Chapter 1

Introduction

1.1. Introduction

During the last few decades, a number of modern techniques/methods have been developed in solving the decision making problems arising in the management of large systems of men, machines, materials and money in different private and government sectors. The collection of these techniques/methods is called *Operations Research* (OR). From its development, this subject has been defined in various ways as ‘the art of giving bad answers to the problem while worse answers are given’, ‘Scientific approach to decision making problems’, ‘an art of winning the war without actually fighting it’.

According to Churchman et al. (1957), OR is defined as the applications of scientific methods, techniques and tools to decision making problems involving the operations of systems so as to provide these in the control of the system with optimum solutions to the problem.
The subject ‘Operations Research (OR)’ came into existence in the context of military in England (U.K.) during the Second World War. At that time, the authority of military department of that country engaged several inter-disciplinary teams of scientists to undertake scientific research into strategic and tactical military operations in air and land simultaneously. Their mission was to develop specific plans and proposals for aiding the military department to arrive at the decisions on the optimal utilization of very limited military resources and efforts and also to implement those decisions in the most effective manner. Accordingly, that group of scientists suggested some plans and proposals to the British military. Using those plans and proposals, they received remarkable progress in the said war. As the term was dealing with research work on military operations, the work of the group of scientists was called Operations Research in England.

After the successful applications of the encouraging results obtained by British OR team, the military departments of different countries are quickly motivated to start the similar activities in their countries.

After the end of War, the success of OR teams attracted the industrial executives to solve their complex problems. In this field, the first mathematical technique (Simplex Method of Linear Programming) was developed in the year 1947 by George B. Dentzig, an American mathematician. Since then, a large number of techniques have been developed to solve the different types of problems with the joint effort and cooperations of interested individuals in academic institutions and industry both. Apart from military and business applications, the OR activities have been seen in the different sectors like transportation system, libraries, hospitals, city planning, financial
institutions, etc. At the same time, researchers in educational institutes formulated and solved different types of real-life problems. Among these, the problem of assigning jobs/tasks to machines/agents is known as classical assignment problem. This problem is discussed in almost every text book for introductory courses of either Operations Research or Management Science. Basically, the problem is to find a one-to-one matching between $n$ jobs/tasks and $n$ machines/agents in such a way that the total cost/time is minimized. Incorporating various realistic situations, a number of assignment problems has been formulated. These problems have been classified as follows:

- Assignment problems with at most one job/task per machine/agent
- Assignment problems with multiple jobs/ tasks per machine/agent
- Multi-dimensional assignment problems
- Crew-scheduling problems
- Traveling Salesman problems.

1.2. Genetic Algorithm for 0-1 Programming Problems

In solving optimization problems, particularly the decision making problems in the field of Operations Research, either single or multi-objective functions (linear/non-linear) of single/several variables with/without constraints are optimized and the optimum values of the decision variables are obtained. For this purpose, various search and optimization methods developed after the Second World War are available in the literature.

Traditional search and optimization methods can be classified into two groups:

(a) direct method
(b) gradient-based method.

In direct method, only objective function and constraints are used to guide the search strategy. On the other hand, in case of gradient-based methods, the first and/or second order derivatives of the objective function and/or constraints are used in the search process. As the derivative information is not used in the direct search methods, these methods are usually slow, require many function evaluations for convergence. As a result, these methods can be applied to many problems without a major change of the algorithm. On the other hand, gradient-based methods quickly converge to an optimal solution, but are not efficient for non-differentiable or discontinuous problems.

In addition, there are some common difficulties observed in the traditional direct and gradient-based methods. These are as follows:

(i) Convergence to an optimal solution is dependent on the chosen initial solution.
(ii) Most of the algorithms get stuck to a sub-optimal solution.
(iii) An algorithm efficient in solving one type of optimization problem may not be efficient in solving a different type.
(iv) Algorithms are not efficient in handling problems with other variables except continuous one.
(v) Algorithms cannot be efficiently used on a parallel machine.

To overcome the difficulties mentioned above, during the last few decades, attempts have been made to develop new optimization techniques in terms of algorithms based on natural evolution and natural genetics. Such algorithms have some selection process based on the fitness of individuals and some operations
equivalent to natural genetics. Recently, a number of such type of algorithms, viz. Genetic Algorithm (GA), Differential Evolution (DE), Simulated Annealing (SA), Tabu search, Ant Colony optimization, etc., which are known as soft computing methods, have been proposed. Among these methods, Genetic Algorithm is very popular. The theory and applicability of this algorithm was first conceived by J. H. Holland, University of Michigan, Ann Arbor in 1965, who can be considered as the pioneer of this subject. Since then, this subject has witnessed a tremendous development. In this area, most of the initial research works have been published in several conference proceedings. However, at present, there exist several text books like Goldberg (1989), Mitchell (1996), Michalewicz (1996), Gen and Cheng (1997), Sakawa (2002), Davis (1991) and a few journals dedicated to publishing research papers in this subject. Besides, most application oriented research works can also be found in the domain-specific journals.

The same technique with certain modifications can be used for solving discrete optimization problems particularly for solving 0-1 programming problems, viz. assignment problems, job scheduling, time-tabling, etc.

**Motivation from the Nature**

Creationists occasionally charge that evolution is useless as a scientific theory because it produces no practical benefits and has no relevance in daily life. However, the biological phenomenon shows that this claim is not true. There are numerous natural phenomena for which evolution gives us a sound theoretical underpinning. Concisely stated, Genetic Algorithm (GA) is a technique that mimics biological evolution as a problem-solving strategy. As the name itself indicates, GA is the outcome of the
imposition of a concept of genetic science on mathematical optimization. It is based on the well known principle, viz. ‘survival of the fittest’ due to the famous biologist Charles Darwin. In most situations, the nature ruthlessly follows two simple principles:

(i) If an above-average offspring is created by any genetic processing, it is going to survive longer than an average individual and thus there is more opportunity to produce offspring having some of its traits than an average individual.

(ii) On the other hand, if a below-average offspring is created, it does not survive longer and thus it may eliminated from the population with higher probability.

With the help of ‘survival of the fittest’ principle, the renowned biologist Richard Dawkins explains many evolutionary facts in his seminal works. According to his argument, the tall trees that exist in the mountains were only a foot tall during early ages of evolution. By genetic processing if a tree had produced an offspring one inch taller than all other trees, that offspring enjoyed more sunlight and rain and attracted more insects for pollination than all other trees. With extra benefits, that lucky offspring had an increased life and had produced more offspring like it (with tall feature) than others. In this way, it occupies most of the mountain with trees having its genes and the competition of survival now continues with other trees, since the available resource (land) is limited. However, if a tree had produced an offspring smaller than others, it was less fortunate to enjoy all the facilities than neighbouring trees. Thus, that offspring could not survive longer.

In a similar way, if we study the human civilization, then the journey from the time immemorial when the first human being came on this planet to the modern
civilization reflects the improvement and refinement of the human race. Those individuals who could fight against the environmental forces better, were able to survive in nature and take part to create the better individuals by the birth process. At the same time, the characteristics of their gene changed continuously in the consecutive generations. On the other hand, comparatively weak individuals could not fight in the environment and got abolished. This has lead to the constant refinement and betterment of the qualities of human race.

These features of natural evolution and genetics have been introduced in an algorithm called genetic algorithm (GA). This algorithm is used for determining the optimum solution of an optimization problem. For solving a specific problem, the input to the GA is a set of potential solutions (plays the role of the initial population of GA) to that problem, encoded in some fashion, and a fitness function (plays the role of the environmental forces) that allows each individual (solution) to be quantitatively evaluated to judge its capability (as the individual in the case of human civilization) with respect to the fitness function (i.e., the environmental forces). The GA then evaluates each individual according to the fitness function. The individuals for which the fitness function produces better results are ranked higher than those for which the fitness function produces inferior results. Highly ranked individuals are then kept (i.e., permitted to survive) into the population and inferior individuals are deleted (just as inferior individuals get abolished) from the population. Then a certain percentage of these existing promising individuals are allowed to reproduce improved offspring. These offspring are then go on to the next generation, forming a new pool of individuals (solutions). Finally, to incorporate the genetic diversity into the population,
so that the information lost in the previous generations may be regained, mutation operation is applied to a single chromosome of the population.

Advantages of Genetic Algorithm

Genetic Algorithm has a lot of advantages as follows:

This algorithm

(i) optimizes with continuous, discrete, permutation and mixed variables.

(ii) performs well in solving optimization problems where the fitness function is discontinuous, noisy, changes over time, or has many local optima.

(iii) able to manipulate many parameters simultaneously and produce multiple equally good solutions to the same problem, possibly with one solution optimizing one parameter and another optimizing a different one.

(iv) does not require the derivative information of the objective function as well as constraint functions.

(v) deals with problems where the space of all potential solutions is truly huge – too vast to search exhaustively in any reasonable amount of time.

(vi) produces either global or near to global optimum and does not stick to local optimum.

(vii) knows nothing about the problem as well as the variables it is deployed to solve.

(viii) gives alternative solution.

(ix) works not only with the analytical functions, but also works with experimental data.
works with a set of solutions instead of single solution in each iteration/generation.

Limitations of Genetic Algorithm

Although GA has proven to be an efficient and powerful problem-solving strategy, it has certain limitations which are as follows:

(i) The first and the most important consideration in using a GA is defining a representation for the problem. The language used to specify candidate solutions must be robust, so that it must be able to tolerate random changes such that fatal errors or nonsense do not consistently result.

(ii) The problem of how to write the fitness function must be considered carefully so that higher fitness is attainable and actually does equate to a better solution for the given problem. If the fitness function is chosen poorly or defined vaguely, the GA may be unable to find a solution at all, or may end up solving the wrong problem.

(iii) In addition to making a good choice of fitness function, the other parameters of GA – the population size, the probabilities of crossover and mutation and the type of selection as well as its suitability – must also be chosen with care. If the population size is too small, the GA may not explore enough of the solution space and might yields substandard solutions. Again, if the rate of genetic change is too high or the selection scheme is not chosen properly, beneficial features may be disrupted and the
population may enter error catastrophe, changing too fast for selection to ever bring about convergence.

(iv) GA is difficult for problems with "deceptive" fitness functions, where the locations of improved points give misleading information about where the global optimum is likely to be found.

(v) One well-known problem that can occur with a GA is the premature convergence when an individual that is fitter than most of its competitors emerges early in the course of the run, it may reproduce so abundantly that it drives down the population's diversity too soon, leading the algorithm to converge to the local optimum rather than searching thoroughly enough to find the global optimum. This is a common problem for small populations, where even variations in reproduction rate may cause one genotype to become dominant over others.

(vi) Finally, several researchers (like Holland (1992), Forrest (1993), Haupt and Haupt (1998)) advised against using GAs to problems those can be solved using analytical methods. It is not that GAs cannot find good solutions to such problems; it is merely that traditional analytical methods take lesser time and computational effort than GAs and unlike GAs, usually produce one mathematically guaranteed exact solution. Of course, since there is no such thing as a mathematically perfect solution to any problem of biological adaptation, this issue does not arise in nature.
1.3. Historical Review of the Assignment Problems

The Assignment Problem (AP), the problem of finding the cost effective assignment of a set of tasks/jobs to a set of facilities, is one of the first fundamental problems in the area of combinatorial optimization. The problem was investigated by Monge (1784), who was motivated by transporting earth, which he considered as the discontinuous, combinatorial problem of transporting molecules. There are two areas of equal acreage, one filled with earth, the other empty. The question is to move the earth from the first area to the second, in such a way that the total transportation distance is as small as possible. The total transportation distance is the distance through which all the molecules moved. Hence it is an instance of the assignment problem, obviously with an enormous cost matrix. The first algorithm for the assignment problem might have been published by Easterfield (1946) who seems to have worked without knowledge of the existing literature and described a primal-dual type method for the assignment problem. A breakthrough in solving the assignment problem came when Dantzig (1951) showed that the assignment problem can be formulated as a linear programming problem and has an integer optimum solution. So the assignment problem can be solved with the simplex method. Thorndike (1950) presented three different heuristics for the assignment problem, the Method of Divine Intuition, the Method of Daily Quotas, and the Method of Predicted Yield. Other heuristic and geometric methods for the assignment problem were proposed by Lord (1952), Votaw and Orden (1952), Tornqvist (1953), and Dwyer (1954). In a talk in the Princeton University Game Seminar on October 26, 1951, Neumann (1951) showed that the assignment problem can be reduced to finding an optimum column
strategy in a certain two-person zero-sum game and that it can be found by a method given by Brown and Neumann (1950). The basic combinatorial (non-simplex) method for the assignment problem is the Hungarian method. The method was developed by Kuhn (1955, 1956), based on the work of Egerváry (1931), whence Kuhn introduced the name Hungarian method for it. In recognition of the significance of Kuhn’s article, *Naval Research Logistics* reprinted it in honor of its 50-th anniversary in 2005, along with a statement by Kuhn about the development of the Hungarian method and a tribute by Frank (2005) explaining the relationship between Kuhn’s technique and its antecedents in the papers by two Hungarian mathematicians. Over the past 50 years, many variations on the AP have been proposed. Among them, some well-known are AP with side constraints (APSC), Generalized AP (GAP), Quadratic AP (QAP), Multi-criteria AP (MCAP), Multi-dimensional AP (MDAP), Semi-AP and a variety of others.

In this thesis, classical AP, AP with single side constraint (APSC) and Generalized AP (GAP) have been discussed considering realistic situations. In the area of classical AP, one may refer to the recent works of Dowsland and Thomson (2000), Aickelin and Dowsland (2004), Harper et al. (2005) and others.

During the last 30 years, the assignment problems with side constraints (APSC) have been studied by several researchers dealing with real-life assignment problems subject to capacities of one or more resources. In this area, the works of Mazzola and Neebe (1986), Aboudi and Nemhauser (1991), Rosenwein (1991), Kennington and Mohammadi (1994), Punnen and Aneja (1995), Foulds and Wilson (1999) and Leishout and Volgenant (2007) are worth mentioning.
In GAP, the assignment cost/time is minimized for assigning \( n \) jobs to \( m \) agents \((n > m)\) such that at least one job is assigned to one agent, subject to a capacity constraint for that agent. This problem has several real-life applications in job scheduling, resource scheduling, production planning and storage space allocation, computer and communication networks, vehicle routing, facility location, etc. To solve these problems, several exact and heuristic methods have been proposed. The well known exact methods are tree search, Branch-and-bound, Branch-and-cut and Branch-and-price algorithms. In this case, one may refer to the works of Ross and Soland (1975), Martello and Toth (1981, 1987, 1990), Savelsbergh (1997), Nauss (2003) and others. On the other hand, to solve the same problem, Klastorin (1979), Fisher et al. (1986), Trick (1992), Cattrysse and Van Wassenhove (1992), Amini and Racer (1994), Cattrysse et al. (1994), Osman (1995), Lorena and Narciso (1996), Wilson (1997), Chu and Beasley (1997), Narciso and Lorena (1999), Yagiura et al. (1998, 2002, 2004), Lourenço and Serra (2002), Alfandari et al. (2001, 2002), Felth and Raidl (2004), Haddadi and Ouzia (2001, 2004) have proposed heuristic methods.

1.4. Historical Review of the Crew-scheduling Problems

Due to globalization of our market economy, the crew-scheduling problem of a transport industry plays an important role to fulfill the goal of that industry in the present days’ market situations. The goal of the transport industries is to assign a number of crews for their round-trips (i.e., departure and arrival) in such a way that the total cost/time is minimized. These types of problems are seen in surface/land
transport as well as in air transport. In this area, most of the researchers studied airline crew-scheduling problems and devised different methods to solve them.

In the year 1969, Arabeyre et al. (1969) had done the survey of different approaches studied by a number of airlines in the past few years to find the optimal allocation of crews to flights. Marsten and Shepardson (1981) decomposed the crew-scheduling problem as the set partitioning problem and proposed a new solution technique to solve the same employing Lagrangian relaxation and subgradient optimization. Gershkoff (1989) formulated flight crew-scheduling problem as an optimization problem and then he solved the same by reducing it as a set-partitioning problem where the rows represent flights to be covered and the columns represent candidate crew trips. Anbil et al. (1992) proposed a global approach in solving crew-pairing optimization in airline flight planning; finding tours of duty (pairings) which are legal and cover every flight leg at minimum cost. After Anbil et al. (1992), in this area, the works of Hoffman and Padberg (1993), Desrosiers et al. (1993), Levine (1996), Moudani et al. (2001), Zeghal and Minoux (2006) and Schaefer et al. (2005) are worth mentioning.

1.5. Historical Review of the Traveling salesman Problems

In very primitive societies, finding shortest paths for searching of foods was essential for primitive men and even animals. Since then, the notion of the traveling salesman problem (TSP) cropped up. The problem in that sense is a relatively old problem. The origin of the name “traveling salesman problem” is a bit of mystery (Applegate et al. (2006)). There does not appear to be any authoritative documentation pointing out the creator of the name and we have no good guesses as to when it first came into use.
Most probably, in 1759, Euler first gave the instance of the TSP whose problem was to move a knight to every position on a chess board exactly once. Mathematical problems related to TSP were treated in the 1800s by the Irish mathematician Sir William Rowan Hamilton and by the British mathematician Thomas Penyngton Kirkman. This problem first gained fame in a book by a German salesman BF Voigt in 1832 on how to be a successful traveling salesman. The solution techniques of the TSP are obscure. The general form of the TSP has been first proposed by mathematician and economist Karl Menger in 1920s among his colleagues in Vienna and Harvard. It seemed that the TSP has spanned at least 9 decades. The problem was later studied by Hassler Whitney and Merrill M. Flood of Princeton University and the RAND Corporation in the 1930s. A detailed treatment of the connection between Menger and Whitney and the growth of the TSP as a topic of study can be found in Alexander Schrijver's paper “On the history of combinatorial optimization (till 1960)”. In 1940s, there were some papers on TSP that study the problem in a different context. During the decade 1940-1950, P.C. Mahalanobis, Jessen, Gosh and Marks investigated the problem in connection with an agricultural application. In the mid-1950s, different solution methods were developed with a debate regarding the nomenclature of traveling salesman problem. Dantzig, Fulkerson, and Johnson (1954) referred to the “traveling-salesman problem”, Heller (1953) used “travelling salesman's problem” and Morton and Land (1955) preferred “the travelling salesman problem”. Except for these small variations in spelling and punctuation, the name “traveling salesman problem” was in wide use from the mid-
1950s. Among these, the works of Flood (1956), Barachet (1957), Bocks (1958), Dantzig et al. (1959) are worth mentioning.

There are two types of traveling salesman problems (TSPs), viz. symmetric TSPs (STSPs) and asymmetric TSPs (ATSPs). STSPs are those where the distance/cost from the $i$-th city to the $j$-th city is the same as that from the $j$-th city to the $i$-th city. On the other hand, for ATSPs, the said distances/costs are not equal.

For solving STSPs, several exact and heuristic approaches are found in the literature. Exact methods include cutting plane, Branch and Bound (B&B) based on assignment problem relaxation (Eastman (1958), Branch and Bound (B&B) (Pedberg and Rinaldi (1987); Held and Karp (1971); Smith et al. (1977); Carpaneto and Toth (1980); Balas & Christofides (1981)), Branch and Cut (B&C) (Crowder and Padberg (1980); Padberg and Hong (1980); Grötschel and Holland (1991)) and dynamic programming (Bellman (1963); Ergan and Orlin (2006)). However, very small size problems can be solved by exact methods. On the other hand, large size problems have been solved using heuristic and probabilistic methods like 2-opt (Croes (1958); Lin and Kernighan (1973); Johnson (1987)), Markov chain (Martin et al. (1991)), metaheuristic algorithms like Tabu Search (Knox (1989); Fiechter (1990); Glover (1990)), neural networks (Hopfield and Tank (1985)), simulated annealing (Kirkpatrick et al. (1983); Bonomi & Lutton (1984); Kirkpatrick and Toulouse (1985); Golden and Skiscim (1986); Lam (1988); Nahar et al. (1989); Lo and Hus (1998)) and genetic algorithms (GAs) (Grefenstette et al. (1985); Oliver et al. (1987); Goldberg (1989); Jog et al. (1989); Whitley et al. (1989); Braun (1991)). Comprehensive review
of the methods developed for STSPs can be found in Lawler et al. (1985), Laporte (1992), Johnson & McGeoch (1997).

On the other hand, for ATSPs, many heuristic algorithms devised by several researchers like Cirasella et al. (2001), Burke et al. (2001), Johnson et al. (2002) and Choi et al. (2003) are worth mentioning.

1.6. Motivation and objective of the thesis

The Assignment Problem, the problem of finding the cost effective assignment of a set of tasks/jobs to a set of facilities, is one of the first fundamental problems in the area of combinatorial optimization. Despite its historical roots, the problem has tremendous importance to date, due to its numerous real-life applications like facility location, personnel scheduling, job scheduling, production planning, project assignment, task assignment, time-tableing, vehicle routing, storage space allocation, etc. The assignment problem is also significant because of the insights it provides to more complex 0-1 programming problems.

The objectives addressed in this thesis are

• to develop different types of assignment and related problems like Assignment Problems (AP) with single side constraint (APSSCs), Generalized Assignment Problems (GAPs), Crew-scheduling Problems and Asymmetric Traveling Salesman Problems (ATSPs) with interval objectives, which have not been referred so far in the existing literature,

• to develop Genetic Algorithm (GA) appropriately for solving the above problems with interval objectives with the help of interval arithmetic and order
relations between interval valued numbers and also to improve the existing one suitably for solving the above mentioned problems with fixed objectives,

- to analyze and compare the performance of these methods on a large set of test problems.

According to the literature, there is a number of exact methods like cutting plane, tree search, branch-and-bound (B&B), B&B with bounding strategies (Linear Programming (LP) relaxation and Lagrangian relaxation (LR)), Branch-and-cut, Branch-and-price, etc. for solving 0-1 programming problems. However, in these methods, the computational effort is relatively high due to the application of branching procedure, bounding technique and next to solve the reduced problem(s). To overcome these drawbacks, Genetic Algorithm (GA), a relatively new efficient computerized heuristic search and optimization method, is used to find the global or near to the global solution of the problems concerned. By this method, a large variety of problems having fixed objective functions have been solved by several researchers. However, none has attempted to solve the earlier mentioned problems with interval objectives.

From the existing literature, it is observed that several researchers developed different methods including genetic algorithms for solving different types of assignment problems as well as some special cases like manpower scheduling, nurse-scheduling, employee-scheduling, project assignment problems. In their genetic algorithms, initialization, crossover and mutation processes have been reported differently.
Moreover, among all the aforesaid works, to the best of our knowledge, fixed (deterministic) real numbers have been used in effective matrices of the concerned assignment problems. However, in real-life situations, the elements of the effective matrix should be imprecise number instead of fixed real numbers as because time/cost for doing a job by a facility (machine/person) might vary due to different reasons. To solve the problem with such imprecise numbers generally stochastic, fuzzy and fuzzy-stochastic approaches have been used in the literature. However, in stochastic approach, the coefficients/parameters are viewed as random variables and it is assumed that their probability distributions are known. On the other hand, in fuzzy approaches, the parameters, constraints and goals are viewed as either fuzzy sets with known membership function or fuzzy numbers appropriately. In fuzzy-stochastic approaches, the combination of stochastic and fuzzy approaches is used. However, it is not always easy task for a decision maker to specify the appropriate membership function/fuzzy numbers or the probability distribution of a parameter exactly. To avoid these drawbacks, one may use the interval numbers to represent the imprecise numbers as interval number representation is the best representation among others. 

Till now, none has considered assignment problems with interval valued cost(s)/time(s) and solved with the help of GA.

Crew-scheduling for public transport concerns the assignment of a number of crew staff for their round-trips (i.e., departure and arrival) for all the public transports (e.g., train, bus or air bus) at a minimum cost/time for transport industry. In the past, several researchers developed different exact methods and efficient heuristics for solving different types of crew-scheduling problems. To the best of our knowledge, all
the aforesaid scheduling problems have been solved with the assumption that the coefficients or cost parameters are specified in a precise way, i.e., in other words, fixed (deterministic) real numbers have been used in the effective matrices of the concerned assignment problems. However, in real-life, there may be many diverse situations due to traffic jam, bad condition of road/railway track, rainy/foggy/cloudy weather, etc. for which the time taken by a transport vehicle for a trip from one place to another will be imprecise. Thus, in real-life situations, the time parameters are flexible in nature and their values lie within some intervals. *Till now, none has developed public transport crew-scheduling problem with interval valued rest time as well as the service time (including rest time) of each crew and solved the same using GA (and also with tournamenting).*

Sometimes there exist some real-life APs that contain an additional side constraint due to budgetary limitation, or time restriction, etc. Only Lieshout and Volgenant (2007) studied this type of problems and solved the same by Branch-and-bound method. In their work, fixed (deterministic) real numbers have been used as the cost/time parameters in the concerned assignment problems. However, in real-life situations, these numbers might not be fixed rather be imprecise as cost/time for performing a task/job by a facility (machine/person) may vary due to different reasons as have been stated earlier. Thus, real-life problems involve flexible cost/time parameters in general and their values lie within intervals. In this way, an Assignment Problem with Single Side Constraint (APSSC) with interval cost/time parameters can be formulated. Moreover, to solve constrained optimization problems using GAs or classical optimization methods, penalty function techniques are the most popular. *Till*
now, none has considered APSSCs with interval valued cost(s)/time(s) and solved with the help of GA using penalty (Big-M Penalty and Parameter Free Penalty) techniques.

The Generalized Assignment Problem (GAP) is a minimum cost/time assignment problem that allows multiple jobs to be assigned to an agent subject to a capacity constraint for that agent. In the past, a number of researchers developed different exact and heuristic methods to solve small-sized as well as large GAPs. Among all the aforesaid works, fixed (deterministic) real numbers have been used as the cost/time parameters for assigning a job to an agent. However, in real-life situations, these numbers might not be fixed rather be imprecise as cost/time to perform a job by a facility (machine/operator) may vary due to different reasons arising from sudden power failure/machine disorder/price fluctuation in market etc. Thus, real-life problems involve flexible cost/time parameters in general and their values lie within intervals and so a GAP with interval valued objective can be framed. Till now, none has solved the GAP with interval valued costs/times by GA with interval fitness function.

The Traveling Salesman Problem (TSP) deals with determining the closed route of the shortest length or of the minimum cost (or time) passing through a given set of cities where each city is visited exactly once. In this area, a large number of approaches both by exact and heuristic methods (including genetic algorithms (GAs)) have been developed by several researchers for solving symmetric TSPs (STSPs). Among these, most of the heuristic methods cannot be applied efficiently to solve asymmetric TSPs (ATSPs). Moreover, as most of the TSP applications are of
asymmetric nature, further research is necessary for developing good heuristic algorithms for ATSPs. Again, to the best of our knowledge, generally, the distance/cost parameters have been used in the concerned TSPs of the aforesaid works and they were specified precisely by fixed real numbers. However, due to the competitive market situation of present-day scenario, consideration of travel times between two cities will be more appropriate instead of distance/cost parameters. Moreover, in real-life situations of the third-world countries, the travel times from one city to another would be imprecise instead of precise (fixed) numbers due to several diverse situations arising from traffic jam, bad condition of road/railway track, rainy/foggy weather etc. for which there prevails a poor transport system. Thus, in real-life considerations, the time parameters are flexible in nature and their values lie within intervals and so an interval ATSP (I-ATSP) can be framed. Till now, none has solved the ATSP with interval valued time parameters by GA with interval fitness function.

In this thesis, we have tried to fill up the gaps of different types of assignment and related problems as mentioned above.

In Chapter 3, an assignment problem has been solved by a new approach using elitist genetic algorithm (EGA). Here, initially, the problem has been formulated as 0-1 programming problem and then EGA has been developed to solve the problem. In our proposed EGA, new methodologies for GA initialization, crossover and mutation have been developed instead of existing methodologies.

For the first time, an interval valued assignment problem has been developed in Chapter 4, where the coefficient of the objective function has been considered as
interval valued number. The aforesaid problem has been formulated using the interval arithmetic and solved using elitist genetic algorithm (EGA) where the existing definitions of the order relations between interval numbers have been used with respect to optimistic as well as pessimistic decision makers' point of view.

In Chapter 5, two different models of realistic crew-scheduling problem have been formulated of which the first one is an airline crew-scheduling problem considering imprecise total service time (including rest time) of each crew assumed to lie within some intervals while the second one is a public transport day-to-day crew-scheduling problem assuming both the rest and service times (including rest times) of crews as intervals. Here, initially, both the problems have been formulated as multi-objective (interval valued) assignment problems and then converted into multi-objective assignment problems with crisp objectives considering the corresponding centre and width values of intervals using the existing revised definition of order relation (for pessimistic case). Then the problem of the first model has been solved by our proposed two different methods based on elitist genetic algorithm (EGA). On the other hand, the problem of the second model has been solved using tournament genetic algorithm (TGA).

In Chapter 6, Assignment Problems with Single Side Constraint (APSSCs) for both deterministic and interval objectives have been investigated and solved using elitist genetic algorithm (EGA). Here, to formulate the problems with interval objectives, interval arithmetic have been used. Then, to solve the same using EGA, existing interval order relations for minimization problems have been used with respect to optimistic as well as pessimistic decision makers' point of view. To evaluate
the fitness of infeasible solutions in terms of the violation of the side constraint, two different penalty techniques, viz. Big-M Penalty (BMP) and Parameter Free Penalty (PFP) techniques have been considered.

In Chapter 7, for the first time, the Generalized Assignment Problem (GAP) with interval valued costs/times has been formulated and solved using hybrid advanced genetic algorithm (HAGA). As a special case, the GAP with fixed costs/times has been solved considering the upper and lower bounds of the interval valued costs/times the same. Here, to solve the interval valued problems, only pessimistic decision makers' preference of interval valued numbers for minimization problems have been considered. The fitness of an infeasible solution (due to the violation of the capacity constraints) has been framed in such a way that this solution is simply thrown out during comparison.

In Chapter 8, a realistic asymmetric traveling salesman problem has been developed based on the concept of time minimization, where the inter-city travel times are considered as interval valued numbers and solved the same using GA with interval valued fitness function.

1.7. Organisation of the thesis

The proposed thesis has been divided into four parts (excluding Chapter-I and Chapter-2), on the basis of different types of problems. Part-I contains Chapter-3 and Chapter-4 (Classical Assignment Problems), Part-II contains Chapter-5 (Crew-scheduling Problems), Part-III contains Chapter-6 and Chapter-7 (Assignment Problems with resource constraints) and Part-IV contains Chapter-8 (Traveling Salesman Problems). So, the whole thesis has been divided into eight Chapters.
Chapter-1: Introduction.

Chapter-2: Solution Methodology

Part-I: Classical Assignment Problems

Chapter-3: Elitist genetic algorithm approach for Assignment Problem

Chapter-4: Elitist genetic algorithm for assignment problem with imprecise goal

Part-II: Crew-scheduling Problems

Chapter-5: Genetic Algorithm Approach for Crew-scheduling Problem with imprecise travel time

Part-III: Assignment Problems with resource constraints

Chapter-6: Penalty approaches for assignment problem with single side constraint via Genetic Algorithms

Chapter-7: Hybrid Advanced Genetic Algorithm for Generalized Assignment Problem

Part-IV: Traveling Salesman Problem

Chapter-8: Genetic Algorithm for asymmetric traveling salesman problem with imprecise travel times

In Chapter-1, a general introduction, the basic ideas of Genetic Algorithm (GA) with its advantages and limitations as well as the history of development of assignment problem, assignment problem with resource constraints, crew-scheduling problem and traveling salesman problem have been discussed.

In Chapter-2, some definitions and preliminary ideas on finite interval arithmetic, order relations between interval valued numbers and the working principle
of Genetic Algorithm used/developed for solving the different types of assignment and related problems have been presented.

The part-wise and chapter-wise descriptions of the different problems have been presented below:

**PART-I**
This part consists of the application of GA in assignment problems with fixed and interval valued cost(s)/time(s).

*Chapter 3*

**Elitist genetic algorithm approach for Assignment Problem**
In this work, a new approach for solving an assignment problem (balanced) has been proposed with the help of an elitist genetic algorithm (EGA) and also its computational behaviour has been reported. The mathematical formulation of the problem indicates that this problem is a 0-1 programming problem. To solve this problem, an EGA with new types of initialization, crossover, mutation and existing rank-based selection has been developed. As special cases, different types of assignment problems like unbalanced assignment problem, maximization assignment problem, restricted assignment problem have been reported and illustrated with some numerical examples.

*Chapter 4*

**Elitist genetic algorithm for assignment problem with imprecise goal**
The purpose of this work is to solve an assignment problem with imprecise cost(s)/time(s) instead of precise one by elitist genetic algorithm (EGA). Here, the
impreciseness of cost(s)/time(s) has been represented by interval valued numbers. To solve these types of problems, an EGA has been developed with interval valued fitness function. In this developed GA, the existing definitions of the order relations between two interval valued numbers from the point of view of two types of decision making, viz. optimistic decision making and pessimistic decision making have been used in the selection process for selecting better chromosomes/individuals for the next generation and in finding the best as well as the worst chromosomes/individuals in each generation. Here two new crossover schemes and two new mutation schemes have been introduced. In order to maintain the feasibility with crossover operations, a repair algorithm has been suggested. Extensive comparative computational studies based on different parameters of our developed algorithm on one illustrative example have also been reported.

PART-II

This part consists of the application of GA in solving crew-scheduling problems with imprecise rest and/or service times.

Chapter 5

Genetic Algorithm approach for Crew-scheduling Problems with imprecise travel time

In this chapter, two crew-scheduling models have been developed considering interval valued travel time.

Model – I: Application of Genetic Algorithm for solving Airline Crew-scheduling Problem with imprecise travel time
In this model, an well-known airline crew-scheduling problem which has been formulated considering the day-to-day assignment of the technical crew members to their legal round-trip rotations for all the scheduled flights that will minimize the overall service times (including rest times) of all the crews. Here, the service times of crews from their starting airport to another airport have been assumed imprecise numbers which are represented by intervals. For solving this problem, two different methods have been proposed:

(a) an elitist genetic algorithm (EGA) with interval valued fitness function

and

(b) EGA approach after converting it into a multi-objective assignment problem with crisp objectives considering the centre and width values of the corresponding intervals.

The experimental results of the proposed methods to a realistic airline crew-scheduling problem have been compared. Finally, the effect of changes of different genetic parameters on success rate of both the methods, computation times and function evaluations has been observed by sensitivity analyses taking one at a time.

Model – II: Application of Genetic Algorithm for solving a multi-objective Crew-scheduling Problem with imprecise rest and service time

In this model, a crew-scheduling problem has been formulated considering the daily assignment of a set of crew staff to their round-trips for all the public transports (e.g., train, bus or air bus) that will minimize the rest times and total service times (including rest times) of all crews separately. Here, both the rest times as well as the service times of crews have been taken as intervals. Initially, the problem has been
formulated as a multi-objective (interval valued) assignment problem and then converted into a multi-objective assignment problem with crisp objectives considering the corresponding centre and width values of intervals using the existing revised definition of order relation with respect to the pessimistic decision maker's preference between interval valued times. Finally, the reformulated problem has been converted to a single objective optimization problem using Global Criterion Method and then solved the same by tournament genetic algorithm (TGA). Again, the experimental results of a realistic bus crew-scheduling problem have been reported for this model.

PART-III
This part consists of the application of GA in assignment problems with resource constraints.

Chapter 6
Penalty approaches for assignment problem with single side constraint via Genetic Algorithms
The goal of this work is to investigate the applicability of Genetic Algorithms (GAs) in solving Assignment Problem with Single Side Constraint (APSSC) due to either time restrictions or budgetary restrictions, etc. For this purpose, two different models of APSSC have been formulated – one for deterministic cost/time parameters and another for imprecise cost/time parameters. To handle the side constraint in solving each of these models, the original constrained optimization problems have been reduced to two different unconstrained optimization problems with the help of two different penalty function techniques. Then the reduced problems have been solved
using elitist genetic algorithm (EGA). In this algorithm, some new features on initialization, pair-wise careful comparison among feasible and infeasible solutions using tournament selection in conjunction with two heuristic operators one for making feasible solution from the infeasible one and the other for improving feasible solution have been employed. To illustrate the models, a set of test problems generated randomly have been solved and the computational statistics of each model regarding the objective function values, generations, computational times and number of objective function evaluations have been compared.

Chapter 7

Hybrid Advanced Genetic Algorithm for Generalized Assignment Problem

This work deals with an application of hybrid advanced genetic algorithm (HAGA) to the generalized assignment problem (GAP), a problem that allows multiple jobs to be assigned to an agent subject to a capacity constraint for the agent. Here, the GAP with interval valued costs/times has been formulated and solved considering the interval ranking with respect to only pessimistic decision makers’ preference between interval valued costs/times. Again, the GAP with fixed costs/times has been solved as a special case of interval valued GAP considering the upper and lower bounds for the interval valued costs/times the same. The HAGA has some new features on initialization, tournament selection for handling infeasible solutions carefully, crossover, mutation and four heuristic improvement operators for improving feasibility and optimality both. Computational results with comparative performance of the algorithm have been presented on 6 randomly generated interval valued test problems and 12 existing
standard (fixed valued) test problems of type A to D from the OR-library with very small population size and predetermined maximum number of generations.

**PART-IV**

This part consists of the application of GA in asymmetric traveling salesman problems with imprecise travel times.

*Chapter 8*

**Genetic Algorithm for asymmetric traveling salesman problem with imprecise travel times**

In this work, a variant of the asymmetric traveling salesman problem (ATSP) has been investigated where the traveling time between each pair of cities has been represented by an interval of values (wherein the actual travel time is expected to lie) instead of a fixed (deterministic) value as in the classical ATSP. Here the ATSP (with interval objective) has been formulated using the interval arithmetic. To solve the interval ATSP (I-ATSP), a genetic algorithm with interval valued fitness function has been proposed. For this purpose, the existing revised definition of order relations between interval valued numbers for the case of pessimistic decision making has been used. The proposed algorithm is based on a previously published work and includes as new features of the basic genetic operators. To analyze the performance and effectiveness of the proposed algorithm and different genetic operators, computational studies of the proposed algorithm on some randomly generated test problems have been reported.