Chapter 7

Half car models with passengers seat suspensions
Model-1: Vibration optimization of a passive suspension system via Evolutionary Algorithm

7.1.1 Introduction

To provide maximum comfort to the passengers during ride is the utmost goal of the vehicle designers. Although ride quality depends upon environmental factors like, air resistance, noise, vibration, temperature, humidity and road surface etc., the perception of comfort is primarily related to the seat design and the suspension design parameters. Recently, attention has increasingly focused on finding ways to enhance the suspension components in order to provide the greatest possible comfort and road holding ability (Nagai, 1993; Hrovat, 1997; Sun, 2002). A broad variety of vehicle mathematical models ranging from quarter car (Foag et al., 1987; Gobbi et al., 2001), half car (Tamboli et al., 1999; Feng et al., 2003; Marzbanrad et al., 2004) to full car (Güçlü, 2003) have been developed to provide reliable models for computer aided automotive design and to test vehicle performance. However, such models did not permit to evaluate the passenger’s performance because passenger’s seats were not taken into account. To measure more straightforwardly the response of the passenger’s during vehicle motion some researchers (Stein et al., 1991; Rakheja et al., 1994; Güçlü, 2003) incorporated driver’s seat suspension into the vehicle model. These models are found to helpful in predicting the response of the front occupant/ passenger but do not permit to evaluate the rear passenger’s response. However, studying the response of both the passengers during vehicle motion is an important factor in the design of the suspension system. It helps in accessing the merits of suspension system.

So far most of the studies dealing with the design and analysis of suspension systems adopt a linear suspension model. In order to improve the suspension performances and balance
between ride comfort, road holding ability and constraints of suspension travel it is necessary to consider non-linearity of the suspension system. Taking care of the above conditions in this chapter, a half car model described under model-I of chapter 5 has been modified by introducing two passengers' seat suspensions, one at the front and the other at the rear position of the vehicle. On each seat one passenger is seated. The non-linearity of suspension spring and damper, which are the most important characteristics of the suspension, has been taken into account in order to validate the model to real applications. The non-linear cubic polynomial has been used to describe the spring characteristic whereas a quadratic polynomial has been used to describe the damper characteristic. The coefficients of each polynomial represent the design parameters of the suspension system and are to be determined. To find these parameters we have formulated a non-linear constrained optimization problem in which the bouncing transmissibility of the sprung mass at the center of mass has been minimized with respect to technological constraints and the constraints which satisfy the performance as per ISO 2631 standards. The GA has been used to solve this problem in time domain and the results obtained have been compared to those obtained using the existing design (ED) parameters. The objective function and the constraints have been evaluated by simulating the vehicle model over two roads with multiple bumps at uniform velocity.

7.1.2 Dynamical model of a half car with passengers seat and non-linear suspensions

The following is a description of the model which will be used in the simulation analysis to design the parameters of its suspension system. This model is the modified form of the well known half car model with passive suspension system. In this model two passenger’s seats (see Figure 7.1); one at the front and the other at the rear position of the vehicle has been included. The model is therefore composed of passenger’s seats, sprung mass (body of the car), unsprung masses (tire with its wheels) and vehicle suspensions (springs and dampers) located at the front and rear between the sprung and unsprung masses. The model has six degrees of freedom represented by the independent generalized coordinates

\[ q = \{q_1(t), q_2(t), q_3(t), q_4(t), q_5(t), q_6(t)\}^T \]

which have been measured from the static equilibrium position.

At any time \( t \), the generalized coordinates \( q_1(t) \) and \( q_3(t) \) represents the vertical and angular motion of the sprung mass at the centre of mass, the coordinates \( q_4(t) \) and \( q_6(t) \) represents the
vertical displacements of the front and rear unsprung masses respectively, the coordinates $q_4(t)$ and $q_6(t)$ represents the vertical motion of the front and the rear passengers.

![Diagram of Half Car Model](image)

**Figure 7.1:** Six degrees of freedom half car model with non-linear springs and non-linear dampers.

The sprung mass have two degrees of freedom i.e., bounce and pitch while the two passenger’s seats and two unsprung masses have one degree of freedom each i.e., vertical oscillations. It has been assumed that the unsprung mass have only the spring features and is in contact with the road terrain. The road terrain serves as an external disturbance input to the system. The road input has been induced into the model through the wheels by the functions $f_1(t)$ and $f_2(t)$. The suspension of the vehicle has been simplified to non-linear spring and non-linear damper. The suspension spring has been assumed to have the following characteristics:

$$f_s = k_1 \Delta x + k_2 \Delta x^2 + k_3 \Delta x^3$$

The above expression is a cubic polynomial representing non-linearity of the springs. Here $f_s$ is the spring force, the constants $k_1$, $k_2$ and $k_3$ are the stiffness coefficients associated with the linear, quadratic and cubic portions of the spring force. The term $\Delta x$ is the deformation of the spring that can be calculated by the displacement of both extremes of the spring. The unit of $\Delta x$ is meter (m) and of $k_1$, $k_2$ and $k_3$ is Newton/ meter (N/m).

The non-linear damping force of the suspension has been assumed to have the form:

$$f_d = \alpha_1 \Delta \dot{x} + \alpha_2 \Delta \ddot{x}^2$$
where \( f_d \) is the damping force, the constants \( \alpha_1 \) and \( \alpha_2 \) are the damping coefficients associated with the linear and quadratic portions of the damping force and \( \Delta \dot{x} \) is the relative velocity of the extremes of the damper.

It has been further assumed that the tire stiffness is linear in their operation ranges and the tire does not leave the ground. The periodic function has been used to describe excitation caused by road surface which we shall discuss in the coming section. The model parameters values and their respective units have been shown in Table 7.1. The parametric values remain the same as in reference Tashirad et al. (1998).

<table>
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<th>Notation</th>
<th>Description</th>
<th>Units</th>
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<td>( k_2 )</td>
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<td>( k_{p2} )</td>
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7.1.2.1 Equations of motion

Assuming that the effect of angular motion \( q_3(t) \), is very small, the equations of motion of the dynamical system i.e., the motion of the unsprung masses, the sprung mass, the angular motion of the sprung mass and also the motion of the passengers have been computed by applying Newton's second law of motion from the free body diagram as follows:

The equation for the vertical motion of the sprung mass has been expressed as:
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\[ m_\phi \ddot{q}_\phi(t) = -F_i + F_2 + F_3 + F_4 - f_\phi^{\text{front}} - f_\phi^{\text{rear}} - f_\phi^{\text{rear}} \]  \hspace{1cm} (7.1a)

and the angular motion of the sprung mass has been expressed as:

\[ J\ddot{q}_s(t) = \{F_1 + F_2\} d_1 - \{F_3 + F_4\} d_2 - \{f_\phi^{\text{front}} + f_\phi^{\text{rear}}\} l_1 + \{f_\phi^{\text{rear}} - f_\phi^{\text{rear}}\} l_2 \]  \hspace{1cm} (7.1b)

where,

\[ F_1 = c_\phi \{\dot{q}_1(t) - \dot{q}_2(t) - d_1 \dot{q}_1(t)\}, \quad F_2 = k_\phi \{q_2(t) - q_4(t) - d_2 \dot{q}_2(t)\} \]
\[ F_3 = c_\phi \{\dot{q}_6(t) - \dot{q}_2(t) + d_2 \dot{q}_2(t)\}, \quad F_4 = k_\phi \{q_8(t) - q_2(t) + d_2 q_3(t)\} \]

\[ f_\phi^{\text{front}} = k_f \Delta x_f + k_f^\prime \Delta x_f^\prime + k_f \Delta x_f^\prime \]
\[ f_\phi^{\text{rear}} = k_r \Delta x_r + k_r^\prime \Delta x_r^\prime + k_r \Delta x_r^\prime \]

\[ f_d^{\text{front}} = \alpha_f^\prime \Delta \dot{x}_f + \alpha_f^\prime \Delta \dot{x}_f^\prime, \quad f_d^{\text{rear}} = \alpha_r^\prime \Delta \dot{x}_r + \alpha_r^\prime \Delta \dot{x}_r^\prime \]

\[ \Delta x_f = \{q_2(t) - q_1(t) + l_1 q_3(t)\}, \quad \Delta x_r = k_n \{q_8(t) - q_4(t) - l_2 q_4(t)\} \]

\[ \Delta \dot{x}_f = \{\dot{q}_2(t) - \dot{q}_1(t) + l_1 \dot{q}_3(t)\} \quad \text{and} \quad \Delta \dot{x}_r = \{\dot{q}_8(t) - \dot{q}_4(t) - l_2 \dot{q}_4(t)\} \]

In the above expressions, \( f_\phi^{\text{front}} \) and \( f_\phi^{\text{rear}} \) denotes the front and rear suspensions spring force, \( \Delta x_f \) and \( \Delta x_r \) denotes the static deformation of the front and rear suspension springs with stiffness coefficients \( k_f, k_f^\prime \) and \( k_r, k_r^\prime \). The damping force at the front and rear suspensions are \( f_d^{\text{front}} \) and \( f_d^{\text{rear}} \), and \( \Delta \dot{x}_f \) and \( \Delta \dot{x}_r \) is the relative velocity of the extremes of the front and the rear dampers with damping coefficients \( \alpha_f^\prime, \alpha_f^\prime \); and \( \alpha_r^\prime, \alpha_r^\prime \).

The equations of motion of the front and the rear unsprung masses are expressed as:

\[ m_\phi \ddot{q}_\phi(t) = f_\phi^{\text{front}} + f_d^{\text{front}} - k_\phi \{q_1(t) - f_\phi(t)\} \]  \hspace{1cm} (7.1c)

\[ m_n \ddot{q}_n(t) = f_\phi^{\text{rear}} + f_d^{\text{rear}} - k_n \{q_4(t) - f_\phi(t)\} \]  \hspace{1cm} (7.1d)

The equations of motion of the front and the rear passengers are expressed as:

\[ m_p \ddot{q}_p(t) = -F_i - F_2 \]  \hspace{1cm} (7.1e)
\[ m_p \ddot{q}_p(t) = -F_3 - F_4 \]  \hspace{1cm} (7.1f)

Eq. (7.1a to 7.1f) are a system of non-linear second orders coupled simultaneous equations, where single and double dots represent first and second order derivatives with respect to time respectively. Letting \( z_i = \dot{q}_i, i=1 \text{ to } 6 \), the above system of equations can be reduced to a system of twelve non-linear first order simultaneous equations. These equations have been solved in the time domain numerically considering equilibrium state as initial conditions.
7.1.3 Road excitation

The model simulation has been carried on two distinct road conditions, with three repetitive bumps. One road has bumps in the form of triangular wave while the other one has trapezoidal wave type of bumps. All these bumps have the same peak height (c) and span (w) of an assumed spatial profile and they induce periodic excitation to the vehicle. These periodic excitations have been transferred into harmonic excitations and are embedded in the dynamics of the vehicle model by the inclusion of the certain harmonic terms resulting from the Fourier series expansion of the time varying function of periodic bumps. The time varying function have been constructed from the space functional representation of repetitive bumps by introducing the vehicle speed. The mathematical expressions of harmonic excitations generated from the considered road bumps given below.

Case 1: Triangular wave type of continuous bumps

As a function of time the triangular wave type of three consecutive road bumps have been expressed as:

\[
E(t) = \begin{cases} 
\frac{2c}{T_i} t, & (q - 1)T_i \leq t \leq \frac{(2q - 1) T_i}{2} \\
\frac{2c}{T_i} (T_i - t), & \frac{(2q - 1)}{2} T_i \leq t \leq qT_i 
\end{cases}
\]

where \( q = 1, 2, 3 \) (7.2a)

The graphical representation of this function is shown in Figure 7.2.

![Figure 7.2: Repetitive triangular wave type road bumps.](image)

This function is periodic with period \( T_i \) and it can be represented by a series of sinusoids of suitable frequencies, amplitudes and phases. However, for periodic functions, the domain over which the approximation is required is only one period of the periodic function. The rest of the function is taken care of by the definition of periodicity in the function. Therefore, expanding
(7.2a) by Fourier series taking into account only one cycle of the wave i.e., \( q = 1 \) (considering one bump) we have

\[
E(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[ a_n \cos(n\omega t) + b_n \sin(n\omega t) \right]
\]

(7.2b)

The coefficients \( a_0, a_n \) and \( b_n \) are Fourier constants defined by

\[
a_0 = \frac{2}{T_1} \int_0^{T_1} f(t) dt, \quad a_n = \frac{2}{T_1} \int_0^{T_1} f(t) \cos(n\omega t) dt \quad \text{and} \quad b_n = \frac{2}{T_1} \int_0^{T_1} f(t) \sin(n\omega t) dt
\]

where \( \omega = \frac{2\pi}{T_1} \) is the frequency of excitation which decreases with increase in \( T_1 \).

Using (7.2a) and (7.2b) we have

\[
a_0 = c, \quad a_n = \begin{cases} 0, & \text{when } n \text{ is even} \\ -\frac{4c}{n^2\pi^2}, & \text{when } n \text{ is odd} \end{cases}
\]

and \( b_n = 0, \) for all \( n \).

Hence (7.2b) reduces to

\[
E(t) = \frac{c}{2} - \frac{4c}{\pi^2} \left( \cos(\omega t) + \frac{1}{3^2} \cos(3\omega t) + \frac{1}{5^2} \cos(5\omega t) + \frac{1}{7^2} \cos(7\omega t) + ... \right)
\]

(7.2c)

The above expression shows that the bump induces large magnitude of excitation at low harmonic and small magnitude of excitation at high harmonic. Since the magnitude of excitation decreases with increase in harmonic we therefore neglect the higher harmonic terms and approximate the above function by considering its first four terms. The approximated function is given below:

\[
E(t) = \frac{c}{2} - \frac{4c}{\pi^2} \left( \cos(\omega t) + \frac{1}{3^2} \cos(3\omega t) + \frac{1}{5^2} \cos(5\omega t) \right)
\]

(7.2c)

This function serves as the road input for the front wheel i.e., \( f_1(t) = E(t) \). The rear wheel also has the same road input but with time delay which depends upon the velocity of the car. The road input for the rear wheel has been expressed as:

\[ f_2(t) = E(t - L/v), \]

where \( L \) is the distance between the front and the rear wheels and \( v \) is the velocity of the car.

The series of continuous bumps as a function of time have been therefore obtained by repetitive use of the function (7.2c).
Case 2: Trapezoidal wave type of continuous bumps

As a function of time the trapezoidal wave type of three consecutive road bumps have been expressed as

$$E(t) = \begin{cases} 
\frac{3c}{T_i} t, & \frac{(q-1)}{3} T_i \leq t \leq \frac{(3q-2)}{3} T_i \\
\frac{3c}{T_i} (T_i - t), & \frac{(3q-1)}{3} T_i \leq t \leq qT_i 
\end{cases}$$

(7.3a)

where $q = 1, 2, 3$

Graphical representation of this function is shown in Figure 7.3.

![Figure 7.3: Repetitive trapezoidal wave type road bumps.](image)

As this function is periodic with period $T_i$ and it has been represented by a series of sinusoids of suitable frequencies, amplitudes and phases by applying Fourier series. As in Case 1, for one cycle of the wave i.e., $q = 1$ The Fourier constants $a_0$, $a_n$ and $b_n$ of this function are given below:

$$a_0 = \frac{4c}{3}, \quad a_n = \frac{3c}{n^2 \pi^2} \left\{ (-1)^{n} \cos \left( \frac{n\pi}{3} \right) - 1 \right\}, \text{ for all } n$$

and $b_n = 0$, for all $n$.

Hence the Fourier series expansion of the original function (7.3a) can be expressed as:

$$E(t) = \frac{2c}{3} + \frac{3c}{\pi^2} \sum_{n=1}^{\infty} \left\{ (-1)^{n} \cos \left( \frac{n\pi}{3} \right) - 1 \right\} \cos(n \omega t)$$

By neglecting the higher harmonic terms the above function can be approximated as

$$E(t) = \frac{2c}{3} + \frac{3c}{\pi^2} \sum_{n=1}^{3} \left\{ (-1)^{n} \cos \left( \frac{n\pi}{3} \right) - 1 \right\} \cos(n \omega t)$$

(7.3b)
This function serves as the road input for the front wheel. The rear wheel also has the same road input but with time delay which depends upon the velocity of the car. The series of continuous bumps have been introduced into the model by repetitive use of the function (7.3b).

### 7.1.4 Optimization problem

In order to find the optimal value of the parameters of the suspension system with respect to ride comfort, suspension deflection and road holding we have consider the following objective function:

$$
\text{Minimize } f(x), \quad f(x) = \max_{0 \leq t \leq T} \left| q_2(t) \right| / \text{Amp}
$$

where,

$$
x = \{ k_1', k_2', k_3', k_1', k_2', k_3', \alpha_1', \alpha_2', \alpha_3', \alpha_4' \} \text{ and } a_1 \leq k_1' \leq b_1, a_2 \leq k_2' \leq b_2, a_3 \leq k_3' \leq d_3, c_1 \leq k_1' \leq d_1, c_2 \leq k_2' \leq d_2, c_3 \leq k_3' \leq d_3,
$$

$$
e_1 \leq \alpha_1' \leq f_1, e_2 \leq \alpha_2' \leq f_2, g_1 \leq \alpha_3' \leq h_1, g_2 \leq \alpha_4' \leq h_2
$$

The discomfort over road bumps has been evaluated by computing the bouncing transmissibility ($q_2(t)/\text{Amp}$) at the center of mass of the sprung mass, where Amp is the maximum excitation amplitude transferred to the vehicle by the road bumps during its motion. This discomfort characteristic is very meaningful and has therefore been expressed in the objective function.

As per ISO standards human body feel comfortable if the maximum allowable jerk experienced by the passengers does not exceed $18 \text{ m/s}^3$ (Griffin, 2003 and Gillespie, 2003).

To meet this requirement we have included this condition as constraints in the optimization problem.

$$
g_1(x) = \max_{t} \left| \ddot{q}_2(t) \right| - 18 \text{ m/s}^3 \leq 0 \quad \text{and} \quad g_2(x) = \max_{t} \left| \ddot{q}_6(t) \right| - 18 \text{ m/s}^3 \leq 0.
$$

The relative displacement or the suspension deflection between the wheel and the vehicle body should be kept smaller than the mechanical structure which holds the finite space under the car body and the unsprung mass. Exceeding the limit will deteriorate passengers comfort and even cause structural damage. This condition has been taken care of by the inclusion of the following constraints in the optimization problem.

$$
g_3(x) = \left| q_2(t) - q_1(t) \right| - f_{\text{def}} \leq 0 \quad \text{and} \quad g_4(x) = \left| q_2(t) - q_2(t) \right| - r_{\text{def}} \leq 0
$$
where \( f_{\text{def}}\text{max} \) and \( r_{\text{def}}\text{max} \) are maximum allowable suspension deflection near the front wheel and the rear wheel respectively.

In order to reduce the risk of losing the contact between the wheels and the road (good road holding) we have introduce additional constraints; the loss of contact is possible when the dynamic tire deflection exceeds the static tire deflection. Static deflection of the tire spring is the ratio between the total load acting on the tire and the spring constant of the tire. Therefore we have the constraints

\[
\begin{align*}
    g_4(x) &= |q_1(t) - f_1(t)| - \frac{9.81 \times (m_s + m_{fs})}{k_f} \leq 0 \quad \text{and} \\
    g_6(x) &= |q_3(t) - f_3(t)| - \frac{9.81 \times (m_s + m_{ms})}{k_r} \leq 0
\end{align*}
\]

In order to enhance ride quality it is necessary to keep the natural frequency of the sprung mass lower than the exciting frequency. This has also been taken care of by allowing the tire stiffness to be larger than the suspension spring stiffness. A large value of tire stiffness also allows the tire to maintain its geometry under carload. To meet these requirements we therefore have add the following constraints to the optimization problem.

\[
\begin{align*}
    g_7(x) &= S_f - k_f \leq 0 \quad \text{and} \\
    g_8(x) &= S_r - k_r \leq 0
\end{align*}
\]

where \( S_f \) and \( S_r \) represents the front and the rear suspension spring stiffness.

During the motion of the vehicle sudden bump disturbance, may cause the natural frequency of the sprung mass to go beyond the natural frequency of the unsprung mass resulting in sharp increase in the amplitude of vibration of the vehicle body. To take care of this we have used the following constraints which will help in reducing the peak values of the vehicle body vibration caused by bumps/irregular road excitations.

\[
\begin{align*}
    g_9(x) &= \sqrt{\frac{S_f}{m_s}} - \sqrt{\frac{k_f}{m_{fs}}} \leq 0 \quad \text{and} \\
    g_{10}(x) &= \sqrt{\frac{S_r}{m_s}} - \sqrt{\frac{k_r}{m_{sr}}} \leq 0
\end{align*}
\]

where \( S_f \) and \( S_r \) represents the front and the rear suspension spring stiffness.

In accordance with the aforementioned requirements, we solve the following non-linear constrained optimization problem for designing the parameters of the suspension system.
Minimize \( f(x) \)
subject to \( g_k(x) \leq 0, k = 1, 2, \ldots, 10; x = \{k_1', k_2', k_3', k_1, k_2, k_3, \alpha_1, \alpha_2, \alpha_3\} \)  
and \( a_s \leq k_s' \leq b_s, c_s \leq k_s' \leq d_s, e_s \leq \alpha_s' \leq f_s, \ g_s \leq \alpha_s' \leq h_s \); \( i = 1, 2, 3 \) and \( j = 1, 2 \).  

7.1.5 Solution methodology

To find the optimal or near to optimal values of the parameters of the suspension system viz. springs stiffness \( (k_1', k_2', k_3') \) for front; \( k_1, k_2, k_3 \) for rear) and dampers coefficients \( (\alpha_1', \alpha_2', \alpha_3') \) for front; \( \alpha_1, \alpha_2, \alpha_3 \) for rear), the non-linear optimization problem (7.5) have been solved by applying real coded GA. This has been carried out performing the following step wise procedure:

ALGORITHM 7.1

\textbf{Begin}
\begin{enumerate}
\item Set \( t \leftarrow 0 \) [\( t \) represents the current generation].
\item Create an initial population \( P(t) \), of popsize number of chromosomes/individuals whose genes are the parameters of the suspension system chosen randomly from its bounds.
\item Evaluate the objective function (7.4) and the constraints \( g_k(x) \leq 0, k = 1, 2, \ldots, 10 \); by simulating the vehicle model in time domain, with the fourth order Runge-Kutta method taking initial conditions \( q_i(0) = 0 \) and \( \dot{q}_i(0) = 0 \) (\( i = 1, 2, \ldots, 6 \)) where \( t \epsilon [0, T] \).
\item Evaluate the fitness value of each chromosome using penalty method (SFP)
\item Evaluate the output parameters (1. peak and Root-Mean-Square (RMS) values of velocity, acceleration and jerk of sprung mass and the two passengers; 2. peak and RMS values of displacement, velocity and acceleration of unsprung masses) of each chromosome.
\item \textbf{Repeat}
\begin{enumerate}
\item Increase \( t \) by unity for next generation.
\item Select \( P(t) \) from \( P(t-1) \) by tournament selection operator of size four.
\item With probability \( p_c \), which decreases with increase in generation number apply multiparent whole arithmetical crossover operator.
\item With probability rate \( p_m \), which decreases with increase in generation number apply whole non-uniform mutation operator.
\item Evaluate the fitness value and the value of the output parameters of the problem.
\item Apply Elitism of size three.
\end{enumerate}
\item \textbf{Until} \( t \) reaches maximum generation.
\end{enumerate}
7.1.6 Computational results and discussions

Here we shall discuss about the results obtained after solving the suspension design problem by applying GA. Then we shall compare the GA results with the results obtained after simulating the vehicle model with the existing design (ED) parameters of the suspension system.

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<th>Notations</th>
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<th>Value</th>
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<tr>
<td>$k_2'$</td>
<td>N/m</td>
<td>0.0</td>
</tr>
<tr>
<td>$k_3'$</td>
<td>N/m</td>
<td>0.0</td>
</tr>
<tr>
<td>$k_4'$</td>
<td>N/m</td>
<td>18615.00</td>
</tr>
<tr>
<td>$k_5'$</td>
<td>N/m</td>
<td>0.0</td>
</tr>
<tr>
<td>$k_6'$</td>
<td>N/m</td>
<td>0.0</td>
</tr>
<tr>
<td>$\alpha_1'$</td>
<td>Ns/m</td>
<td>1190.00</td>
</tr>
<tr>
<td>$\alpha_2'$</td>
<td>Ns/m</td>
<td>0.0</td>
</tr>
<tr>
<td>$\alpha_3'$</td>
<td>Ns/m</td>
<td>1000.00</td>
</tr>
<tr>
<td>$\alpha_4'$</td>
<td>Ns/m</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Note: OD implies optimal design and ED implies existing design.

The optimal or near to optimal values of the suspension design parameters and the corresponding output parameters obtained after optimizing the suspension design problem using GA have been shown in Table 7.2, Table 7.3 and Table 7.4. These results have been obtained by simulating the vehicle model for 15 seconds using the following assumptions: the vehicle travels with uniform velocity of 50 km/hr, the model is excited by fundamental frequency of 174.53 Hz at 50 km/hr by there repetitive bumps of span 0.5 m and an elevation of 8.0 cm and the maximum suspension deflection allowed in this study is 8.0 cm. The GA is run with a 350 individuals population size ($p_{size}$) during 200 generations. The crossover and mutation rate where decreasing in the interval [0.8, 0.9] and [0.15, 0.2] respectively.
Table 7.3. Best found fitness value with its output parameters describing ride comfort.

<table>
<thead>
<tr>
<th>Measurements</th>
<th>Case 1</th>
<th>Case 2</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OD</td>
<td>ED</td>
<td>OD</td>
<td>ED</td>
</tr>
<tr>
<td><strong>Sprung mass</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bounce</td>
<td>PV</td>
<td>0.1429</td>
<td>0.1457</td>
<td>0.1744</td>
</tr>
<tr>
<td></td>
<td>RMS</td>
<td>0.0573</td>
<td>0.0373</td>
<td>0.0375</td>
</tr>
<tr>
<td>Velocity (m/s)</td>
<td>PV</td>
<td>0.0423</td>
<td>0.0407</td>
<td>0.0853</td>
</tr>
<tr>
<td></td>
<td>RMS</td>
<td>0.0116</td>
<td>0.0080</td>
<td>0.0174</td>
</tr>
<tr>
<td>Acceleration (m/s²)</td>
<td>PV</td>
<td>1.2887</td>
<td>0.8961</td>
<td>1.2851</td>
</tr>
<tr>
<td></td>
<td>RMS</td>
<td>0.0995</td>
<td>0.0889</td>
<td>0.1526</td>
</tr>
<tr>
<td>Jerk (m/s³)</td>
<td>PV</td>
<td>61.5603</td>
<td>44.8549</td>
<td>48.1686</td>
</tr>
<tr>
<td></td>
<td>RMS</td>
<td>3.2217</td>
<td>2.7609</td>
<td>3.6640</td>
</tr>
<tr>
<td><strong>Front passenger</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bounce</td>
<td>PV</td>
<td>0.1626</td>
<td>0.1535</td>
<td>0.3114</td>
</tr>
<tr>
<td></td>
<td>RMS</td>
<td>0.0475</td>
<td>0.0341</td>
<td>0.0646</td>
</tr>
<tr>
<td>Velocity (m/s)</td>
<td>PV</td>
<td>0.0877</td>
<td>0.0724</td>
<td>0.1864</td>
</tr>
<tr>
<td></td>
<td>RMS</td>
<td>0.0168</td>
<td>0.0148</td>
<td>0.0390</td>
</tr>
<tr>
<td>Acceleration (m/s²)</td>
<td>PV</td>
<td>0.9452</td>
<td>0.6666</td>
<td>1.8359</td>
</tr>
<tr>
<td></td>
<td>RMS</td>
<td>0.1831</td>
<td>0.1691</td>
<td>0.3971</td>
</tr>
<tr>
<td>Jerk (m/s³)</td>
<td>PV</td>
<td>15.7927</td>
<td>12.3771</td>
<td>21.8566</td>
</tr>
<tr>
<td></td>
<td>RMS</td>
<td>2.5888</td>
<td>2.3565</td>
<td>5.0329</td>
</tr>
<tr>
<td><strong>Rear passenger</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bounce</td>
<td>PV</td>
<td>0.3344</td>
<td>0.2858</td>
<td>0.2656</td>
</tr>
<tr>
<td></td>
<td>RMS</td>
<td>0.1123</td>
<td>0.0691</td>
<td>0.0449</td>
</tr>
<tr>
<td>Velocity (m/s)</td>
<td>PV</td>
<td>0.0693</td>
<td>0.0983</td>
<td>0.1237</td>
</tr>
<tr>
<td></td>
<td>RMS</td>
<td>0.0229</td>
<td>0.0150</td>
<td>0.0216</td>
</tr>
<tr>
<td>Acceleration (m/s²)</td>
<td>PV</td>
<td>0.8405</td>
<td>1.1436</td>
<td>1.5640</td>
</tr>
<tr>
<td></td>
<td>RMS</td>
<td>0.1364</td>
<td>0.1539</td>
<td>0.2253</td>
</tr>
<tr>
<td>Jerk (m/s³)</td>
<td>PV</td>
<td>12.5219</td>
<td>17.2996</td>
<td>26.545</td>
</tr>
<tr>
<td></td>
<td>RMS</td>
<td>1.9644</td>
<td>2.3487</td>
<td>3.2241</td>
</tr>
</tbody>
</table>

Note: OD implies optimal design and ED implies existing design.

In Table 7.3 we see that under OD, the crest factor (ratio of peak value (PV) and RMS value (RMS)) values of the front and rear passengers over Case 1 and Case 2 road conditions are: 5.16, 6.16 and 3.94, 7.43 respectively. Regarding such crest factor values that are less than 9, we shall evaluate the influence of vibration on passengers comfort using ISO 2631: 1997 standards.
PART II. APPLICATIONS OF ADVANCED REAL CODED GENETIC ALGORITHM

Table 7.4. Output parameters describing road holding performance.

<table>
<thead>
<tr>
<th>Measurements</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OD</td>
<td>ED</td>
</tr>
<tr>
<td>Front Unsprung mass</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Displacement (m)</td>
<td>PV 0.0644</td>
<td>0.0420</td>
</tr>
<tr>
<td></td>
<td>RMS 0.0049</td>
<td>0.0040</td>
</tr>
<tr>
<td>Velocity (m/s)</td>
<td>PV 1.6154</td>
<td>1.0961</td>
</tr>
<tr>
<td></td>
<td>RMS 0.1313</td>
<td>0.0786</td>
</tr>
<tr>
<td>Acceleration (m/s²)</td>
<td>PV 83.2116</td>
<td>72.4208</td>
</tr>
<tr>
<td></td>
<td>RMS 5.0608</td>
<td>3.8699</td>
</tr>
<tr>
<td>Rear Unsprung mass</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Displacement (m)</td>
<td>PV 0.0751</td>
<td>0.0592</td>
</tr>
<tr>
<td></td>
<td>RMS 0.0109</td>
<td>0.0050</td>
</tr>
<tr>
<td>Velocity (m/s)</td>
<td>PV 1.9136</td>
<td>1.3761</td>
</tr>
<tr>
<td></td>
<td>RMS 0.2864</td>
<td>0.1224</td>
</tr>
<tr>
<td>Acceleration (m/s²)</td>
<td>PV 56.7138</td>
<td>48.9551</td>
</tr>
<tr>
<td></td>
<td>RMS 8.0152</td>
<td>3.8169</td>
</tr>
</tbody>
</table>

Note: OD implies optimal design and ED implies existing design.

Performance evaluation

Here the ride comforts of the passengers have been evaluated with respect to ISO 2631: 1997 standards. As per ISO 2631, if the crest factor value is less than 9 the passengers comfort can be evaluation from Table 7.5 which is based on the RMS of weighted acceleration.

Table 7.5. Perception of ride comfort according to ISO 2631.

<table>
<thead>
<tr>
<th>RMS vibration level</th>
<th>Perception</th>
</tr>
</thead>
<tbody>
<tr>
<td>Less than 0.315 m/s²</td>
<td>Not uncomfortable</td>
</tr>
<tr>
<td>0.315 m/s² to 0.63 m/s²</td>
<td>A little uncomfortable</td>
</tr>
<tr>
<td>0.5 m/s² to 1.0 m/s²</td>
<td>Fairly uncomfortable</td>
</tr>
<tr>
<td>0.8 m/s² to 1.6 m/s²</td>
<td>Uncomfortable</td>
</tr>
<tr>
<td>1.25 m/s² to 2.5 m/s²</td>
<td>Very uncomfortable</td>
</tr>
<tr>
<td>Greater than 2 m/s²</td>
<td>Extremely uncomfortable</td>
</tr>
</tbody>
</table>

In Table 7.3, we see that the vertical weighted RMS acceleration of the passengers lies within the comfortable zone “Less than 0.315 m/s²”. Also, we see in the table that the maximum jerk experienced by the passengers is less than 18 m/s³. Therefore, according to ISO 2631 the
passengers are enjoying comfortable ride. From the above observations it is clear that optimal values of the suspension parameters do not deteriorate ride comfort. The peak and RMS values of displacement, velocity and acceleration of unsprung masses under OD in Table 7.4 shows that there is small deflection of unsprung masses emphasizing that the tire is in contact with the ground throughout the simulation giving good road holding. The tire and the suspension deflections in time domain over the road bumps have been shown graphically in Figure 7.4 and Figure 7.5 respectively.

Figure 7.4: Tire deflection during the motion of the vehicle over the roads.

Figure 7.5: Suspension deflection during the motion of the vehicle over the roads.
The curve drawn in Figure 7.6 and Figure 7.7 shows the behavior of the non-linear suspension spring. It is clearly seen that when a large force is exerted during vehicle motion, the spring becomes nonlinear. The non-linearity is caused by either stiffening or weakening of the spring increasing elongation or compression. The vibration levels of the passengers during the motion of the vehicle over bumpy roads at different instant of time in shown in Figure 7.8 and Figure 7.9, plotted taking amplitude of vibration verses time. The time domain spectrum of the vibration levels of the passengers are converted to frequency domain using Fast Fourier Transform (FFT) and plotted taking amplitude of vibration verses frequency in Figures 7.10.

Figure 7.6: Behavior of suspension spring during vehicle motion over Case 1 road condition.

Figure 7.7: Behavior of suspension spring during vehicle motion over Case 2 road condition.
Figure 7.8: Bounce and velocity of passengers in time domain over different road conditions during vehicle motion.
Figure 7.9: Acceleration and jerk of passengers in time domain over different road conditions during vehicle motion.
In order to verify the validity of the GA results, these results have been compared to those obtained by using the existing design (ED) parameters given in Table 7.2. For this purpose the model has been simulated for 15 seconds over the same road conditions keeping vehicle velocity constant at 50 km/hr. The simulation results have been shown in Table 7.3 and Table 7.4. From Table 7.3 (see under ED) it can be seen that the RMS values of the vertical accelerations of the rear passenger are below $0.315 \text{ m/s}^2$ and lies in the zone "Not
uncomfortable" and the RMS values of the vertical accelerations of the front passenger are above 0.315 m/s² and lies in the zone "A little uncomfortable". The peak value of jerk of both the passengers over the two roads exceeds the comfortable level 18 m/s³ (Griffin, 2003 and Gillespie, 2003). The above observations indicate that the ED parameters slightly deteriorate passengers comfort. Hence, in respect to ride comfort the GA parameters are found to be better than the ED parameters. Passenger’s vibrations in time domain during vehicle motion are shown in Figure 7.8 and Figure 7.9. The passenger’s response in frequency domain is shown in Figure 7.9. Now, in respect to road holding performance comparing the results under OD and ED in Table 7.4 it is seen that with ED parameters the unsprung mass displacement, velocity and acceleration are slightly lower to those obtained with the OD parameters. As can be seen, the ED parameters shows better road holding performance than the OD parameters but, the OD parameters also do not deteriorate road holding performance. The tire and suspension deflection during the motion of the vehicle over the road bumps are shown in Figure 7.4 and Figure 7.5. Behavior of the suspension spring is shown in Figure 7.6 and Figure 7.7. Based on the above discussions, tabulated results and graphical representations it can be said that the optimal design can serve as an efficient design of the suspension system.

7.1.7 Conclusion

In this chapter GA has been applied to estimate the optimal values of the passive suspension parameters of the vehicle model. These parameters represent the coefficients of the cubic polynomial characterizing non-linear spring stiffness and that of quadratic polynomial characterizing non-linear damping. The parameters have been obtained by solving the formulated non-linear constrained optimization problem which satisfies performance with respect to ride comfort, road holding and passenger’s comfort as per ISO 2631 standards. The performance of the optimal or near to optimal values of the suspension parameters have been verified by comparing the GA results with that of simulation results obtained by simulating the model with the existing suspension parameters under the same conditions. Comparison shows better ride comfort and road holding performance with the GA results. From the simulation results it has been observed that, with the existing suspension parameters the peak value of jerk of the passenger’s exceeds the comfortable level 18 m/s³ and also the RMS values of vertical accelerations of both the passenger’s are not below 0.315 m/s². With respect to ISO standards the above results therefore show passengers discomfort in moving vehicle. With the optimal values of the suspension parameters the peak value of jerk as well as the RMS values of vertical accelerations of both the passenger’s are found to be less than 18 m/s³ and below 0.315 m/s² respectively. This further proves that the optimal suspension parameters have better potential to improve ride comfort. Responses of the passengers have also been compared graphically in time and frequency domain taking the optimal and existing suspension parameters.
Model-2: On optimal design of passive suspension using GA based heuristic method

7.2.1 Introduction

In this chapter, a real coded GA with variable rate of crossover and mutation has been applied to find the suspension parameters of the half car model with two passengers which satisfy the performance as per ISO 2631 standards. Analysis of the prior research shows that the vehicle model with passenger's dynamics has rarely been used in the literature in vehicle suspension design (Stein et al., 1991; Rakheja et al., 1994; Güçlü, 2003; Tashirad et al., 1998). However, inclusion of passenger's dynamics in the model is important as it helps in analyzing passenger's response to vibration in moving vehicle. In this chapter, we have developed the half car passenger model which to some extent is similar to the model which was developed by Taghirad et al. (1998). The dissimilarity lies in the nature of suspension system used. Their model has active suspension system. Although, active suspension systems can provide superior control performance over wide frequency range of excitations induced by the road irregularities beyond that attainable by passive suspensions, hindered by its construction complexity and possible stability problems when components fail, its high cost, and its power consumption have yet to be accepted for conventional use. Therefore, active suspensions remain limited to expensive vehicle. Passive suspension systems are simple, reliable, and cheap and are dominant in the marketplace (Elbeheiry et al., 1996). To design a cheaper model we have replaced the active suspension of their model by passive one and reformulated the equation of motions of the dynamical system. The passive suspensions are assumed to have linear springs and dampers. The performance of the constructed model have been studies on the basis of ISO 2631 standards by simulating the vehicle model over different road conditions at uniform
velocity taking initial values of the suspension parameters. In order to improve ride comfort and road holding performance, we have formulated a non-linear constraint optimization problem and the problem has been solved using GA to search for the better suspension design parameters. The objective function considered is the maximum bouncing transmissibility of the sprung mass at the center of mass of the vibrating vehicle during its uniform motion over the road. The roads considered are of deterministic and probabilistic type. For deterministic type of road we have developed a periodical waveform of multiple road bumps of saw-tooth type using Fourier series. The motivation for applying Fourier series is based on the fact that any periodic function can be represented by a series of sinusoids of suitable frequency, phase and amplitude. Further, to emphasize on the fact that most of the random variables are assumed to follow Gaussian distribution we have assumed probabilistic road as Gaussian. To verify the validity, the results obtained by GA are compared with those obtained with the existing suspension parameters in time domain and it has been found that GA results show better performance.

7.2.2 Dynamical model of a half car with passengers seat and linear suspensions

In this Chapter, we have considered the two-dimensional model of a car with passenger's dynamics as shown in Figure 7.1 with passive suspension. This type of model was earlier developed for active suspension by Taghirad et al., (1998). In their work, they examined the performance of active suspension which is generally used in expensive vehicle (Elbeheiry et al., 1996), over random road condition. In our work a new model has been developed by replacing the active suspension by passive ones. Here our intension is to determine passive suspension parameters in order to optimize passengers comfort over two types of road conditions viz., deterministic and probabilistic type.

Here it has been assumed that the suspension spring stiffness and the damping are represented by linear elements. In reality, suspension motion together with the characteristics of the spring and damper are all non-linear. However, for small suspension deflection motion linear representation can be acceptable.

7.2.2.1 Formulating the equations of motion

In this model the number of generalized coordinates is six which are equal to the number of degrees of freedom of the model. Thus the model will be described by six second order differential equations. For the above model, the equations of motion have been derived from a
set of free body diagrams of the masses. The equations of motion have been then determined by applying Newton’s second law to each free body mass considering the effect of angular motion \( q_3(t) \), to be very small.

Applying Newton’s second law to the sprung mass, the vertical and angular motion of the sprung mass are given by

\[
m\ddot{q}_3(t) = F_1 + F_2 + F_3 + F_4 - F_5 - F_6 - F_7 - F_8 \quad (7.6a)
\]

\[
J\ddot{q}_3(t) = (F_1 + F_2)d_1 - (F_3 + F_4)d_2 - (F_5 + F_6)l_1 + (F_7 + F_8)l_2 \quad (7.6b)
\]

Similarly, by applying Newton’s second law to the front and rear unsprung masses, the vertical motion of the unsprung masses are given by

\[
m_{f_1}\ddot{q}_1(t) = F_3 + F_6 - F_9 \quad (7.6c)
\]

\[
m_{f_2}\ddot{q}_6(t) = F_7 + F_8 - F_{10} \quad (7.6d)
\]

In similar manner the equations of motion of the front and the rear passengers are given by

\[
m_{p_1}\ddot{q}_5(t) = -F_1 - F_2 \quad (7.6e)
\]

\[
m_{p_2}\ddot{q}_8(t) = -F_3 - F_4 \quad (7.6f)
\]

Where,

\[
F_1 = k_p \{ \dot{q}_3(t) - (q_2(t) + d_1\dot{q}_3(t)) \}, \quad F_2 = c_p \{ \ddot{q}_3(t) - (\dot{q}_2(t) + d_1\ddot{q}_3(t)) \},
\]

\[
F_3 = k_p \{ \dot{q}_6(t) - (q_2(t) - d_2\dot{q}_3(t)) \}, \quad F_4 = c_p \{ \ddot{q}_6(t) - (\dot{q}_2(t) - d_2\ddot{q}_3(t)) \},
\]

\[
F_5 = \alpha_p \{ (\dot{q}_2(t) + l_1q_3(t)) - q_1(t) \}, \quad F_6 = \alpha \{ (\dot{q}_2(t) + l_1\dot{q}_3(t)) - \dot{q}_1(t) \},
\]

\[
F_7 = \alpha_\dot{q}_1 \{ (\dot{q}_2(t) - l_2q_3(t)) + q_4(t) \}, \quad F_8 = \alpha \{ (\dot{q}_2(t) - l_2\dot{q}_3(t)) + \dot{q}_4(t) \},
\]

\[
F_9 = k_p \{ \dot{q}_1(t) - f_1(t) \} \quad \text{and} \quad F_{10} = k_p \{ q_4(t) - f_4(t) \}
\]

Eq. (7.6a to 7.6f) is a system of linear second orders coupled simultaneous equations, where single and double dots represent first and second order derivatives with respect to time respectively. Letting \( z_p = \dot{q}_p, \ p = 1 \ to \ 6 \), the above system of equations have been reduced to a system of linear first order simultaneous equations (7.7). In model simulation these equations have been solved in the time domain numerically using fourth order Runge-Kutta method. The above system of equations has now been expressed as:

\[
\dot{X}(t) = AX(t) + B(t) \quad (7.7)
\]

where
\[ \dot{X}(t) = \left[ \dot{z}_1(t), \dot{z}_2(t), \dot{z}_3(t), \dot{z}_4(t), \dot{z}_5(t), \dot{q}_1(t), \dot{q}_2(t), \dot{q}_3(t), \dot{q}_4(t), \dot{q}_5(t) \right]^T, \]

\[ X(t) = \left[ z_1(t), z_2(t), z_3(t), z_4(t), z_5(t), q_1(t), q_2(t), q_3(t), q_4(t), q_5(t) \right]^T, \]

\[
A = \begin{bmatrix}
-a_{11} & a_{11} & a_{12} & 0 & 0 & 0 & -\frac{(k_p + k_n)}{m_p} & \frac{k_p}{m_p} & \frac{k_n}{m_n} & 0 & 0 & 0
\end{bmatrix}
\]

\[
h(t) = \begin{bmatrix}
\alpha_f & -\alpha_f & 0 & 0 & 0 & -\frac{k_p}{m_p} & -\frac{k_n}{m_n} & -\frac{(k_p + k_n)}{m_p} & 0 & 0
\end{bmatrix}
\]

where,

\[
a_{22} = \frac{-(c_{pl} + c_{p2} + \alpha_f + \alpha_s)}{m_i},
\]

\[
a_{23} = \frac{-(c_{pi} d_1 - c_{p2} d_2 + \alpha_f l_1 - \alpha_f l_2)}{m_i},
\]

\[
a_{25} = \frac{-(k_p d_1 - k_p d_2 + k_{pl} l_1 - k_{pl} l_2)}{m_i},
\]

\[
a_{32} = \frac{-(c_{pi} d_1 - c_{p2} d_2 + \alpha_f l_1 - \alpha_f l_2)}{m_i},
\]

\[
a_{33} = \frac{-(c_{pi} d^2_1 + c_{p2} d^2_2 + \alpha_f l^2_1 + \alpha_f l^2_2)}{m_i},
\]

\[
a_{35} = \frac{-(k_p d^2_1 - k_p d^2_2 - k_{pl} l_1^2 + k_{pl} l_2^2)}{m_i},
\]

\[
a_{39} = \frac{-(k_p d^2_1 - k_p d^2_2 - k_{pl} l_1^2 + k_{pl} l_2^2)}{m_i},
\]

and \( B(t) = \begin{bmatrix} k_p f_1(t) & 0 & 0 \end{bmatrix} \)

As the rear wheel travel over the same path as the front wheel; hence \( f_2(t) \) is a delayed version of \( f_1(t) \) and is expressed as

\[ f_2(t) = f_1 \left( t - \frac{L}{v} \right), \]

\( L \) being the axle distance and \( v \) the uniform velocity of the car.
7.2.3 Description of road conditions

The amount of vibration induced into the vehicle depends upon the nature of the road profile over which the vehicle moves and upon the suspension system used. Often, intensive vibration of the vehicle creates discomfort for the passengers. Therefore, the response of the passenger's to road input is an important factor to be considered in the design of vehicle suspension. When designing vehicle suspension generally, the road inputs considered for vehicle dynamic simulation are: deterministic road (periodic or non-periodic) inputs, random/probabilistic type inputs and discrete type of road inputs with bumps and potholes (Baumal et al., 1998). As far as deterministic inputs are considered, a variety of periodic waves can be used, such as sine waves (Caponetto et al., 2003), square waves or triangular waves. Although not realistic, these are useful in studying the performance of the suspension design both in time and frequency domain. The random road input gives the most real representation of road profile. The power spectral density (PSD) (Uys et al., 2007; Marzbanrad et al., 2004) and ISO 2631 standards for road roughness are the most common way to characterize the random road surface. In this chapter, model simulation has been done over both deterministic and probabilistic/random type of road condition. For deterministic type of road we have developed a periodical waveform of multiple road bumps of saw-tooth type using Fourier series. To generate random road we have used Gaussian distribution function. The descriptions of these road conditions are given below:

Case 1: Deterministic type

As a function of time the saw-tooth wave type of three successive road bumps are expressed as:

\[ E(t) = \frac{c}{T_1} t, \quad (q-1)T_1 \leq t \leq qT_1 \quad \text{where} \quad q = 1, 2, 3 \]  

(7.8)

where \( c \) is the height of the bump and \( T_1 \) its cycle time. Graphical representation of this function is shown in Figure 7.11.

![Graphical representation of saw-tooth wave type of road bumps](image)

Figure 7.11: Successive saw-tooth wave type of road bumps.
This function is periodic with period $T_1$ and it can be represented by a series of sinusoids of suitable frequencies, amplitudes and phases. However, for periodic functions, the domain over which the approximation is required is only one period of the periodic function. Therefore, expanding (7.8) by Fourier series taking into account only one cycle of the wave i.e., $q = 1$ (considering one bump) we have

$$E(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[ a_n \cos(n\omega t) + b_n \sin(n\omega t) \right]$$  \hspace{1cm} (7.9)

The coefficients $a_0$, $a_n$ and $b_n$ are Fourier constants defined by

$$a_0 = \frac{2}{T_1} \int_0^{T_1} f(t) dt, \quad a_n = \frac{2}{T_1} \int_0^{T_1} f(t) \cos(n\omega t) dt, \quad b_n = \frac{2}{T_1} \int_0^{T_1} f(t) \sin(n\omega t) dt$$

where $\omega = \frac{2\pi}{T_1}$ is the fundamental frequency of excitation.

Using (7.8) and (7.9) we have

$$a_0 = c, \quad a_n = 0 \quad \text{and} \quad b_n = -\frac{c}{n\pi} \quad \text{for all} \quad n$$

Hence (7.9) reduces to

$$E(t) = \frac{c}{2} - \frac{c}{\pi} \left( \sin(\omega t) + \frac{1}{2} \sin(2\omega t) + \frac{1}{3} \sin(3\omega t) + ... \right)$$

The above expression shows that the bump induces large magnitude of excitation at low harmonic and small magnitude of excitation at high harmonic. Since the magnitude of excitation decreases with increase in harmonic we therefore neglect the higher harmonic terms and approximate the above function by considering its first four terms. The approximated function is given below:

$$E(t) = \frac{c}{2} + \frac{c}{\pi} \left( \sin(\omega t) + \frac{1}{2} \sin(2\omega t) + \frac{1}{3} \sin(3\omega t) \right)$$  \hspace{1cm} (7.10)

This function serves as the road input for the front wheel i.e., $f_f(t) = E(t)$. The rear wheel also has the same road input but with time delay which depends upon the velocity of the car. The road input for the rear wheel is expressed as $f_r(t) = E(t - L/v)$, where $L$ is the distance between the front and the rear wheels and $v$ is the velocity of the car. The series of three continuous bumps as a function of time are obtained by repetitive use of the function (7.10).

The road inputs which are incorporated into the vehicle model consists of three repetitive bumps each of span $w = 0.5$ m and of maximum height $c = 10$ cm. When the vehicle
travels over the road with uniform velocity $v = 50$ km/hr the road inputs are analyzed in the form of wave as shown in Figure 7.12.

![Figure 7.12: Road input at the front and rear tire.](image)

**Case 2: Probabilistic type**

The probabilistic type of road condition is shown in The Figure 7.13. This road condition has been created artificially following the same procedure which has discussed in the previous chapter. In this figure the peak and valleys show the irregular road surface of varying amplitude formed by RMS of random numbers.

![Figure 7.13: Random input from the ground.](image)

### 7.2.4 Optimization problem

In order to find the optimal value of the suspension parameters with respect to ride comfort, suspension deflection and road holding we have consider the following optimization problem.

The objective function is defined as
Minimize \( f(x) = \max_{0 \leq t \leq T} \left| q_2(t) \right| \)  

(7.11)

where,

\[ x = \left( k_f, k_n, \alpha_f, \alpha_n \right)^T \in \mathbb{R}^4 \] and 
\[ \alpha \leq k_f \leq b, \ \alpha \leq k_n \leq d, \ \alpha \leq \alpha_f \leq f, \ \alpha \leq \alpha_n \leq h \]

This function minimizes the maximum bouncing transmissibility of the sprung mass at the center of mass, resulting from the vibrating vehicle during its uniform motion over the road.

The constraints considered have been defined as following:

(i) \( g_1(x) = \max |\ddot{q}_2(t)| - 18 \text{ m/s}^3 \leq 0 \) and \( g_2(x) = \max |\ddot{q}_3(t)| - 18 \text{ m/s}^3 \leq 0 \).

As per ISO standards human body feel comfortable if the maximum allowable jerk experienced by the passengers does not exceed 18 m/s\(^3\) (Griffin, 2003; Gillespie, 2003). To meet this requirement we include this condition as constraints in the optimization problem.

(ii) \( g_5(x) = |q_2(t) - q_1(t)| - \text{fdef}_{\text{max}} \leq 0 \) and \( g_6(x) = |q_2(t) - q_3(t)| - \text{rdef}_{\text{max}} \leq 0 \)

where \( \text{fdef}_{\text{max}} \) and \( \text{rdef}_{\text{max}} \) are maximum allowable suspension deflection near the front wheel and the rear wheel respectively. The suspension deflection between the wheel and the vehicle body should be kept smaller than the mechanical structure which holds the finite space under the car body and the unsprung mass. Exceeding the limit will deteriorate passengers comfort and even cause structural damage. This condition is taken care of by considering these two constraints in the optimization problem.

(iii) \( g_5(x) = k_f - k_n \leq 0 \) and \( g_7(x) = k_n - k_n \leq 0 \)

These constraints assure that the spring stiffness does not exceed the tire stiffness, and keeps the natural frequency of the sprung mass less than the exciting frequency. Moreover, a large value of tire stiffness maintains the circular shape of the tire under carload and improves the ride quality of the vehicle.

(iv) \( g_7(x) = \sqrt{\frac{k_f}{m_s}} - \sqrt{\frac{k_n}{m_n}} \leq 0 \) and \( g_8(x) = \sqrt{\frac{k_n}{m_s}} - \sqrt{\frac{k_n}{m_n}} \leq 0 \)

These constraints always prevent the sprung mass natural frequency from exceeding the unsprung mass natural frequency. The sprung mass natural frequency is the frequency with which the body of the vehicle vibrates. The unsprung mass natural frequency is the frequency
with which the wheel and the suspension vibrate on the tire. So, if the natural frequency of the sprung mass is lower, the bouncing transmissibility of the sprung mass caused by the road disturbances will be kept at a lower value and will therefore provide a more comfortable ride to the passengers.

\[ g_5(x) = \left| q(t) - f(t) \right| - \frac{9.81 \times (m_s + m_w)}{k_b} \leq 0 \quad \text{and} \quad g_{10}(x) = \left| q_s(t) - f_z(t) \right| - \frac{9.81 \times (m_s + m_w)}{k_n} \leq 0 \]

In order to reduce the risk of losing the contact between the wheels and the road (good road holding) we have introduce the above two constraints. It is to be noted that the loss of contact is possible when the dynamic tire deflection exceeds the static tire deflection.

Taking into account the above assumptions we therefore formulate the optimization problem as a non-linear constrained optimization problem as follows:

\[
\begin{align*}
\text{Minimize } f(x) \\
\text{subject to } g_k(x) \leq 0, \quad k = 1, 2, \ldots, 10; \quad x = \left( k_f, k_n, \alpha_f, \alpha_s \right)^T \in \mathbb{R}^4 \\
\text{and } X = \left\{ a \leq k_f \leq b, \quad c \leq k_n \leq d, \quad e \leq \alpha_f \leq f, \quad g \leq \alpha_s \leq h \right\}
\end{align*}
\]

(7.12)

In order to find the optimal or near to optimal values of the suspension parameters \((k_f, k_n, \alpha_f, \alpha_s)\), the optimization problem (7.12) has been solved by applying real coded GA.

### 7.2.5 Fitness function

The fitness value of each chromosome has been evaluated from the fitness function. The fitness function has been computed using Superiority of Feasible point Penalty method (SFP) (Kaisa et al., 2003).

### 7.2.6 Solution methodology

To find the optimal or near to optimal values of the parameters of the suspension system viz. springs stiffness \((k_f, k_n)\) and dampers coefficients \((\alpha_f, \alpha_s)\), the non-linear constrained optimization problem (7.12) has been solved by applying advanced real coded GA. This has been carried out by performing the following steps:
ALGORITHM 7.3

Begin
Set $t \leftarrow 0$ [t represents the current generation].
Create an initial population $P(t)$, of popsize number of chromosomes/individuals whose genes are the suspension parameters chosen randomly from its bounds.
Evaluate the objective function (7.11) and the constraints $g_k(x) \leq 0$, $k = 1, 2, \ldots, 10$; by simulating the vehicle model in time domain taking initial conditions as $q_p(0) = 0$ and $\dot{q}_p(0) = 0$ ($p = 1, 2, \ldots, 6$)
where $t \in [0, T]$.
Evaluate the fitness value of each chromosome.
Evaluate the output parameters (1. peak and Root-Mean-Square (RMS) values of velocity, acceleration and jerk of sprung mass and the two passengers; 2. peak and RMS values of displacement, velocity and acceleration of unsprung masses) of each chromosome.
Repeat
(i) Increase $t$ by unity for next generation.
(ii) Select $P(t)$ from $P(t-1)$ by tournament selection operator of size four.
(iii) With probability rates $p_c$ and $p_m$ which decreases with increase in generation number apply multiparent whole arithmetical crossover operator and whole non-uniform mutation operator respectively.
(iv) Evaluate the fitness value and the value of the output parameters of the problem.
(v) Apply Elitism of size three.
Until $t$ reaches maximum generation.
Return the best found solution, the corresponding fitness function value and its output parameters.
End

7.2.7 Computational results and discussions

This section has been divided into two parts. In the first part we present the simulation results obtained with the existing design parameters, whereas, in the second part we present the results obtained after optimizing the suspension design problem using GA. Finally, we compare the two results. Model simulation and GA optimization have been done by coding the corresponding programs in C-language and implementing it on Pentium IV PC.

7.2.7.1 Results with numerical simulation

In order to verify the validity of the GA results, model simulation has been initially done taking the existing values of the design parameters (Taghirad et al., 1998) which are given in Table
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7.6 under ED. For this purpose the vehicle model has been simulated for 15 seconds (in time domain) during its uniform motion (at 50 km/hr) over both deterministic and probabilistic type of road conditions. The simulation results have been shown in Table 7.7 and Table 7.8.

Table 7.6. Comparison between the existing design (ED) and the optimal design (OD) parameters.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Units</th>
<th>ED</th>
<th>OD Case 1 (A)</th>
<th>OD Case 2 (B)</th>
<th>Convex [0.9A + (1 - 0.9)B]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(k_p)</td>
<td>N/m</td>
<td>66824.2</td>
<td>6133.25</td>
<td>34938.39</td>
<td>9013.77</td>
</tr>
<tr>
<td>(k_c)</td>
<td>N/m</td>
<td>18615</td>
<td>2436.21</td>
<td>8689.77</td>
<td>3061.56</td>
</tr>
<tr>
<td>(\alpha_f)</td>
<td>N/m</td>
<td>1190</td>
<td>889.22</td>
<td>1121.5</td>
<td>912.44</td>
</tr>
<tr>
<td>(\alpha_r)</td>
<td>N/m</td>
<td>1000</td>
<td>459.19</td>
<td>927.09</td>
<td>505.98</td>
</tr>
</tbody>
</table>

Table 7.7. Comparison between the simulation and GA results with respect to ride comfort.

<table>
<thead>
<tr>
<th>Measurements</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ED</td>
<td>OD</td>
</tr>
<tr>
<td>Bounce</td>
<td>PV</td>
<td>0.2183</td>
</tr>
<tr>
<td></td>
<td>RMS</td>
<td>0.0469</td>
</tr>
<tr>
<td>Bounce</td>
<td>PV</td>
<td>0.1327</td>
</tr>
<tr>
<td></td>
<td>RMS</td>
<td>0.0273</td>
</tr>
<tr>
<td>Velocity</td>
<td>PV</td>
<td>1.9666</td>
</tr>
<tr>
<td></td>
<td>RMS</td>
<td>0.2373</td>
</tr>
<tr>
<td>Acceleration</td>
<td>PV</td>
<td>86.5896</td>
</tr>
<tr>
<td></td>
<td>RMS</td>
<td>5.7004</td>
</tr>
<tr>
<td>Sprung mass</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bounce</td>
<td>PV</td>
<td>0.3894</td>
</tr>
<tr>
<td></td>
<td>RMS</td>
<td>0.0808</td>
</tr>
<tr>
<td>Bounce</td>
<td>PV</td>
<td>0.2912</td>
</tr>
<tr>
<td></td>
<td>RMS</td>
<td>0.0610</td>
</tr>
<tr>
<td>Acceleration</td>
<td>PV</td>
<td>2.8668</td>
</tr>
<tr>
<td></td>
<td>RMS</td>
<td>0.6202</td>
</tr>
<tr>
<td>Jerk</td>
<td>PV</td>
<td>34.0740</td>
</tr>
<tr>
<td></td>
<td>RMS</td>
<td>7.8570</td>
</tr>
<tr>
<td>Front</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bounce</td>
<td>PV</td>
<td>0.3329</td>
</tr>
<tr>
<td></td>
<td>RMS</td>
<td>0.0564</td>
</tr>
<tr>
<td>Bounce</td>
<td>PV</td>
<td>0.1934</td>
</tr>
<tr>
<td></td>
<td>RMS</td>
<td>0.0338</td>
</tr>
<tr>
<td>Acceleration</td>
<td>PV</td>
<td>2.4456</td>
</tr>
<tr>
<td></td>
<td>RMS</td>
<td>0.3521</td>
</tr>
<tr>
<td>Jerk</td>
<td>PV</td>
<td>41.3649</td>
</tr>
<tr>
<td></td>
<td>RMS</td>
<td>5.0325</td>
</tr>
</tbody>
</table>

Note: OD, ED and CC implies optimal design, existing design and convex combination of OD.
These results will be helpful in evaluating the performance of the suspension design parameters on the basis of ride comfort and road holding over the two roads.

Regarding ride comfort, the results have been evaluated according to the ISO 2631: 1997 standards (see Table 7.5). It is seen from Table 7.7 (see under ED) that the RMS values of the vertical accelerations of the front passenger over case 1 and case 2 road conditions are 0.6202 m/s² and 0.4904 m/s² respectively (range 0.315 m/s² to 0.63 m/s²), equivalent to "a little uncomfortable" in Table 7.5, giving an indication of a little uncomfortable ride for this passenger. Again it is seen from the same table (see under ED) that the RMS values of the vertical accelerations of the rear passenger over case 1 and case 2 road conditions are 0.3521 m/s² and 0.2672 m/s² respectively. These values lies in the range "0.315 m/s² to 0.63 m/s²" and "Less than 0.315 m/s²" respectively and are equivalent to "a little uncomfortable" and "Not uncomfortable" in Table 7.5, giving an indication that the rear passenger is feeling a little uncomfortable over Case 1 road condition whereas comfortable over Case 2 road condition.

When comfort of the passengers are measured on the basis of absolute peak value of jerk of the passengers, it is seen that the peak value of jerk of both the passengers over both the road conditions exceeds the limiting value of comfort which is 18 m/s³ as per ISO standards. From the above information it is evident that the vibration causes discomfort for the passengers.

Table 7.8. Comparison between the simulation and GA results with respect to road holding.

<table>
<thead>
<tr>
<th>Measurements</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ED</td>
<td>OD</td>
</tr>
<tr>
<td>Displacement (m)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PV</td>
<td>0.0519</td>
<td>0.0773</td>
</tr>
<tr>
<td>RMS</td>
<td>0.0050</td>
<td>0.0062</td>
</tr>
<tr>
<td>Velocity (m/s)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PV</td>
<td>1.3910</td>
<td>2.1273</td>
</tr>
<tr>
<td>RMS</td>
<td>0.0956</td>
<td>0.1679</td>
</tr>
<tr>
<td>Acceleration (m/s²)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PV</td>
<td>90.8640</td>
<td>91.5882</td>
</tr>
<tr>
<td>RMS</td>
<td>4.6008</td>
<td>6.1887</td>
</tr>
<tr>
<td>Displacement (m)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PV</td>
<td>0.0732</td>
<td>0.0899</td>
</tr>
<tr>
<td>RMS</td>
<td>0.0063</td>
<td>0.0102</td>
</tr>
<tr>
<td>Velocity (m/s)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PV</td>
<td>1.7006</td>
<td>2.2234</td>
</tr>
<tr>
<td>RMS</td>
<td>0.1510</td>
<td>0.2564</td>
</tr>
<tr>
<td>Acceleration (m/s²)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PV</td>
<td>61.8503</td>
<td>66.7083</td>
</tr>
<tr>
<td>RMS</td>
<td>4.6710</td>
<td>7.1683</td>
</tr>
</tbody>
</table>

Note: OD, ED and CC implies optimal design, existing design and convex combination of OD; PV and RMS implies absolute peak value and root mean square value.
Trend of vibration of the passengers over the two roads during vehicle motion have been shown in Figure 7.14 and Figure 7.15. These figures have been plotted by taking amplitude of vibration verses time. The road holding performances have been measured from Table 7.8. Figure 7.16 and Figure 7.17 shows deflection of the suspension and the tire during the motion of the vehicle over different types of roads.

7.2.7.2 Results with GA application

The optimal or near to optimal values of the suspension parameters and the corresponding output parameters obtained after optimizing the suspension design problem using GA are shown in Table 7.6, Table 7.7 and Table 7.8. These results have been obtained by simulating the vehicle model for 15 seconds using the following assumptions:

(i) The vehicle is traveling with uniform velocity at 50 km/hr.
(ii) The model is excited by fundamental frequency of 174.53 Hz at 50 km/hr by there repetitive bumps of span 0.5 m and peak height 10.0 cm under case 1 type of road. It is excited by uneven road surface of peak height 5.0 cm and span 3.0 m under case 2 type of road.
(iii) The search space \( X \) is defined as

\[
X = \{2000 \leq k_p \leq 68000, 2000 \leq k_r \leq 19000, 400 \leq \alpha_f \leq 1200, 400 \leq \alpha_r \leq 1200\}.
\]

(iv) The suspension deflections in case 1 and case 2 roads are considered as:

\[
\begin{align*}
&f_{def_{\text{max}}} \leq 0.08 \text{ (meters)}, \quad r_{def_{\text{max}}} \leq 0.085 \text{ (meters)} \quad \text{and} \\
&f_{def_{\text{max}}} \leq 0.03 \text{ (meters)}, \quad r_{def_{\text{max}}} \leq 0.025 \text{ (meters)} \quad \text{respectively.}
\end{align*}
\]

(v) The GA is run with a 350 individuals population size \((p_{\text{size}})\).
(vi) The algorithm is terminated after 400 generations.
(vii) Crossover rate is taken as decreasing function within the interval [0.8, 0.9].
(viii) Mutation rate is a decreasing function within the interval [0.15, 0.20].

It is seen from Table 7.7 (see under OD) that during vehicle motion over case 1 and case 2 road conditions the computed RMS values of the vertical accelerations of the front passenger are 0.1457 m/s\(^2\) and 0.2973 m/s\(^2\) respectively, whereas, the same values for the rear passenger are 0.1278 m/s\(^2\) and 0.1724 m/s\(^2\) respectively. All these values are well within the comfort level “Less than 0.315 m/s\(^2\)” as per ISO 2631 and respectively indicates “Not uncomfortable” zone. This means that the passengers are not affected by the vibration of the moving vehicle.
Moreover, the absolute peak values of jerks perceived by the passengers on both the roads are found to be less than 18 m/s$^3$.

Again, comparing the values under ED and OD of Table 7.8 it is seen that there are insignificant difference in the peak and RMS values of the displacement, velocity and acceleration of the sprung and unsprung masses. This indicates that the optimal design parameters do not deteriorate road handling performance.

Now the convex combination of the optimal values of the design parameters (see Table 7.6) obtained over different roads are used as design parameters of the suspension system. To verify the validity of the parameters the vehicle model has been simulated over the same roads and with same assumptions for 15 seconds. The simulation results are shown in Table 7.7 and Table 7.8 (see under CC). It is seen from Table 7.7 (see under CC) that the RMS value of vertical accelerations of the passengers lies in the comfortable zone “Less than 0.315 m/s$^2$”. Also, the jerk experienced by the passengers does not exceed the allowable limit 18 m/s$^3$. This indicates that the vehicle vibration do not deteriorate passengers comfort.

On examining the ride comfort and road handling performances it is seen that both the optimal design parameters and parameters obtained by the convex combination show similar ride comfort and road handling performance. Moreover, both these parameters show better performance than compared to the existing design parameters. Again, comparing the results under OD and CC it is seen that, the convex combination parameters increases the maximum bouncing transmissibility of the sprung mass (objective function) from 0.0917 to 0.1058 over Case 1 road whereas decreases the maximum bouncing transmissibility of the sprung mass from 0.3437 to 0.2035 over Case 2 road. In the later case it is seen that $f_{def_{max}} \leq 0.0374$ (m) and $r_{def_{max}} \leq 0.0371$ (m) which exceeds the assumed value and hence violates the constraints $g_3(x)$ and $g_4(x)$. Although, the convex combination parameters violates the two constraints they do not deteriorate road holding performance as because the suspension deflection values are much smaller than the mechanical structure which holds the finite space under the car body and the unsprung mass. In view of the above analysis it can be said that the convex combination parameters can be considered as the design parameters of the suspension system. These parameters are better than the existing design parameters and can provide more comfortable ride to the passengers over both the roads.

Levels of vibration of the passengers over the two roads during vehicle motion have been shown in Figure 7.14 and Figure 7.15. These figures have been plotted by taking amplitude of vibration verses time. The suspension and tire deflection during the motion of the vehicle over different types of roads have been shown in Figure 7.16 and Figure 7.17.
Figure 7.14: Front passengers comfort over different roads in time domain.
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Figure 7.15: Rear passengers comfort over different roads in time domain.

Figure 7.16: Suspension deflection during motion of the vehicle over the roads.
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7.2.8 Conclusion

In this chapter, the half car model with passive suspension system, including passenger's dynamics is constructed and the suspension parameters which satisfy performance as per ISO standards are determined by applying real coded GA. The suspension parameters are obtained by solving the formulated nonlinear constrained optimization problem. For validity, the GA results are compared with the existing design parameters by simulating the vehicle model over two different road conditions. The performance of the suspension parameters are evaluated in time domain with respect to ISO 2631 standards. Results show that GA suspension parameters have better potential to improve ride comfort and road holding. Simulation results further show that if the convex combinations of the suspension parameters obtained by GA application over different roads are used as suspension parameters then it will again show better performance than the existing design parameters over all roads and will also satisfies performance as per ISO standards.