Chapter-1

Introduction to Market Research
1. INTRODUCTION TO MARKET RESEARCH

1.1 Role of Shares in Market research

Investments in share market have attractive returns, but at the same time, they involve hidden risk factors. Hence one has to think carefully and properly before investing (or) purchasing a company share. A well experienced investor can predict the prosperity of a share based on the company's past history, fiscal policies of the Government, investment patterns in the past and in the future and so on. These type of predictions are not scientific ones and are often may go wrong because those predictions/decisions are based on the past experience of the investor (which are subjective ones), but not done, using scientific methods. Scientific predictions are also based on the past data but analysis is through sophisticated Statistical techniques. Here also, there is a possibility of these predictions/decisions to go wrong, but the chances are very remote, say 5% (or) 1% (or) still less (based on the techniques used). In this connection, it is worth to mention that use of more recent Statistical techniques are very essential and helpful to take appropriate decisions by a common investor. Thus there is a necessity to apply Statistical techniques to analyse the market value of different shares collected through proper resources. This leads to a new field of research, known as "Market Research" [3], in broader sense.

Market Research, now-a-days, occupies a vital place in modern research, because it provides solutions to business problems through an appropriate model. In brief, a model is a replica of a real situation. If a business problem is represented through a mathematical equation, then it is
called 'Business Mathematical Model (B.M.M)'. Thus Business Mathematical Models play an important role in Market Research and are mainly based on the assumptions considered by the researcher. Such assumptions are necessary and inevitable to apply well known Mathematical tools and techniques, through which one can get solutions to the model proposed. The solutions thus obtained to the proposed models are to be applied in 'reality' to set fruitful results. But in reality, these assumptions, some times may not be true. For instance, generally one can assume a linear relationship between demand and price, but this linearity may not hold good for all the items in the market. For example, L.P.Gas cylinder's price may not increase depending upon the demand. If demand is more, supply of the cylinders may perhaps be delayed, but cost may not increase. Thus the basic assumption of linearity is invalid here. Similarly, in some situations non-linear relationships may hold good for the variables under consideration. Hence, proposed models are to be modified by incorporating such new and practically valid assumptions into the models, so that these modified models are closer to the reality. In Market Research probabilistic models proposed by Researchers are to be updated, because the variables under consideration inherently have Stochastic nature. Thus Stochastic models are more appropriate to deal with problems arising in Market Research, than probabilistic models. This is very essential particularly to determine the behaviour of the Market Value of a Share [3,15].

Market value of a share is very sensitive and depends on many factors, their interrelationships, and fiscal policies taken by the Government/ Reserve Bank of India (RBI)/Stock Exchange Board of India (SEBI) and so on. Thus Stochastic modelling is very essential in Market Research to detect the
stochastic behaviour of the market value of a share [15,17]. In Stochastic modelling, we assume that the parameters of the distribution under study are functions of time, denoted by the variable ‘t’, where as; these parameters are assumed as *constants* in *Probabilistic models*.

1.2 Problems in determining the Market value of a share

(1.2a). An Introduction to Share market and its behaviour

The *Capital of a company* is divided into a number of equal parts and these parts are known as Shares [3].

I. Definitions

1. According to section 2(46) of Indian companies Act, 1956 "a share is the share capital of a Company and includes stock except where a distinction between stock and shares is expressed or implied".

Kinds of Shares

Prior to passing of companies Act, 1956, public Companies used to issue three types of shares, namely;

i. Preference shares

ii. Equity shares and

iii. Deferred shares.

However, issuing last type of shares is now, prohibited by the Company Act. 1956. Hence, public companies in recent times, can issue only the remaining two, which are explained as follows:

(l). Preference Shares

As the name suggests, these shares have certain preferences, as compared to other shares. They are
(a). Preference in the payment of dividend and
(b). Preference in the payment of capital at the time of Companies liquidation.

Preference Shares are further divided into the following eight categories, namely:

1. Cumulative preference shares,
2. Non-cumulative preference shares,
3. Redeemable preference shares,
4. Non-redeemable preference shares,
5. Participating preference shares,
6. Non-participating preference shares,
7. Convertible preference shares and
8. Non-convertible preference shares.

However, at present, the Companies are not issuing preference shares to public, though permitted by the Act.

(ii). Equity Shares

Equity Shares are popularly known as "Ordinary shares" or simply "Shares". These are ownership instruments, which means that "the holders of these shares are the real owners of the company". Equity shareholders are paid dividend only after paying it to the preference shareholders. The rate of dividend on these shares is floating from time to time and depends mainly on the profits of the company. Some times, the rate of dividend is high and some times it is as low as possible which is equal to zero. Thus investment in equity shares, involves two types of risks namely:

(i). Risk of capital and
(ii). Risk of dividend.
The equity shareholders cannot demand their capital back during the life time of the company. Here, in this thesis, for simplicity sake the *equity shares* are referred simply as *shares/securities*.

II. Procedure for issue of Shares [3,15].

When a public company wants to raise capital, either for its fixed or working capital needs, it has the following options/sources, namely;

(i) Issue of shares,

(ii) Issue of Debentures/Bonds and

(iii) Mobilisation of capital through public deposits.

Among the above three sources the *first* one is the *cheapest* and most *popular one*, which enriches the 'Permanent capital' of the company.

Following are basic steps involved in issuing shares by a company:

(i) Public Ltd., Companies issue prospectus along with application to the public, informing about the issue of shares.

(ii) Interested public fill up these applications and send them to company along with application money.

(iii) The company opens a Bank account and the whole application money is deposited there in.

(iv) Prepare a list of applications among with the details of shares applied (by the company secretary) and the same is presented to the Board of Directors (B.O.D).

III. Allotment and Listing of Securities (Shares)

The B.O.D. follow these guide lines to allot the shares to the applicants:
(i) The BOD of company along with the officers of concerned stock exchange and officials of SEBI may sit together and prepare a basis of allotment of shares (i.e. a list).

(ii) The company secretary sends the letter of allotment along with request to pay the allotment money to **allottees**.

(iii) If any calls are there, the secretary also sends 'call money notices' to the shareholders.

(iv) When application money/allotment money/call money is paid by the shareholders in full, then the shares are called fully paid, if not, partly paid.

After the completion of issue and allotment of shares, it is the duty of a company to provide liquidity to the shares. The public company after allotment of shares, shall take permission either from 'The Regional Stock Exchange/The Bombay Stock Exchange/The National Stock Exchange' to **enlist shares** for the purpose of trading, by paying *listing fee* to the concerned **Stock Exchange**. The inclusion of shares of a company in the official list of a Stock Exchange is called "**Listing of shares**". Once the listing is completed company's shares are traded in the **Stock Exchange**, which is explained as follows:

**Stock Exchange**

Stock Exchange is a place where securities Industrial securities, Government securities, securities of mutual funds are traded. A Stock Exchange provides facilities for trading but it can neither buy, nor sell a share of any type.

**Leading Stock Exchanges**

*The Bombay Stock Exchange (B.S.E.)* and *the National Stock Exchange (N.S.E.)* are the leading Stock Exchanges in India, of which the former stands first.
Regional Stock Exchanges

Almost all states have Stock Exchanges, these are called, Regional Stock Exchanges. For instance, the Hyderabad Stock Exchange Ltd., the Bangalore Stock Exchange Ltd., the Chennai Stock Exchange Ltd., and so on. Some States have two Stock Exchanges. For instance, in Maharashtra one at Mumbai and another one at Pune. Similarly in Tamilnadu one is at Chennai and the other one at Coimbatore.

At present there are '23' Stock Exchanges all over India of which '21' are Regional Stock Exchanges, which are listed below:

(1). The Bombay Stock Exchange Ltd.,
(2). The Kolkata Stock Exchange Ltd.,
(3). The Chennai Stock Exchange Ltd.,
(4). The Delhi Stock Exchange Ltd.,
(5). The Ahmedabad Stock Exchange Ltd.,
(6). The Ludhiana Stock Exchange Ltd.,
(7). The Kanpur Stock Exchange Ltd.,
(8). The Jaipur Stock Exchange Ltd.,
(9). The Indore stock Exchange Ltd.,
(10). The Pune Stock Exchange Ltd.,
(11). The Hyderabad Stock Exchange Ltd.,
(12). The Bangalore Stock Exchange Ltd.,
(13). The Cochin Stock Exchange Ltd.,
(14). The Guwahati Stock Exchange Ltd.,
(15). The Mangalore Stock Exchange Ltd.,
(16). The Bhubaneswar Stock Exchange Ltd.,
(17). The Patna Stock Exchange Ltd.,
(18). The Rajkot Stock Exchange Ltd.,
(19). The Baroda Stock Exchange Ltd.,
(20). The Coimbatore Stock Exchange Ltd., and
(21). The Meerut Stock Exchange Ltd.
Two other Stock Exchanges are:

(1) National Stock Exchange (N.S.E) and
(2) Over The Counter Exchange of India (O.T.C.E.I.).

Securities traded in Stock Exchanges:

The following are different types of shares/securities dealt in Stock Exchanges.

1. Shares (both equities and preference shares) of public companies,
2. Debentures,
3. Government securities and
4. Shares/Units issued by Mutual funds.

IV. Operators in Stock Exchanges

Persons involved in Stock Exchanges are classified into the following categories [15], namely:

1. Brokers/Members

Brokers/Members, are the persons who act as intermediaries between the buyers and sellers of securities. They are appointed by the concerned Stock Exchange in consultation with SEBI after taking their integrity, financial strength, infrastructural facilities and so on, into consideration. They charge commission from both buyers as well as sellers of the shares. Some times they also act as underwriters to the public issue of shares of public companies, where underwriters means giving assurance to the portion for which public does not subscribe.
2. Bulls

*Bulls* are, one type of operators in Stock Exchanges. They always expect a rise in price of shares and go on buying the shares. These people are *optimistic in nature* and when their expectation comes true they make profit and vice versa. They are called *Bulls* because a bull throws a person *up into the air*. So they expect a rise in the prices of shares.

3. Bears

*Bears* are another type of operators in Stock Exchanges. They expect a fall in the prices of shares and go on selling the shares. These people are *pessimistic in nature* and when their expectation comes true, they make profit and vice versa. They are called *Bears* because the bear *throws a person down to the ground*. So they expect a depreciation in the prices of shares.

4. Jobbers

*Jobbers* are also one of the types of operators in Stock Exchanges. They buy and sell shares not for and on behalf of the investing public but in their own name and for the purpose of earning a profit for themselves. A *Jobber* is a specialized operator in a few selected and specialized shares. He offers to buy and sell some particular securities. He quotes the price for buying and also for selling for the same shares. The difference between the prices of buying and selling is called *"JOBBERS SPREAD"* and this Jobbers spread is his profit.

5. Lame duck

When a Bear is trapped by the Bulls, the position of the bear is called "*Lame duck*", where Lame duck means unable to find a way.
Generally the **Bulls** and **Bears** operate actively in Stock Exchanges. Based on the available information and situations they buy and sell the shares. Such information or situations if goes wrong they may find it difficult to find the way or trapped. Bulls also may be trapped by bears but always there is a way for them to come out. But it is not in the case of bears.

6. **Stag**

A **Stag** is an operator, in primary market. He is a cautious **operator**. He applies for shares in a public company and sells them for a profit after allotment. He is also an **active** player in ‘**KERB DEALS**’ or unofficial trading. The market for Kerb deals is known as ‘**GREY MARKET**’. Thus the speculator, who applies for shares in public company and sells them, after a profit is known as **Stag**.

Apart from these operators, there are Institutional Investors/Operators who are classified into two categories, namely:

(i). **Indian Institutional Investors**.

For example, Life Insurance Corporation of India (LIC), Unit Trust of India (UTI), General Insurance Corporation (GIC), Indian Mutual Funds, Non-Banking companies and so on.

(ii). **Foreign Institutional Investors**.

Several Foreign institutions operating in their country's stock markets come to India to make investments in Indian securities.

For example, Franklin Templeton of India, DSP Morillinch, and so on.

Companies issue shares to the public at a **Face value** of Rs.10, but this price may go up or come down in Stock Exchanges due to certain factors. These factors may be internal or external to the company. The effect of these factors on the **face value** determines the **Market value** of the share. The following are the general classification of these factors.

A: General Factors and their effects:

These factors are further sub-divided into the following variables, namely;

1. **Financial position of the company**

   A sound financial position of a company result

   (i). Good sales,
   (ii). Good profit,
   (iii). Good dividend,
   (iv). Issue of bonus shares and so on.

2. **Demand and supply position**

   The demand and supply position of the shares of a company may have the effect on market value of a share in the following pattern. More demand/Less supply of shares in the market may increase the market value of a share. Similarly, Less demand/More supply may decrease the market value of a share.

3. **Bank Rate**

   Bank Rate is the most crucial factor in determining the market value of a share. An increase in Bank rate, reduce the share prices and vice versa.
4. Interest of Financial Institutions

Market value of a share also depends on the interest/tendency to buy shares by Financial Institutions, which plays a vital role. If they show interest in purchase of a company share it will increase the market value of a share and vice versa.

5. Change in the Composition of BOD

Board of Directors of a company have major contributing role in determining the market value of a share; because, the fiscal policies of the company are taken by the B.O.D.

6. Government policy

Role of rules/policies made by the Government also has an equally important role in fixing market value of a share. For instance, imposition of fresh taxes on corporate sectors will decrease the market value of a share and vice versa. Similarly presentation of Budget, decisions taken by the Ministry of Finance also influence the market value of a share.

7. Political Developments-National and International

The political scenario (both National and International) has a powerful impact on the market value of a share. Changes in Government, out break of civil war, announcement of General elections, Political parties announcing their policies, Educational background of elected Leaders/Policy makers also have influence on the market value of a share.

8. Industrial Relations

Relation between managements, workers and agents of a company also has an equally effective role in the market value of a share. Peaceful, calm and good relations among these people will increase the market value of a share where as, Lock-outs/Strikes/Irregularities/Mismanagement of raw material or finance will decrease market value of a share.
9. Trade Cycles

Trade cycles/Business cycles also have inherent effect in the determination of market value of a share. For instance, *Boom/Recovery* period will *increase* the market value of a share, whereas, *Depression/Recession* periods will *decrease* the market value of a share.

10. Other Factors

Apart from above factors, climatic conditions, Natural calamities like floods, droughts, earth quakes, epidemics, wars and so on also take an important role in determining market value of a share. Industrial combination, psychology of the public, cash flow in the country play an equally important role in fixing *market value of a share*.

B. Technical Factors

Apart from general factors, explained above, there are some technical factors which are listed below influencing the market value of a share.

- Equity Base
- Dividend pay outs
- Earning Per Share (EPS)
- Profit Earning ratio
- Extent and volume of buying
- Extent of short selling and so on.

1.3. Objectives of the thesis and collection of data

The above listed factors are not completely exhaustive, but there are some other inherently hidden factors influencing the market value of a share. Thus in order to determine the market value of a share, latest and more
appropriate techniques are to be used. This is because of the fact that the market value of a share is so sensitive and is influenced by many variables which are Stochastic in nature. Keeping the past experience in view shares are broadly classified into three categories, namely;

(1). High value shares,

(2). Medium value shares and

(3). Low value shares.

In the light of those, problems arising in market as explained in section (1.2), and the role played by shares in Market Research and market value of these shares explained in section (1.1), the thesis aims at the following objectives:

1. To identify best performing shares in three categories namely,

   (i) High Value Shares,

   (ii) Medium Value Shares and

   (iii) Low Value Shares

   using "Analysis of Means (ANOM)" technique.

2. To propose a Markov Chain model applicable in determining the behaviour of best performing Shares selected, as shown in the above step and to obtain steady-state behaviour of these shares.

3. To apply Simulation models to determine the future behaviour of these shares and compare these Simulated results with the reality.

4. To propose a Moving Average model by viewing the share prices as ‘Time Series’ model and predict the future market value of shares are under consideration.
Keeping the above four objectives in view, data relating to ‘40’ different company shares are collected from 1\textsuperscript{st} March-2004 to 31\textsuperscript{st} March-2006 from \textit{REUTERS INDIA (P) Ltd., Bangalore}. Since the data collected is voluminous, only best selected share values for the above period namely, from 1\textsuperscript{st} March-2004 to 31\textsuperscript{st} March-2006 for

(i) Infosys Tech., in category-I,
(ii) Thomas Cook in category-II
(iii) Biocon in category-II and
(iv) LIC Housing Finance in category-III

is given in Appendix (Table-A3).

1.4. Methodology.

In this thesis, the following Statistical tools and techniques are used and a brief descriptions of these tools and techniques are given in this section.

Tools and Techniques used in this Thesis:

(a) Analysis of Means (ANOM),
(b) Markov Chain Models,
(c) Simulation techniques and
(d) Time Series Models.

1.4a. Analysis of Means (ANOM)

The Analysis of Means (ANOM) is a useful alternative to the Analysis of Variance (ANOVA) as a method of comparing a group of treatments [23,25]. Attractive attributes of ANOM include the inherent ease of interpretation and graphical presentation. An ANOM chart, conceptually
similar to a control chart, portrays decision lines, so that magnitude differences and Statistical significance of the treatments may be assessed simultaneously [24].

The ANOM technique was developed by Prof. Ott (1967) for comparing a group of treatment means to see if any one of them differs significantly from the overall mean [23]. This technique was extended by Schilling (1973) to what he called the analysis of means for treatments effects (ANOME)[30]. It is important to note that the Analysis of Means procedure is appropriate for factors involving fixed effects but is inappropriate for factors involving Random effect. For fixed effects the model assumes factor level means are constant. However, for random effects, the factor level means are random variables and in that case the aim is estimation of variance rather than mean effects [20].

Variables Data

The one-way classification model results when an experimenter obtains 'k' independent random samples of size \( n_i \) \((i = 1, 2, 3, \ldots, k)\) each from a different population. These 'k' populations might, for example, represent 'k' treatments, 'k' methods of production, or 'k' groups. The data consist of a quantitative measurement of some characteristic for each experimental unit sampled from the different populations.

For comparison of the mean responses by the ANOM procedure let us consider the simplest case of 'k' groups of equal size 'n'. The 'k' means \( \bar{X}_i \) are assumed to be from normally distributed populations with common variance \( \sigma^2 \). Let \( \bar{X} \) represent the grand mean and \( s^2 \) the pooled estimate of the common but unknown variance. These quantities are defined mathematically by
\[
\overline{X} = \frac{1}{k} \sum_{i=1}^{k} \overline{X}_i
\]  \hspace{1cm} \text{...(1.4.1)}

\[
s^2 = \frac{1}{k} \sum_{i=1}^{k} s^2_i
\]  \hspace{1cm} \text{...(1.4.2)}

where \( \overline{X}_i = \frac{\sum_{j=1}^{n} X_{ij}}{n} \)  \hspace{1cm} \text{...(1.4.3)}

\[
s^2_i = \frac{\sum_{j=1}^{n} (X_{ij} - \overline{X}_i)^2}{(n-1)}
\]  \hspace{1cm} \text{...(1.4.4)}

\( X_{ij} \) = \( j^{th} \) observation from \( i^{th} \) population.

Other estimates of \( \sigma^2 \) have been considered acceptable. In particular, Ott (1967)[23] and Schilling (1973)[30] use one based on ranges. However, \( s^2 \) shall be preferred in the applications that follow.

The steps to carry out ANOM are:

1. Compute the group means, \( \overline{X}_i, (i = 1, 2, 3, \ldots, k) \).

2. Compute the grand mean, \( \overline{X} \), using equation (1.4.1).

3. Compute \( s^\prime \), an estimate of the standard deviation of an individual observation. This is the square root of \( s^2 \) is computed using equations (1.4.2) and (1.4.4).

4. Obtain the value \( h_\alpha \) from the table in L.S. Nelson (1983a) for Type I risk level \( \alpha \), number of means \( k \), and degrees of freedom \( (n-1)k \) [20] [Vide Tables A1 & A2 of Appendix].

5. Compute the upper and lower decision lines, UDL and LDL, where,
UDL = $\bar{X} + h s \sqrt{(k-1)/kn}$ \hspace{1cm} (1.4.5)

C.L. = $\bar{X}$

LDL = $\bar{X} - h s \sqrt{(k-1)/kn}$ \hspace{1cm} (1.4.6)

6. Plot the means against the decision lines. If any mean falls outside the decision lines conclude there is a statistically significant difference among the means.

**ANOM with Unequal Sample Sizes**

For sets of means each of which is based on the sample size, equations (1.4.5) and (1.4.6) together with the tabled values provided here will give exact results. When the means are based on unequal sample sizes their deviations from the grand mean are no longer equicorrelated and decision limits, the following equation (1.4.7) should be computed using critical values that are upper bounds on the true but not available values of $h$. The decisions limits in this case are

$\bar{X} \pm h^* s \sqrt{(N-n)/N_n}$ \hspace{1cm} (1.4.7)

where

$N = \sum_{i=1}^{k} n_i = \text{total number of observations}$

$n_i = \text{number of observations in the } i^{th} \text{ mean}.$

Following Sidak (1967)[33], the necessary values of $h^*$ can be calculated as the upper $\alpha^* / 2$ percentage points of a ‘t-distribution’, where
\[ \alpha^* = 1 - (1 - \alpha)^{1/k} \] ....(1.4.8)

\( \alpha = \) desired significance level

\( k = \) number of means.

The upper bounds obtained by the use of equation (1.4.8) are slightly less than the factors given by L.S.Nelson (1974)[19]. Remember that in the use of equation (1.4.7) it is necessary to calculate as many pairs of decision lines as there are different sample sizes.

**Advantages of graphical Analysis of Means** [25]:

Analysis of Means provides a **direct study** of possible effects of the factors by dealing with **means** instead of variances. The Analysis of Means thus provides a comparison of the relative importance and magnitude of the factors, as well as their Statistical significance.

It provides a **graphical comparison of effects**. A primary function of industrial experimentation is not only to obtain information, but to present it in a way which will be accepted as a basis for decision and action by appropriate technical administrative personnel. The graphical presentation encourages the translation of conclusions into scientific action; this is a critical advantage.

Analysis of Means provides a **pin-pointing** of source of non-randomness.

A graphical presentation of data is almost a necessity when interpreting the meaning of interactions whose presence have been indicated by an Analysis of variance.
The Analysis of Means frequently provides a bonus by suggesting the unsuspected presence of certain types of non-random variability; these suggestions can then be included in subsequent experiments for study.

The graphical Analysis of Means has frequently indicated errors in calculation in an Analysis of Variance. These errors are often apparent in a graphical presentation even to the untrained [5,23,24,25].

In literature Analysis of Means (ANOM) was applied to the wide variety of data as illustrated below:

ANOM was applied to the data from Walpole and Mayers (1972) [38]. The data resulted from an experiment which studied how mean absorption of moisture in concrete is affected by different aggregates. Five different concrete aggregates were studied by exposing six samples of each to moisture for 48 hours [38].

Schilling (1979) incorporated the ANOM technique in the grand-lot scheme of Simon to verify the homogeneity of a grand lot. This resulting approach simplifies application of the grand-lot scheme can be applied to attributes or variable data, is easy to use, and provides high levels of protection economically and a reduction in sample size of 80% [31].

Enrick (1976) applied ANOM to wire-life data obtained from a three way factorial design [6].

Ott and Snee (1973) applied the ANOM method to identify the differences in a circular multiple-head speed machine, which runs continuously. All the heads are intended to perform the same operation, which may be filling a bottle, can, or box to a desired average or minimum [24].
Tomlison and Lavigna (1983) gave an application of ANOM for percent defective data obtained from *silicon crystal growing*, the first processing step in semiconductor manufacturing [35].

Ullman (1989) has expanded the area of application by providing factors for ANOM on ranges suitable for use in the analysis of *Taguchi signal-to-noise ratio* [36].

Parra and Loaiza (2003) applied the ANOM technique to a case study data from *chemical and pharmaceutical industries* and demonstrated the value of the application of ANOM as a powerful visualization and communication tool, to complement the conventional analysis of nested designs [26].

**General Applications**


**1.4 b. Markov Chain models**

Loosely speaking, the mathematical description of a random phenomenon as it changes in time is known as *Stochastic Process* [1].

**Definition**

A Stochastic process is a family of random variables \( \{ X_t \} \), where 't' takes values in the index set 'T' (some times called a parameter set or time set). The values of \( X_t \) are called the state space and will be denoted by 'S'.
If 'T' is countable then the Stochastic Process is called a **Stochastic sequence** (or discrete parameter stochastic process). If 'S' is countable then the Stochastic Process is called a **discrete state process**.

If 'S' is subset of the real line the stochastic process is called a **real valued process**.

If 'T' takes continuously uncountable number of values like 
\((0,\infty)\) or \((-\infty,\infty)\) the stochastic process is called a **continuous time process** [17].

**Different types of Stochastic Processes**

Following are the most important types of stochastic processes we come across [28]:

1. **Independent stochastic sequence (discrete time process).** Here 
   \(T = \{1, 2, 3, \ldots\}\) and \(\{X_t, t \in T\}\) are independent random variables.

2. **Renewal process (Discrete time process).**

   Here \(T = \{0, 1, 2, 3, \ldots\}\), \(S = [0, \infty]\).

   If \(X_n\) are i.i.d. non-negative random variables and \(S_n = X_1 + X_2 + \ldots + X_n\) then \(\{S_n\}\) forms a discrete time (renewal process).

3. **Independent increment process (Continuous time process)**

   Here \(T = [t_0, \infty]\), where \(t_0\) be any real number (positive or negative).

   For every \(t_0 < t_1 < \ldots < t_n, t_i \in T, i = 1, 2, 3, \ldots, n\) \(\ldots(1.4.9)\)
if \( X_k, X_h - X_t(i, X_{ti} - X_h, \ldots, X_{i} - X_{i-1} \) are independent for all possible choices of (1.4.8), then the stochastic process \( \{X_t, t \in T\} \) is called independent increment stochastic process.

4. Markov process

Definition

A Stochastic system is called a Markov Process if the occurrence of a future state depends on the immediately preceding state and only on it. In other words, future depends only on present and present depends only on the immediate past. This property is popularly known as "Memory Less" property [17].

Here \( S \) = a countable set, \( T = \{0, 1, 2, 3, \ldots\} \).

The Stochastic Process \( \{X_n, n = 0, 1, 2, 3, \ldots\} \) is called a Markov Chain, if, for \( j, k, j_1, \ldots, j_{n-1} \in N \) (or any subset of \( I \)),

\[
P\{X_n = k / X_{n-1} = j, X_{n-2} = j_1, \ldots, X_0 = j_{n-1}\} =
\]

\[
P\{X_n = k / X_{n-1} = j\} = p_{jk} \quad \text{(say)} \quad \ldots(1.4.10)
\]

when ever the first member is defined.

The outcomes are called the states of the Markov Chain; if \( X_n \) has the outcome \( 'j' \) \( (i.e. X_n = j) \), the process is said to be at state \( 'j' \) at \( n^{th} \) trial. To a pair of states \( (j, k) \) at the two successive trials (\( \text{say, } n^{th} \) and \( (n+1)^{th} \) trials) there is an associated conditional probability \( P_{jk} \). It is the probability of transition from the state \( 'j' \) at \( n^{th} \) trial to the state \( 'k' \) \( (n+1)^{th} \) trial. The transition probabilities \( p_{jk} \) are basic to the study of the structure of the Markov Chain.
The transition probability may or may not be independent of 'n'. If the transition probability $P_{jk}$ is independent of 'n', the Markov Chain is said to be homogeneous. If it is dependent on 'n', the chain is said to be non-homogeneous. The transition probability $P_{jk}$ refers to the states $(j,k)$ at two successive trials (say, $n^{th}$ and $(n+1)^{th}$ trial); the transition is one-step and $P_{jk}$ is called one-step transition probability. In the more general case, we are concerned with the pair of states $(j,k)$ at two non-successive trials, say, state 'j' at the $n^{th}$ trial and state 'k' at the $(n+m)^{th}$ trial. The corresponding transition probability is then called m-step transition probability and is denoted by $p^{(m)}_{jk}$, i.e.,

$$ p^{(m)}_{jk} = \Pr\{X_{n+m} = k \mid X_n = j\}. \quad (1.4.11) $$

5. Martingale or fair game process [1]

If $E[X_{t_{m+1}} / X_{t_0}, X_{t_1}, \ldots, X_{t_m}] = a_n$

i.e. $E[X_{t_{m+1}} / X_{t_0}, X_{t_1}, \ldots, X_{t_m}] = X_{t_m} \text{ a.s.}$ for all choices of the partition (1.4.9), then $\{X_t, t \in T\}$ is called a Martingale process.

6. Stationary process [17].

If the joint distribution of $(X_{t_1}, \ldots, X_{t+h})$ are the same for all $h > 0$ and $t_1 < t_2 < \ldots < t_n, t_i \in T, t_i + h \in T$ then $\{X_t, t \in T\}$ is called a stationary process (strictly stationary process).
7. Point process [1].

When a countable set or sets of points randomly distributed on the real line or any arbitrary sets we call the family of random variables governed by the distribution of those random points as **point processes**.

**Transition Matrix**

The transition probabilities $p_{jk}$ satisfy $p_{jk} \geq 0$, $\sum_k p_{jk} = 1 \forall j$.

\[
P = \begin{pmatrix}
p_{11} & p_{12} & \cdots \\
p_{21} & p_{22} & \cdots \\
\vdots & \vdots & \ddots
\end{pmatrix}
\]  

...(1.4.12)

This is called the transition probability matrix (t.p.m.) of the Markov Chain. $P$ is a **Stochastic matrix** [28] i.e., a square matrix with non-negative elements and unit row sums. Apart from unit rows sums if it has unit column sums also then ‘$P$’ is called **Doubly Stochastic Matrix** [17].

**Order of a Markov Chain**

**Definition**

A Markov Chain $\{X_n\}$ is said to be of order ‘$s$’ ($s = 1, 2, 3, \ldots$), if, for all

\[
\Pr\{X_n = k \mid X_{n-1} = j, X_{n-2} = j_1, \ldots, X_{n-s} = j_{s-1}, \ldots\} = \\
\Pr\{X_n = k \mid X_{n-1} = j, X_{n-2} = j_1, \ldots, X_{n-s} = j_{s-1}\}
\]

...(1.4.13)

whenever the l.h.s. is defined [17].
Higher Transition Probabilities

Chapman-Kolmogorov equations [2]

\( p_{jk} \) gives the probability of unit step transition from the state ‘j’ at a trial to the state ‘k’ at the next following trial. The m-step transition probability is denoted by

\[
p_{jk}^{(m)} = \Pr \{ X_{n+m} = k \mid X_n = j \}
\]
gives the probability that from the state ‘j’ at \( n^{th} \) trial, the state ‘k’ is reached at \((m+n)^{th}\) trial in m-steps, i.e. the probability of transition from state ‘j’ to the state ‘k’ in exactly ‘m’ steps.

Let \( P = (p_{jk}) \) denote the transition matrix of the unit-step transitions and \( P^{(m)} = (p_{jk}^{(m)}) \) denote the transition matrix of the m-step transitions. For \( m = 2 \), we have the matrix \( P^{(2)} = P \cdot P = P^2 \).

Similarly, \( P^{(m+1)} = P^{(m)} \cdot P = P^{(m+1)} \) and

\[
p^{(m+n)} = P^m \cdot P^n = P^m \cdot P^n .
\]...

(1.4.14)

Classification of States and Chains [2]

The states ‘j’, \( j = 0,1,2,3,\ldots \) of a Markov Chain \( \{X_n, n \geq 0\} \) can often be classified in a distinctive manner according to some fundamental properties of the system. By means of such classification it is possible to identify certain types of chains.

Communication Relations

If \( p_{ij}^{(n)} > 0 \) for some \( n \geq 1 \), then we say that state ‘j’ can be reached or state ‘j’ is accessible from state ‘i’; the relation is denoted by \( i \to j \).

Conversely, if for all \( n \), \( p_{ij}^{(n)} = 0 \), then \( j \) is not accessible from ‘i’; in notation \( i \not\to j \).
If two states 'i' and 'j' are such that each is accessible from the other then we say that the two states communicate; it is denoted by \( i \leftrightarrow j \); then there exist integer 'm' and 'n' such that

\[
p_{ij}^{(n)} > 0 \text{ and } p_{ij}^{(m)} > 0.
\]

The relation \( \rightarrow \) is transitive, i.e. if \( i \rightarrow j \) and \( j \rightarrow k \) then \( i \leftrightarrow k \).

From Chapman-Kolmogorov equation [17]

\[
p_{ik}^{(m+n)} = \sum_{r} p_{ir}^{(m)} p_{rk}^{(n)}
\]

we get

\[
p_{ik}^{(m+n)} \geq p_{ij}^{(m)} p_{jk}^{(n)}
\]

**Class Property**

A class of states is a subset of the state space such that every state of the class communicates with every other and there is no other state outside the class which communicates with all other states in the class. A property defined for all states of a chain is a class property if its possession by one state in a class implies its possession by all states of the same class. One such property is the periodicity of a state [17].

**Periodicity**

State 'i' is return state if \( p_{ii}^{(n)} > 0 \) for some \( n \geq 1 \). The period \( d_i \) of a return to the state 'i' is defined as

\[
d_i = \text{G.C.D.} \{ m : p_{ii}^{(m)} > 0 \};
\]

state 'i' is said to be aperiodic if \( d_i = 1 \) and periodic if \( d_i > 1 \). Clearly state 'i' is aperiodic if \( p_{ii} \neq 0 \).
Classification of Chains

If \( C \) is a set of states such that no state outside \( C \) can be reached from any state in \( C \), then \( C \) is said to be closed [13]. If \( C \) is closed and \( j \in C \) while \( k \notin C \), then \( p_{jk}^{(n)} = 0 \) for all, i.e. \( C \) is closed iff \( \sum_{j \in C} p_{ij} = 1 \) for every \( i \in C \).

A closed set may contain one or more states. If a closed set contains only one state \( 'j' \) then the state \( j \) is said to be absorbing; \( j \) is absorbing iff \( p_{jj} = 1, p_{jk} = 0, k \neq j \).

A Markov chain is irreducible if there is only one communicating class, i.e. if all states communicate with each other or every state can be reached from other state [17].

Classification of states: Transient and Persistent (Recurrent) States

Suppose that a system starts with the state \( j \). Let \( f_{jk}^{(n)} \) be the probability that it reaches the state \( k \) for the first time at the \( n^{th} \) step and let \( p_{jk}^{(n)} \) be the probability that it reaches state \( k \) (not necessarily for the first time) after \( n \) transitions. Let \( \tau_k \) be the first passage time to state \( k \), i.e. \( \tau_k = \min\{n \geq 1, X_n = k\} \) and \( \{f_{jk}^{(n)}\} \) be the distribution of \( \tau_k \) given that the chain starts at state \( j \) [17].

First Entrance Theorem

Whatever be the states \( j \) and \( k \)

\[
p_{jk}^{(n)} = \sum_{r=0}^{n} f_{jk}^{(r)} p_{ik}^{(n-r)}, \quad n \geq 1
\]

with \( p_{kk}^{(0)} = 1, f_{jk}^{(0)} = 0, f_{jk}^{(1)} = p_{jk} \)
First Passage Time Distribution

Let $F_{jk}$ denote the probability that starting with state $j$ the system will ever reach state $k$. Clearly

$$F_{jk} = \sum_{n=1}^{\infty} f^{(n)}_{jk}$$  

\[ (1.4.15) \]

we have $\sup_{n\geq 1} p^{(n)}_{jk} \leq F_{jk} \leq \sum_{n=1}^{\infty} p^{(n)}_{jk} \forall n \geq 1$.

The mean (first passage) time from state $k$ is given by

$$\mu_{jk} = \sum_{n=1}^{\infty} nf^{(n)}_{jk}$$  

\[ (1.4.16) \]

In particular, when $k = j$, $\{f^{(n)}_{jj}, n = 1, 2, \ldots\}$ will represent the distribution of the recurrence times of $j$; and $F_{jj} = 1$ will imply that the return to the state $j$ is certain. In this case

$$\mu_{jj} = \sum_{n=1}^{\infty} nf^{(n)}_{jj}$$  

\[ (1.4.17) \]

is known as the mean recurrence time for the state $j$.

**Definitions**

A state $j$ is said to be **persistent (or recurrent)** if $F_{jj} = 1$ (i.e. return to state $j$ is certain) and **transient** if $F_{jj} < 1$ (i.e. return to state $j$ is uncertain).

A persistent state $j$ is said to be **null persistent** if $\mu_{jj} = \infty$, i.e. if the mean recurrence time is infinite, and is said to be **non-null (or positive) persistent** if $\mu_{jj} < \infty$.

A persistent non-null and aperiodic state of a Markov chain is said to be **ergodic** [17].
Ergodicity

The behaviour in which sample averages formed from a process converge to some underlying parameter of the parameter of the process is termed ergodic. To make inference about the underlying laws governing an ergodic process, one need not observe separate independent replications of entire processes or sample paths. Instead, one need only observe a single realization of the process, but over a sufficiently long span of time. Thus, it is an important practical problem to determine conditions that leads to a stationary process being ergodic [17]. The theory of stationary processes has a primal goal the classification of ergodic behaviour and the prediction problem for process falling in the wide range of extremities.

Ergodic Theorem

For a finite irreducible, aperiodic chain with t.p.m. \( P = (p_{jk}) \), the limits

\[
\nu_k = \lim_{n \to \infty} p_{jk}^{(n)}
\]

(1.4.18)

exists and are independent of the initial state \( j \). The limits \( \nu_k \) are such that

\[ v_k \geq 0, \sum v_k = 1, \text{ i.e. the limits } \nu_k \text{ define a probability distribution [17].} \]

1.4 c. Simulation Techniques

A Simulation is the imitation of the operation of a real-world process or system over time. It is a numerical technique for conducting experiments that involve certain types of Mathematical and logical relationships necessary to describe the behaviour and structure of a complex real world system over an extended period of time [32].
Simulation is the process of generating values using random numbers without really conducting experiments [11]. Experiments may be undertaken before the real system is operational so as to aid in its design, or to see how the system might react to change in its operating or to evaluate the system, response to change in its structure.

When To Use Simulation

The reasons for selecting simulation technique other than the known mathematical techniques are

(i). It is impossible to develop a mathematical solution.
(ii). Actual observation of a system may be too expensive.
(iii). Simulation may be the only method available because it is difficult to observe the actual environment.
(iv). There may not be able to wait for a long period to study the system extensively.
(v). Actual operation and observation of a real system may be too disruptive.

Methodology of Simulation

The methodology developed for simulation process consists of the following steps [12]:

(i) Identify and clearly define the problem.
(ii) List the statement of objectives of the problem.
(iii) Construct an appropriate mathematical model of the given problem.
(iv) Ensure that the model represents the real situation.
(v) Make experiment with the model constructed, i.e., obtain a consistent set of values (or states) for the variables—a sample of what could happen in reality.
(vi) Analyse the results of simulation activity, i.e., use the sample obtained in (v) above to calculate the value of the decision criterion, by actually following the relationships among the variables for each of the alternative decisions.

(vii) Make changes in the model, or the sample, and repeat the process until a sufficient number of samples are available.

(viii) Tabulate the various values of the decision criterion and choose the best policy.

**Simulation Models**

A simulation model may be a physical or mathematical model, conceptual or a combination of them. Since physical models are relatively expensive to build, mathematical models often preferred [7].

Broadly, the simulation models can be classified into the following four categories [14]:

(i) **Simulation of Deterministic Models.** In the case of these models, the input and output variables are not permitted to be random variables and models are described by exact functional relationships.

(ii) **Simulation of Probabilistic Models.** In such cases, method of random sampling is used. The technique used for solving these models is termed as “Monte-Carlo Technique”.

(iii) **Simulation of Static Models.** These models do not take variable time into consideration.

(iv) **Simulation of Dynamic Models.** These models deal with time varying interaction.

**Monte-Carlo Simulation [14]**

Mante-Carlo method of simulation was developed by the two mathematicians John Von Neumann and S.M.Ulam. This method is generally used to solve problems which cannot be adequately represented by the
mathematical models, or where the problems are too expensive for experimental solutions. This model involves random sampling from a known probability distribution and yields a solution which will be very close to the number of simulated trials should be infinity. Monte-Carlo simulation is generally computer oriented and performed as per steps given below.

Step-1: Identify the objectives of the problem and clearly define the problem.

Step-2: Construct an appropriate model and clearly specify the variables, parameters and the manner in which the time will change etc.
Define the relationship between the variables and parameters.

Step-3: Prepare the model for experimentation by specifying the number of runs of simulation to be made.

Step-4: Determine the probability distribution for each variable in step 2 and establish the cumulative distribution function.

Step-5: Set up the table and assign tag numbers, with the help of cumulative distribution function.

Step-6: Generate random numbers and choose the corresponding tag number. Then select the variable value corresponding to the tag number.

Step-7: Generate random numbers at many number of trials (given in the problem) and compute the values for different trials. The optimal solution will be the average values of different trials.

Note: When the number of simulated runs are more, the obtained solution from this will be closer to the optimal.
Generation of Random Numbers [7]

For solving all discrete problems, it is essential to generate random numbers. Most computer languages have a subroutine or function that will generate random numbers [32]. In carrying out Monte-Carlo simulation, one needs to generate random numbers to obtain random observations from a probability distribution. Random number is a sequence of numbers whose probability of occurrence is the same as that of any other number in the sequence. The sequence of numbers must have two important Statistical properties, uniformity and independence. Each random number is an independent sample drawn from a continuous uniform distribution between 0 and 1. The probability density function (p.d.f.) is given by

\[ f(x) = \begin{cases} 1, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases} \]

Random numbers can be generated by any of the three methods.

(i) Manually generated random numbers

(ii) Random numbers are selected from the Random number table.

(iii) Computer-recursive algorithm.

Properties of Random numbers

1. Random numbers should be uniformly distributed.
2. It must be Statistically independent.
3. They must have sufficiently long period.
4. The numbers must be generated at high speed.
5. They must occupy less memory.
6. There should not be a cyclic.
Tests for Random Numbers [12]

In literature there are many tests available to test the desirable properties of Random Numbers—uniformity and independence. Some of the important tests are given below. They are

1. **Frequency test:** Uses the Kolmogorov-Smirnov or the chi-square test to compare the distribution of the set of numbers generated to a uniform distribution.

2. **Run test [10]:** Tests the runs above or below the median by comparing the actual values to expected values.

   Let $x_1, x_2, \ldots, x_n$ be the set of observations arranged in the order in which they occur. Then, for each of the observations, we see if it is above or below the value of the median of the observations and write 'A' if the observation is above and 'B' if it is below the median value. Thus we get a sequence of A's and B's of the type, (say),

   ABBAAABABBAAABAAAA 

   **Definition of Run**

   A run is defined as a sequence of letters of one kind surrounded by a sequence of letters of the other kind.

   **Null hypothesis**

   $(H_0)$ that the set of numbers is random, the number of runs 'U' is a random variable with
\[ E(U) = \frac{n + 2}{2} \]  
\[ \text{and } \text{Var}(U) = \frac{n(n - 2)}{2(n - 1)} \]

For large \( n \) (say, \( > 25 \)), \( U \) may be regarded as asymptotically normal.

Under \( (H_0) \) the test statistic is given by,

\[ Z = \frac{U - E(U)}{\sqrt{\text{Var}(U)}} \sim N(0,1) \]

If calculated \( |Z| \leq Z_\alpha \), we accept \( H_0 \) at \( \alpha\% \) level of significance otherwise we reject \( H_0 \) and conclusion can be drawn accordingly.

3. **Autocorrelation test**: Tests the correlation between numbers and compares the sample correlation to the expected correlation of zero.

4. **Gap test**: Counts the number of digits that appear between repetitions of a particular digit and then uses the Kolmogorov-Smirnov test to compare with the expected size of gaps.

5. **Poker test**: Treat numbers grouped together as a poker hand. Then the hands obtained are compared to what is expected using the chi-square test.

In testing for uniformity, the hypotheses are as follows:

\[ H_0 : R_i \sim U[0,1] \]

\[ H_1 : R_i \not\sim U[0,1] \]
The null hypothesis $(H_0)$, reads that the numbers are distributed uniformly on the interval $[0,1]$. Failure to reject the null hypothesis means that no evidence of non-uniformity has been detected on the basis of this test. This does not imply that further testing of generator for uniformity is unnecessary.

In testing for independence, the hypotheses are as follows:

$$H_0 : R_i \sim \text{independently}$$

$$H_1 : R_i \not\sim \text{independently}$$

This null hypothesis $(H_0)$, reads that the numbers are independent. Failure to reject the null hypothesis means no evidence of dependence has been detected on the basis of this test. This does not imply that further testing of the generator for independence is unnecessary.

Advantages and Limitations of Simulation [7,12]

Advantages

Simulation is sometimes described as indirect experimentation. In Operations Research we represent the system under study constructing a model. Simulation is the process of experimenting on the model rather than on the operation which the model represents. Following are some of the advantages of Simulation:

1. The study of very complicated system or sub-system can be done with the help of simulation. Simulation has been described as ‘what to do when all else fail’.
2. By using simulation, we can investigate the consequences for a system of possible changes in parameters in terms of the model.

3. The knowledge of a system obtained in designing and conducting the simulation is very valuable.

4. It is a teaching aid, e.g., in business games, case studies, etc.

5. It enables us to assess the possible risks involved in a new policy before actually implementing it.

6. The simulation of complicated systems helps us to locate which variables have the important influences on system performance.

7. This can be used to experiment unfamiliar systems to prepare routine and extreme eventualities.

8. Simulation methods are easier to apply than pure analytical methods.

Limitations of Simulation

Use of simulation in place of other techniques, like everything else, involves a trade-off, and we should be aware of the disadvantages involved in the simulation approach. These include:

1. Simulation is not precise. It is not an optimization process and does not yield an answer but merely provides a set of the system's responses to different operating conditions. In many cases, this lack of precision is difficult to measure.

2. A good simulation model may be very expensive. Often it takes years to develop a usable corporate planning model.

3. Not all situations can be evaluated using simulation. Only situations involving uncertainty are considered, because without a random component, all simulated experiments would produce the same answer.
4. Simulation generates a way of evaluating solutions but it does not generate the solution techniques. Managers must still generate all of the solution approaches they want to test.

5. Even when you spend the resources to build a simulation model that makes sense in some real-world context, it is often difficult for the people who built it to understand that they are still not looking at reality, but at best, an abstraction of the real world.

6. Simulation is a **time-consuming exercise**.

Simulation is indeed a versatile tool. It provides only Statistical estimates rather than exact results and it only compares the alternatives rather than generating an optimal one. It is a slow and costly way to study a problem. Despite limitations it is an invaluable tool.

"*When all else fails.....Simulation*".

1.4 d. **Time Series models** [17]:

A typical Time series consists of the following four components.

1. **Trend**

   A smooth long-term movement covering a number of years reflecting the general tendency of the series. Growth curves are good examples of time series exhibiting trend. Naturally, characteristics of the process depend very much on the time of observation. One method of eliminating the trend factor from a time series is the method of **Moving Averages** [9].

II. **Cyclic Component (Seasonal Component)**

   The result of periodic movement in definite time periods and can be represented by a strictly periodic function of time. Fourier analysis techniques
may be employed in the investigation of known periodic movement like a seasonal variation. Periodogram analysis is a method based on the harmonic analysis of the Fourier representation [9].

III. Oscillatory Component

It is the Irregular periodic movement of a time series.

IV. Random Component

Random disturbance term is the for the Stochastic process representation of the time series.

Difference between (2) and (3)

In cyclic series, apart from the disturbances due to superimposition of the random element, the maximum and minimum occur at equal intervals. Moreover, the amplitude also remains the same.

Moving Average (MA) Process [17,29]

A time series \( \{X_t, t \in T\} \) is called a moving average process of order 'p' if it can be expressed in the form

\[
X_t = a_0 e_{t} + a_1 e_{t-1} + \ldots + a_p e_{t-p},
\]  

(1.4.22)

\( a_i \)'s are constants, and where \( \{ e_t \} \) is a white noise with \( E(e_t) = 0 \) and \( V(e_t) = \sigma^2 \).

This is simply a moving average of a random series generating a stationary process with \( E(X_t) = 0 \) for all \( t \) and \( V(X_t) = \sigma^2 \sum_{i=0}^{p} e_i^2 \) for all \( t \).
\[\gamma(k) = \text{Cov}(X_t, X_{t+k})\]

\[= \text{Cov}(a_0e_t + \ldots + a_pe_{t-p}, a_0e_{t+k} + \ldots + a_pe_{t+k-p})\]

\[
\left\{ \begin{array}{ll}
0, & \text{if } k > p \\
\sigma^2 \sum_{i=0}^{p-k} a_i a_{i+1}, & \text{if } k \leq p \text{ for all } t \\
\gamma(-k), & \text{if } k < 0
\end{array} \right.
\]

Hence this process \(\{X_t\}\) is covariance stationary.

Also

\[
p(k) = \left\{ \begin{array}{ll}
0, & \text{if } k > p \\
1, & \text{if } k = 0 \\
\sum_{i=0}^{p-k} a_i a_{i+1} \sum_{i=0}^{p} a_i^2, & \text{if } k = 1, 2, 3, \ldots, q
\end{array} \right.
\]

and \(p(k) = p(-k)\) if \(k < 0\).

In particular, a linear form of the 3-day Moving Averages model is given as follows:

\[X_t = a_1X_{t-1} + a_2X_{t-2} + a_3X_{t-3}\] \hspace{1cm} \ldots (1.4.23)

where, \(a_1, a_2, a_3\) are unknown constants.

According to principle of 'Least Squares'[10] to determine these unknown constants, we have three normal equations. They are,
Solving these normal equations we will get estimates unknown constants.

Therefore, the fitted 3-day moving average model to the data is

\[ \hat{X}_t = \hat{a}_1 X_{t-1} + \hat{a}_2 X_{t-2} + \hat{a}_3 X_{t-3} \]  \hspace{1cm} (1.4.27)

**Linear form of the 5-day Moving Averages model is**

\[ X_t = a_1 X_{t-1} + a_2 X_{t-2} + a_3 X_{t-3} + a_4 X_{t-4} + a_5 X_{t-5} \]  \hspace{1cm} (1.4.28)

where \( a_1, a_2, a_3, a_4, a_5 \) are unknown constants.

According to principle of ‘Least Squares' to determine these unknown constants, we have five normal equations. They are,

\[ a_1 \sum X_{t-1}^2 + a_2 \sum X_{t-1} X_{t-2} + a_3 \sum X_{t-1} X_{t-3} + a_4 \sum X_{t-1} X_{t-4} + a_5 \sum X_{t-1} X_{t-5} = \sum X_t X_{t-1} \]  \hspace{1cm} (1.4.29)

\[ a_1 \sum X_{t-1} X_{t-2} + a_2 \sum X_{t-2}^2 + a_3 \sum X_{t-2} X_{t-3} + a_4 \sum X_{t-2} X_{t-4} + a_5 \sum X_{t-2} X_{t-5} = \sum X_t X_{t-2} \]  \hspace{1cm} (1.4.30)

\[ a_1 \sum X_{t-1} X_{t-3} + a_2 \sum X_{t-2} X_{t-3} + a_3 \sum X_{t-3}^2 + a_4 \sum X_{t-3} X_{t-4} + a_5 \sum X_{t-3} X_{t-5} = \sum X_t X_{t-3} \]  \hspace{1cm} (1.4.31)

\[ a_1 \sum X_{t-1} X_{t-4} + a_2 \sum X_{t-2} X_{t-4} + a_3 \sum X_{t-3} X_{t-4} + a_4 \sum X_{t-4}^2 + a_5 \sum X_{t-4} X_{t-5} = \sum X_t X_{t-4} \]  \hspace{1cm} (1.4.32)
Solving these normal equations we will get estimates of unknown constants. Therefore, the fitted 5-day moving average model to the data is

\[ \hat{X}_t = a_1 X_{t-1} + a_2 X_{t-2} + a_3 X_{t-3} + a_4 X_{t-4} + a_5 X_{t-5} \]  

... (1.4.34)

Control Limits For Moving Averages Chart [5]

Let \( \overline{X}_i \) = Moving Average of \( i^{th} \) day and \( R_i \) = Moving Range of \( i^{th} \) day then the control limits for moving average charts are given by

Upper Control Limit (U.C.L.) = \( \overline{X} + A_2 \overline{R} \)  

... (1.4.35)

Central Line (C.L.) = \( \overline{X} \)  

... (1.4.36)

Lower Control Limit (L.C.L.) = \( \overline{X} - A_2 \overline{R} \)  

... (1.4.37)

Where \( \overline{X} = \frac{\sum_{i=1}^{k} X_i}{k} \),

\[ \overline{R} = \frac{\sum_{i=1}^{k} R_i}{k}, \quad k = \text{number of groups}. \]

The Mean Square Error [4,18]

In order to compare a biased estimator with an unbiased estimator, or two estimators with different amounts of bias, a useful criterion is the Mean Square Error (M.S.E.) of the estimate, measured from the population value that is being estimated. Formally,
\[
MSE\left( \hat{X}_i \right) = E\left( \hat{X}_i - X_i \right)^2 = \frac{1}{n} \sum_{i=1}^{n} (\hat{X}_i - X_i)^2 \quad \ldots (1.4.38)
\]

Use of the MSE as a criterion of accuracy of an estimator amounts to regarding two estimates that have the same MSE as equivalent. This is not strictly correct because the frequency distributions of errors \( \left( \hat{X}_i - X_i \right) \) of different sizes will not be the same for the two estimates if they have different amounts of bias [18].

1.5 Chapter summaries

Chapter – 1

This chapter is an introductory in nature where, a brief introduction is given to Market Research, role played by shares in market, market value of a share are introduced. After giving brief introduction to the above points, objectives of the thesis and source of data for the analysis is explained. Further a brief account of tools and techniques used in this thesis are introduced and finally discussed with chapter summaries.

Chapter – 2

In this chapter, using the data collected on '40' shares, four top performing companies based in their performances are selected from each category using ANOM technique. This technique is applied for daily price values as well as monthly average price values and ANOM charts are drawn for each category of shares and conclusions are drawn based on these charts.
Chapter – 3

In this chapter, a \textit{Markov chain model} is proposed for those company shares selected in Chapter-2 and \textit{transition probability matrices} are formed for these company shares, \textit{Steady-state solutions} are obtained using \textit{Chapman-Kolmogorov equations}. \textit{Further, independence, stationarity} (\textit{Time homogeneity}) and \textit{ergodicity properties} are discussed. Conclusions are drawn based on the results obtained.

Chapter – 4

In this chapter, a \textit{Simulation Model} is proposed to study the future behaviour of these selected company shares. For the purpose of study five sets of \textit{Random Numbers} are generated and using steady-state probabilities. These five sets of Random Numbers are tested for "Randomness" using "Run Test". Simulation results are obtained using these five sets of Random Numbers representing ‘103’ days market values. The \textit{Simulation results} obtained by us are compared with the \textit{actual results} for the months April, May, June and July 2006. Conclusions are drawn based on the results obtained.

Chapter – 5

In this chapter, a \textit{Moving Average Model} is proposed to predict the market value of the share by considering the face values of a share as \textit{‘Time Series’}. A ‘3’ day and a ‘5’ day Moving Average models are fitted using \textit{Method of Least Squares} and predictions are made using these fitted models. Closeness between the actual values and predicted values are
calculated by using *Mean Square Error (M.S.E.)* concept and conclusions are drawn about the closeness of the model *proposed* and the *reality*. Finally the chapter is concluded with *further scope* of the work.

Finally, the thesis is Appended with the list of *References* and *Appendix* containing six tables explained as follows:

Table A1. *Exact Critical Values* $k_{0.05}$ *for the Analysis of Means*

Table A2. *Exact Critical Values* $k_{0.01}$ *for the Analysis of Means*

Table A3. *Market Values of Best Performing Company Shares From Different Categories*

[The data representing market value of best performing four Company shares selected. For brevity's sake, other '36' Company market values are not appended].

Table A4: *Monthly Averages of Company Shares of Category-I*

Table A5: *Monthly Averages of Company Shares of Category-II*

Table A6: *Monthly Averages of Company Shares of Category-III*