Chapter 6

Half car models without passengers seat suspensions
Model-1: A heuristic approach for optimization of vibration of a passive suspension system under different road conditions

6.1.1 Introduction

In recent years the demand for better ride comfort and vehicle safety has motivated many automobile industries for designing good suspension system. The suspension parameters are optimally designed by studying the dynamic response of the vehicle which is modeled as a complex multi-body dynamic system. Many studies have been carried out on the dynamic response and the vibration control using mathematical model with linear suspensions (Alkhatib et al., 2004; Sun et al., 2007; Shirahatt et al., 2008). But in most of the practical problems suspensions are non-linear. The study on the non-linear vehicle suspension began in the early 1990s (Hrovat, 1997; Gordon et al., 1991; Kim et al., 1998; Li et al., 2006). Those include mainly four aspects: non-linear damping, non-linear spring stiffness, non-linear tire and the non-linear control strategies.

In this chapter, the non-linear properties of the suspension spring have been introduced in the half car model. A cubic non-linear model with three parameters has been used to characterize the non-linear spring stiffness of the suspension. However, the damping characteristics of the suspension are assumed to be linear. The dynamic response of the vehicle have been studied via simulation with initial suspension parameters (Tashirad et al., 1998), under periodic and random excitations by making the vehicle run with uniform velocity over periodic and random type of road conditions. Here the mathematical expressions have been developed for the road conditions which are series of repetitive bumps in the form of triangular and trapezoidal waves at equal intervals. Due to the periodic nature of the bumps Fourier series have been applied to those expressions to represent them as a combination of series of
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sinusoids of suitable frequency, phase and amplitude. In order to determine the better values of
the suspension parameters (spring stiffness and damper coefficient) a non-linear constrained
optimization problem characterizing ride comfort as objective function and road holding and
suspension travel as constraints has been formulated. The formulated problem is then
minimized in different road conditions using GA. The ride comfort is evaluated from the
bouncing transmissibility of the sprung mass at the center of mass of the running vehicle. So,
the bouncing transmissibility is to be minimized to increase passengers comfort. The optimal
values of the suspension parameters so obtained have been compared with the existing
suspension parameters. Again, the corresponding output parameters obtained after optimization
under different road conditions have been compared with the simulated results. This is done to
verify the validity of the GA results.

6.1.2 Dynamical model of a half car with non-linear passive suspension
system

We refer to a block diagram of a complete passive suspension with four degrees of freedom as
shown in Figure 6.1. In this figure, \( m_s \) is the sprung mass, which represents the car body; \( m_{fu} \)
and \( m_{ru} \) are the front and the rear unsprung masses respectively, which represent the mass of
the wheel assembly; \( k_f \) and \( k_r \) the front and the rear tire stiffness; \( J \) the polar moment of inertia
of the car. The stiffness of the suspension springs is non-linear and the damping is linear. At
time \( t \), variables \( q_1(t) \) and \( q_4(t) \) are the vertical displacements of the front and rear unsprung
masses measured with respect to the static equilibrium position. The vehicle body motion at
time \( t \) is described by the vertical position \( q_2(t) \) of the centre of mass (again with respect to the
static equilibrium position) and the angle of rotation about the centre of mass, \( q_3(t) \). The
distance between the front and the rear axels measured in the horizontal direction is \( L \). As the
car runs over the road the functions \( f_1(t) \) and \( f_2(t) \) at any time \( t \), induces an excitations due to
irregularities in road surface at the front and rear wheel respectively. To obtain the wheel base
preview information it has been assumed that the rear wheel travels over the same path as the
front wheel, except for time delay. The excitation at the front wheel has been defined as:
\[
f_1(t) = E(t)
\]
As \( f_2(t) \) is a delayed version of \( f_1(t) \), excitation at the rear wheel has been defined as:
\[
f_2(t) = E(t-L/v)
\]
where \( L/v \) represents the time delay between the front and rear wheel axis, separated by a
distance \( L \) and \( v \) is the forward uniform velocity of a car.
The suspension of a vehicle has been simplified to non-linear springs and linear dampers. The suspension spring has been assumed to have the following characteristics:

\[ f_s = k_1 \Delta x + k_2 \Delta x^2 + k_3 \Delta x^3 \]

The above expression is a cubic polynomial representing non-linearity of the springs. Here \( f_s \) is the spring force, the constants \( k_1, k_2 \) and \( k_3 \) are the stiffness coefficients associated with the linear, quadratic and cubic portions of the spring force. The term \( \Delta x \) is the deformation of the spring that can be calculated by the displacement of both extremes of the spring. The unit of \( \Delta x \) is meter (m) and of \( k_1, k_2 \) and \( k_3 \) is Newton/meter (N/m).

The linear damping force of suspension has been given by:

\[ f_d = \alpha \Delta \dot{x} \]

Where \( f_d \) is the damping force and \( \Delta \dot{x} \) is the relative velocity of the extremes of the damper.

The model parameters values and their respective units are shown in Table 6.1. The parametric values remain the same as in reference Tashirad et al. (1998).

### Table 6.1. Half car model parameters.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
<th>Units</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_s )</td>
<td>Sprung mass</td>
<td>Kg</td>
<td>1794.4</td>
</tr>
<tr>
<td>( m_{f_{ru}} )</td>
<td>Front unsprung mass</td>
<td>Kg</td>
<td>87.15</td>
</tr>
<tr>
<td>( m_{ru} )</td>
<td>Rear unsprung mass</td>
<td>Kg</td>
<td>140.04</td>
</tr>
<tr>
<td>( k_f )</td>
<td>Front tire stiffness</td>
<td>N/m</td>
<td>10115.0</td>
</tr>
<tr>
<td>( k_r )</td>
<td>Rear tire stiffness</td>
<td>N/m</td>
<td>10115</td>
</tr>
<tr>
<td>( J )</td>
<td>Moment of inertia</td>
<td>Kg m²</td>
<td>3443.05</td>
</tr>
<tr>
<td>( l_1 )</td>
<td>Front axle length</td>
<td>m</td>
<td>1.271</td>
</tr>
<tr>
<td>( l_2 )</td>
<td>Rear axle length</td>
<td>m</td>
<td>1.713</td>
</tr>
</tbody>
</table>
6.1.2.1 Equations of motion

Assuming that the effect of pitch motion, $q_3$, is very small, the equations of motion of the dynamical system are given below. These equations have been computed in the time domain by applying the Newton's second law of motion on a mass body which is based on the following formula.

\[
\text{mass} \times \text{acceleration} = \text{net forces acting on the mass object.}
\]

The equations governing the vertical motion of the sprung mass have been expressed as:

\begin{align*}
\label{6.1a}
m \ddot{q}_2(t) &= -f_s^{\text{front}} - \alpha_f \{ \dot{q}_2(t) - \dot{q}_1(t) + l \dot{q}_3(t) \} - f_s^{\text{rear}} - \alpha_r \{ \dot{q}_2(t) - \dot{q}_4(t) - l_2 \dot{q}_3(t) \} \\
&= l_2 \left[ f_s^{\text{front}} + \alpha_f \{ \ddot{q}_2(t) - \ddot{q}_1(t) - l_2 \ddot{q}_3(t) \} \right] - l_1 \left[ f_s^{\text{rear}} + \alpha_r \{ \ddot{q}_2(t) - \ddot{q}_4(t) + l_2 \ddot{q}_3(t) \} \right]
\end{align*}

and the angular motion of the sprung mass as:

\begin{align*}
\label{6.1b}
J \ddot{q}_3(t) &= l_2 \left[ f_s^{\text{front}} + \alpha_f \{ \dot{q}_2(t) - \dot{q}_1(t) - l_2 \dot{q}_3(t) \} \right] - l_1 \left[ f_s^{\text{rear}} + \alpha_r \{ \dot{q}_2(t) - \dot{q}_4(t) + l_2 \dot{q}_3(t) \} \right]
\end{align*}

where,

\[
\begin{align*}
 f_s^{\text{front}} &= k_f \Delta x_f + k_f' \Delta x_f^2 + k_f'' \Delta x_f^3, \\
f_s^{\text{rear}} &= k_r \Delta x_r + k_r' \Delta x_r^2 + k_r'' \Delta x_r^3, \\
\Delta x_f &= \{ q_2(t) - q_1(t) + l_2 q_3(t) \} \quad \text{and} \quad \Delta x_r = k_n \{ q_2(t) - q_4(t) - l_2 q_3(t) \}
\end{align*}
\]

In the above expressions, $f_s^{\text{front}}$ and $f_s^{\text{rear}}$ denotes the front and rear suspensions spring force, $\Delta x_f$ and $\Delta x_r$ denotes the static deformation of the front and rear suspension springs with stiffness coefficients $k_f'$, $k_f''$, and $k_f'''$; and $k_r'$, $k_r''$, and $k_r'''$. The damping coefficients for the front and the rear suspensions are $\alpha_f$ and $\alpha_r$ respectively.

Again by applying Newton's second law the equations of motions of the front and the rear unsprung masses have been formulated as:

\begin{align*}
\label{6.1c}
m_i \ddot{q}_i(t) &= f_s^{\text{front}} + \alpha_f \{ \dot{q}_2(t) - \dot{q}_1(t) + l_2 \dot{q}_3(t) \} - k_f \{ q_i(t) - f_s(t) \} \\
\label{6.1d}
m_i \ddot{q}_i(t) &= f_s^{\text{rear}} + \alpha_r \{ \dot{q}_2(t) - \dot{q}_4(t) - l_2 \dot{q}_3(t) \} - k_r \{ q_i(t) - f_s(t) \}
\end{align*}

Eqs. (6.1a to 6.1d) is a system of non-linear second orders coupled simultaneous equations, where single and double dots represent first and second order derivatives with respect to time respectively. Letting $z_i = \dot{q}_i$, $i=1,2,3,4$, the above system of equations have been rewritten to a system of eight non-linear first order simultaneous Eqs. (6.2a to 6.2h). These equations have been solved numerically by the Runge-Kutta method of fourth order in time domain. The above system of equations is now expressed as:
where the functions $f_i(t)$ and $f_2(t)$ are road excitations as functions of time in the front and rear tire, respectively. For excitation the following three different types of road conditions have been used.

### 6.1.3 Road conditions for road excitation input

Road profiles play a very important role in controlling the speed limits of the vehicles. Further it prevents the road accidents, with possible loss of life and damage of the vehicle. In this work we have studied the performance of the vehicle over three different types of road conditions. Each of these roads may be interpreted as a realistic road that might be encountered during driving. The description of each of the road conditions are shown in Table 6.2. The parameters values of case 1 and case 2 road conditions are given in Table 6.3.

<table>
<thead>
<tr>
<th>Case</th>
<th>Description</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Triangular wave type road bumps placed in series at equal intervals</td>
<td>Deterministic</td>
</tr>
<tr>
<td>2</td>
<td>Trapezoidal wave type road bumps placed in series at equal intervals</td>
<td>Deterministic</td>
</tr>
<tr>
<td>3</td>
<td>Random road surface</td>
<td>Probabilistic</td>
</tr>
</tbody>
</table>
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Table 6.3. The parameters of road bump

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Parameters</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1$</td>
<td>0.25 m</td>
<td>0.15 m</td>
<td>$T_1$</td>
<td>$(w + d)/v$ s</td>
<td>$(w + d)/v$ s</td>
</tr>
<tr>
<td>$w_2$</td>
<td>0.25 m</td>
<td>0.20 m</td>
<td>$T_1$</td>
<td>$T_1/18$ s</td>
<td>$T_1/30$ s</td>
</tr>
<tr>
<td>$w_3$</td>
<td>0.15 m</td>
<td>0.15 m</td>
<td>$t_1$</td>
<td>$T_1/9$ s</td>
<td>$(7T_1)/90$ s</td>
</tr>
<tr>
<td>$w$</td>
<td>0.50 m</td>
<td>0.50 m</td>
<td>$t_2$</td>
<td>$T_1/9$ s</td>
<td>$(7T_1)/90$ s</td>
</tr>
<tr>
<td>$d$</td>
<td>4.00 m</td>
<td>4.00 m</td>
<td>$t_3$</td>
<td>-</td>
<td>$T_1/9$ s</td>
</tr>
<tr>
<td>$c$</td>
<td>0.15 m</td>
<td>0.15 m</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The triangular and trapezoidal bumps have same peak height ($c$) and span ($w$), of an assumed spatial profile. Also the effective distance ($d$) between the bumps have been considered the same. These series of repetitive bumps induces periodic excitations to the vehicle. The space functional representation of the series of repetitive bumps at equal intervals has been converted to a time function depending upon the velocity of the car at the time of movement of negotiation. The time functional representation is also periodic and is characterized as periodic functions with time period. All the road conditions have been characterized mathematically and are discussed below.

Case 1. Triangular wave type of road bumps

The time functional representation of a series of periodic triangular wave type of road bumps have been expressed mathematically as:

$$
E(t) = \begin{cases} 
\frac{c}{t_1} & 0 \leq t \leq t_1 \\
\frac{c}{t_1 - t_2} (t - t_2) & t_1 \leq t \leq t_2 \\
0 & t_2 \leq t \leq T_1 
\end{cases} \quad \forall t \text{ and } q = 1,2,...,N \tag{6.3a}
$$

where $c$ is a constant representing peak height of the bump and $N$ is a constant representing a finite number of repetitive bumps. The variable $t$ represents the time taken to complete the movement of the wheels of the vehicle over the bumps and the road surface as the vehicle travels over it with velocity $v$. Graphical representation of this function together with its periodic extension is shown in Figure 6.2.
Figure 6.2: A series of triangular wave type road bumps

As the function is periodic with period $T_1$, it has been represented by a series of sinusoids of suitable frequencies, amplitudes and phases. However, for periodic functions, the domain over which the approximation is required is only one period of the periodic function. The rest of the function is taken care of by the definition of periodicity in the function. Therefore, expanding (6.3a) by Fourier series taking into account only one cycle of the wave i.e., $q = 1$ we have

$$E(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos(n\omega t) + b_n \sin(n\omega t)]$$

(6.3b)

The coefficients $a_0$, $a_n$ and $b_n$ are Fourier constants defined by

$$a_0 = \frac{2}{T_1} \int_0^{T_1} E(t) dt, \quad a_n = \frac{2}{T_1} \int_0^{T_1} E(t) \cos(n\omega t) dt, \quad b_n = \frac{2}{T_1} \int_0^{T_1} E(t) \sin(n\omega t) dt$$

where $\omega = \frac{2\pi}{T_1}$ is the fundamental frequency.

Using (6.3a) and (6.3b) we have

$$a_0 = \frac{ct}{T_1}, \quad a_n = \frac{2c}{T_1 \omega^2} \left[ \frac{1}{T_1} \cos(n\omega t_1) + \frac{1}{T_1} \left( \frac{1}{n^2} \left( \cos(n\omega t_2) - \cos(n\omega t_1) \right) - \frac{1}{n^2} \right) \right], \quad \text{for all } n$$

and $b_n = \frac{2c}{T_1 \omega^2} \left[ \frac{1}{T_1} \sin(n\omega t_1) + \frac{1}{T_1} \left( \frac{1}{n^2} \left( \sin(n\omega t_2) - \sin(n\omega t_1) \right) \right) \right], \quad \text{for all } n$.

Hence (6.3b) reduces to

$$E(t) = \frac{ct}{2T_1} + \frac{2c}{T_1 \omega^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \left[ \frac{1}{T_1} \cos(n\omega t_1) + \frac{1}{T_1} \left( \frac{1}{n^2} \left( \cos(n\omega t_2) - \cos(n\omega t_1) \right) - \frac{1}{n^2} \right) \right] \cos(n\omega t)$$

$$+ \frac{2c}{T_1 \omega^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \left[ \frac{1}{T_1} \sin(n\omega t_1) + \frac{1}{T_1} \left( \frac{1}{n^2} \left( \sin(n\omega t_2) - \sin(n\omega t_1) \right) \right) \right] \sin(n\omega t)$$

which is a function of infinite series. Considering the bump parameters as given in Table 6.3, it has been seen that with the increase in the values of $n$, the amplitudes ($a_n$ and $b_n$) of the
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Harmonic terms \((a_n \cos(n\omega t) + b_n \sin(n\omega t))\) decreases. So, a finite number of terms has been considered which approximates the function \(E(t)\). Figure 6.3 show graph of an approximated function \(E(t)\). This function serves as the road input for the front wheel. The rear wheel also have the same road input but with time delay as shown in the same figure. This road input will help to induce forced vibration to the moving vehicle through its wheels.

![Figure 6.3: Road input for the front and rear wheel.](image)

Case 2. Trapezoidal wave type of road bumps

This wave has been defined by the function \(E(t)\) as follows:

\[
E(t) = \begin{cases} 
\frac{c}{t_1}, & 0 \leq t \leq t_1 \\
\frac{c}{t_1-t_3}(t-t_3), & t_2 \leq t \leq t_3 \\
0, & t_3 \leq t \leq T_1 
\end{cases}
\]

and \(E(t + qT) = E(t)\), \(q = 1,2,\ldots, N\)

where \(c\) is a constant representing peak height of the bump and \(N\) is a constant representing a finite number of repetitive bumps. The variable \(t\) represents the time taken to complete the movement of the wheels of the vehicle over the bumps and the road surface as the vehicle travels over it with velocity \(v\). Graphical representation of this function together with its periodic extension is shown in the following figure.

![Figure 6.4: A series of trapezoidal wave type road bumps](image)
As the function $E(t)$ is periodic with period $T_1$, it has been represented by a series of sinusoids of suitable frequencies, amplitudes and phases. As in Case 1 the Fourier series of this wave for $q = 1$ has been expressed as:

$$E(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[ a_n \cos(nwtt) + b_n \sin(nwtt) \right]$$

where the Fourier constants are given below:

$$a_0 = \frac{c}{T_3} (-t_1 + t_2 + t_3),$$

$$a_n = \frac{2c}{T_1 n^2 w^2} \left[ \frac{1}{t_1} \cos(nwtt) + \frac{1}{(t_2 - t_3)} \left\{ \cos(nwtt) - \cos(nwtt_3) \right\} - \frac{1}{t_3} \right], \text{ for all } n$$

and

$$b_n = \frac{2c}{T_1 n^2 w^2} \left[ \frac{1}{t_1} \sin(nwtt) + \frac{1}{(t_2 - t_3)} \left\{ \sin(nwtt_3) - \sin(nwtt_2) \right\} \right], \text{ for all } n$$

$\omega = \frac{2\pi}{T_1}$ is the fundamental frequency.

Taking the bump parameters as given in Table 6.3, it has been seen that with the increase in the values of $n$, the amplitudes ($a_n$ and $b_n$) of the harmonic terms ($a_n \cos(nwtt) + b_n \sin(nwtt)$) gradually becomes smaller and therefore a finite number of terms is considered which approximates the function $E(t)$. Figure 6.5 shows $E(t)$ as approximated by the sum of the first thirty terms of the series. This function serves as the road input for the front wheel. The rear wheel also have same road input but with time delay as shown in the same figure.

![Figure 6.5: Road input for the front and rear wheel.](image)

**Case 3. Random road surface**

Here Root-Mean-Square (RMS) values of excitations have been used to generate artificial random road surface. In general, under random excitation the magnitude of excitation acting as an input of the dynamical system at a given time cannot be predicted. In this case a large
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collection of records of the excitation may exhibit some statistical regularity. It is possible to estimate the average such as mean, mean square values or root mean square values of excitation \{x_i, x_2, ..., x_n\}. To produce a random excitation \(x_i\) \((i=1, 2, ..., n)\), a standard random number generator like the Gaussian distribution function is generally used. This distribution function is defined by the following probability density function:

\[
p(m - \lambda \sigma \leq x_i \leq m + \lambda \sigma) = \frac{1}{\sqrt{2\pi}\sigma} \int_{m-\lambda \sigma}^{m+\lambda \sigma} e^{-\frac{1}{2} \left(\frac{x_i - m}{\sigma}\right)^2} \, dx_i,
\]

(6.4)

\[\lambda = \sqrt{-2 \times \log(y_1) \cos(2\pi y_2)}\]

where \(m\) and \(\sigma\) denotes the mean and standard deviation of \(x_i\) and \(\lambda\) a real number defining the deviation from the mean. A very important property of this distribution function is that it is invariant with respect to linear transformation. This means that if the excitation is a Gaussian distribution function, the response will generally be a different distribution function but still a normal one. The only changes are that the magnitude of the mean and standard deviation of the response are different from those of excitation.

In this graph we have indicated the area within 1, 2, and 3 times of standard deviations to the mean as equal, respectively, to 68.27%, 95.45%, and 99.73% of the total area. Applying the fundamental transformation law of probabilities a transformation function represented by (6.4) with zero mean and unit standard deviation is defined as follows:

\[x_i = m + \lambda \sigma\]

Figure 6.6: Gaussian distribution function
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The random input \((x_{RMS})\) at any time \(t\), is produced by taking root mean square of a collection of \(n\) excitations \(\{x_1, x_2, ..., x_n\}\) as defined below:

\[
x_{RMS}(t) = \sqrt{\frac{1}{n} \sum_{i=1}^{n} x_i^2}
\]

The algorithm developed for generating a random excitation \(x_i\) \((i=1, 2, ..., n)\), is given as follows:

**ALGORITHM 6.1**

begin
Input the range i.e., the upper and lower bounds \(\bar{x}(t)\) and \(\underline{x}(t)\) respectively for the \(N(m, \sigma)\) distribution function, from which the variable \(x_i\) will be chosen at random.

Assign \(m \leftarrow \frac{\bar{x}(t) + \underline{x}(t)}{2}\) and \(\sigma \leftarrow \frac{\bar{x}(t) - m}{3}\), so as to cover 99.73% of the specified region, \(m\) and \(\sigma\) being the mean and the standard deviation respectively.

repeat
Compute \(\lambda \leftarrow \text{sqrt}(-2 \times \log(y_1)) \times \cos(2\pi y_2)\), where \(y_1\) and \(y_2\) are random numbers generated within \((0, 1)\).

until \((\lambda > -3.0\) or \(\lambda < 3.0\))

Assign \(x(t) \leftarrow m + \sigma \lambda\).

return \(x\).
end

The random input for a series of discrete time \(t\) measured in seconds and for \(\bar{x}(t) = 0\) meters and \(\underline{x}(t) = 0.05\) meters is shown in Figure 6.7.

![Figure 6.7: Disturbance input from the ground](image-url)
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6.1.4 GA based solution approach for suspension design

The general scheme of the GA for solving the suspension design problem is give as follows:

6.1.4.1 Solution representation and population initialization
The values of the suspension design parameters viz., springs stiffness (\(k^f_i\), \(k^r_i\) and \(k^f_i\) for front; \(k^r_i\), \(k^r_i\) and \(k^f_i\) for rear) and dampers coefficients (\(\alpha_f\), \(\alpha_r\)) have been used as decision variables. An initial parent population \(V_1, V_2, \ldots, V_{\text{popsize}}\) of popsize number of chromosomes is created randomly using uniform distribution function, in which the decision variables are chosen from respective bounds. The chromosome designating the \(j\)th individual of the population is given as below:

\[ V_j = [k^f_1, k^r_1, k^f_1, k^r_1, k^f_1, k^r_1, \alpha_f, \alpha_r] \]

Thus, the chromosome \(V_j\) is encoded in the form of an array of real numbers in which each gene represents a suspension design parameters. These chromosomes are directly used for evaluating the fitness function.

6.1.4.2 Objective function and constraints
The performance characteristics which are of most interest while designing the vehicle suspension are passenger’s ride comfort, road holding and suspension travel. The ride comfort is related to how much comfortable the passenger feels while riding in the vehicle. The ride comfort can be inferred by analyzing the sprung mass body dynamics. Several factors can adversely affect the ride comfort. The factors include large vertical sprung mass acceleration, large vertical sprung mass displacement, the sprung mass jerk (the time derivative of acceleration) and bounce. The suspension travel is related to relative displacement between the unsprung mass and the sprung mass and road handling is related to tire displacement.

In the optimization process the value of the design parameters, the spring stiffness and the damper coefficients have been determined by minimizing the bouncing transmissibility of the sprung mass, with respect to the set of technological constraints. The bouncing transmissibility is the ratio of the maximum deflection amplitude of the sprung mass to the excited displacement amplitude (Amp) imposed during motion. It has been computed after solving (6.2a to 6.2h) by applying fourth order Runge Kutta method in time domain with the initial conditions that the system is in equilibrium state initially. The objective function has been therefore expressed as:
Minimize $f(x), f(x) = \max_{0 \leq t \leq T} \left| q_1(t) \right|$

where $x = \{k_1', k_2', k_3', k_4', k_5', \alpha_f, \alpha_r\}$ and $a_1 \leq k_1' \leq b_1, a_2 \leq k_2' \leq b_2, a_3 \leq k_3' \leq b_3, c_1 \leq k_4' \leq d_1, c_2 \leq k_5' \leq d_2, c_3 \leq k_6' \leq d_3, e \leq \alpha_f \leq f, g \leq \alpha_r \leq h$

The constraints we have considered are as follows:

(i) $g_1(x) = \left| q_2 - q_1 + l_1q_3 \right| - f_{\text{def max}} \leq 0$; $g_2(x) = \left| q_2 - q_4 - l_2q_3 \right| - r_{\text{def max}} \leq 0$

where $f_{\text{def max}}$ and $r_{\text{def max}}$ are maximum allowable suspension deflection near the front wheel and the rear wheel respectively. These constraints are considered so that at any time the suspension will not hit the car body.

(ii) $g_3(x) = S_f - k_f \leq 0$ and $g_4(x) = S_r - k_r \leq 0$

These constraints assure that the spring stiffness does not exceed the tire stiffness and keep the natural frequency of the sprung mass less than the exciting frequency. Moreover, large values of tire stiffness maintain the circular shape of the tire under carload and improve the ride quality of the vehicle. Here $S_f$ and $S_r$ represents the front and the rear suspension spring stiffness. The spring stiffness can be calculated as

Spring stiffness = \frac{\text{Spring force}}{\text{Spring deformation}}

(iii) $g_5(x) = \sqrt{S_f m} - \sqrt{k_p m} \leq 0$ and $g_6(x) = \sqrt{S_r m} - \sqrt{k_r m} \leq 0$

These constraints always prevent the sprung mass natural frequency from exceeding the unsprung mass natural frequency. These keeps the bouncing transmissibility of the sprung mass caused by the road disturbances at a lower value and therefore provides a more comfortable ride to the passengers. Here $S_f$ and $S_r$ represents the front and the rear suspension spring stiffness.

(iv) $g_7(x) = \left| q_1 - f_1 \right| - \frac{9.81 \times (m_f + m_p)}{k_p} \leq 0$ and $g_8(x) = \left| q_4 - f_2 \right| - \frac{9.81 \times (m_r + m_p)}{k_r} \leq 0$

In order to ensure a firm uninterrupted contact of wheels to road, the dynamic tire deflection should not exceed the static tire deflection. The static deflection of the tire spring is defined as follows:

Static deflection of the tire spring = \frac{\text{Total load on the tire spring}}{\text{Tire spring constant}}
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Using the given parameter values (see Table 6.1) the static deflections of the front and the rear tire springs are found to be 0.182545 and 0.187676 respectively.

In relation to the above conditions, the optimization problem has therefore been formulated as a non-linear constrained optimization problem, which is defined as follows:

\[
\begin{align*}
\text{Minimize } & f(x) \\
\text{subject to } & g_i(x) \leq 0, k = 1, 2, \ldots, 8; \ x = \{k'_1, k'_2, k'_3, k'_4, \alpha_f, \alpha_r\} \\
\text{and } & a_i \leq k'_i \leq b_i, \ c_i \leq k'_i \leq d_i, \ e \leq \alpha_f \leq f, \ g \leq \alpha_r \leq h; \ i = 1, 2, 3
\end{align*}
\]

6.1.4.3 Fitness evaluation

During each generation, the chromosomes are evaluated using some measures of fitness. In case of constraint optimization problems, generally, fitness value is calculated by adding a penalty term to the objective function. To handle those constraints we have applied the Superiority of Feasible point Penalty method (SFP) (Kaisa et al., 2003). Based on the SFP method the fitness value of the chromosome is calculated from the fitness function, which is defined as

\[
\text{Minimize } \tilde{f}(x) = f(x) + R \left( \sum_{j=1}^{8} \max[0, g_j(x)] \right) + \theta(x), \quad (6.6)
\]

\(f(x)\) is the objective function considered, \(g_j(x) \ (j = 1, 2, \ldots, 8)\) are the constraints, \(R\) the penalty coefficient and \(\theta(x)\) is a function defined as

\[
\theta(x) = \begin{cases} 
0 & \text{if } X \cap S = \emptyset \text{ or } x \in S \\
\alpha & \text{otherwise}
\end{cases}
\]

where \(\alpha = \max \left[ 0, \max_{y \in X \cap S} f(y) - \min_{z \in X \setminus S} \left( f(z) + R \left( \sum_{j=1}^{8} \max[0, g_j(z)] \right) \right) \right] \)

Here the feasible region \(S \neq \emptyset\) is bounded by box constraints

\(X= (a_i \leq k'_i \leq b_i, \ c_i \leq k'_i \leq d_i, \ e \leq \alpha_f \leq f, \ g \leq \alpha_r \leq h; \ i = 1, 2, 3)\)

and inequality constraints

\(g_j(x) \leq 0 \quad \text{for } j = 1, 2, \ldots, 8\)

and is defined as

\[
S = \{ x = (k'_1, k'_2, k'_3, k'_4, \alpha_f, \alpha_r) \in \mathcal{Y}: x \leq \bar{x} \text{ and } g_j(x) \leq 0 \text{ for } j = 1, 2, \ldots, 8 \}
\]
6.1.4.4 Selection

Selection operator follows the rule of the nature's survival of the fittest mechanism. Here fitter solutions survive while weaker ones perish. In GA, the fittest solution receives a higher number of offspring and thus has a higher chance of surviving in the subsequent generation. In this work, we adopt the tournament selection scheme with replacement. This selection process is strictly based on the following choices, taking two chromosomes at a time.

6.1.4.5 Crossover and mutation

In our work we have applied multi-parent whole arithmetical crossover operation with variable rate (decreasing in sense) dependent on the age of population. For mutation operation whole nonuniform mutation operator is applied with variable rate (decreasing in sense) dependent on the age of population.

6.1.4.6 Steps of GA for solving the suspension design problem

**ALGORITHM 6.2**

*Begin*

Set $t \leftarrow 0$ [t represents the current generation].

Create an initial population $P(t)$, of popsize number of chromosomes whose genes are chosen randomly from its bounds.

Evaluate the objective function (6.5) and the constraints after solving the set of dynamical equations (6.2a to 6.2h) by the fourth order Runge-Kutta method, considering static equilibrium conditions as initial conditions i.e., $q_i(0) = 0$ and $q_{ii}(0) = 0$ (i=1, 2, 3, 4) where $t \in [0, T]$.

Evaluate the fitness value (6.6) and the value of the output parameters (1 peak and Root-Mean-Square values of displacement, velocity, acceleration and jerk of sprung mass; 2. peak and Root-Mean-Square values of displacement, velocity and acceleration of unsprung masses) of each chromosome.

*Repeat*

(i) Increase $t$ by unity for next generation.

(ii) Select $P(t)$ from $P(t-1)$ by tournament selection operator of size four.

(iii) With probability $p_c$, which decreases with increase in generation number apply multiparent whole arithmetical crossover operator.

(iv) With probability rate $p_m$, which decreases with increase in generation number apply whole non-uniform mutation operator.

(v) Evaluate the fitness value and the value of the output parameters of the problem.

(vi) Apply Elitism of size three.
Until \( t \) reaches maximum generation.
Return the best found solution together with the corresponding fitness function value and its output parameters.
End

6.1.5 Simulation results and analysis

Here simulation results and analysis have been carried out in two different parts. In the first part, numerical simulation (NS) has been done while in the other part GA optimization has been carried out to determine the spring stiffness and damper coefficients of the front and the rear suspensions. The main objective in this study is to improve the performance of the suspension system. For this purpose, two different performance indexes; ride comfort and road handling have been evaluated. The ride comfort characteristic has been inferred by analyzing the sprung mass vibration behavior whereas, the road handling characteristic has been inferred by analyzing the unsprung mass vibration behavior.

NS has been done with data's taken from Taghirad et al., (1998) and is listed in Table 6.1 and Table 6.4. Here the system of Eqs. (6.2) has been solved by applying Runga-Kutta method of fourth order, considering static equilibrium conditions as initial conditions i.e., \( q_i(0) = 0 \) and \( \dot{q}_i(0) = 0 \) (\( i = 1, 2, 3, 4 \)) where \( t \in [0.0, 15.0] \) and is measured in seconds. For the road excitation functions \( f_f(t) \) and \( f_r(t) \), defined in equation (6.2f & 6.2h) three different road conditions (Case 1 to Case 3) have been considered. In Case 3 type of road condition the random road roughness is taken as 3.0 m wide. The car velocity is taken as 50 km/hr. Table 6.5, shows the output parameters obtained after simulation over the three road inputs. The output parameters consist of the absolute peak and RMS values of bounce, displacement, velocity, acceleration and jerk of the sprung mass and absolute peak and RMS values of displacement, velocity and acceleration of the unsprung masses. The motion of the vehicle over these road conditions during the entire simulation, which is carried for 15.0 sec has been shown graphically in Figures 6.8, 6.10 and 6.12. These graphs show bounce, jerk, displacement, velocity and acceleration of the sprung mass plotted in time domain. In Figures 6.9, 6.11 and 6.13 the curves of the tire deflection have been plotted, here the negative deflection means that the tire is in contact with the road and the figures reflects road handling performance. In these figures, \((q_1 - f_1)\) and \((q_2 - f_2)\) represents plots of the front and rear tire deflection respectively. In the same figures, \((q_2 - q_1 + l_1(q_3))\) and \((q_2 - q_1 - l_2(q_3))\) represents plots of
Chapter 6. Half car models without passengers seat suspensions

the front and rear suspension deflection under different road conditions. These results have been later compared with those of GA optimized solutions.

When solving the suspension design problem by applying GA optimization technique, the model parameters used are given in Table 6.1. The GA parameters that we have used are given below:

(i) Initial population size \( p_{\text{size}} \): 120

(ii) Maximum number of generation \( m_{\text{gen}} \): 150

(iii) Crossover rate is a decreasing function \( p_c(t) = p_c(0)\exp(-\alpha t) \), within the interval \([0.8, 0.9]\), where \( \alpha = \frac{1}{m_{\text{gen}}} \log \left( \frac{p_c(m_{\text{gen}})}{p_c(0)} \right) \).

(iv) Mutation rate is a decreasing function \( p_m(t) = p_m(0)\exp(-\beta t) \), where \( \beta = \frac{1}{m_{\text{gen}}} \log \left( \frac{p_m(m_{\text{gen}})}{p_m(0)} \right) \) within the interval \([0.15, 0.2]\).

(v) Tournament selection of size four and elitism of size three.

Here, equations of motion (6.2a to 6.2h) of the dynamical system have been solved numerically in time interval \([0.0, 15.0]\) which is measured in seconds. The same road conditions used in NS (Case 1 to Case 3) have been considered as road excitation functions. The car velocity also remains the same. The design parameters \( k'_f, k'_r, k'_f, k'_r, k'_f, k'_r, \alpha_f \) and \( \alpha_r \) have been initialized from the domain \([0, 68000], [0, 68000], [0, 68000], [0, 19000], [0, 19000], [0, 19000], [0, 1200], [0, 1200] \) randomly. The front deflection \( f_{\text{defmax}} \) and rear deflection \( r_{\text{defmax}} \) are limited to 0.053 meters, 0.073 meters and 0.032 meters over Case 1, Case 2 and Case 3 road conditions respectively. The optimal solutions so obtained are presented in Table 6.4 and have been also compared with the design parameters used in NS. The fitness value (bounce of sprung mass) and the corresponding output parameters so obtained have been shown in Table 6.5. The output parameters have been analyzed as follows: (i) the peak absolute values of displacement, velocity, acceleration and jerk of the sprung mass and the RMS values of bounce, displacement, velocity, acceleration and jerk of the sprung mass and (ii) the peak absolute values and the RMS values of displacement, velocity and acceleration of the front and rear unsprung masses. The curve shown in Figure 6.14, 6.15 and 6.16 for Case 1, Case 2 and Case 3 road conditions respectively illustrates the behavior obtained for non-linear spring. In these figures the spring force \( f'_{\text{fmax}} \) and \( f'_{\text{rmax}} \) have been plotted against the suspension deflection \( \Delta x_f \) and \( \Delta x_r \) respectively.
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### Table 6.4. Design parameters of suspension systems

<table>
<thead>
<tr>
<th>Notations</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{1}$</td>
<td>N/m</td>
<td>66824.2</td>
</tr>
<tr>
<td>$k_{2}$</td>
<td>N/m</td>
<td>0</td>
</tr>
<tr>
<td>$k_{3}$</td>
<td>N/m</td>
<td>0</td>
</tr>
<tr>
<td>$k_{4}$</td>
<td>N/m</td>
<td>18615</td>
</tr>
<tr>
<td>$k_{5}$</td>
<td>N/m</td>
<td>0</td>
</tr>
<tr>
<td>$k_{6}$</td>
<td>N/m</td>
<td>0</td>
</tr>
<tr>
<td>$k_{7}$</td>
<td>N/m</td>
<td>1190</td>
</tr>
<tr>
<td>$k_{8}$</td>
<td>N/m</td>
<td>1000</td>
</tr>
</tbody>
</table>

### Table 6.5. Ride comfort and Road handling comparisons between NS and GA output parameters under different road conditions.

<table>
<thead>
<tr>
<th>Measurements</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bounce</td>
<td>PV</td>
<td>RMS</td>
<td>PV</td>
</tr>
<tr>
<td>Disp (m)</td>
<td>0.078939</td>
<td>0.016612</td>
<td>0.100113</td>
</tr>
<tr>
<td>Vel (m/s)</td>
<td>0.072317</td>
<td>0.016228</td>
<td>0.096065</td>
</tr>
<tr>
<td>Acc (m/s²)</td>
<td>0.110559</td>
<td>0.023277</td>
<td>0.139695</td>
</tr>
<tr>
<td>Jerk (m/s³)</td>
<td>0.012172</td>
<td>0.022976</td>
<td>0.136144</td>
</tr>
</tbody>
</table>

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</tr>
<tr>
<td>Jerk (m/s³)</td>
<td>0.012172</td>
<td>0.022976</td>
<td>0.136144</td>
</tr>
</tbody>
</table>

Note: PV and RMS imply absolute peak and root mean square values respectively.
With the obtained solutions the curves showing the levels of vibrations of bounce, jerk, displacement, velocity and acceleration of the sprung mass verses time over different road conditions have been plotted in Figures 6.8, 6.10 and 6.12. The curves showing suspensions and tire deflections have been plotted in Figures 6.9, 6.11 and 6.13. From these figures the performance of the NS and GA design parameters has been then compared. The convergence graph showing the evolution of GA in different generations for different road input conditions have been shown in Figure 6.17. In these graph two curves is plotted, one showing the evolution of best found solution and the other showing the evolution of average solution in the population with increase in generation number.

For solving this problem the algorithms for NS and GA optimization have been coded in C programming and implemented on a Pentium (IV), 2.66 GHZ with 512 MB RAM PC in Linux environment.

Figure 6.8: Comparison between sprung mass vibration levels in NS and GA results under Case 1 road excitation in time domain.
Figure 6.9: Suspension and tire deflection over Case 1 road excitation in time domain with NS and GA optimal solutions.
Figure 6.10: Sprung mass vibration levels over Case 2 road excitation in time domain with NS and GA results.
Figure 6.11: Suspension and tire deflection over Case 2 road excitation in time domain with NS and GA optimal solutions.
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<table>
<thead>
<tr>
<th>NS</th>
<th>GA</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Bounce" /></td>
<td><img src="image" alt="Bounce" /></td>
</tr>
<tr>
<td><img src="image" alt="Jerk" /></td>
<td><img src="image" alt="Jerk" /></td>
</tr>
<tr>
<td><img src="image" alt="Disp" /></td>
<td><img src="image" alt="Disp" /></td>
</tr>
<tr>
<td><img src="image" alt="Vel" /></td>
<td><img src="image" alt="Vel" /></td>
</tr>
<tr>
<td><img src="image" alt="Acc" /></td>
<td><img src="image" alt="Acc" /></td>
</tr>
</tbody>
</table>

Figure 6.12: Sprung mass vibration levels over Case 3 road excitation in time domain with NS and GA results.
Figure 6.13: Suspension and tire deflection over Case 3 road excitation in time domain with NS and GA optimal solutions.

Figure 6.14: The cubic non-linear stiffness of the suspension spring over case 1 road condition.
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Figure 6.15: The cubic non-linear stiffness of the suspension spring over case 2 road condition.

Figure 6.16: The cubic non-linear stiffness of the suspension spring over case 3 road condition.
6.1.6 Conclusion

In this chapter, half car passive suspension model with non-linear suspension characteristics has been considered. In order to find the suitable values of the suspension parameters viz., coefficients of the cubic polynomial representing spring stiffness and damper coefficients, the formulated non-linear constrained optimization problem has been solved by applying GA in time domain over three different road conditions. Design objective such as; maximum bouncing transmissibility of the sprung mass, suspension deflection, road holding and tire deflection has been introduced in the formulated problem for improving ride comfort and road holding. The suspension parameters obtained has been compared with the existing ones. In order to verify the validity, GA results have been further compared with those obtained by simulating the vehicle model with the existing suspension parameters over the same roads and with same uniform velocity. Comparisons show that GA results are better than the simulated results. Robustness of the GA has been proved by the convergence graph, showing the evolution of the best and the average objective value of the individuals in the population in consecutive generations. This means that the suspension parameters obtained may be either global or near to the global solution. Further, by simulating the vehicle model it can be proved that if the average of the suspension parameters obtained over different roads is used, ride will be comfortable on any type of road conditions. However, more comfort will be experienced if the optimized design parameters obtained over a particular road is used for that particular road.

Figure 6.17: A typical evolving process of the fitness function for different road input conditions.
Model-2: Application of Real Coded Genetic Algorithm in optimum suspension design

6.2.1 Introduction

In this chapter, the half car model with linear suspension springs and linear dampers has been considered to study the vibration behavior of a passively suspended vehicle, running with a uniform velocity over different types of road conditions. Over the years, studies have been carried out on vehicle suspension optimization by researches under artificially generated road conditions with sine-wave holes (Caponetto et al., 2003), single sinusoidal bump (Baumal et al., 1998; Shirahatt et al., 2008; Lin et al., 1997; Hač et al., 1992), double sinusoidal bumps of different amplitudes (Liu et al., 2006), Power Spectral Density function (Uys et al., 2007; Marzbanrad et al., 2004), Gaussian white noise etc. These road excitations induce uncomfortable vibration in the vehicle encouraging researchers in analyzing the dynamic response of the vehicle by computer modeling and simulation.

In the present study, periodic and random excitations have been incorporated into vehicle model as road conditions for designing the suspension parameters. The periodic excitations have been superposed into harmonic excitations by taking the Fourier series expansion of the series of bumps which are in the form of triangular and trapezoidal waves installed at equal intervals. These types of road conditions may exist in real life. However, random excitation is induced taking the root mean square (RMS) of random numbers generated from the Gaussian distribution function. The optimal values of the suspension parameters have been obtained over each road by minimizing the bouncing transmissibility of the sprung mass satisfying technological constraints in time domain by applying GA. The suspension parameters of the car model have been than characterized by taking the average values of the suspension parameters obtained over these three types of roads. In order to validate the

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suspension parameters, simulation has been done in time domain over variety of road conditions of periodic, random and sinusoidal types. These results have been then compared with the simulation results obtained using the existing design parameters of the vehicle model (Taghirad et. al., 1998). Comparisons have been also done graphically in time and frequency domain.

6.2.2 Dynamical model of a half car with linear passive suspension

The two-dimensional model of a car with completely passive suspension systems with linear springs and dampers has been considered here and is shown in Figure 6.18.

![Block diagram of a half car model with passive suspension with linear springs and dampers.](image)

Figure 6.18: A block diagram of a half car model with passive suspension with linear springs and dampers.

The model has four degrees of freedom with the following characteristics:

(i) The front and rear wheel unsprung masses are under vertical motion.

(ii) The sprung mass has bounce and pitch.

The model parameters values and their respective units have been shown in Table 6.1.

6.2.2.1 Equations of motion

Assuming that the characteristics of all passive suspension elements are linear and that the effect of pitch motion, \( q_3 \), is very small, the equations of motion of the dynamical system are given below. These equations have been computed in the time domain by applying the Newton's second law of motion.
The equation for the vertical motion of the sprung mass at the centre of gravity of the vehicle has been expressed as:

\[ m_1 \ddot{q}_1(t) = -F_1 - F_2 - F_3 - F_4 \]  

(6.7a)

The equation for the pitch/ angular motion of the sprung mass has been expressed as:

\[ J_1 \ddot{\beta}(t) = l_2 \{ F_3 + F_4 \} - l_1 \{ F_1 + F_2 \} \]  

(6.7b)

The equations of motion for the front and rear unsprung masses have been expressed as:

\[ m_2 \ddot{q}_2(t) = F_1 + F_2 - F_3 \]  

(6.7c)

and 

\[ m_3 \ddot{q}_4(t) = F_3 + F_4 - F_6 \]  

(6.7d)

where

\[ F_1 = k_f \{ q_2(t) - q_1(t) + l_1 q_3(t) \}, \quad F_2 = \alpha_f \{ q_2(t) - \dot{q}_1(t) + l_1 \dot{q}_3(t) \}, \quad F_3 = k_r \{ q_2(t) - q_3(t) - l_2 q_3(t) \}, \quad F_4 = \alpha_r \{ q_2(t) - q_4(t) - l_2 \dot{q}_3(t) \}, \quad F_5 = k_f \{ q_1(t) - f_1(t) \} \]

and \( F_6 = k_r \{ q_4(t) - f_2(t) \} \)

Eqs. (6.7a to 6.7d) is a system of linear second orders coupled simultaneous equations, where single and double dots represent first and second order derivatives with respect to time respectively. Letting \( z_i = \dot{q}_i \), \( i=1,2,3,4 \), the above system of equations have been reduced to a system of linear first order simultaneous equations (6.8). These equations have been solved by Runge-Kutta method of fourth order. The above system of equations can now be expressed as:

\[ \dot{X}(t) = AX(t) + B(t) \]  

(6.8)

where

\[ \dot{X}(t) = \begin{bmatrix} \dot{z}_1(t), \dot{z}_2(t), \dot{z}_3(t), \dot{z}_4(t), \dot{q}_1(t), \dot{q}_2(t), \dot{q}_3(t), \dot{q}_4(t) \end{bmatrix}^T, \]

\[ X(t) = \begin{bmatrix} z_1(t), z_2(t), z_3(t), z_4(t), q_1(t), q_2(t), q_3(t), q_4(t) \end{bmatrix}^T, \]

\[ A = \begin{bmatrix} -\alpha_f & \frac{\alpha_f}{m_1} & \frac{\alpha_f}{m_2} & 0 & -\frac{(k_f + k_r)}{m_3} & \frac{k_r}{m_3} & \frac{k_f}{m_3} & 0 \\
\frac{\alpha_f}{m_1} & -\frac{(\alpha_f + \alpha_r)}{m_1} & \frac{\alpha_f}{m_2} & \frac{\alpha_f}{m_1} & -\frac{(k_f + k_r)}{m_2} & \frac{(k_r l_2 - k_f l_1)}{m_2} & \frac{k_f l_1}{m_2} & 0 \\
\frac{\alpha_f}{m_2} & \frac{\alpha_f}{m_1} & -\frac{(\alpha_f + \alpha_r)}{m_2} & \frac{\alpha_f}{m_1} & -\frac{(k_f + k_r)}{m_2} & \frac{(k_r l_2 + k_f l_1)}{m_2} & \frac{k_f l_1}{m_2} & 0 \\
0 & \frac{\alpha_f}{m_1} & \frac{\alpha_f}{m_2} & -\frac{\alpha}{m_1} & \frac{k_r}{m_3} & \frac{-k_r l_2}{m_3} & \frac{-k_f l_1}{m_3} & 0 \\
0 & 0 & 0 & \frac{\alpha_f}{m_2} & \frac{k_r}{m_3} & \frac{-k_r l_2}{m_3} & \frac{-k_f l_1}{m_3} & 0 \\
0 & 0 & 0 & 0 & \frac{k_f}{m_3} & \frac{-k_f l_2}{m_3} & \frac{-k_r l_1}{m_3} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{\alpha}{m_3} & \frac{\alpha}{m_3} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{\alpha}{m_3} & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \]
and \[ B(t) = \begin{bmatrix} k_n f_1(t) & 0 & 0 & k_n f_2(t) & 0 & 0 & 0 \end{bmatrix}^T. \]

The functions \( f_1(t) \) and \( f_2(t) \) at any time \( t \), describe the ground motion at the two tire positions due to variations of the road profile. We consider two such functions as sinusoidal and random uneven surface. The different types of roads to be used in the study are discussed below.

6.2.3 Road conditions for ride analysis

The road surface is a natural changing condition for vehicle. There are many possible ways to analytically describe the road input, which can be classified as shock or vibration (Hrovat, 1997). Shocks are the discrete events of relatively short duration and high intensity, e.g. a pronounced bump or pothole on road. Vibrations, on the other hand, are characterized by prolonged and consistent excitations that are called rough roads. This section discusses the different types of road conditions which we shall use as road inputs while designing the suspension parameters using GA. These roads induce shock and high vibration levels in car during it motion over them. These different amplitudes of road inputs conditions will also be used in the simulation study to test the performance of the obtained suspension parameters.

Road 1. Triangular wave type of road bumps

The time functional representation of a series of periodic triangular wave type of road bumps have been expressed mathematically as

\[
E(t) = \begin{cases} 
\frac{c}{t_1} t, & 0 \leq t \leq t_1 \\
\frac{c}{t_1 - t_2} (t - t_2), & t_1 \leq t \leq t_2 \\
0, & t_2 \leq t \leq T_1 
\end{cases}
\]

and \( E(t + qT_1) = E(t) \forall t \) and \( q = 1, 2, \ldots, N \)

where \( c = 15 \) cm, is a constant representing peak height of the bump and \( N \) is a constant representing a finite number of repetitive bumps. The frequency content of periodic road input has been obtained by using a Fourier series expansion of the displacement function \( E(t) \).

Taking \( t_1 = \frac{T_1}{18} \) seconds and \( t_2 = \frac{T_1}{9} \) seconds, the Fourier series expansion is given as:

\[
E(t) = \frac{c}{18} + \frac{9c}{\pi^2} \sum_{n=1}^{N} \frac{1}{n^2} \left\{ 2 \cos \left( \frac{2n\pi}{18} \right) - \cos \left( \frac{2n\pi}{9} \right) - 1 \right\} \cos(n\omega t)
\]
where $\omega = \frac{2\pi}{T_1}$ is the excitation frequency and $T_1 = \frac{w + d}{v}$ (w = 0.5 m is the bump width and d = 4.0 m the distance between two bumps).

Road 2. Trapezoidal wave type of road bumps

This wave has been defined by the function $E(t)$ as follows:

$$E(t) = \begin{cases} 
  c, & 0 \leq t \leq t_1 \\
  \frac{c}{t_1 - t_3}(t - t_3), & t_1 \leq t \leq t_2 \\
  0, & t_2 \leq t \leq t_3 
\end{cases}$$

and $E(t + qT_1) = E(t), \quad q = 1, 2, \ldots, N$

where $c = 15$ cm, is a constant representing peak height of the bump and $N$ is a constant representing a finite number of repetitive bumps. As the function $E(t)$ is periodic with period $T_1$, the frequency content of road input has been obtained by using a Fourier series expansion of the displacement function $E(t)$. Taking $t_1 = \frac{T_1}{30}$, $t_2 = \frac{7T_1}{90}$ and $t_3 = \frac{T_1}{9}$ seconds, the Fourier series expansion is given as:

$$E(t) = \frac{9c}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \left\{ 2 \cos \left( \frac{2n\pi}{18} \right) - \cos \left( \frac{2n\pi}{9} \right) \right\} \sin(n \omega t)$$

where $\omega = \frac{2\pi}{T_1}$ is the excitation frequency and $T_1 = \frac{w + d}{v}$ (w = 0.5 m is the bump width and d = 4.0 m the distance between two bumps).

Road 3. Random road surface

The road condition has been given by

$$E(t) = \begin{cases} 
  r, & \text{if } 0 \leq t \leq 0.22 \\
  0, & \text{otherwise} 
\end{cases}$$

Where, $r \in [0, 5]$ (cm) is the RMS of a set of random numbers generated from the Gaussian distribution function and $t$ is in seconds.
Road 4. A sinusoidal shape of road profile

This type of road consists of two successive depressions of depth $h = 0.05$ m, length $\lambda = 20$ m (Baumal et al., 1998; Shirahatt et al., 2008) and is as shown in Figure 6.19. As a function of time, the road condition is given by

$$E(t) = \begin{cases} 
\frac{h}{2}(1 - \cos(\omega t)), & \text{if } 0 \leq t \leq \frac{2\lambda}{v} \\
0 & \text{otherwise}
\end{cases}$$

where, $\omega = \frac{2\pi v}{\lambda}$ is the frequency of excitation and $v$ is the vehicle velocity.

The front wheel follows this trajectory and the rear wheel follows the same with a time delay ($L/v$). This road input will help to induce bounce, pitch and roll motion simultaneously.

![Figure 6.19: A sinusoidal shape of road profile with two successive depressions](image)

Road 5. The uneven terrain

The uneven road terrain (Chen et al., 2005) shown in Figure 6.20 is composed of a positive bump followed by a negative bump. Sinusoidal disturbances are also superimposed on the road profile to simulate the rough road surface. Therefore, the terrain disturbance input as a function of time has the form

$$E(t) = \begin{cases} 
-0.0592(t-3.5)^3 + 0.1332(t-3.5)^2 + d(t) & \text{if } t \in [3.5, 5), \\
0.0592(t-6.5)^3 + 0.1332(t-6.5)^2 + d(t) & \text{if } t \in [5, 6.5), \\
0.0592(t-8.5)^3 - 0.1332(t-8.5)^2 + d(t) & \text{if } t \in [8.5, 10), \\
-0.0592(t-11.5)^3 - 0.1332(t-11.5)^2 + d(t) & \text{if } t \in [10, 11.5), \\
d(t) & \text{otherwise}
\end{cases}$$

where, $d(t) = 0.002 \sin(2\pi t) + 0.002 \sin(7.5\pi t)$ is the sinusoidal disturbance.
This function serves as the road input for the front wheel. The excitation on the rear wheel has a time lag \((L/v)\).

\[
\begin{align*}
E(t) &= a(1 - \cos(8\pi t)) \quad \text{if} \quad 0.5 \leq t \leq 0.75 \quad \text{and} \\
&= 0 \quad \text{otherwise}
\end{align*}
\]

where, \(a = 0.055\) correspond to bump of peak magnitude 11 cm.

\[
E(t) = \begin{cases} 
  h [1 - \cos 20\pi (t - 0.3)], & \text{if} \quad 0.3 \leq t \leq 0.4 \\
  0, & \text{otherwise}
\end{cases}
\]

where, \(h = 10.0\) is the bump height in cm and \(t\) time in seconds.
6.2.4 Problem formulation

The passive suspension parameters, springs stiffness ($k_p, k_s$) and dampers coefficients ($\alpha_f, \alpha_s$) have been determined by optimizing the following optimization problem using GA.

6.2.4.1 Formulation of objective function and constraints

Most of the criteria used in objective function for optimization of a vibratory system are based on acceleration, jerk and displacement. The reduction of the absolute acceleration is important
in the optimization of suspensions since it measures the transmitted force to the sprung mass (Shieh. N-C et al., 2005; Gao et al., 2006). However, relative displacement transmissibility at the center of mass also called bouncing transmissibility is another significant quantity to be taken into consideration. It measures the ratio of the relative deflection amplitude at the center of mass of the sprung mass to the excited displacement amplitude imposed during motion. Based on the later aspect, we have therefore formulated the objective function as follows:

\[
\text{Minimize } f(x), \quad f(x) = \max_{\text{Amp}} \frac{|q_2(t)|}{Amp} \tag{6.9}
\]

where \( x = \{k_p, k_r, \alpha_f, \alpha_r\} \) and \( a \leq k_p \leq b, \quad c \leq k_r \leq d, \quad e \leq \alpha_f \leq f, \quad g \leq \alpha_r \leq h \)

The function \( f(x) \) represents the bouncing transmissibility and is the ratio of the maximum deflection amplitude \( q_2(t) \) of the sprung mass to the excited displacement amplitude (Amp) imposed during motion. We shall compute its value after solving (6.8) by applying fourth order Runga Kutta method in time domain \([0, T]\) (measured in seconds), with the initial conditions that the system is in equilibrium state initially.

For obtaining the better road handling performance and ride comfort the above objective function has been subjected to technological constraints \( g_j(x), \quad j = 1, 2, ..., 6 \). These constraints have been formulated as follows:

(i). \( g_1(x) = |q_2 - q_1| - f_{\text{def max}} \leq 0 \) and \( g_2(x) = |q_2 - q_3| - r_{\text{def max}} \leq 0 \)

where \( f_{\text{def max}} \) and \( r_{\text{def max}} \) are maximum allowable suspension deflection near the front wheel and the rear wheel respectively. Structural features of the vehicle place a hard limit on the amount of suspension to reduce vertical acceleration of the car body. Hitting the deflection limit results in not only considerable passenger’s discomfort, but increases the wear of the vehicle.

(ii). \( g_3(x) = k_p - k_{fi} \leq 0 \) and \( g_4(x) = k_r - k_{ri} \leq 0 \)

These constraints assure that the spring stiffness does not exceed the tire stiffness, and keep the natural frequency of the sprung mass less than the exciting frequency. Moreover, large values of tire stiffness maintain the circular shape of the tire under carload and improve the ride quality of the vehicle.

(iii). \( g_5(x) = \frac{k_p}{m_i} - \frac{k_p}{m_{fi}} \leq 0 \) and \( g_6(x) = \frac{k_r}{m_r} - \frac{k_r}{m_{ri}} \leq 0 \)
These constraints always prevent the sprung mass natural frequency from exceeding the unsprung mass natural frequency. This keeps the bouncing transmissibility of the sprung mass caused by the road disturbances at a lower value and therefore provide more comfortable ride to the passenger's.

In relation to the above conditions, the optimization problem has therefore been constructed as a non-linear constrained optimization problem, which is defined as follows:

\[
\begin{align*}
\text{Minimize } f(x) \\
\text{subject to } & g_k(x) \leq 0, \ k = 1, 2, ..., 6 \text{ and } x = \{k', k, \alpha_f, \alpha_r\} \\
& a \leq k_p \leq b, \ c \leq k_r \leq d, \ e \leq \alpha_f \leq f, \ g \leq \alpha_r \leq h
\end{align*}
\]  
(6.10)

### 6.2.4.2 Fitness function

In order to evaluate the fitness function we have applied the Superiority of Feasible point Penalty method (SFP) (Kaisa et al., 2003). This method has already been discussed in Chapter 5 of Part I.

### 6.2.5 Solution methodology

To find the optimal / near to optimal values of the suspension parameters a non-linear constrained optimization problem (6.10) has been solved using GA. Different steps of GA for solving the suspension design problem are given below:

**ALGORITHM 6.3**

**Begin**

Set \( t \leftarrow 0 \) [\( t \) represents the current generation].

Create an initial population \( P(t) \), of \( p \) size number of chromosomes / individuals randomly from the search space, where each individual contains four design variables \( k_p, k_r, \alpha_f \) and \( \alpha_r \).

Solve the set of dynamical equations (6.8) by the fourth order Runge-Kutta method, with initial conditions \( q_i(0) = 0 \) and \( \dot{q}_i(0) = 0 \) (\( i = 1, 2, 3, 4 \)) where \( t \in [0, T] \).

Evaluate the objective function (6.9) and the constraints.

Evaluate the fitness value and the output parameters (1. peak and Root-Mean-Square values of displacement, velocity, acceleration and jerk of sprung mass; 2. peak and Root-Mean-Square values of displacement, velocity and acceleration of unsprung masses) of each chromosome.

**Repeat**

\( (i) \) Increase \( t \) by unity for next generation.
Chapter 6. Half car models without passengers seat suspension

(ii) Select $P(t)$ from $P(t-1)$ by tournament selection operator of size four.
(iii) With probability $p_o$, which decreases with increase in generation number apply multiparent whole arithmetical crossover operator.
(iv) With probability rate $p_m$, which decreases with increase in generation number apply whole non-uniform mutation operator.
(v) Evaluate the fitness value and the value of the output parameters of the problem.
(vi) Apply Elitism of size three.

Until $t$ reaches maximum generation.

Return the best found solution with the corresponding fitness function value and output parameters.

End

6.2.6 Computational results and discussions

This section has been divided into two parts. The first part presents the computational results obtained after solving the suspension design problem using GA over different road input conditions (Road 1, Road 2 and Road 3). The second part deals with the model simulation considering the average value of the design parameters obtained in the first part, and comparison of those results with the simulated results obtained taking existing design parameters of the car model (Taghirad et. al., 1998). The simulation has been carried out over eight different road input conditions (Road 1 to Road 8). The profiles of the roads on which the vehicle performance has been tested present numerous irregularities, with size and shape.

6.2.6.1 Results with GA application

The GA parameter values used in the optimization are: $p_{size} = 120$, $m_{gen} = 150$, $p_e \in [0.8, 0.9]$ and $p_m \in [0.15, 0.20]$. The search space $X$ has been defined as

$$X = (0 \leq k_h \leq 68000, 0 \leq k_r \leq 19000, 0 \leq c_f \leq 1200, 0 \leq c_r \leq 1200)$$

The penalty coefficient $r$ of the fitness function is taken as 500. The maximum suspension deflections ($fdef_{\text{max}}$ and $rdef_{\text{max}}$) considered for the different road conditions are shown in Table 6.6.

<table>
<thead>
<tr>
<th></th>
<th>Road 1</th>
<th>Road 2</th>
<th>Road 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$fdef_{\text{max}}$</td>
<td>0.024</td>
<td>0.020</td>
<td>0.054</td>
</tr>
<tr>
<td>$rdef_{\text{max}}$</td>
<td>0.044</td>
<td>0.074</td>
<td>0.061</td>
</tr>
</tbody>
</table>
The vehicle has been assumed to be running straight ahead with a uniform velocity of 50 km/hr (= 13.89 m/s). The vibrations analysis has been done for 15 seconds in time domain.

In view of the probabilistic nature of the GA, the optimization problem has been run 20 times over the three road cases. The statistical results of 20 independent runs over the three road cases are shown in Table 6.7. It is evident from the statistical analysis that the design parameters obtained may be the optimal solution (either global or near to the global) of the problem.

### Table 6.7. Statistical results over different roads.

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Road 1</th>
<th>Road 2</th>
<th>Road 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best</td>
<td>0.077462</td>
<td>0.107897</td>
<td>0.410985</td>
</tr>
<tr>
<td>Worst</td>
<td>0.078974</td>
<td>0.110695</td>
<td>0.416846</td>
</tr>
<tr>
<td>Mean</td>
<td>0.077877</td>
<td>0.108974</td>
<td>0.412210</td>
</tr>
<tr>
<td>median</td>
<td>0.077926</td>
<td>0.108844</td>
<td>0.411630</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>4.096133 $\times 10^{-4}$</td>
<td>0.001138</td>
<td>0.001561</td>
</tr>
<tr>
<td>Variance</td>
<td>1.677830 $\times 10^{-7}$</td>
<td>1.294529 $\times 10^{-6}$</td>
<td>2.436385 $\times 10^{-6}$</td>
</tr>
</tbody>
</table>

The values of the suspension parameters (solution of the problem) so obtained have been shown in Table 6.8. These values have been then compared with the existing ones (Taghirad et al., 1998) on the same table. The corresponding best found fitness values of bouncing transmissibility of sprung mass and the other output parameters so obtained have been shown in Table 6.9. The output parameters includes: (i) the peak absolute values of displacement, velocity, acceleration and jerk of the sprung mass and the RMS values of bounce, displacement, velocity, acceleration and jerk of the sprung mass and (ii) the peak absolute values and the RMS values of displacement, velocity and acceleration of the front and rear unsprung masses. The convergence graph showing the evolution of GA in different generations for different road input conditions have been shown in Figure 6.24. In these graphs two curves are plotted one showing the evolution of best found solution and the other showing the evolution of average solution in the population with increase in generation number.

### Table 6.8. Design parameters of the suspension systems.

<table>
<thead>
<tr>
<th>Not.</th>
<th>Units</th>
<th>Existing design</th>
<th>Optimal design</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{f}$</td>
<td>N/m</td>
<td>66824.20</td>
<td>67100.89</td>
</tr>
<tr>
<td>$K_{s}$</td>
<td>N/m</td>
<td>18615.00</td>
<td>13597.43</td>
</tr>
<tr>
<td>$\alpha_{f}$</td>
<td>N/m</td>
<td>1190.00</td>
<td>1183.56</td>
</tr>
<tr>
<td>$\alpha_{r}$</td>
<td>N/m</td>
<td>1000.00</td>
<td>1199.99</td>
</tr>
</tbody>
</table>
Table 6.9. The best found fitness value with its output parameters over three road conditions.

<table>
<thead>
<tr>
<th>Measurements</th>
<th>Road 1</th>
<th>Road 2</th>
<th>Road 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PV</td>
<td>RMS</td>
<td>PV</td>
</tr>
<tr>
<td>Bounce (m)</td>
<td>0.077462</td>
<td>0.017054</td>
<td>0.107897</td>
</tr>
<tr>
<td>Disp (m)</td>
<td>0.011619</td>
<td>0.002558</td>
<td>0.016185</td>
</tr>
<tr>
<td>Vel (m/s)</td>
<td>0.100330</td>
<td>0.016204</td>
<td>0.139699</td>
</tr>
<tr>
<td>Acce (m/s²)</td>
<td>2.567244</td>
<td>0.190701</td>
<td>3.565026</td>
</tr>
<tr>
<td>Jerk (m/s³)</td>
<td>131.467327</td>
<td>7.277342</td>
<td>150.785174</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sprung mass</th>
<th>Road 1</th>
<th>Road 2</th>
<th>Road 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PV</td>
<td>RMS</td>
<td>PV</td>
</tr>
<tr>
<td>Bounce (m)</td>
<td>0.054683</td>
<td>0.0000002</td>
<td>0.075978</td>
</tr>
<tr>
<td>Disp (m)</td>
<td>2.066199</td>
<td>0.0000008</td>
<td>2.783468</td>
</tr>
<tr>
<td>Vel (m/s)</td>
<td>121.483433</td>
<td>0.000078</td>
<td>150.543263</td>
</tr>
</tbody>
</table>

Note: PV and RMS imply absolute peak and root mean square values respectively.

Figure 6.24: Evolving process of the fitness function for different road input conditions.

6.2.6.2 Results with Numerical Simulation (NS)

The average values of the design parameters (see Table 6.8) obtained after applying GA are now used in the suspension system of the car model. In order to substantiate the model, simulation has been done with the Runge-kutta method of fourth order in time domain for 15 seconds, over variety of road input conditions (Road 1 to Road 8) defined in section 6.2.3.
Simulation results will help to test the performance of the suspension system and to analyze the dynamic response of the vehicle model as it runs with uniform velocity over different roads. The results obtained have been shown in Table 6.10 to Table 6.12. These results have been then compared with the simulation results obtained using the existing design of the vehicle model in the same table. Examining the ride comfort and handling performances over all the road conditions, it can be concluded that the average values of design parameters obtained by applying GA are better than those of the existing ones. The road handling performances are measured from the motions of unsprung masses.

Table 6.10. Comparison between the simulation results of the average optimal design (AD) and the existing design (ED)

<table>
<thead>
<tr>
<th>Measurements</th>
<th>Road 1</th>
<th>Road 2</th>
<th>Road 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PV</td>
<td>RMS</td>
<td>PV</td>
</tr>
<tr>
<td>Bounce</td>
<td>AD</td>
<td>0.076073</td>
<td>0.017123</td>
</tr>
<tr>
<td></td>
<td>ED</td>
<td>0.078939</td>
<td>0.016612</td>
</tr>
<tr>
<td>Disp (m)</td>
<td>AD</td>
<td>0.011411</td>
<td>0.002569</td>
</tr>
<tr>
<td></td>
<td>ED</td>
<td>0.011841</td>
<td>0.002492</td>
</tr>
<tr>
<td>Bounce</td>
<td>AD</td>
<td>0.100338</td>
<td>0.016115</td>
</tr>
<tr>
<td></td>
<td>ED</td>
<td>0.100113</td>
<td>0.015485</td>
</tr>
<tr>
<td>Sprung mass</td>
<td>Vel (m/s)</td>
<td>2.567011</td>
<td>0.190036</td>
</tr>
<tr>
<td></td>
<td>ED</td>
<td>2.564957</td>
<td>0.188562</td>
</tr>
<tr>
<td>Jerk (m/s²)</td>
<td>AD</td>
<td>131.418562</td>
<td>7.252375</td>
</tr>
<tr>
<td></td>
<td>ED</td>
<td>131.556773</td>
<td>7.281815</td>
</tr>
<tr>
<td>Front unsprung mass</td>
<td>Disp (m)</td>
<td>AD</td>
<td>0.054688</td>
</tr>
<tr>
<td></td>
<td>ED</td>
<td>0.054666</td>
<td>0.000002</td>
</tr>
<tr>
<td>Vel (m/s)</td>
<td>AD</td>
<td>2.066448</td>
<td>0.000008</td>
</tr>
<tr>
<td></td>
<td>ED</td>
<td>2.065180</td>
<td>0.000007</td>
</tr>
<tr>
<td>Acce (m/s²)</td>
<td>AD</td>
<td>121.497824</td>
<td>0.000077</td>
</tr>
<tr>
<td></td>
<td>ED</td>
<td>121.423843</td>
<td>0.000073</td>
</tr>
<tr>
<td>Rear unsprung mass</td>
<td>Disp (m)</td>
<td>AD</td>
<td>0.055940</td>
</tr>
<tr>
<td></td>
<td>ED</td>
<td>0.054888</td>
<td>0.000000</td>
</tr>
<tr>
<td>Vel (m/s)</td>
<td>AD</td>
<td>1.549362</td>
<td>0.000001</td>
</tr>
<tr>
<td></td>
<td>ED</td>
<td>1.540676</td>
<td>0.000001</td>
</tr>
<tr>
<td>Acce (m/s²)</td>
<td>AD</td>
<td>85.788201</td>
<td>0.000002</td>
</tr>
<tr>
<td></td>
<td>ED</td>
<td>85.459176</td>
<td>0.000003</td>
</tr>
<tr>
<td>with AD fdefₘₕ ≤</td>
<td>0.021384</td>
<td>0.004608</td>
<td>0.063497</td>
</tr>
<tr>
<td>with AD rdefₘₕ ≤</td>
<td>0.021384</td>
<td>0.004608</td>
<td>0.063497</td>
</tr>
</tbody>
</table>

Note: PV and RMS imply absolute peak and root mean square values respectively.
Table 6.11. Comparison between the simulation results of the average optimal design (AD) and the existing design (ED)

<table>
<thead>
<tr>
<th>Measurements</th>
<th>Road 4</th>
<th>Road 5</th>
<th>Road 6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PV</td>
<td>RMS</td>
<td>PV</td>
</tr>
<tr>
<td>Bounce (m)</td>
<td>AD 1.248695</td>
<td>0.340918</td>
<td>1.170303</td>
</tr>
<tr>
<td></td>
<td>ED 1.295274</td>
<td>0.364820</td>
<td>1.145252</td>
</tr>
<tr>
<td>Displacement (m)</td>
<td>AD 0.062435</td>
<td>0.017046</td>
<td>0.117030</td>
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<td>ED 4.123600</td>
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Note: PV and RMS imply absolute peak and root mean square values respectively.

Figures 6.25 to 6.27 show the vibration behavior of the sprung mass obtained with the average optimal design and the existing design under the different road excitation. Figures 6.28 to 6.29 and Figures 6.30 to 6.31 show the suspension deflection and tire deflection respectively, under the different road excitation in time domain system. On comparing each figure it is seen that in most of the cases similar level of vibrations are obtained with both the design parameters and in some cases the levels of vibrations are lower with the average optimal design than those with the existing ones. The frequency response characteristics of the sprung mass acceleration with the average optimal design and the existing design have been shown in Figure 6.31. The graphs
in this figure have been plotted taking the Fast Fourier Transforms (FFTs) of recorded time domain vibrations of the sprung mass acceleration.

Table 6.12. Comparison between the simulation results of the average optimal design (AD) and the existing design (ED)

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Note: PV and RMS imply absolute peak and root mean square values respectively.
Figure 6.25: Bouncing transmissibility of sprung mass in time domain over different road conditions.

Figure 6.26: Sprung mass acceleration in time domain over different road conditions.
Figure 6.27: Sprung mass jerk in time domain over different road conditions.
Figure 6.28: Front suspension deflection in time domain over different road conditions.
Figure 6.29: Rear suspension deflection in time domain over different road conditions.
Figure 6.30: Front tire deflection in time domain over different road conditions.
Figure 6.31: Rear tire deflection in time domain over different road conditions.
Figure 6.32: Sprung mass acceleration in frequency domain over different road conditions.
6.2.7 Conclusion

In this chapter, the optimal or near to optimal values of the suspension parameter of the half car model with linear suspension characteristics have been determined by minimizing the formulated non-linear constrained optimization problem using GA. Optimization has been done in time domain at uniform vehicle velocity over three different road conditions. Results show that the parameters so obtained satisfy ride comfort and road holding performance. Suspension parameters of the vehicle model have been constructed by taking the average of the three sets of optimal parameters obtained with three different road conditions. The performance of the constructed suspension parameters have be examined with respect to ride comfort and road holding performance by simulating the vehicle model in time domain over eight different types of road conditions. These roads are characterized as periodic bumps, random road surface, single and double sinusoidal bumps of varying peak. The above results have been further compared with the simulation results obtained over the same roads using existing suspension parameters under same assumptions for validity. Comparisons show better performance with the constructed suspension parameters.