problems in magneto-hydrodynamics taking Hall effects into account. An exact solution of the initial value problem is obtained by the Laplace transform technique. Both the steady and the transient components of velocity field are explicitly obtained with their implication. The effects of Hall current on the hydrodynamic boundary layers and shear stress are discussed.
In this chapter, we discuss the Hall effects on steady hydro magnetic flow of couple stress fluid in a rotating parallel plate channel bounded by a porous bed on one side. The exact solutions for the velocities in the clean fluid region and porous medium have been obtained analytically. Their behaviour has been discussed computationally with reference to the various governing parameters. The shear stresses on the boundaries and the mass flux are also obtained analytically discussed computationally.
5.2. Formulation and Solution of the problem:

We consider an incompressible viscous and electrically conducting couple stress fluid in a rotating parallel plate channel bounded by a porous bed on the lower side. Both the fluid and the plates are in state of rigid rotation with uniform angular velocity $\Omega$ about z-axis normal to the plates. The entire flow is subjected to strong uniform transverse magnetic field normal to the plate in its own plane. Equation of motion along x-direction the x-component current density $\mu_x J_x H_0$ and the y-component current density $-\mu_x J_y H_0$.

We chose a Cartesian system $O(x,y,z)$ such that the boundary walls are at $z=0$ and $z=1$. Z-axis being the axis of rotation of the plates the fluid medium consists of two zones namely zone 1 and zone 2. Zone 1 consists of clean fluid governed by Navier - Stokes equations and zone 2 corresponds to the flow through porous bed governed by Brinkman equations at the interface the fluid satisfies the continuity condition of velocity and stress. The steady hydro magnetic equations governing the couple stress fluid in zone 1 under the influence of a transverse magnetic field with reference to a frame rotating with a constant angular velocity $\Omega$ are

\[
-2\Omega \nu = -\frac{1}{\rho} \frac{\partial \rho}{\partial x} + \nu \frac{d^2 u}{dz^2} + \mu_x J_y H_0 u - \frac{\eta}{\rho} \frac{d^4 u}{dz^4} \tag{5.2.1}
\]

\[
2\Omega \nu = -\frac{1}{\rho} \frac{\partial \rho}{\partial y} + \nu \frac{d^2 v}{dz^2} - \mu_x J_x H_0 v - \frac{\eta}{\rho} \frac{d^4 v}{dz^4} \tag{5.2.2}
\]
The Brinkman equations governing flow through porous medium with respect to the rotating frame in zone 2 are

\[
-2\Omega v_p = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu_{\text{eff}} \frac{d^2 u_p}{dz^2} + \mu_e J_x H \mu_p - \frac{v}{k} u_p - \frac{\eta}{\rho} \frac{d^4 u}{dz^4}
\]

\[
2\Omega u_p = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu_{\text{eff}} \frac{d^2 v_p}{dz^2} - \mu_e J_x H \nu_p - \frac{v}{k} v_p - \frac{\eta}{\rho} \frac{d^4 v}{dz^4}
\]

(5.2.3) (5.2.4)

Where, (u,v) and (u_p,v_p) are the velocity components along O(x,y) directions respectively. \( \rho \) is the density of the fluid, \( \mu_e \) is the magnetic permeability, \( \nu \) is the coefficient of kinematic viscosity, \( \nu_{\text{eff}} \) the coefficient is the effective kinematic viscosity, \( k \) is the permeability of the medium, \( H_0 \) is the applied magnetic field. When the strength of the magnetic field is very large, the generalized Ohm’s law is modified to include the Hall current, so that

\[
J + \frac{\omega_e \tau_e}{H_0} J x H = \sigma (E + \mu_e q x H)
\]

(5.2.5)

Where, \( q \) is the velocity vector, \( H \) is the magnetic field intensity vector, \( E \) is the electric field, \( J \) is the current density vector, \( \omega_e \) is the cyclotron frequency, \( \tau_e \) is the electron collision time, \( \sigma \) is the fluid conductivity and, \( \mu_e \) is the magnetic permeability.
In equation (5.2.5) the electron pressure gradient, the ion-slip and thermo-electric effects are neglected. We also assume that the electric field \( E = 0 \) under assumptions reduces to

\[
J_x + m J_y = \sigma \mu_i H_0 v \quad \text{(5.2.6)}
\]

\[
J_y + m J_x = -\sigma \mu_i H_0 u \quad \text{(5.2.7)}
\]

where \( m = \omega_e \tau_e \) is the Hall parameter.

On solving equations (5.2.6) and (5.2.7) we obtain

\[
J_x = \frac{\sigma \mu_e H_0}{1 + m^2} (v + mu) \quad \text{(5.2.8)}
\]

\[
J_y = \frac{\sigma \mu_e H_0}{1 + m^2} (mv - u) \quad \text{(5.2.9)}
\]

Using the equations (5.2.8) and (5.2.9), the equations of the motion with reference to rotating frame are given by

\[
- 2\Omega \ v = -\frac{1}{\rho} \frac{\partial p}{\partial x} + v \frac{d^2 u}{dz^2} + \frac{\sigma \mu_i^2 H_0^2}{\rho (1 + m^2)} (mv - u) - \frac{\eta}{\rho} \frac{d^4 u}{dz^4} \quad \text{(5.2.10)}
\]

\[
2\Omega \ u = -\frac{1}{\rho} \frac{\partial p}{\partial y} + v \frac{d^2 v}{dz^2} - \frac{\mu_z^2 H_0^2}{\rho (1 + m^2)} (v + mu) - \frac{\eta}{\rho} \frac{d^4 v}{dz^4} \quad \text{(5.2.11)}
\]
and the equations of motion governing flow through a porous medium with respect to a rotating frame are given by

\[-2\Omega v_p = -\frac{1}{\rho} \frac{\partial p}{\partial x} + v_{\text{eff}} \frac{d^2 u_p}{dz^2} + \frac{\sigma \mu_e^2 H_0^2}{\rho(1 + m^2)} (\rho v_p - u_p) - \frac{\eta}{\rho} \frac{d^4 u_p}{dz^4} - \frac{\nu}{k} u_p\]  

(5.2.12)

\[2\Omega u_p = -\frac{1}{\rho} \frac{\partial p}{\partial y} + v_{\text{eff}} \frac{d^2 v_p}{dz^2} - \frac{\sigma \mu_e^2 H_0^2}{\rho(1 + m^2)} (v_p + mu) - \frac{\eta}{\rho} \frac{d^4 v_p}{dz^4} - \frac{\nu}{k} v_p\]  

(5.2.13)

Let \( q = u + iv, \ q_p = u_p + iv_p, \ \xi = x - iy \)

Now combining the equations (5.2.10) and (5.2.11), we obtain

\[\frac{\eta}{\rho} \frac{d^4 q}{dz^4} - 2i\Omega q = -\frac{1}{\rho} \frac{\partial p}{\partial \xi} + v_{\text{eff}} \frac{d^2 q}{dz^2} + \frac{\sigma \mu_e^2 H_0^2}{\rho(1 + m^2)} (1 + im)q\]  

(5.2.14)

and combining equations (5.2.12) and (5.2.13), we obtain,

\[\frac{\eta}{\rho} \frac{d^4 q_p}{dz^4} + 2i\Omega q_p = -\frac{1}{\rho} \frac{\partial p}{\partial \xi} + v_{\text{eff}} \frac{d^2 q_p}{dz^2} - \frac{\sigma \mu_e^2 H_0^2}{\rho(1 + m^2)} (1 + im)q_p - \frac{\nu}{k} q_p\]  

(5.2.15)

The boundary conditions are

\[q_p = 0, \quad \text{at} \quad z = 0\]  

(5.2.16)

\[q = 0, \quad \text{at} \quad z = l\]  

(5.2.17)

\[\frac{d^2 q_p}{dz^2} = 0, \quad \text{at} \quad z = 0\]  

(5.2.18)

\[\frac{d^2 q}{dz^2} = 0, \quad \text{at} \quad z = l\]  

(5.2.19)
The interfacial conditions are

\begin{align*}
q &= q_p \\
\nu \frac{dq}{dz} &= \nu_{\text{eff}} \frac{dq_p}{dz} \\
\nu \frac{d^2 q}{dz^2} &= \nu_{\text{eff}} \frac{d^2 q_p}{dz^2} \\
\nu \frac{d^3 q}{dz^3} &= \nu_{\text{eff}} \frac{d^3 q_p}{dz^3}
\end{align*}
\tag{5.2.20}

at \ z = h

We introduce the non-dimensional variables

\begin{align*}
z^* &= \frac{z}{l}, \quad q^* &= \frac{q}{l}, \quad q_p^* &= \frac{q_pl}{l}, \\
\rho^* &= \frac{pl^2}{\rho v^2}, \quad h^* &= \frac{h}{l}, \quad \xi^* &= \frac{\xi}{l},
\end{align*}

The governing non-dimensional equations are (dropping asterisks)

\begin{align*}
S \frac{d^4 q}{dz^4} - \frac{d^2 q}{dz^2} + 2iE^{-1} + \frac{M^2 (1 + i m)}{1 + m^2} \int q = P \tag{5.2.21}
\int
\end{align*}

and

\begin{align*}
S \frac{d^4 q_p}{dz^4} - \beta \frac{d^2 q_p}{dz^2} + 2iE^{-1} + \frac{M^2 (1 + i m)}{1 + m^2} + D^{-1} \int q_p = P \tag{5.2.22}
\int
\end{align*}

where,

\[ M^2 = \frac{\sigma \mu \tau_e^2 H_0^2 l^2}{\rho v} \] is the Hartmann number,

\[ m = \omega \tau_e \] is the Hall Parameter,

\[ D^{-1} = \frac{l^2}{k} \] is the Inverse Darcy parameter,

\[ E = \frac{v}{\Omega l^2} \] is the Eckmann number,

\[ S = \frac{n}{\rho l^2 v} \] is the Couple stress parameter,

\[ P = -\frac{\partial p}{\partial \xi} \] is the imposed pressure gradient.

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Corresponding boundary conditions are

\[ q_p = 0, \quad \text{at} \quad z = 0 \quad (5.2.23) \]
\[ q = 0, \quad \text{at} \quad z = 1 \quad (5.2.24) \]
\[ \frac{d^2 q_p}{dz^2} = 0, \quad \text{at} \quad z = 0 \quad (5.2.25) \]
\[ \frac{d^2 q}{dz^2} = 0, \quad \text{at} \quad z = 1 \quad (5.2.26) \]

The interfacial conditions are

\[ q = q_p, \quad \text{at} \quad z = h \quad (5.2.27) \]
\[ \frac{dq}{dz} = \beta \frac{dq_p}{dz}, \quad \text{at} \quad z = h \quad (5.2.28) \]
\[ \frac{d^2 q}{dz^2} = \beta \frac{d^2 q_p}{dz^2}, \quad \text{at} \quad z = h \quad (5.2.29) \]
\[ \frac{d^3 q}{dz^3} = \beta \frac{d^3 q_p}{dz^3}, \quad \text{at} \quad z = h \quad (5.2.30) \]

where \( \beta = \frac{v_{eff}}{v} \)

Solving the equations (5.2.21) and (5.2.22)

\[ q = \left\{ \begin{array}{c} (A e^{a_1 z} + C e^{-a_1 z}) \cos b_1 z + (B e^{a_2 z} + D e^{-a_2 z}) \cos b_2 z \\ + i \left\{ (A e^{-a_1 z} - C e^{a_1 z}) \sin b_1 z + (B e^{-a_2 z} - D e^{a_2 z}) \sin b_2 z \right\} \end{array} \right\} \quad (5.2.31) \]
\[ q_p = \left\{ \begin{array}{c} (L e^{a_1 z} + G e^{-a_1 z}) \cos (b_1 z) + (F e^{a_2 z} + H e^{-a_2 z}) \cos b_3 z \\ + i \left\{ (L e^{-a_1 z} - G e^{a_1 z}) \sin (b_1 z) + (F e^{-a_2 z} - H e^{a_2 z}) \sin b_3 z \right\} \end{array} \right\} \quad (5.2.32) \]
where,

\[
\begin{align*}
  a_1 &= \left\{ \left[ \frac{1 + r^{1/2} \cos(\theta/2)}{2S} \right]^2 + \left[ \frac{r^{1/2} \sin(\theta/2)}{2S} \right]^2 \right\}^{1/4} \cos(\phi/2) \\
  b_1 &= \left\{ \left[ \frac{1 + r^{1/2} \cos(\theta/2)}{2S} \right]^2 + \left[ \frac{r^{1/2} \sin(\theta/2)}{2S} \right]^2 \right\}^{1/4} \sin(\phi/2) \\
  a_2 &= \left\{ \left[ \frac{1 - r^{1/2} \cos(\theta/2)}{2S} \right]^2 + \left[ \frac{r^{1/2} \sin(\theta/2)}{2S} \right]^2 \right\}^{1/4} \cos(\phi/2) \\
  b_2 &= \left\{ \left[ \frac{1 - r^{1/2} \cos(\theta/2)}{2S} \right]^2 + \left[ \frac{r^{1/2} \sin(\theta/2)}{2S} \right]^2 \right\}^{1/4} \sin(\phi/2) \\
  a_3 &= \left\{ \left[ \frac{\beta + r^{1/2} \cos(\theta'/2)}{2S} \right]^2 + \left[ \frac{r^{1/2} \sin(\theta'/2)}{2S} \right]^2 \right\}^{1/4} \cos(\psi/2) \\
  b_3 &= \left\{ \left[ \frac{\beta + r^{1/2} \cos(\theta'/2)}{2S} \right]^2 + \left[ \frac{r^{1/2} \sin(\theta'/2)}{2S} \right]^2 \right\}^{1/4} \sin(\psi/2) \\
  a_4 &= \left\{ \left[ \frac{\beta - r^{1/2} \cos(\theta'/2)}{2S} \right]^2 + \left[ \frac{r^{1/2} \sin(\theta'/2)}{2S} \right]^2 \right\}^{1/4} \cos(\psi/2) \\
  b_4 &= \left\{ \left[ \frac{\beta - r^{1/2} \cos(\theta'/2)}{2S} \right]^2 + \left[ \frac{r^{1/2} \sin(\theta'/2)}{2S} \right]^2 \right\}^{1/4} \sin(\psi/2) \\
  r^{1/2} &= \left\{ \left[ \frac{1 - 4S \frac{M^2 (1 + im)}{1 + m^2}}{1 + m^2} \right]^2 + \left[ 8SE^{-1} \right]^2 \right\}^{1/4} \\
  r_{1/2} &= \left\{ \left[ \frac{\beta^2 - 4S \left[ \frac{M^2 (1 + im)}{1 + m^2} + D^{-1} \right]}{1 + m^2} \right]^2 + \left[ 8SE^{-1} \right]^2 \right\}^{1/4}
\end{align*}
\]
\[
\varphi = \cos \left[ \frac{1}{2} \tan^{-1} \left( \frac{r^{1/2} \sin(\theta/2)}{1 + r^{1/2} \cos(\theta/2)} \right) \right],
\]

\[
\psi = \cos \left[ \frac{1}{2} \tan^{-1} \left( \frac{r_1^{1/2} \sin(\theta_1/2)}{\beta + r_1^{1/2} \cos(\theta_1/2)} \right) \right],
\]

\[
\cos(\theta/2) = \frac{(1 + N^2)^{1/2} + 1}{2(1 + N^2)^{1/2}}, \quad \sin(\theta/2) = \frac{(1 + N^2)^{1/2} - 1}{2(1 + N^2)^{1/2}},
\]

\[
\cos(\theta_1/2) = \frac{(1 + N_1^2)^{1/2} + 1}{2(1 + N_1^2)^{1/2}}, \quad \sin(\theta_1/2) = \frac{(1 + N_1^2)^{1/2} - 1}{2(1 + N_1^2)^{1/2}},
\]

\[
N = \frac{8SE^{-1}}{1 - 4S \frac{M^2(1 + im)}{1 + m^2}}, \quad N_1 = \frac{8SE^{-1}}{\beta^2 - 4S \frac{M^2(1 + im)}{1 + m^2} + D^{-1}}.
\]

\[
A = - \left[ \frac{P}{k_3} e^{-m_i} + Be^{-m_i - m_0} + Ce^{-2m_i} + De^{-m_i - m_1} \right]
\]

\[
B = \frac{Pe^{-m_0} m_i^2}{k_3(m_i^2 - m_i^2)} - De^{-2m_0}
\]

\[
C = \frac{1}{e^{-m_i} - e^{-m_0(1-h)}} \left\{ \frac{P}{k_3} \left( 1 + e^{-m_0(1-h)} \right) - \frac{P}{k_4} \right\}
\]

\[
- \frac{pm_i^2}{k_3(m_i^2 - m_i^2)} \left( e^{-m_i(1-h)} - e^{-m_0(1-h)} \right) - \left[ e^{-m_i} - e^{-m_0(1-h)} \right] D - e^{-m_i} \left[ \frac{P}{k_4} + \frac{pm_i^2}{k_4[m_0^2 - m_0^2]} + \frac{G_1}{G_2} \right] +
\]

\[
e^{-m_i} \left[ \frac{pm_i^2}{k_4[m_0^2 - m_0^2]} H_1 \right] e^{-m_i} \left[ \frac{G_1}{G_2} \right] + e^{-m_i} \left[ \frac{H_1}{H_2} \right]
\]
\[ D = \frac{1}{e^{m_1 h} - e^{-m_1 (l-h)}} \left[ \frac{P}{k_1} \left( m_1 e^{-m_1 (l-h)} \right) \right] - \]

\[ - \frac{pm_1^2}{k_3 (m_2^2 - m_1^2)} \left[ m_2 e^{-m_1 (l-h)} - m_1 e^{-m_1 (l-h)} \right] + \]

\[ + m_1 k_1 \left\{ \frac{P}{k_3} \left[ 1 + e^{-m_1 (l-h)} \right] - \frac{P}{k_3} \left[ m_2 e^{-m_1 (l-h)} - m_1 e^{-m_1 (l-h)} \right] \right\} \]

\[ - \left\{ \beta m_6 e^{-m_6 h} + m_1 k_1 e^{m_1 h} \right\} \left\{ \frac{pm_1^2}{k_4} + \frac{pm_5^2}{k_4 (m_6^2 - m_5^2)} + \left[ \frac{G_1}{G_2} \right] \right\} + \]

\[ + \left\{ \beta m_6 e^{-m_6 h} + m_1 k_1 e^{m_1 h} \right\} \left\{ \frac{pm_5^2}{k_4 (m_6^2 - m_5^2)} + \left[ \frac{H_1}{H_2} \right] \right\} \]

\[ - \frac{G_1}{G_2} \left\{ \beta m_6 e^{-m_6 h} - m_1 k_1 e^{m_1 h} \right\} - \frac{H_1}{H_2} \left\{ \beta m_6 e^{-m_6 h} - m_1 k_1 e^{m_1 h} \right\} \}

\[ L = - \left[ \frac{P}{k_4} + \frac{pm_5^2}{k_4 (m_6^2 - m_5^2)} + \frac{G_1}{G_2} \right], \]

\[ F = \frac{pm_5^2}{k_4 (m_6^2 - m_5^2)} \frac{H_1}{H_2}, \]

\[ G = \frac{G_1}{G_2}, \]

\[ H = \frac{H_1}{H_2}, \]

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\[ G_1 = \left\{ \frac{pm_1^2}{k_4^2} - \frac{pm_1^2}{k_3^2} \left[ I + e^{-m_2(t-h)} \right] \left[ \frac{m_2^2 - m_1^2}{m_1 k_i - m_2} \right] \right\} \frac{P}{k_3} m_i e^{-m_i(t-h)} - \]

\[ - \frac{pm_1^2}{k_3 (m_2^2 - m_i^2)} \left[ m_2 e^{-m_2(t-h)} - m_1 e^{-m_i(t-h)} \right] + \]

\[ + m_1 k_i \left\{ \frac{P}{k_3} \left[ I + e^{-m_1(t-h)} \right] - \frac{P}{k_4^2} \frac{pm_1^2}{k_3^2} \left[ e^{-m_2(t-h)} - e^{-m_i(t-h)} \right] \right\} \]

\[ - \frac{P}{k_4} \left[ (m_i^2 - \beta m_5^2) e^{m_h} + (m_2^2 - m_i^2) e^{m_h} \right] \left( \beta \frac{m_5 + m_1 k_i}{m_1 k_i - m_2} \right) \]

\[ - \frac{pm_2^2}{k_4 (m_6^2 - m_5^2)} \{ m_1^2 (e^{m_h} - e^{m_h}) + \beta (m_5^2 e^{m_h} - m_6^2 e^{m_h}) \} + \]

\[ + \left[ \frac{m_2^2 - m_1^2}{m_1 k_i - m_2} \right] \left\{ \beta e^{m_h} (m_6 e^{m_h} - m_5 e^{m_h}) + m_1 k_i (e^{m_h} - e^{m_h}) \right\} \]

\[ - \frac{H_1}{H_2} \left\{ (m_i^2 - \beta m_6^2) \left( e^{-m_h} - e^{m_h} \right) + \right\] \]

\[ + \left[ \frac{m_2^2 - m_1^2}{m_1 k_i - m_2} \right] \left[ - \beta m_6 e^{-m_h} + e^{m_h} \right] + m_1 k_i \left( e^{-m_h} - e^{m_h} \right) \}

\[ G_2 = \left\{ \frac{m_i^2 - \beta m_5^2}{e^{-m_h} - e^{m_h}} \right\} + \]

\[ + \left[ \frac{m_2^2 - m_1^2}{m_1 k_i - m_2} \right] \left[ - \beta m_5 e^{-m_h} + e^{m_h} \right] + m_1 k_i \left( e^{-m_h} - e^{m_h} \right) \]
\[ H = \left\{ \frac{P}{k_3} m_i^3 e^{-m_i(l-h)} - \frac{pm_i^2}{k_3(m_i^2 - m_e^2)} \left[ m_i^3 e^{-m_i(l-h)} - m_i^3 e^{-m_i(l-h)} \right] + \right. \]

\[ + m_i^3 k_3 \left\{ \frac{P}{k_3} \left[ l + e^{-m_i(l-h)} \right] - \frac{pm_i^2}{k_3(m_i^2 - m_e^2)} \left[ e^{-m_i(l-h)} - e^{-m_i(l-h)} \right] \right\} - \]

\[ - \frac{k_2}{\left[ e^{-m_i h} - e^{-m_i(l-h)} \right]} \left[ e^{-m_i h} - e^{-m_i(l-h)} \right] \left[ m_i^2 e^{-m_i(l-h)} - m_i e^{-m_i(l-h)} \right] + \]

\[ + m_i k_1 \left\{ \frac{P}{k_3} \left[ l + e^{-m_i(l-h)} \right] - \frac{pm_i^2}{k_3(m_i^2 - m_e^2)} \left[ e^{-m_i(l-h)} - e^{-m_i(l-h)} \right] \right\} + \]

\[ + \frac{P}{k_4} \left\{ - \beta m_5^3 e^{m_i h} - m_i^3 e^{m_i h} k_1 + \frac{k_2}{\left[ e^{-m_i h} - e^{-m_i(l-h)} \right]} \left[ \beta m_i e^{m_i h} + m_i k_i e^{m_i h} \right] \right\} - \]

\[ - \frac{pm_5^3}{k_4(m_5^2 - m_e^2)} \left\{ \beta \left( m_5 e^{m_i h} - m_5 e^{m_i h} \right) + m_i k_i \left( e^{m_i h} - e^{m_i h} \right) + \right. \]

\[ m_i \left\{ \frac{k_2}{\left[ e^{-m_i h} - e^{-m_i(l-h)} \right]} \left( m_i k_i - m_2 \right) \left( e^{-m_i h} - e^{-m_i(l-h)} \right) \right\} \left\} \right\} \]

\[ \left\{ m_1^3 - \beta m_5^3 \right\} \left( e^{-m_i h} - e^{m_i h} \right) + \frac{m_1^3 - m_2^3}{m_i k_i - m_2} \]

\[ - \beta m_5 \left( e^{-m_i h} + e^{m_i h} \right) + m_i k_i \left( e^{-m_i h} - e^{m_i h} \right) \right] - \]

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\[
- \left( \frac{p m_i^2}{k_4} - \frac{p m_i^2}{k_3} \right) \left[ 1 + e^{-m_i(l-h)} \right] - \frac{m_2^2 - m_i^2}{m_1 k_i - m_2} \left\{ \frac{p}{k_3} m_i e^{-m_i(l-h)} - \right.
\]
\[
\left. \frac{p m_i^2}{k_3 (m_2^2 - m_i^2)} \right) \left( m_2 e^{-m_2(l-h)} - m_i e^{-m_i(l-h)} \right) +
\]
\[
+ m_i k_i \left\{ \frac{p}{k_3} \left[ I + e^{-m_i(l-h)} \right] - \frac{p m_i^2}{k_3 (m_2^2 - m_i^2)} \right) \left( e^{-m_2(l-h)} - e^{-m_i(l-h)} \right) \right\} +
\]
\[
+ \frac{p}{k_4} \left[ (m_i^2 - \beta m_1^2) e^{m_i h} + \left( m_2^2 - m_i^2 \right) e^{m_2 h} \left\{ \frac{\beta m_3 + m_i k_i}{m_1 k_i - m_2} \right\} -
\]
\[
- \frac{p m_1^2}{k_4 (m_6^2 - m_2^2)} \left\{ m_i^2 \left( e^{m_i h} - e^{m_2 h} \right) + \beta \left( m_3 e^{m_3 h} - m_2 e^{m_2 h} \right) +
\]
\[
+ \frac{m_2^2 - m_i^2}{m_1 k_i - m_2} \left\{ \beta \left( m_3 e^{m_3 h} - m_2 e^{m_2 h} \right) + m_i k_i \left( e^{m_i h} - e^{m_2 h} \right) \right\} \right) \right\} \right]
\]
\[
\left[ \beta m_5 \left[ e^{-m_2 h} + e^{m_i h} \right] + m_2^3 k_j \left( e^{m_2 h} - e^{-m_1 h} \right) + \left\{ - \beta m_5 \left[ e^{-m_2 h} + e^{m_i h} \right] +
\]
\[
+ m_i k_i \left[ e^{-m_i h} - e^{m_2 h} \right] \right\} \right) \left\{ \frac{k_5}{\left( e^{-m_2 h} - e^{-m_1 (2-h)} \right) \left( m_1 k_i - m_2 \left( e^{-m_2 h} - e^{-m_1 (2-h)} \right) \right)} \right) \right] \right]
\]
\[ H_2 = \left[ \beta m_6^3 (e^{-m_6 h} + e^{m_6 h}) + m_1 k_i (e^{m_1 h} - e^{-m_1 h}) + \right. \]
\[ + \left\{ - \beta m_6 (e^{-m_6 h} + e^{m_6 h}) + m_1 k_i (e^{-m_1 h} - e^{m_1 h}) \right\} + \]
\[ \frac{k_2}{(e^{-m_2 h} - e^{-m_2(2-h)})(m,k_i - m_2)} \left( e^{-m_2 h} - e^{-m_2(2-h)} \right) \]
\[ - \left[ \beta m_3^3 (e^{-m_3 h} + e^{m_3 h}) + m_1 k_i (e^{-m_3 h} - e^{m_3 h}) \right] + \]
\[ - \left[ \beta m_5^3 (e^{-m_5 h} + e^{m_5 h}) + m_1 k_i (e^{-m_5 h} - e^{m_5 h}) \right] + \]
\[ + \left\{ - \beta m_6 (e^{-m_6 h} + e^{m_6 h}) + m_1 k_i (e^{-m_1 h} - e^{m_1 h}) \right\} + \]
\[ \frac{k_2}{(e^{-m_2 h} - e^{-m_2(2-h)})(m,k_i - m_2)} \left( e^{-m_2 h} - e^{-m_2(2-h)} \right) \]
\[ - \frac{m_2^2 - m_1^2}{m_1 k_i - m_2} \]
\[ \left\{ - \beta m_6 (e^{-m_6 h} + e^{m_6 h}) + m_1 k_i (e^{-m_1 h} - e^{m_1 h}) \right\} \]
\[ K_1 = \frac{e^{-m_1 h} + e^{-m_1 (2-h) - m_1 h}}{e^{-m_1 h} - e^{-m_1 (2-h) - m_1 h}} \]

\[ K_2 = m_1^3 \left( e^{-m_1 h} + e^{-m_1 (2-h) - m_1 h} \right) \left( e^{-m_1 h} - e^{-m_1 (2-h) - m_1 h} \right) \]

\[ -m_2^3 \left( e^{-m_2 h} + e^{-m_2 (2-h) - m_2 h} \right) \left( e^{-m_2 h} - e^{-m_2 (2-h) - m_2 h} \right) \]

\[ K_3 = 2iE^{-1} + M^2 \left( \frac{1 + im}{1 + m^2} \right) \]

\[ K_4 = 2iE^{-1} + M^2 \left( \frac{1 + im}{1 + m^2} \right) + D^{-1} \]

\[ m_1 = \sqrt{1 + \sqrt{1 - 4S(2iE^{-1} + M^2 (1 + iM))}} \]

\[ 2S \]

\[ m_2 = \sqrt{1 - \sqrt{1 - 4S(2iE^{-1} + M^2 (1 + iM))}} \]

\[ 2S \]

\[ m_5 = \frac{\beta + \sqrt{\beta^2 - 4S(2iE^{-1} + M^2 (1 + iM)) + D^{-1}}}{2S} \]

\[ m_6 = \frac{\beta - \sqrt{\beta^2 - 4S(2iE^{-1} + M^2 (1 + iM)) + D^{-1}}}{2S} \]
The shear stresses on the upper plate and lower plate are given by

\[ \tau_U = A m_1 e^{m_1} + B m_2 e^{m_2} - C m_1 e^{-m_1} - D m_2 e^{-m_2} \]

and

\[ \tau_L = (L - G) m_2 + (F - H) m_6 \]

To determine the mass flux by the formula

\[ Q_x + iQ_y = \int q \, dz \]

the mass flux

\[ Q = \sqrt{Q_x^2 + Q_y^2} \]

\[ Q_x + iQ_y = \frac{A}{m_1} [e^{m_1} - e^{m_1,h}] + \frac{B}{m_2} [e^{m_2} - e^{m_2,h}] \]

\[- \frac{C}{m_1} [e^{-m_1} - e^{-m_1,h}] - \frac{D}{m_2} [e^{-m_2} - e^{-m_2,h}] \]
5.3. DISCUSSION:

We now discuss the behaviour of the flow field, the shear stresses and the mass flux with reference to the variations in the governing parameters $S$, the couple stress parameter, $E$ the Eckmann number, $D^{-1}$ the inverse Darcy parameter, $M$ the Hartmann number, and $m$ the Hall parameter, in either cases of small and larger thickness of porous bed. The velocity components for different variations of governing parameters have been plotted in figures (1-40).

We observe that, in case of small thickness of porous bed ($h=0.2$) both the velocity components enhances with increase in the couple stress parameter $S$ and hence the resultant velocity also enhances with $S$ (Fig 1, 11). From figures (2 and 12), we find that an increase in the Eckmann number $E$ reduces the resultant velocity in the porous bed while enhances in the clean fluid region. The velocity component $v$ reduces with increase in $E$ in the entire flow region. Keeping the thickness of the porous bed small we notice that lower the permeability lesser the fluid speed in the both clean and porous regions. It is evident from the fact that the individual velocity components reduce their magnitude for increase in $D^{-1}$ (Fig 3&13). Fixing the other parameters an increase in $M$ reduces the fluid speed in the clean fluid region and enhances in the porous region. This is in according as in a fact that the magnetic field reduces a retarding force of the clean fluid flow (Fig 4&14). The effect of the Hall parameter of the flow field may be observed from figures (5 &15). It is interesting to note that an increase in Hall parameter $m$ reduces the velocity $u$ and enhances the velocity $v$ in the entire flow field. However the resultant velocity indicates the retardation in the clean fluid and acceleration in the porous bed for increase in $m$. 
Figures (6-10 and 16-20) correspond to the velocity profiles $u$ and $v$ when the thickness of the porous bed slightly increases ($h=0.3$). The influence of the thickness of the porous bed through smaller values does not affect the flow behaviour in general. Although it affects the individual velocity components. In fact as the couple stress parameter $S$ increases $u$ slightly retards in the clean fluid while enhances in the porous bed. But the resultant velocity continues to enhance in the entire fluid region (6 and 16). Likewise the velocity components $u$ and $v$ retards with increase in $E$ while enhance in the clean fluid region. Thus increase in the rotation parameter $E$ enhances the resultant velocity in the clean fluid region while reduces in the porous bed (Fig 7 & 17). The retardation in the velocity components as well as the resultant velocity with reference to increase in $D^{-1}$ (Fig 8 and 18) shows lesser the permeability of the porous medium lower the fluid speed in the flow region. The influence of the magnetic field on the flow is indicated (Figures 9 and 19). As $M$ increases $u$ increases in the entire flow region except in the vicinity the boundary, $v$ reduces however in the entire flow field. The resultant velocity shows and appreciation in the clean fluid region while it reduces in the porous region. A slight increase in the thickness of porous bed does not affect the variations of $u$ and $v$ in the Hall parameter, its behaviour remains similar with fluid retarding in the fluid while accelerating the porous bed (Fig 10 & 20).

Fig (21-30 & 31-40) correspond to the behaviour of the velocity components $u$ and $v$ when the thickness of the porous bed is large ($h=0.5$). In contrast to the earlier case the velocity components and their resultant experiences a retardation for increase in the couple stress parameter $S$ (Fig 21 & 31). As a rotation parameter $E$ increases $u$ experiences a retardation in the entire flow field except near the lower
boundary. But $v$ reduces in the entire flow region except in the mid layer. The resultant velocity however reduces in the clean fluid region and enhances in the porous region (Fig 22 & 32). The fluid moves with reduced velocities in porous beds with lesser permeability irrespective of the thickness of the beds (Fig 23 & 33). An increase in $M$ enhances $u$ in the entire flow field while it enhances $v$ in the clean fluid and retards in the porous region. The fluid speed reduces in the clean fluid while accelerate in the porous bed for increase in the intensity of the magnetic field (Fig 24 & 34). The behaviour of the flow with reference to the Hall parameter independent of the thickness of the porous bed and is evident from (Fig 25 & 35). As in the earlier cases the fluid speed enhances in the clean fluid and retards in the porous region for an increase in the Hall parameter.

Similar behaviour of the fluid velocity with reference to the variations of the governing parameters is observed even the thickness of the porous bed sufficiently large ($h=0.8$) Fig (26-30 and 36-40). The velocity components $u$ or $v$ however exhibits a slight variation with reference to the parameters $E$, $M$ and $m$ as the thickness of the porous bed sufficiently increases. For example in case of $h=0.5$ in the clean fluid region $v$ increases with $E$ where as reduces with $E$ in the same region where $h=0.8$. A reversal behaviour is observed with reference to $u$ in the porous bed (Fig 27 & 37). With reference to variation in $M$, $v$ experiences a retardation in the porous bed where $h=0.5$ while enhances in the porous bed when $h=0.8$ (Fig 29 & 39). With reference to Hall parameter the influence of the thickness of the porous bed is clearly observed. In contrast to $h=0.5$ $u$ reduces in the clean fluid while enhances in the same region when $h=0.8$. The behaviour of $v$ in case of $h=0.5$ is totally in contrast to its behaviour when $h=0.8$, $v$
enhances every where in the flow region in case \( h=0.5 \) while reduces in case \( h=0.8 \) (Fig 30 & 40).

The shear stresses are calculated on upper and lower plates and are tabulated in tables (1-4). We notice that on the upper plate \( \tau_x \) reduces with increase in the couple stress parameter \( S \) irrespective of the thickness of the porous bed. For an increase \( E \), \( \tau_x \) increases when the thickness of the porous bed is small and reduces when the thickness of the porous bed is large. Lesser the permeability of the medium lower the stresses irrespective thickness of the porous bed. The variation of \( \tau_x \) with reference to \( M \) is in contrast to the variations in \( E \) with \( \tau_x \) reducing in case of small thickness while increasing in case of large thickness of the porous bed. Similar variation of \( \tau_x \) is observed with increase in the Hall parameter (Table 1). \( \tau_y \) increases with increase in couple stress parameter \( S \) in case of small thickness while reduces with \( S \) in case of the large thickness of porous bed with reference to the variations in \( E \), we notice that \( \tau_y \) reduces irrespective of the thickness with increasing the rotation parameter \( E \). Likewise reduces with increase \( D^{-1} \) for all thickness of the porous bed. With reference to \( M \) the behaviour of \( \tau_y \) similar to that of \( \tau_x \) with \( \tau_y \) reducing in case of small thickness of the bed enhancing with \( M \) in case of large thickness of the bed. A reversal behaviour is observed with reference to variation in \( m \) (Table 2).

On the lower plate \( \tau_x \) and \( \tau_y \) exhibits similar behaviour with reference to increasing the couple stress parameter \( S \), The Eckmann number \( E \) as well as the Hartmann number \( M \). Lower the permeability of the medium \( \tau_y \) reduces where as \( \tau_x \)
exhibits as increasing when the thickness of the porous bed is sufficiently large. An increase in the Hall parameter reduces $\tau_x$ while enhancing $\tau_y$, irrespective of the thickness of the porous bed (Table 3 and 4).

The mass flux has been evaluated and tabulated in the table 5. We find that the Mass flux increases with increase in $S$, $E$, $D^{-1}$ and $m$ where as reduces with $M$ fixing the other parameters.
I. Profiles of $u$ when the thickness of the porous bed ($h=0.2$) is small.

Fig 1: The velocity profile for $u$ with $S$.
$\beta=1.2, h=0.2, M=5, D^{-1}=1000, m=1.5, S=1, E=0.01$. 

Fig 2: The velocity profile for $u$ with $E$.
$\beta=1.2, h=0.2, M=5, D^{-1}=1000, m=1.5, S=1$. 
Fig 3: The velocity profile for $u$ with $D^{-1}$.
$\beta=1.2$, $h=0.2$, $E=0.01$, $M=5$, $m=1.5$, $S=1$.

Fig 4: The velocity profile for $u$ with $M$.
$\beta=1.2$, $h=0.2$, $E=0.01$, $D^{-1}=1000$, $m=1.5$, $S=1$. 
Fig 5: The velocity profile for $u$ with $m$.
$\beta=1.2$, $h=0.2$, $E=0.01$, $D^{-1}=1000$, $M=5$, $S=1$.

II. Profiles of $u$ when the thickness of the porous bed ($h=0.3$) is small.

Fig 6: The velocity profile for $u$ with $S$.
$\beta=1.2$, $h=0.3$, $M=5$, $D^{-1}=1000$, $m=1.5$, $E=0.01$. 
Fig 7: The velocity profile for $u$ with $E$.
$\beta=1.2$, $h=0.3$, $M=5$, $D^{-1}=1000$, $m=1.5$, $S=1$.

Fig 8: The velocity profile for $u$ with $D^{-1}$.
$\beta=1.2$, $h=0.3$, $E=0.01$, $M=5$, $m=1.5$, $S=1$. 
Fig 9: The velocity profile for $u$ with $M$.

$\beta=1.2$, $h=0.3$, $E=0.01$, $D^{-1}=1000$, $m=1.5$, $S=1$.

Fig 10: The velocity profile for $u$ with $m$.

$\beta=1.2$, $h=0.3$, $E=0.01$, $D^{-1}=1000$, $M=5$, $S=1$. 
III. Profiles of $v$ when the thickness of the porous bed ($h=0.2$) is small.

Fig 11: The velocity profile for $v$ with $S$.
$\beta=1.2$, $h=0.2$, $M=5$, $D^{-1}=1000$, $m=1.5$, $E=0.01$.

Fig 12: The velocity profile for $v$ with $E$.
$\beta=1.2$, $h=0.2$, $M=5$, $D^{-1}=1000$, $m=1.5$, $S=1$. 
Fig 13: The velocity profile for $v$ with $D^{-1}$.
$\beta=1.2$, $h=0.2$, $E=0.01$, $M=5$, $m=1.5$, $S=1$

Fig 14: The velocity profile for $v$ with $M$.
$\beta=1.2$, $h=0.2$, $E=0.01$, $D^{-1}=1000$, $m=1.5$, $S=1$. 
IV. Profiles of \( v \) when the thickness of the porous bed (\( h=0.3 \)) is small.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig15.png}
\caption{The velocity profile for \( v \) with \( m \).
\( \beta=1.2, h=0.2, E=0.01, D^{-1}=1000, M=5, S=1. \)}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig16.png}
\caption{The velocity profile for \( v \) with \( S \).
\( \beta=1.2, h=0.3, M=5, D^{-1}=1000, m=1.5, E=0.01. \)}
\end{figure}
Fig 17: The velocity profile for $v$ with $E$.

$\beta=1.2$, $h=0.3$, $M=5$, $D^{-1}=1000$, $m=1.5$, $S=1$.

Fig 18: The velocity profile for $v$ with $D^{-1}$.

$\beta=1.2$, $h=0.3$, $E=0.01$, $M=5$, $m=1.5$, $S=1$. 
Fig 19: The velocity profile for $v$ with $M$. 
$\beta=1.2$, $h=0.3$, $E=0.01$, $D^{-1}=1000$, $m=1.5$, $S=1$.

Fig 20: The velocity profile for $v$ with $m$. 
$\beta=1.2$, $h=0.3$, $E=0.01$, $D^{-1}=1000$, $M=5$, $S=1$. 
I. Profiles of $u$ when the thickness of the porous bed ($h=0.5$) is large.

![Fig 21: The velocity profile for $u$ with $S$.]

$\beta=1.2, h=0.5, M=5, D^{-1}=1000, m=1.5, E=0.01$.

![Fig 22: The velocity profile for $u$ with $E$.]

$\beta=1.2, h=0.5, M=5, D^{-1}=1000, m=1.5, S=1$. 
Fig 23: The velocity profile for $u$ with $D^{-1}$.
$\beta=1.2$, $h=0.5$, $E=0.01$, $M=5$, $m=1.5$, $S=1$.

Fig 24: The velocity profile for $u$ with $M$.
$\beta=1.2$, $h=0.5$, $E=0.01$, $D^{-1}=1000$, $m=1.5$, $S=1$. 
Fig 25: The velocity profile for $u$ with $m$.
\[ \beta = 1.2, \ h = 0.5, \ E = 0.01, \ D = 1000, \ M = 5, \ S = 1. \]

II. Profiles of $u$ when the thickness of the porous bed ($h=0.8$) is large.

Fig 26: The velocity profile for $u$ with $S$.
\[ \beta = 1.2, \ h = 0.8, \ M = 5, \ D = 1000, \ m = 1.5, \ E = 0.01. \]
Fig 27: The velocity profile for $u$ with $E$.
$\beta=1.2$, $h=0.8$, $M=5$, $D^{-1}=1000$, $m=1.5$, $S=1$.

Fig 28: The velocity profile for $u$ with $D^{-1}$.
$\beta=1.2$, $h=0.8$, $E=0.01$, $M=5$, $m=1.5$, $S=1$.
Fig 29: The velocity profile for $u$ with $M$.
$\beta=1.2$, $h=0.8$, $E=0.01$, $D^{-1}=1000$, $m=1.5$, $S=1$.

Fig 30: The velocity profile for $u$ with $m$.
$\beta=1.2$, $h=0.8$, $E=0.01$, $D^{-1}=1000$, $M=5$, $S=1$. 
III. Profiles of $v$ when the thickness of the porous bed ($h=0.5$) is large.

Fig 31: The velocity profile for $v$ with $S$.
$\beta=1.2$, $h=0.5$, $M=5$, $D^{-1}=1000$, $m=1.5$, $E=0.01$.

Fig 32: The velocity profile for $v$ with $E$.
$\beta=1.2$, $h=0.5$, $M=5$, $D^{-1}=1000$, $m=1.5$, $S=1$.
Fig 33: The velocity profile for $v$ with $D^{-1}$.
$\beta = 1.2$, $h = 0.5$, $E = 0.01$, $M = 5$, $m = 1.5$, $S = 1$.

Fig 34: The velocity profile for $v$ with $M$.
$\beta = 1.2$, $h = 0.5$, $E = 0.01$, $D^{-1} = 1000$, $m = 1.5$, $S = 1$. 
IV. Profiles of $v$ when the thickness of the porous bed ($h=0.8$) is large.

**Fig 35:** The velocity profile for $v$ with $m$.

$\beta=1.2$, $h=0.5$, $E=0.01$, $D^{-1}=1000$, $M=5$, $S=1$.

**Fig 36:** The velocity profile for $v$ with $S$.

$\beta=1.2$, $h=0.8$, $M=5$, $D^{-1}=1000$, $m=1.5$, $E=0.01$. 
Fig 37: The velocity profile for $v$ with $E$.
$\beta = 1.2$, $h = 0.8$, $M = 5$, $D^{-1} = 1000$, $m = 1.5$, $S = 1$.

Fig 38: The velocity profile for $v$ with $D^{-1}$.
$\beta = 1.2$, $h = 0.8$, $E = 0.01$, $M = 5$, $m = 1.5$, $S = 1$. 

$D^{-1} = 1000$, $D^{-1} = 2000$, $D^{-1} = 3000$, $D^{-1} = 4000$. 

$E = 0.01$, $E = 0.02$, $E = 0.03$, $E = 0.04$. 

$z$ $v$
Fig 39: The velocity profile for \( v \) with \( M \).
\( \beta=1.2, h=0.8, E=0.01, D^{-1}=1000, m=1.5, S=1. \)

Fig 40: The velocity profile for \( v \) with \( m \).
\( \beta=1.2, h=0.8, E=0.01, D^{-1}=1000, M=5, S=1. \)
Table 1

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The shear stresses ($\tau_x$) on the upper plate.
Table 2

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The shear stresses ($\tau_y$) on the upper plate.
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The shear stresses ($\tau_z$) on the lower plate.
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The shear stresses \( (\tau_y) \) on the lower plate.
Table 5

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- The mass flux (Q).
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