4.1 General

Masonry is extensively used in India as infill walls in reinforced concrete buildings. For the linear static analysis and design of buildings with masonry requires the material properties of masonry. And in the case of detailed non-linear analyses such as static pushover analysis stress-strain curves of masonry are required. The stress-strain curves of masonry are not available easily because of insufficient controlled experimental studies and significant geographical variation in the material properties.

Initially the masonry infill behavior is isotropic up to elastic stage, beyond this stage, cracks start to develop perpendicular to tension diagonal. Once cracks start and propagate further, the isotropic nature of material suddenly changes to orthotropic nature. Due to non-linear nature of this infill wall behavior, it is difficult to get exact strength of the masonry infill subjected to lateral loads. (Crasifulli, 1997)

4.2 Modeling of Infills

4.2.1 Diagonal Strut Action

Non-integral infill frame subjected to lateral load behaves like diagonally braced frame with an equivalent strut as discussed in the previous chapter. Infill acts as a diagonal strut, an infill wall can be replaced by an equivalent strut in the analysis model and analogous structure is shown in Fig.4.2.

4.2.2 Equivalent Width of Infill Strut

Prior to cracking the elastic in-plane stiffness of a solid un-reinforced masonry infill panel shall be represented with an equivalent diagonal compression strut of width, $Z$, as in equation (4.1). The thickness and modulus of elasticity of the equivalent strut shall be the same as the infill panel (FEMA 306, 1998).
\[
Z = 0.175(\lambda_i h)^{0.4} d_m
\]  
\[(4.1)\]

Where, \[\lambda_i = \left[ \frac{E_m t_m \sin 2\theta}{4E_f I_{\text{col}} h_m} \right]^\frac{1}{2} \]  
\[(4.2)\]

- \(h\) = column height between centerlines of beams in cm,
- \(h_m\) = height of masonry infill panel in cm,
- \(E_f\) = modulus of elasticity of frame material in MPa,
- \(E_m\) = modulus of elasticity of infill material in MPa,
- \(I_{\text{col}}\) = moment of inertia of column in \(\text{cm}^4\),
- \(l_m\) = length of infill panel in cm,
- \(d_m\) = diagonal length of infill panel in cm,
- \(t_m\) = thickness of infill panel and equivalent strut in cm,
- \(\theta\) = angle whose tangent is the infill height-to-length aspect ratio in radians,
- \(\lambda_i\) = coefficient used to determine equivalent width of infill strut.

Fig.4.1 Masonry Infill Frames
4.2.3 Lateral Strength of Infill Frames (Stiffness of Masonry Infill)

In estimating the lateral strength of an infill frame it is necessary to find the weakest of the various modes of failure of the frame and infill. The infill frame strength may be governed by strength of the frame members or the masonry infills.

4.2.4 Strength of the Frame

At larger lateral loads, the failure may occur in either frame or the infill. The failure mode in the frame is -

a) Tension failure of windward column,

b) Flexural or shear failure of the column or beam.

An approximate method to determine the strength of these modes is to analyze the forces in the equivalent frame subjected to the known horizontal force, assuming the infill to be replaced by diagonal struts, as shown in Figure 4.2. The calculated tensile load in the windward column and the shearing components of the load in the diagonal struts may then be compared with the respective strength of the columns, beams and connections. If the frame strength is sufficient to prevent collapse by one of these modes, then infill wall crushing will lead to failure.

Fig.4.2 Equivalent Frame

In the Figure 4.2, \( h_1, h_2, h_3 \) = height of storey, \( L \) = bay width, \( F \) = applied force, \( K_1, K_2, K_3 \ldots \) = member forces.
4.2.5 Shear Strength of the Infill Masonry Walls

There are several potential failure modes for infill masonry walls (Paulay and Priestley 1992) including -

1. Horizontal sliding shear failure of masonry walls,
2. Compression failure of diagonal strut,
3. Diagonal tensile cracking,
4. Tension failure mode (flexural) which is not a usually a critical failure mode for infill walls.

The strength of the masonry infill may be taken as minimum of the failure of

a) Shear cracking along the bed joints (i.e. Sliding shear failure)

b) Local crushing of the masonry at one of the compression corners as governed by one of the failure modes.

a) Sliding Shear Failure: In ductile RC frames, failure of masonry infill with weak mortar joints and strong brick units may take place by sliding through the horizontal bed joint in the masonry infill panel. Initial sliding shear capacity of masonry infill panel is calculated by the Mohr-Coulomb failure. The maximum shear strength for this kind of failure mechanism is given by (Mostafaei, 2004) -

\[
\tau_f = \tau_0 + \mu \sigma_N
\]

Where,

\( \tau_o \) - Cohesive capacity of the mortar bed joint,
\( \mu \) - Sliding friction coefficient along the bed joint,
\( \sigma_N \) - Vertical compression stress in the infill walls.

After the cohesive bond in a mortar bed joint is destroyed, as a result of cyclic loading, i.e. (\( \tau_o = 0 \)), masonry infill still has some ability to resists sliding through shear friction in the bed joints. If the lateral deformation is small, then cohesive capacity due to sliding shear failure is zero, (\( \tau_o = 0 \)). The vertical stresses in wall may
only result from the self weight of the panels, as the inter storey drift becomes large, and then bounding column impose a vertical load due to shortening of strain $\varepsilon$ in the panel.

The maximum horizontal shear force in the infill wall is given by $V_f$

$$V_f = \tau_\nu t_m l_m + \mu N$$  (4.4)

Where, $t_m$ - Infill wall thickness,

$l_m$ - Length of infill panel,

$N$ - Vertical load in infill walls.

$N$ is caused by vertical shortening strain in the panel due to the lateral drifts -

$$N = \varepsilon L_m l_m E_m r^2$$  (4.5)

Where, $r$ - The inter story drift angle.

In this study $N$ may be estimated as the sum of applied vertical load on the panel and the vertical component of the diagonal compression force $R_c$ as shown in the Fig. 4.3.

$$\theta \mu \tau = \sin \theta$$

Substituting the value of $R_c \cos \theta$ in the above equation

The horizontal component of shear force

$$R_c \cos \theta = \tau_o t_m l_m + \mu R_c \sin \theta$$  (4.7)
\[
R_c = \frac{\tau_{o}\ell_m t_m}{1 - \mu \tan \theta}
\]

\[
\therefore V_f = \tau_{o}\ell_m t_m \left[ 1 + \frac{\mu \tan \theta}{1 - \mu \tan \theta} \right]
\]

\[
V_f = \frac{\tau_{o}\ell_m t_m}{(1 - \mu \tan \theta)}
\]

For the purpose of analysis, \(\tau_o\) is within the range of \(0.1 \leq \tau_o \leq 1.5\) MPa

\[
\tau_o = 0.04 f_m', \quad & \mu = 0.5 \quad \text{(Paulay and Priestley, 1992)}
\]

\(\mu\) is determined from experimental values from the equation of Chen (2003). The values range between 0.5 to 0.68 and the values of \(\tau_o\) is within the range \(0.1 \leq \tau \leq 1.5\) MPa.

b) Compression failure:

Failure of masonry infill may takes place by compression failure of the equivalent diagonal strut if the mortar joints and brick units are strong and RC frames are sufficiently ductile. Horizontal force required for the failure of equivalent diagonal strut is given as per (FEMA 306, 1998) recommendations as

\[
V_c = Zt_m f_m' \cos \theta
\]

Where, \(V_c\) - Shear at the compression failure of equivalent diagonal strut,

\(Z\) - Width of equivalent diagonal compression strut of infill panel,

\(t_m\) - Thickness of infill panel and equivalent strut,

\(f_m'\) - Compressive strength of masonry,

\(\theta\) - Angle whose tangent is the infill height-to-length aspect ratio in radians.

4.3 Analytical Modeling of Masonry Infill Panel

According to Madan et al. (1997) a panel system can be replaced by two diagonal masonry compression struts, for global building analysis purposes. These struts may be placed concentrically across the diagonals of the frame, effectively
forming a concentrically braced frame system (Fig.4.4). In this configuration, however, the forces imposed on columns of the frame by the infill are not represented. To account for these effects, compression struts may be placed eccentrically within the frames as per guidelines given in (FEMA 306, 1998). If the analytical models incorporated by eccentrically located compression struts, the results should yield infill effects on columns directly. Alternatively, global analyses may be performed using concentric braced frame models and the infill effects on columns may be evaluated at a local level by applying the strut loads onto the columns (Mostafaei, 2004).

The lateral force-deformation relationship for the structural masonry infill is assumed to be a smooth curve bounded by a bilinear strength envelope with an initial elastic stiffness until the yield force $V_y$ and there on a post yield degraded stiffness till the maximum force $V_m$ is reached. After which the post-peak residual shear force $V_p$ will be reached as shown in Fig.4.4.

![Fig.4.4 Strength Envelopes for Masonry Infill Panel (Mostafaei, 2004)](image)

Where,

$V_m, U_m$ - Maximum shear force and corresponding displacement respectively,

$V_y, U_y$ - Shear force at yielding and corresponding displacement respectively,

$V_p, U_p$ - Post-peak residual shear force and corresponding displacement respectively,

$K_O$ - Initial Stiffness of the infill panel,

$\alpha$ - The ratio of the post-yield stiffness to the pre-yield stiffness.
Maximum lateral strength $V_m$, should be estimated considering the two critical failure modes, sliding shear mode and compression failure mode. The maximum displacement at the maximum lateral force is

$$ U_m = \frac{\varepsilon'_m d_m}{\cos \theta}; \varepsilon'_m = \frac{f'_m l}{E_m} $$(4.12)

Where, $U_m$, Maximum displacement at the maximum lateral force,

$\varepsilon'_m$ - Masonry compression strain at the maximum compression stress.

The initial stiffness of the infill panel is given by

$$ K_0 = \frac{2V_m}{U_m} \text{ (Madan et al 1997)} $$ (4.13)

### 4.4 Determination of Stiffness

Since basically the frame is assumed to be shear frame with the horizontal rigid element, only the vertical elements (columns) are emphasized. Rigidity of beams makes it easy to calculate the stiffness of the storey as direct sum of the stiffness of the columns.

Unless the span of adjacent beams is very different; the earthquake-induced axial forces will normally affect only the outer columns in a frame. Moreover the frame elements are assumed to respond elastically throughout the time history of the earthquake. Hence the following average moments of inertia are adopted for the exterior and interior columns (0.75 for columns and 0.5 for beams).

Stiffness contribution of columns at a storey level,

$$ Kf = \frac{12E_f}{h^3} \sum I_{col} $$ (4.14)

and the initial stiffness, $K_0$, of the masonry infill is

$$ K_0 = 2\left( \frac{V_m}{U_m} \right) $$ (4.15)
Hence the total stiffness, $K$, at the floor level,

$$K = K_f + K_o$$

(4.16)

Where,

- $K_f$ - The sum of the stiffness of columns in a storey,
- $\sum I_{col}$ - The sum of moments of inertia of exterior and interior columns at a storey,
- $V_m$ - Maximum Shear Force,
- $U_m$ - Maximum displacement.

### 4.5 Numerical Example

Example 4.4.1.: Determination of equivalent strut width and shear forces at sliding shear and compression failures for the panel aspect ratio (L/H) of 2.0.

$$E_f = 22360 \text{ MPa} ; \quad E_m = 2035 \text{ MPa} ; \quad f_m = 3.5 \text{ MPa}$$

Column size $= 400 \times 400 \text{ mm}$  \quad Beam size $= 300 \times 400 \text{ mm}$

$h = 3000 \text{ mm}$  \quad Panel size $= 6000 \times 3000 \text{ mm}$

With reference to

$h_m = 3000 - 400 = 2600 \text{ mm}$  \quad $l_m = 6000 - 400 = 5600 \text{ mm}$

$t_m = 120 \text{ mm}$

Equivalent strut width:

$$I_{col} = I_g = \frac{400^4}{12} = 21.33\times10^8 mm^4$$

$$d_m = \sqrt{h_m^2 + l_m^2} = 6216.9 mm$$

$$\theta = \arctan(h_m / l_m) = 24.90^0$$

$$\lambda_j = \left[ \frac{E_m t_m \sin 20\theta}{4E_f I_{col} h_m} \right]^{\frac{1}{2}} = 0.000868 mm^{-1}$$
\[ Z = 0.175(\lambda_i h) - 0.4d_m = 73.92\text{mm} \]

Shear force at sliding shear failure.

\[ \tau_o = 0.04f'_m = 0.0175 \]

\[ \therefore V_f = \frac{\tau_o f'_m d_m}{1 - \mu \tan \theta} = \frac{0.175 \times 230 \times 5600}{1 - 0.5 \times \tan(24.90^\circ)} \times 10^{-3} = 180.00\text{KN} \]

Example 4.4.2.: Determination of initial shear force and displacement at yielding from the previous example,

\[ f_m = 3.5 \text{ MPa}; E_m = 2035 \text{ MPa}; \ \theta = 24.90^\circ; \ d_m = 6216.9\text{mm} \]

\[ V_m \text{ is the minimum of } Vf / \text{ and } Vc, \ V_m = 180.00 \text{ kN}. \]

\[ \alpha = 0.01 - \text{Default value} \]

\[ \varepsilon'_m = \frac{f'_m l_m}{E_m \cos \theta} = 0.00146 \Rightarrow U_m = \frac{\varepsilon'_m d_m}{\cos \theta} = 1.00\text{cm} = 10\text{mm} \]

\[ K_0 = 2 \left( \frac{V_m}{U_m} \right) = 30628\text{KN} / \text{m} \]

\[ V_y = \frac{V_m - U_m ak_0}{1 - a} = 151.59\text{KN} \Rightarrow U_y = \frac{V_y}{K_0} = 4.95\text{mm} \]

The above examples indicate the procedure used to determine the stiffness parameters. It has been summarized in an Excel sheet for various bay widths in Table B of Appendix.

4.6 Parametric Investigation

Natural period and free vibration mode shapes are the basic dynamic characteristics of any structure. For evaluating the dynamic response it was proposed to evaluate natural frequencies and natural periods of multistoreyed frames considering effect of infill walls. The results obtained were compared with the results of the bare frame and with that given in IS: 1893 (Part I)-2002 provisions.
Using the natural periods thus obtained the design lateral forces were evaluated by pseudo static method as presented in IS: 1893 (Part I)-2002. The structure was analyzed for lateral forces for a bare frame and infilled frame. Effect of arrangements of infill on the behavior of frames were highlighted.

4.6.1 Validation of Results for Single Bay, Single Storey Bare Frame

The computer programming package – ETABS 9.5 (Computer and Structures) was utilized for the analysis of infilled and bare frames. The results are first validated for a single bay, single storey frame shown in Fig 4.1. For the analysis, first stiffness matrix and mass matrix were derived using direct stiffness approach and then the Eigen value problems are solved for the above two matrices. The natural period thus obtained is compared with free vibration analysis obtained by ETABS 9.5 (Computer and Structures)

\[
\text{w/m run}
\]

![Fig.4.5 Single Bay, Single Storey Bare Frame](image)

<table>
<thead>
<tr>
<th>Material</th>
<th>Modulus of elasticity (E) kN/m(^2)</th>
<th>Unit Weight (W) kN/m(^3)</th>
<th>Poisson’s ratio ((\mu))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>22.36068 x 10(^6)</td>
<td>25.0</td>
<td>0.20</td>
</tr>
<tr>
<td>Brick Masonry Infill</td>
<td>2 x 10(^6)</td>
<td>20.0</td>
<td>0.15</td>
</tr>
</tbody>
</table>
Table 4.2 Member Properties

<table>
<thead>
<tr>
<th>Member</th>
<th>Length (m)</th>
<th>Size (m)</th>
<th>Moment of Inertia (m$^4$)</th>
<th>Mass Kg/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column</td>
<td>3</td>
<td>0.3 x 0.3</td>
<td>6.75 x 10$^{-4}$</td>
<td>229.358</td>
</tr>
<tr>
<td>Beam</td>
<td>6</td>
<td>0.15 x 0.3</td>
<td>3.375 x 10$^{-4}$</td>
<td>114.679</td>
</tr>
</tbody>
</table>

Mass of column = 0.3 x 0.3 x 25 x 1000/9.81 = 229.358 kg/m

For 2 m superimposed load acting over a span of the beam

\[ w/m = 229.358 \times 2 – 114.679 = 344.037 \text{ kg/m} \]

\[ w/m = 344.037 \times 9.81/1000 \]

\[ w/m = 3.375 \text{ kN/m} \]

Transformed stiffness matrix for Column members 1 and 3 are

\[
\begin{bmatrix}
\alpha_1 & 0 & -\alpha_2 & \cdots & -\alpha_1 & 0 & -\alpha_2 \\
0 & \alpha_5 & 0 & \cdots & 0 & -\alpha_5 & 0 \\
-\alpha_2 & 0 & \alpha_3 & \cdots & \alpha_2 & 0 & \alpha_4 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
-\alpha_1 & 0 & \alpha_2 & \cdots & \alpha_1 & 0 & \alpha_2 \\
0 & -\alpha_3 & 0 & \cdots & 0 & \alpha_4 & 0 \\
-\alpha_2 & 0 & \alpha_4 & \cdots & \alpha_2 & 0 & \alpha_3
\end{bmatrix}
\]

Where, \( \alpha_1 = \frac{12EI}{L^3} \), \( \alpha_2 = \frac{6EI}{L^2} \), \( \alpha_3 = \frac{4EI}{L} \), \( \alpha_4 = \frac{2EI}{L} \), \( \alpha_5 = \frac{AE}{L} \)

Transformed stiffness matrix for Beam member 2

For the beam member putting the respective values of I = I/2 and L = 6 m

\[
\begin{bmatrix}
\beta_5 & 0 & 0 & \cdots & -\beta_5 & 0 & 0 \\
0 & \beta_1 & -\beta_2 & \cdots & 0 & -\beta_1 & -\beta_2 \\
0 & -\beta_2 & \beta_3 & \cdots & 0 & \beta_2 & \beta_3 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
-\beta_5 & 0 & 0 & \cdots & \beta_5 & 0 & 0 \\
0 & -\beta_1 & \beta_2 & \cdots & 0 & \beta_1 & \beta_2 \\
0 & -\beta_2 & \beta_3 & \cdots & 0 & \beta_2 & \beta_3
\end{bmatrix}
\]

Where, \( \beta_1 = \frac{12EI}{L^3} \), \( \beta_2 = \frac{6EI}{L^2} \), \( \beta_3 = \frac{4EI}{L} \), \( \beta_4 = \frac{2EI}{L} \), \( \beta_5 = \frac{AE}{L} \)
The combined stiffness matrix obtained

\[ K = \begin{bmatrix}
  \alpha_1 + \beta_3 & 0 & \alpha_2 & : & -\beta_3 & 0 & 0 \\
  0 & \alpha_1 + \beta_1 & -\beta_2 & : & 0 & -\beta_1 & -\beta_2 \\
  \alpha_2 & -\beta_2 & \alpha_1 + \beta_1 & : & 0 & \beta_2 & \beta_4 \\
  \cdots & \cdots & \cdots & : & \cdots & \cdots & \cdots \\
  -\beta_3 & 0 & 0 & : & \alpha_1 + \beta_3 & 0 & \alpha_2 \\
  0 & -\beta_1 & \beta_2 & : & 0 & \alpha_1 + \beta_1 & \beta_2 \\
  0 & -\beta_2 & \beta_4 & : & \alpha_2 & \beta_2 & \alpha_1 + \beta_3 \\
\end{bmatrix} \]

Putting the numerical values, the combined stiffness matrix:

\[ K = 1 \times 10^6 \times \begin{bmatrix}
  174.4151 & 0 & 10.06506 & -167.7051 & 0 & 0 \\
  0 & 671.2396 & -1.2578 & 0 & -0.41926 & -1.2578 \\
  10.06506 & -1.2578 & 25.1613 & 0 & 1.2578 & 2.5156 \\
  -167.7051 & 0 & 0 & 174.4151 & 0 & 10.06506 \\
  0 & -0.41926 & 1.2578 & 0 & 671.2396 & 1.2578 \\
  0 & -1.2578 & 2.5156 & 10.06506 & 1.2578 & 25.1613 \\
\end{bmatrix} \]

The mass matrix for column element:

\[ [M_1] = \frac{3 \times m}{2} \begin{bmatrix}
  1 & 0 & 0 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 1 & 0 & 0 \\
  0 & 0 & 0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix} \]

The mass matrix for beam element:

\[ [M_2] = \frac{6 \times m}{2} \begin{bmatrix}
  1 & 0 & 0 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 1 & 0 & 0 \\
  0 & 0 & 0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix} \]

The combined mass matrix obtained from direct approach:
Hence the mass matrix (M) =

\[
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
7.5m & 0 & 0 & 0 & 0 & 0 \\
0 & 7.5m & 0 & 0 & 0 & 0 \\
0 & 0 & 7.5m & 0 & 0 & 0
\end{bmatrix}
\]

\[m = 229.358 \text{ kg/m}\]

Eigen value problems are solved by using mass matrix and stiffness matrix of equation to obtain the natural frequencies

\[
\{ (K) - \omega^2 (M) \} \{ \phi \} = 0
\]  

(4.17)

The natural frequencies obtained are,

\[
\begin{align*}
\omega_1 &= 42.0971 \text{ rad/sec} \\
\omega_2 &= 443.0407 \text{ rad/sec} \\
\omega_3 &= 624.760 \text{ rad/sec} \\
\omega_4 &= 624.476 \text{ rad/sec}
\end{align*}
\]

The natural periods are,

\[
\begin{align*}
T_1 &= 0.14925 \text{ sec.} \\
T_2 &= 0.014182 \text{ sec.} \\
T_3 &= 0.010061 \text{ sec.} \\
T_4 &= 0.010057 \text{ sec.}
\end{align*}
\]

Fig.4.6 Mode Shapes for Single Bay Single Storey Bare Frame
4.6.2 Validation of Result for Single Bay, Single Storey Infill Frame

The contribution of infills to the composite system can be accounted by using the equivalent diagonal strut method presented as by Stafford Smith (1966).

The width of diagonal strut is calculated in accordance with the equation given by (FEMA306, 1998). Putting the values presented in Table 4.1, Width of equivalent strut \( w = 1.509 \) m.

![Fig.4.7 Single Bay Single Storey Infilled Frame](image)

The stiffness matrix for diagonal strut element -

\[
S_{MS} = \frac{AE}{L} \begin{bmatrix}
C^2 & CS & -C^2 & -CS \\
CS & S^2 & -CS & -S^2 \\
-C^2 & -CS & C^2 & CS \\
-CS & -S^2 & -CS & S^2 \\
\end{bmatrix}
\]

\[
S_{MS} = \frac{AE}{L} \begin{bmatrix}
\gamma_1 & -\gamma_2 & \cdots & -\gamma_1 & \gamma_2 \\
-\gamma_2 & \gamma_3 & \cdots & \gamma_2 & -\gamma_3 \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
-\gamma_1 & \gamma_2 & \cdots & \gamma_1 & -\gamma_2 \\
\gamma_2 & -\gamma_3 & \cdots & -\gamma_2 & \gamma_3 \\
\end{bmatrix}
\]
The combined stiffness matrix for above configuration -

\[
K = \begin{bmatrix}
\alpha + \beta + \gamma & -\gamma & \alpha & \ldots & -\beta & 0 & 0 \\
-\gamma & \alpha + \beta + \gamma & -\beta & \ldots & 0 & -\beta & -\beta \\
\alpha & -\beta & \alpha + \beta & \ldots & 0 & \beta & \beta \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
-\beta & 0 & 0 & \ldots & \alpha + \beta & 0 & \alpha \\
0 & -\beta & \beta & \ldots & 0 & \alpha + \beta & \beta \\
0 & -\beta & \beta & \ldots & 0 & \alpha & \alpha + \beta \\
\end{bmatrix}
\]

\[
K = 1 \times 10^6 \times
\begin{bmatrix}
257.2031 & -41.3941 & 10.06506 & -167.7051 & 0 & 0 \\
-41.3941 & 691.9366 & -1.2578 & 0 & -0.41926 & -1.2578 \\
10.06506 & -1.2578 & 25.1613 & 0 & 1.2578 & 2.5156 \\
-167.7051 & 0 & 0 & 174.4151 & 0 & 10.06506 \\
0 & -0.41926 & 1.2578 & 0 & 671.2396 & 1.2578 \\
0 & -1.2578 & 2.5156 & 10.06506 & 1.2578 & 25.1613 \\
\end{bmatrix}
\]

Mass of infill \((m_i) = 19.20 \times 0.23 \times 1.509 \times 1000/9.81 = 679.3394\) kg/m

\[
M_i = \frac{mL}{2} = \frac{m \times 6.708}{2} = 3.354 \ m_i
\]

The ordinate in mass matrix of bare frame is added for first two diagonals to obtain the combined mass matrix as

\[
(M) = \begin{bmatrix}
7.5m + 3.354m_i & 0 & 0 & 0 & 0 & 0 \\
0 & 7.5m + 3.354m_i & 0 & 0 & 0 & 0 \\
0 & 0 & 7.5m & 0 & 0 & 0 \\
0 & 0 & 0 & 7.5m & 0 & 0 \\
0 & 0 & 0 & 0 & 7.5m & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
(M) = \begin{bmatrix}
3998.7573 & 0 & 0 & 0 & 0 & 0 \\
0 & 3998.7573 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1720.185 & 0 & 0 \\
0 & 0 & 0 & 0 & 1720.185 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

The Eigen value problems are solved, to get the natural frequencies and natural period of vibrations using the equation-

\[ ((K) - \omega^2 (M)) \{\phi\} = 0 \] (4.18)
Natural frequencies are
\[
\begin{align*}
\omega_1 &= 120.3048 \text{ rad/sec} \\
\omega_2 &= 382.1242 \text{ rad/sec} \\
\omega_3 &= 418.150 \text{ rad/sec} \\
\omega_4 &= 624.6178 \text{ rad/sec}
\end{align*}
\]

The natural periods are
\[
\begin{align*}
T_1 &= 0.052227 \text{ sec.} \\
T_2 &= 0.016443 \text{ sec.} \\
T_3 &= 0.015026 \text{ sec.} \\
T_4 &= 0.010059 \text{ sec.}
\end{align*}
\]

Fig. 4.8 Mode Shapes for Single Bay Single Storey Infilled Frame

Table 4.3 Comparison of Natural Period of Vibration

<table>
<thead>
<tr>
<th>Mode no.</th>
<th>Bare frame (Sec)</th>
<th>Infill frame (Sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Analytical</td>
<td>ETABS</td>
</tr>
<tr>
<td>1</td>
<td>0.149254</td>
<td>0.149293</td>
</tr>
<tr>
<td>2</td>
<td>0.0141819</td>
<td>0.014184</td>
</tr>
<tr>
<td>3</td>
<td>0.010061</td>
<td>0.010063</td>
</tr>
<tr>
<td>4</td>
<td>0.010057</td>
<td>0.010059</td>
</tr>
</tbody>
</table>
4.7 Numerical Example and Details of the Structural Model

In this study, five different models of an eight storey building symmetrical in the plan are considered. Usually in a building 40% to 60% presence of Masonry infills (MI) are effective as the remaining portion of the Masonry Infills (MI) are meant for functional purpose such as doors and windows openings (Pauley and Priestley 1992). In this study the building is modeled using 40% of Masonry Infills (MI) but arranging them in different ways as shown in the Figure 4.9. The building has four bays in N-S and E-W directions each. The plan dimension is 20 m × 16 m. The storey height is 3.0 m each, in all the models considered. Further inputs include unit weight of the concrete is 25 kN/m$^3$, unit weight of masonry is 20 kN/m$^3$, Elastic modulus of steel is 2×10$^8$ kN/m$^2$, Elastic Modulus of concrete is 22.36×10$^6$ kN/m$^2$, Strength of concrete is 20 N/mm$^2$ (M20), Yield strength of steel is 415 N/mm$^2$ (Fe-415) and Live-load is 3.5 kN/m$^2$. The column sizes are 500 mm × 500 mm and beam size are 230mm × 600mm throughout the study. The modulus of brick masonry and strut width is obtained using FEMA (306, 1998) recommendations.

i.e. $E_m = 550f_m = 2035$ N/mm$^2$ and the width of the strut in longitudinal and transverse direction are $w_1 = 0.68$ m: $w_2 = 0.73$ m respectively.

The Seismic Lateral Forces on the building are found using the provision made in IS 1893 (Part-1): 2002.

$$V_B = A_h W$$  \hspace{1cm} (4.18)

Where, $A_h$ is design horizontal seismic coefficient and $W$ is seismic weight of the building. The design horizontal seismic coefficient $A_h$ for structure shall be determined by the following expression-

$$A_h = \frac{ZIS_{ul}}{2R_g}$$  \hspace{1cm} (4.19)

Where, $Z$ is zone factor, $I$ is importance factor which depends on the functional use of the structure, $R$ is the response reduction factor which depends on the perceived seismic damage performance of the structure, characterized by ductile or brittle
deformations. $Sa/g$ is the average response acceleration coefficient value obtained from response spectrum which depends on the fundamental period of the structure. The fundamental natural period of vibration is given by

$$T_a = 0.075h^{0.75} \quad (4.20)$$

For moment resisting RC frame building without brick infill walls

$$T_a = 0.09h / \sqrt{d} \quad (4.21)$$

For moment resisting RC frame building with brick infill.

Where,

- $h =$ the height of the building, in m,
- $d =$ Base dimension of the building at plinth level, in m, along the considered direction of the lateral forces

The design base shear was obtained by using equation 4.18. The buildings are assumed to be situated in zone-IV of India with an importance factor of 1.0. Response reduction factor of 5, Zone factor of 0.24 and $Sa/g$ for medium soil is taken as 2.5. The design base shear $V_B$ computed from equation 4.18 shall be distributed along the height of the building as per the following expression,

$$Q_i = V_B \frac{W_i h_i^2}{\sum_{j=1}^{n} W_j h_j^2} \quad (4.22)$$

Where, $Q_i$ is the design lateral force at floor $i$, $Wi$ seismic weights of the floor $i$, $h_i$ height of the floor measured from base and 'n' is Number of storeys where the masses are located. Linear Static Analysis was performed for earthquake loading and the structure was checked for the maximum bending moments and shear forces developed. A Strong column and weak beam criterion has been followed. The details of the eight storey building model are
Study on Behavior of Infilled R/C Frames Subjected to Lateral Forces

CHAPTER IV

Fig. 4.9 Plan and Elevation of Eight Storey Reinforced Concrete Building
Model I- Frames modeled as bare frames with wall load in seismic weight calculation.

Model II- Frames have one full masonry infill wall (230 mm thick) in all the storeys including ground storey all along the periphery (with 40% infill).

Model III- The frames do not have masonry infill walls in the first storey which represents a soft storey with full brick masonry infill walls (230 mm thick) in upper storeys (Same as model II).

Model IV- Frames have one full masonry infill walls arranged in alternative span in both X and Y direction (with 40% infill).

Model V- Frames with masonry infill walls are arranged as a lift core all along the corners (with 40% infill).
4.8 Results and Discussions

Table 4.4 Variation of Base Shear (kN)

<table>
<thead>
<tr>
<th>Storey</th>
<th>Model I</th>
<th>Model II</th>
<th>Model III</th>
<th>Model IV</th>
<th>Model V</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bare Frame</td>
<td>Outer Infill</td>
<td>Outer Infill Soft Storey</td>
<td>Inner Infill</td>
<td>Lift Well Infill</td>
</tr>
<tr>
<td>8</td>
<td>171.57</td>
<td>254.07</td>
<td>241.26</td>
<td>254.13</td>
<td>257.91</td>
</tr>
<tr>
<td>7</td>
<td>393.23</td>
<td>582.3</td>
<td>552.96</td>
<td>582.44</td>
<td>591.11</td>
</tr>
<tr>
<td>6</td>
<td>556.08</td>
<td>823.46</td>
<td>781.96</td>
<td>823.66</td>
<td>835.9</td>
</tr>
<tr>
<td>5</td>
<td>669.17</td>
<td>990.92</td>
<td>940.99</td>
<td>991.17</td>
<td>1005.9</td>
</tr>
<tr>
<td>4</td>
<td>741.55</td>
<td>1098.1</td>
<td>1042.77</td>
<td>1098.37</td>
<td>1114.7</td>
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<tr>
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<td>782.26</td>
<td>1158.39</td>
<td>1100.02</td>
<td>1158.67</td>
<td>1175.9</td>
</tr>
<tr>
<td>2</td>
<td>800.36</td>
<td>1185.19</td>
<td>1125.47</td>
<td>1185.48</td>
<td>1203.1</td>
</tr>
<tr>
<td>1</td>
<td>804.88</td>
<td>1191.89</td>
<td>1131.83</td>
<td>1192.18</td>
<td>1209.9</td>
</tr>
</tbody>
</table>

Fig. 4.10 Variation of Base Shear (kN)
### Table 4.5 Variation of Storey Displacement (mm)

<table>
<thead>
<tr>
<th>Storey</th>
<th>Model I Bare Frame</th>
<th>Model II Outer Infill</th>
<th>Model III Outer Infill Soft Storey</th>
<th>Model IV Inner Infill</th>
<th>Model V Lift Well Infill</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>23.0945</td>
<td>15.4672</td>
<td>15.7411</td>
<td>15.4694</td>
<td>15.8059</td>
</tr>
<tr>
<td>5</td>
<td>17.1247</td>
<td>11.483</td>
<td>11.9576</td>
<td>11.4826</td>
<td>11.2428</td>
</tr>
<tr>
<td>2</td>
<td>6.0272</td>
<td>4.3272</td>
<td>5.1552</td>
<td>4.3261</td>
<td>4.0869</td>
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<td>2.5567</td>
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<td>1.6957</td>
</tr>
<tr>
<td>0</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

**Fig. 4.11 Variations of Storey Displacement (mm)**
### Table 4.6 Variation of Storey Drift

<table>
<thead>
<tr>
<th>Storey</th>
<th>Model I Bare Frame</th>
<th>Model I Outer Infill</th>
<th>Model III Outer Infill Soft Storey</th>
<th>Model IV Inner Infill</th>
<th>Model V Lift Well Infill</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0.000372</td>
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<td>0.000239</td>
<td>0.000252</td>
<td>0.000335</td>
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<td>0.000432</td>
<td>0.000455</td>
<td>0.000521</td>
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<td>0.000942</td>
<td>0.000624</td>
<td>0.000592</td>
<td>0.000623</td>
<td>0.000667</td>
</tr>
<tr>
<td>5</td>
<td>0.001134</td>
<td>0.00074</td>
<td>0.000702</td>
<td>0.000738</td>
<td>0.000762</td>
</tr>
<tr>
<td>4</td>
<td>0.001256</td>
<td>0.000811</td>
<td>0.00077</td>
<td>0.000808</td>
<td>0.000812</td>
</tr>
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<td>3</td>
<td>0.001309</td>
<td>0.000847</td>
<td>0.000806</td>
<td>0.000842</td>
<td>0.000826</td>
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<td>2</td>
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<td>0.000849</td>
<td>0.000872</td>
<td>0.000842</td>
<td>0.000806</td>
</tr>
<tr>
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<td>0.000601</td>
<td>0.000573</td>
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<tr>
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<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
</tbody>
</table>

**Fig. 4.12 Variations of Storey Drift**
### Table 4.7 Fundamental Natural Periods of Different Frame Models

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Model</th>
<th>Fundamental Natural Period (Sec.)</th>
<th>Analysis</th>
<th>Codal</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>I- Bare frame</td>
<td>1.5830</td>
<td>0.813</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>II- Infilled frame (infills all along periphery)</td>
<td>1.0509</td>
<td>0.483</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>III – Infilled frame with Soft storey.</td>
<td>1.1022</td>
<td>0.483</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>IV- Infilled frame (infills arranged in alternative span both in X and Y direction)</td>
<td>1.0873</td>
<td>0.483</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>V- Infilled frame (infills arranged as lift core)</td>
<td>1.1596</td>
<td>0.483</td>
<td></td>
</tr>
</tbody>
</table>

### Table 4.8 Stiffness of Different Frame Models

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Model</th>
<th>Stiffness kN/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>I- Bare frame</td>
<td>2105</td>
</tr>
<tr>
<td>2</td>
<td>II- Infilled frame (infills all along periphery)</td>
<td>3367</td>
</tr>
<tr>
<td>3</td>
<td>III – Infilled frame with Soft storey.</td>
<td>3257</td>
</tr>
<tr>
<td>4</td>
<td>IV- Infilled frame (infills arranged in alternative span both in X and Y direction)</td>
<td>3508</td>
</tr>
<tr>
<td>5</td>
<td>V- Infilled frame (infills arranged as lift core)</td>
<td>3404</td>
</tr>
</tbody>
</table>
Natural period of vibration of the buildings depend upon their mass and lateral stiffness. Presence of masonry infill walls in the building increases both the mass and stiffness of buildings; however, the contribution of stiffness is more significant. Consequently, the natural periods of masonry infilled RC frame are normally lower than that of the corresponding bare frame building. Therefore the seismic design forces for masonry infilled frames are generally higher than those for the bare frames. Indian code IS 1893 (Part-I): 2002 explicitly specify the empirical formulae for the calculations of fundamental natural periods. It was observed from the results (Table 4.7) that the natural periods obtained from the codal provisions were underestimated as compared to analytical expressions. From the results it was observed that variations of natural period are insignificant in various infilled models. It was also observed that the natural period of the structures alters with introduction of vertical irregularities in the frames as seen in model III.

The base shear obtained as per IS 1893(Part-I): 2002 equations are 1926.00 kN and 2046.00 kN for bare and infilled frames respectively. Thus it can be observed that the codal method leads to stiffer structures which attracts higher seismic forces and uneconomic in design. The base shear obtained in various infilled frame model were almost 1.5 times compared to bare frame model.

Increase in the stiffness of the structure introduces reduction in top displacement of the structures. Because of addition of infills there was almost 50 % decrease in the displacement in various models compared to bare frame model. It was also observed that the inter storey drifts demand was maximum in the first storey of infilled frame with soft storey (model III).The displacement of this model was 30 % greater than that of bare frame model.