CHAPTER 2
MF radar details and data retrieval

2.1 Introduction

This chapter can be conveniently divided into two sections. The first section of this chapter is devoted to the details of MF radar system installed at Tirunelveli (8.7°N, 77.8°E, 0.3° dip), mode of operation and the method of wind estimation, beginning with the basics of scattering mechanisms. The radar has been installed and operated by the Indian Institute of Geomagnetism. It has been providing nearly continuous wind information of the mesosphere and lower thermosphere region using spaced antenna method. The radar technique utilizes partial reflection from weakly ionized atmospheric layers. A full correlation analysis (FCA) is used to extract wind information from the data [Briggs, 1984] and a brief account on this is given in this chapter.

The interpretation of radar echoes from the higher heights sampled by the radar, in terms of neutral winds is complicated, as the measured drifts at highest heights (above 92 km) probed by the radar are influenced by the electrodynamical processes unique to the equatorial region. There is another problem due to total reflection, as there will be a difference in the true and virtual heights as the transmitted electromagnetic ray approaches the total reflection height, which normally occurs above 94 km. A brief discussion on these aspects is presented in this chapter.

The second section of this chapter describes a few mathematical techniques namely standard Fourier techniques, time-domain filtering and singular and bispectral analyses used for examining the MF radar data.
2.2 Radar scattering mechanisms

2.2.1 Refractive index

Gage and Balsley [1980] has given a formula on refractive index for radio waves propagating through the lower and middle atmosphere as

\[ n-1 = 3.73 \times 10^{-1} e/T^2 + 77.6 \times 10^6 P/T - N_e/2N_c, \]  

(2.1)

where \( e \) is the partial pressure of water vapour in mb, \( T \) is the absolute temperature, \( P \) is the atmospheric pressure in mb, \( N_e \) is the number density of free electrons and \( N_c \) is the critical plasma density. The first term is the contribution due to water vapour, and is dominant in the troposphere due to the presence of relatively large amount of water vapour. The second term is due to dry air and thus dominates above the tropopause. The third term represents neutral turbulence induced electron density fluctuations and is dominant at ionospheric heights (above 50 km).

The scattering/reflection mechanisms responsible for the radar signal return from the atmosphere is broadly classified into coherent scatter and incoherent scatter.

Coherent scatter results from macroscopic fluctuations in refractive index associated with clear air turbulence. In this type of scatter, the echo power depends on the density gradients associated with the coherent scatterers. There are three types of coherent scatter, namely, Bragg scatter, Fresnel reflection and Fresnel scatter that are depicted in Fig. 2.1. We will briefly consider each of these in turn.
Fig. 2.1. Artist’s conception of atmospheric refractivity structure pertinent to Bragg scatter (A), Fresnel scatter (B), and Fresnel reflection (C) [after Gage and Balsley, 1981].
1. Bragg scatter

Bragg scatter or turbulent scatter is generally a coherent scattering phenomenon that results from variations in the refractive index with a scale projected along the line of sight of the radar equal to one-half the radar wavelength. Bragg scatter can be isotropic i.e. without causing a radar aspect sensitivity. The term aspect sensitivity is typically used to describe the decrease of radar return signal power with increasing beam pointing zenith angle; there can also be a smaller variation of radar return signal power as a function of beam azimuth angle. Small aspect sensitivity means that the scatterers are specular with the backscattered power falling rapidly with zenith angle [Rottger, 1980]. This occurs when the turbulent irregularities of refractive index are randomly distributed in the vertical direction but exhibit coherency in the horizontal direction. Bragg scatter can be anisotropic, causing the aspect sensitivity, if the statistical properties of the irregularities, namely their correlation distances, are dependent on direction. Although the aspect sensitivity is different for these two processes, the temporal variations of the radar echoes should be similar because of the randomly fluctuating irregularities. Their Doppler spectrum approximately reveals a Gaussian shape.

2. Fresnel scatter

Fresnel scattering occurs from many closely spaced layers randomly distributed in height. In the case of Bragg scatter, the scattering scales in the horizontal are less than the size of the first Fresnel zone and many scatterers are involved. Therefore, the scatter produced tends to vary randomly in amplitude and phase and is relatively weak. Fresnel scatter produces much stronger and more coherent scattered signals.

3. Fresnel reflection

Fresnel reflection occurs from a sharp vertical gradient in the refractive index that is horizontally coherent over a scale greater than a Fresnel zone. It is same as that of the Fresnel scattering model except that the backscattered signals are from a single extended
layer or sheet. In this case the returned signal may be strong and will be quite steady in amplitude and phase over relatively long periods. Fresnel reflections are responsible for "slow fading" of radio signals backscattered from the mesosphere at medium frequencies. Radar returns from the mesosphere, particularly at long wavelengths, are generally referred to as partial reflections. However, there is observational evidence to suggest that the radar returns are due to both Fresnel reflections from coherent electron density gradients and scattering from turbulent irregularities.

Incoherent scatter is also known as Thomson scatter, or thermal scatter. This type of scatter occurs when the incident electromagnetic field interacts directly with free electrons, which are forced to oscillate by the electric field component. These oscillating electrons re-radiate at a frequency similar to the incident radiation. If the wavelength of the incident radiation is small enough, then the reradiated signal is phase-incoherent. Since the back scattered radar signal arises from small electron density fluctuations due to random thermal motions of the ions and electrons, it has to rely on the Thomson scattering cross section of the electron and hence it is extremely weak. As a consequence of this, incoherent scatter radars must possess relatively large power-aperture products in order to observe these backscattered signals.

2.3 Spaced antenna wind analysis

Earlier, a crude method was designed to calculate the wind vector from time series sampled from spaced antennae [Mitra, 1949]. This method was known as "method of similar fades". Mitra [1949] used this technique to measure the ionospheric drifts from the amplitude fadings of the echoes reflected from the ionosphere. This method is based on the assumption that the features of the fadings recorded at the three antennas are similar but displaced in time with respect to each other. From the time delays, one can easily determine the velocity of the fading pattern over ground, which is twice the velocity of the irregularities of the ionosphere. However, the pattern may change as it moves and add to the signal variation and the method described above is not applicable. Briggs [1950] developed a method called 'full correlation analysis' (FCA) that takes into
account the random change in the diffraction pattern. The analysis calculates the auto- and cross-correlation functions for the three fading records, obtained from the three receiving antennas, at different time lags to extract the background wind vector. This wind vector obtained without considering random changes in the pattern is called apparent velocity and is, in general, an over-estimate of the background wind velocity. It is corrected for random changes in the diffraction pattern to obtain true velocity. Phillips and Spencer [1955] extended the analysis to anisotropic case to find the size and shape of the pattern as well as the velocity parameters corrected for the anisotropy effects. Many discussions of the full correlation analysis in its current form and some of the computational difficulties involved in its implementation are available in the literature [Meek, 1980; Briggs, 1984; Hocking et al., 1989].

In this analysis, a moving pattern over spatially arranged three sensors at the vortices of an equilateral triangle is generally represented by a family of ellipsoids of the forms

\[
\rho(\xi, \eta, \tau) = \rho(A(\xi - V_x \tau)^2 + B(\eta - V_y \tau)^2 + 2H(\xi - V_x \tau)(\eta - V_y \tau)) + K\tau^2
\]

\[
\rho(\xi, \eta, \tau) = \rho(A\xi^2 + B\eta^2 + C\tau^2 + 2F\xi\eta + 2G\eta\tau + 2H\xi\eta),
\]

where \(\rho(\xi, \eta, \tau)\) is the cross-correlation function, which is a function of \(\xi, \eta\), spatial separation between any two receivers in \(x\) and \(y\)-directions respectively and \(\tau\), the time shift with the two receivers. \(V_x\) and \(V_y\) are the component velocities of the moving pattern with respect to a stationary observer. The constants \(A, B, C, F, G\) and \(H\) fully describe the size, shape of the pattern and the velocities. Hence the evaluation of these constants provides the velocity and anisotropy parameters.

Maximum correlation occurs when \(\partial \rho / \partial \tau = 0\) and the corresponding time shift

\[
\tau = -(F/C)\xi - (G/C)\eta.
\]
It is assumed that the cross-correlation between two receivers at zero time shift is equal to the auto-correlation function of the receivers. Mathematically,

\[ \rho(0, 0, \tau) = \rho(\xi, \eta, 0). \]  

(2.5)

Applying the above equation in equation (2.2), we find

\[ \rho(C\tau^2) = \rho(A\xi^2 + B\eta^2 + 2H\xi \eta), \]  

(2.6)

and thus

\[ \tau^2 = (A/C)\xi^2 + (B/C)\eta^2 + (2H/C)\xi \eta \]  

(2.7)

The above equation (2.7) is for a pair of sensors and it gives the time shift at which the mean auto correlation function (averaged over all sensors) has the value equal to the value of cross-correlation between the pair of sensors at zero time shift. Likewise, we get three equations for the three pair of sensors. The coefficients A/C, B/C and H/C can be estimated simultaneously and will in general be over-estimated and by using least squares technique the optimum values and errors can be determined.

By equating the coefficients of \( \xi \) \( \tau \) and \( \eta \) \( \tau \) in equations (2.1) and (2.2), we obtain,

\[ AV_x + HV_y = -F, \]  

\[ BV_y + HV_x = -G. \]  

(2.8)

These equations can be solved simultaneously to obtain the components of true velocity \( V_x \) and \( V_y \). Then the magnitude and direction of the true velocity are given by

\[ |V|^2 = V_x^2 + V_y^2, \]  

(2.9)

\[ \tan \phi = V_x/V_y. \]  

(2.10)

The spatial properties of the pattern itself can be determined using spatial correlation function,
\[ \rho(\xi, \eta, 0) = \rho(A\xi^2 + B\eta^2 + 2H\xi \eta), \quad (2.11) \]

They are usually specified by giving the values of the minor axis of the characteristics ellipse (a particular ellipse for which, \( \rho(\xi, \eta, 0) = 0.5 \)), the axial ratio, and the orientation of the major axis, measured clockwise from north.

The orientation \( \theta \) of the major axis is given by

\[ \tan 2\theta = 2H/(B-A), \text{ clockwise from the y-axis} \quad (2.12) \]

The major and minor axes are respectively,

\[ a = (2Ct_{0.5}^2)\left[\frac{1}{2}(A + B) - \sqrt{(4H^2 + (A - B)^2)}\right] \quad (2.13) \]
\[ b = (2Ct_{0.5}^2)\left[\frac{1}{2}(A + B) + \sqrt{(4H^2 + (A - B)^2)}\right]. \quad (2.14) \]

The axial ratio \( r \) is given by the ratio of the major to the minor axes.

The random changes of the moving pattern are described by a parameter called 'mean lifetime' or 'time-scale'. This is the mean lifetime of the irregularities which cause the diffraction pattern to change randomly and is often taken to be the time lag \( t_{0.5} \) at which the auto-correlation function falls to 0.5. Mathematically,

\[ t_{0.5}^2 = \frac{C}{K}, \quad (2.15) \]

where \( t_{0.5} \) is the time lag for which the directly observed auto-correlation function for a fixed observer falls to 0.5. To find the ratio \( C/K \), the coefficient of \( r^2 \) in equations (2.2) and (2.3) can be equated to obtain

\[ C = AV_x^2 + BV_y^2 + K + 2HV_xV_y. \quad (2.16) \]
Since $V_x$ and $V_y$, and the ratios $A/C$, $B/C$, $H/C$, the above equation (2.16) can be used to find $C/K$.

Some data are rejected, when the assumptions of the analysis are not valid [Briggs, 1984]. The following criteria are found satisfactory for the spaced antenna wind analysis.

1. The mean signal is either very weak or so strong that the receivers are saturated most of the time.
2. The fading is very shallow (signal almost constant). The standard deviation should be at least 2% of the mean signal.
3. The signal to noise ratio is less than 6 dB.
4. The mean auto-correlation function has not fallen to at least 0.5 for the maximum number of lags, which have been computed.
5. The cross-correlation functions have no maxima within the number of lags which have been computed.
6. The cross correlation functions are oscillatory, so that it is impossible to identify the correct maxima.
7. The sum of the time displacements $\tau'_{ij}$ does not sum to zero, as it ideally should, for the data taken in pairs around the triangle of antenna. This condition may be relaxed somewhat to give a more practical criterion: if $|\Sigma \tau'_{ij}|/\Sigma |\tau'_{ij}| > 0.2$ the results are rejected. This quantity has been called the "normalized time discrepancy".
8. The computed value of $V_c^2$ is negative, so that $V_c$ is imaginary. This criterion should not be applied too rigidly, because if the computed $V_c^2$ is only slightly negative, this probably means that the true value is zero, and the small negative value has arisen from statistical fluctuations. Such data may therefore be particularly good, indicating pure drift with negligible random changes.
9. The computed coefficients indicate hyperbolic rather than elliptical contours, i.e. data will be rejected, if $H^2 < AB$
10. The polynomial interpolation procedures break down at any stage.
It is advisable to reject any results for which the correction of the full correlation analysis are very large, i.e. those for which the apparent or true velocities differ greatly in magnitude and or direction. Suitable criteria are: the data will be rejected if $0.5V \geq V_0 \geq 3V_t$ or $|\phi_1 - \phi_0| \geq 40^\circ$.

2.4 About the radar

The radar system is similar to the system installed at Christmas Island [Vincent and Lesicar, 1991]. It consists of three sections.

2.4.1 The transmitting and receiving antennas

The schematic diagram of the MF radar system is shown in Fig. 2.2. The radar has been installed by Indian Institute of Geomagnetism in the year 1992. The system details, mode of operation and method of wind estimation are the same as that installed at Christmas Island [Vincent and Lesicar, 1991]. The transmitting antenna array is arranged in a square, and consists of four centre-fed half-wave dipoles, approximately 75 m in length. One set of dipoles is arranged parallel to the Earth's magnetic field while the other is arranged orthogonal to it. This is because Tirunelveli is situated close to the magnetic equator and the Ordinary (O) and the Extraordinary (E) rays are linearly polarized parallel to, and at right angles to, the magnetic field respectively. The O-mode dipoles are used for daytime transmission while the E-mode dipoles are used at nighttime. This geometrical arrangement also applies to the three receiving antennas. The ideal height of the transmitting antennas is one quarter of the transmitting wavelength above the ground plane, which for an MF radar operating at 1.98 MHz is about 30 m.

The receiving antennas are of the inverted-V type, and are situated at the vertices of an equilateral triangle whose basic spacing is 180 m. The centroids of both the transmitting and receiving arrays coincide, making the radar system a monostatic. The impedance of each receiving antenna is 50 $\Omega$ to match the coaxial cable that takes the signal to the receiving system.
Fig. 2.2. Schematic diagram of the MF radar system at Tirunelveli showing the geometry of the transmitting and receiving antennas.
2.4.2 The transmitter, receiving and data acquisition system, and computer controller

The transmitter, the receiving and data acquisition system (RDAS) and the computer are the main components of the radar system. The transmitting system uses totally solid-state transmitters. It consists of a combiner unit, 10 power amplifier modules, a fan unit, a driver unit and a power supply unit. The transmitter operates at a frequency of 1.98 MHz and is fully phase coherent. The transmitter power is 25 kW, with a pulse length of 30 µs which corresponds to a 4.5 km height resolution. The pulse repetition frequency is 80 Hz with 32 point coherent integration during daytime. This is reduced to 40 Hz during nighttime with 16 point integration, to avoid problems arising from multiple reflections from equatorial spread F. Table 2.1 lists some of the characteristics of the transmitting system.

The specifications of RDAS operating at Tirunelveli is listed in table 2.2. The receiving system is controlled by a pc-based microprocessor, which both controls the transmitter and acquires data. The data are subsequently transferred to a host personal computer for analysis and subsequent storage. The 1.98 MHz transmitter pulses are derived from the master oscillator and frequency synthesis module. The transmitted signal is linearly polarized. The three receivers are of superheterodyne type. In each receiver, the received 1.98 MHz signal is mixed with a 2.475 MHz local oscillator to produce a 495 kHz intermediate frequency (IF), which is then fed to the signal processor modules. The signal processors are phase sensitive: the 495 kHz IF signals are heterodyned with in-phase and quadrature local oscillators to produce in-phase and quadrature components, which are then digitized to 8-bit resolution.
Table 2.1: Characteristics of transmitting system of MF radar

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak Transmit power</td>
<td>25 kW RMS</td>
</tr>
<tr>
<td>Maximum duty cycle</td>
<td>0.25 %</td>
</tr>
<tr>
<td>Operating frequency</td>
<td>1.98 MHz</td>
</tr>
<tr>
<td>Half power pulse width</td>
<td>30 µS</td>
</tr>
<tr>
<td>Pulse rise and fall times</td>
<td>15 µS</td>
</tr>
<tr>
<td>Output impedance</td>
<td>50 Ω unbalanced 30 kHz</td>
</tr>
<tr>
<td>Half power bandwidth</td>
<td>30 kHz</td>
</tr>
<tr>
<td>Harmonics (third, fifth and even harmonics)</td>
<td>-63 dBc, &gt;-70 dBc, &gt;-70 dBc</td>
</tr>
<tr>
<td>Load</td>
<td>An array of four 75Ω dipoles</td>
</tr>
<tr>
<td>Overall efficiency</td>
<td>~70%</td>
</tr>
<tr>
<td>Polarization</td>
<td>Linear</td>
</tr>
<tr>
<td>Height of transmitter towers</td>
<td>30 m</td>
</tr>
</tbody>
</table>

Table 2.2: Characteristics of receiving system of MF radar

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of receivers</td>
<td>3</td>
</tr>
<tr>
<td>Sampling starting height</td>
<td>60-98 km (Day), 70-98 km (Night)</td>
</tr>
<tr>
<td>Sampling interval</td>
<td>2 km</td>
</tr>
<tr>
<td>Pulse repetition frequency</td>
<td>80 Hz (Day), 40 Hz (Night)</td>
</tr>
<tr>
<td>Coherent integrations</td>
<td>32 (Day), 16 (Night)</td>
</tr>
<tr>
<td>Samples per data set</td>
<td>256</td>
</tr>
<tr>
<td>Sampling period</td>
<td>2 minutes</td>
</tr>
<tr>
<td>Centre frequency</td>
<td>1.98 MHz</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>~39 kHz</td>
</tr>
<tr>
<td>Sensitivity</td>
<td>~0.3 µV</td>
</tr>
<tr>
<td>Maximum gain</td>
<td>~10³</td>
</tr>
<tr>
<td>Gain control</td>
<td>Programmable in steps of 1 dB</td>
</tr>
<tr>
<td>Gain control Range</td>
<td>60 dB</td>
</tr>
<tr>
<td>Input impedance</td>
<td>50 Ω</td>
</tr>
</tbody>
</table>
The RDAS acquires and stores in its memory a complete data set. A data set comprises 256 points of 20 consecutive height samples at 2 km intervals, thus providing a 40 km range coverage. Though the radar samples at every 2 km, the radar pulse length of about 30 μs means, however, that the actual height of resolution is about 4.5 km. Each data sample is the result of integrating the digitized data over a number of consecutive transmitter pulses. The transmitter pulses are coherently averaged to produce a mean data point for every 0.4 seconds. A complete data set is therefore acquired in 102.4 seconds.

It is then transferred to the host computer. The coherent integration is done to improve signal-to-noise ratio, which improves the performance of the system particularly during nighttime, when the ionization is low. The other radar system details are given in the tables 2.1 and 2.2.

The recording information is programmable by the host computer. It includes the number of heights per sample, transmitter pulse repetition frequency, integrations (Tx pulses) per sample point, sample points per data set and receiver gains. Different recording configurations are used for daytime and nighttime. The receiver gains are dynamically adjusted by the analysis program before the start of accumulation of each data set.

2.4.3 Computer and analysis software

After each data acquisition run, the computer performs a full correlation analysis on the 256 point complex data set to determine mean winds and various other parameters, namely, pattern decay time, pattern axial ratio, signal-to-noise ratio and the received power of antenna. The system runs continuously for all the days. However the data acquisition is interrupted, while taking data backup and during power failure. The data backup is normally done for every fortnight.
2.5 Statistics of data

The MLT wind data used in the thesis are for the period January 1993-December 2001. The data for the different seasons of the period are grouped into four different height regions namely 60-68 km, 70-78 km, 80-88 km, and 90-98 km and into night (1900-0600 IST) and day times (0700-1800 IST). Time-height distribution of percentage of data accepted for different seasons is given in the tables 2.3a-d.

From the tables, it can be inferred that the percentage of data acceptance is poor in the height region 60-68 km during daytime, with maximum value of only 7% and with mean value of 1.5%. During nighttime, almost no data has been accepted. The maximum rate of data acceptance is relatively higher (~50%) and the mean value is also increased to 16% in the height region 70-78 km during daytime. However, during nighttime, data acceptance rate is being still poor (2%). Below 80 km, due to very low ionization, the percentage of data acceptance is very low during daytime and is insignificant during nighttime.

There has been significant enhancement in the percentage of data acceptance in the height region 80-88 km both during daytime as well as nighttime. The maximum percentage of data acceptance is nearly 80% during daytime and 46% during nighttime and the respective mean values are 33% and 19%. In the height region 90-98 km, the mean value of the percentage of data acceptance is reduced to 21% during daytime is reduced and increased to 24% during nighttime. Since the percentage of data acceptance is relatively larger in the height region 80-98 km both during nighttime and during daytime. the present analysis is, in general, restricted to the height region.

The tables also indicate that there is significant reduction in the acceptance rate with year. For example, the daytime (nighttime) percentage of data acceptance for the height region 80-88 km during winter is decreased from nearly 60% (24%) in the year 1993 to 12% (4%) in the year 2000. Similar is the case for other seasons and for other height regions also. Since the lower percentage of data acceptance in the later years may
affect the gross features revealed by the analysis, the interpretation of these features should be made with extreme care.

There are a few data gaps of two to four week duration during March and May 1993, May, July and October 1994, October 1997, March-June 1998, September-October, 1999 for various reasons. These data gaps do not affect much, when seasonal variabilities of winds with periods greater than four months are taken for study. However, as these data gaps will affect the study of the motions less than seasonal, the data for these segments are not considered for those studies.

Tables 2.3a-d: Time-height distribution of percentage of data accepted for different seasons.

Table 2.3a: Winter solstice (November-February).

<table>
<thead>
<tr>
<th></th>
<th>60-68 km</th>
<th>70-78 km</th>
<th>80-88 km</th>
<th>90-98 km</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Day</td>
<td>Night</td>
<td>Day</td>
<td>Night</td>
</tr>
<tr>
<td>1992-93</td>
<td>5.3</td>
<td>0.0</td>
<td>37.6</td>
<td>1.1</td>
</tr>
<tr>
<td>1993-94</td>
<td>1.4</td>
<td>0.0</td>
<td>20.5</td>
<td>0.4</td>
</tr>
<tr>
<td>1994-95</td>
<td>1.2</td>
<td>0.1</td>
<td>26.0</td>
<td>0.5</td>
</tr>
<tr>
<td>1995-96</td>
<td>0.6</td>
<td>0.0</td>
<td>19.3</td>
<td>0.2</td>
</tr>
<tr>
<td>1996-97</td>
<td>0.3</td>
<td>0.0</td>
<td>5.7</td>
<td>0.2</td>
</tr>
<tr>
<td>1997-98</td>
<td>1.6</td>
<td>0.1</td>
<td>10.6</td>
<td>0.3</td>
</tr>
<tr>
<td>1998-99</td>
<td>0.4</td>
<td>0.0</td>
<td>3.9</td>
<td>0.7</td>
</tr>
<tr>
<td>1999-00</td>
<td>0.1</td>
<td>0.0</td>
<td>2.4</td>
<td>0.2</td>
</tr>
<tr>
<td>2000-01</td>
<td>0.0</td>
<td>0.1</td>
<td>2.6</td>
<td>0.2</td>
</tr>
<tr>
<td>2001-02</td>
<td>0.0</td>
<td>0.1</td>
<td>2.7</td>
<td>0.1</td>
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</table>

Table 2.3b: Spring equinox (March-April).

<table>
<thead>
<tr>
<th></th>
<th>60-68 km</th>
<th>70-78 km</th>
<th>80-88 km</th>
<th>90-98 km</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Day</td>
<td>Night</td>
<td>Day</td>
<td>Night</td>
</tr>
<tr>
<td>1993</td>
<td>5.1</td>
<td>0.0</td>
<td>49.7</td>
<td>0.9</td>
</tr>
<tr>
<td>1994</td>
<td>0.5</td>
<td>0.0</td>
<td>28.3</td>
<td>0.1</td>
</tr>
<tr>
<td>1995</td>
<td>0.6</td>
<td>0.0</td>
<td>29.2</td>
<td>0.3</td>
</tr>
<tr>
<td>1996</td>
<td>0.2</td>
<td>0.0</td>
<td>26.0</td>
<td>0.1</td>
</tr>
<tr>
<td>1997</td>
<td>1.2</td>
<td>0.0</td>
<td>17.5</td>
<td>0.2</td>
</tr>
<tr>
<td>1998</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<tr>
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<td>0.0</td>
<td>0.0</td>
<td>6.0</td>
<td>0.0</td>
</tr>
<tr>
<td>2000</td>
<td>0.0</td>
<td>0.0</td>
<td>3.5</td>
<td>0.2</td>
</tr>
<tr>
<td>2001</td>
<td>0.0</td>
<td>0.0</td>
<td>3.7</td>
<td>0.1</td>
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</table>

40
Table 2.3c: Summer solstice (June-August).

<table>
<thead>
<tr>
<th></th>
<th>60-68 km</th>
<th>70-78 km</th>
<th>80-88 km</th>
<th>90-98 km</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Day</td>
<td>Night</td>
<td>Day</td>
<td>Night</td>
</tr>
<tr>
<td>1993</td>
<td>1.3</td>
<td>0.0</td>
<td>25.4</td>
<td>0.6</td>
</tr>
<tr>
<td>1994</td>
<td>1.3</td>
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Table 2.3d: Winter solstice (September-October).

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2.6 Potential problems in the interpretation of SA drifts

Close to the magnetic equator, the interpretation of the spaced antenna drifts at higher heights (>90 km) in terms of neutral winds is complicated by the presence of equatorial electrojet (EEJ). The EEJ is an intense eastward current system flowing in a narrow latitudinal width of ±3° at an altitude of ~105 km centred around the magnetic equator and is clearly manifested in the ground geomagnetic field variation. At lower heights (below ~90 km), the ionospheric irregularities, which are responsible for partial reflection of radar echoes, are created by neutral turbulence possibly generated by gravity wave breaking. However at heights (above ~90 km) close to the region of EEJ, they are created by neutral turbulence as well as EEJ electric field driven ionospheric plasma
instabilities such as two stream instability (called as type-1) and gradient-drift instability (type-2) [for example, Fejer and Kelley, 1980]. Hence, spaced antenna drift measurements of this region have to be interpreted with caution, since the type-2 irregularities are sufficiently strong at certain times that the intense signals associated with them may overshadow the normal echoes.

The effects of equatorial E region electric fields on the spaced antenna drifts are examined in a case study approach. 15 January, 15 April, 15 July and 15 October are selected for the year 1995, a solar minimum year. All the four days are geomagnetically quiet days with $A_p$ less than or equal to 6. Figs. 2.3a-d show the time variations in the EEJ strength for the days 15 January, 15 April, 15 July and 15 October along with other parameters, which will be discussed shortly. The EEJ strength can be determined by subtracting the H-component of geomagnetic field variation for the off-equatorial station-Alibag from that for the equatorial station-Trivandrum. On both 15 January and 15 October, the field started rising at 0600 hrs. IST (Indian Standard Time) to reach a peak greater than 50 nT at 1300 IST hrs. The field decreases at an equal rate to reach minimum at 1800 IST hrs. The same figures show zonal and meridional drifts at 98 km as determined by the MF radar at Tirunelveli. 15 April 1995 is a counter-electrojet day with morning maximum of nearly 80 nT at 1100 IST and an afternoon depression of nearly 25 nT in the strength of EEJ. Electrojet is relatively not well developed on 15 Jul 1995. The maximum EEJ strength of 25 nT is observed around noon.

On 15 January, at 98 km, eastward drifts prevail in the early morning and late evening hours. In the afternoon hours, one can notice westward speeds nearly reaching 60 m/s at 0900-1000 IST followed by a gradual decrease of westward speed to a minimum at 1800 IST. The Meridional drift shows equatorward motion around noon hours, whereas it shows poleward motions during early morning and late evening hours. On 15 October, eastward drifts prevail in the early morning hours and westward winds prevail in the remaining hours. The westward drift maximum of nearly 40 m/s occurs at 1000-1200 IST followed by a gradual decrease of westward speed. Meridional drifts show poleward motion in the early morning hours and remain equatorward motions in the remaining
Figs. 2.3a-d. Comparison of time variation of the EEJ strength with that of the radar measured parameters, namely, pattern decay time, zonal and meridional drifts for the days 15 January, 15 April, 15 July and 15 October of the year 1995 respectively.
hours. It may be noticed that on both these days, the enhanced westward motion and the enhanced equatorward motion around noon hours occur nearly at the same time when the EEJ strength is maximized. On 15 April 1995 and 15 July 1995, there is no clear relationship between the time variation of zonal and meridional winds, and that of EEJ strength. It may be noted that even though EEJ is well developed in the morning hours, similar variation is not clearly observed in the zonal and meridional drifts. This shows that the spaced antenna drifts measured at 98 km response well in some cases to the variations in the EEJ current system. The figures also show the time variation of pattern decay time, which is a geometrical parameter estimated using full correlation analysis. Since the effects of winds are inherently removed in the analysis as the observer moves with the pattern, the pattern decay time is expected to represent only the random changes of the scattering echoes [Lesicar, 1993]. The daytime values at 98 km on 15 January reveal times are shorter than 2 seconds. In the early morning and late evening hours, it is greater than 2 seconds. Thus the diurnal variation of pattern decay time is like a U-shaped structure that is clearly associated with the variation in the EEJ strength. Though it is not clear as in 15 January, similar U-shaped variation is observed on 15 October also. Since the pattern decay time represents the random changes within the layer, it has been believed that the mechanism responsible for the partial reflection of radar echoes is associated with the plasma instabilities driven by EEJ electric field. However, on the days, 15 April and 15 July, no relationship between the variation of EEJ and pattern decay time is observed. In this case, the irregularities that caused partial reflection might have been generated due to neutral turbulence even at heights close to the region of EEJ current system.

The above observations on different electrodynamical conditions as noticed in the strength of EEJ demonstrate the influence of EEJ on the spaced antenna drifts at 98 km measured by the MF radar situated close to the magnetic equator. Hence interpretation of the MF radar drifts in terms of neutral winds at heights close to the region of EEJ needs to be made with extreme care as they might be due to the combined effects of neutral winds and the electric fields associated with the EEJ.
Fig. 2.4. The column graph for all quiet days (Ap ≤ 6) of December 1996, showing the correlation coefficient (CC) between EEJ strength and the radar measured parameters, namely, the pattern decay time, zonal and meridional drift velocities for different heights (80-98 km).
Fig. 2.4 displays the column graph for all quiet days (Ap ≤ 6) of December 1996 and it shows correlation coefficient (CC) between EEJ strength and the MF radar measured parameters, namely, the pattern decay time (top panel), zonal drift velocities (middle panel), meridional drift velocities (bottom panel) at different heights (80-98 km). Three hourly values centred around noon is used for the purpose. The CC is −0.64 for the zonal drift at 98 km and it increases, as height decreases. The most positive CC of +0.57 can be seen at 82 km. For the meridional drift at 98 km, it is +0.6. There is remarkable anti-correlation between the pattern decay time and EEJ strength. The CC is less than −0.8 for PDT at heights greater than 92 km. These results show that the influence of EEJ associated electric field is greater at the higher heights and it progressively decreases, as height decreases.

One more problem with the MF radar system operating at sites close to the equatorial region, is due to total reflection. The total reflection occurs when plasma frequency is equal to the radar operating frequency. The medium frequency radar at Tirunelveli operates at 1.98 MHz. The plasma frequency equals to this operating frequency at heights greater than 90 km and the height at which this condition is satisfied depends on the solar activity.

Let us assume a model ionosphere in which the electron concentration increases exponentially with height z above the ground so that \( f_0^2 = F^2 e^{\alpha z} \). For a vertically incident wave of frequency f, the true height of total reflection is given by \( z_0 = \frac{2}{\alpha \alpha} \ln(f/F) \) [Budden, 1985]. By fitting an exponential growth function \( e^{\alpha z} \) to the electron density profiles for the heights 80-98 corresponding to 1200 IST of 15th day of January, April, July and October for the years 1995-2001, \( \alpha \) and \( F \) are obtained. Substituting the values of \( \alpha \) and \( F \), the true height of total reflection is obtained. Fig. 2.5 shows the heights of total reflection (bottom panel) along with annually averaged sunspot number (top panel) for the years 1995-2001. The height of total reflection is around 98 km in the years 1995 and 1996. It may be noted that as the sunspot number increases, the total reflection heights decrease monotonically irrespective of season from the year 1997 to 95.5 km in the year 2000. There is again an increase in the height of total reflection in
Fig. 2.5. The heights of total reflection (bottom panel) and the annually averaged sunspot number (top panel) for the years 1995-2001.
the year 2001. This is in accordance with the decrease of sunspot number. These observations show that during sunspot minimum periods, total reflection occurs at heights around 98 km, whereas during sunspot maximum periods, it occurs at heights close to 95 km.

A vertically transmitted wave of frequency \( f \) experiences a decrease of group velocity as soon as the electron density departs from zero. The group retardation is the difference between the virtual height \( h' \) and the real height for any reflection that occurs within the ionosphere [Namboothiri et al. 1993]. It reaches maximum at the height of total reflection for the wave. The virtual height \( h' \) at which a pulse of frequency \( f \) is totally reflected from the ionosphere is given by

\[
h'(f) = \int \mu_0'(f, h) \, dh \tag{2.3}
\]

where \( \mu_0' \) is the group refractive index for the O mode and the limit of integration is the point of reflection. The O mode is totally reflected at a height where the wave frequency equals the plasma frequency \( f_p = \left( \frac{Ne^2}{\pi e^2} \right)^{1/2} \). The term \( \mu_0' \) is related to \( \mu_o \), the wave index of refraction by

\[
\mu_0' = \frac{d(\mu_o f)}{df}, \tag{2.4}
\]

which gives

\[
h'(f) = \int \frac{d(\mu_o f)}{df} \, df = \frac{d}{df} \int \mu_o f \, dh. \tag{2.5}
\]

The value of \( \mu_0 \) is computed from the Appleton-Hartee equation (QL approximation). Assuming the base of the ionosphere at 80 km, the virtual height for a wave frequency \( f \) is given by

\[
H'(f) = \int (d(\mu_o f)/df) \, dh + 80 \tag{2.6}
\]

The value of electron density is obtained from the International Reference Ionosphere (IRI) model [Rawer et al., 1978].
Fig. 2.6. Comparison of real and group retarded heights at 1200 IST for the months January, April, July and October of years 1995-2001.
Fig. 2.6 shows comparison of real and group retarded heights at 1200 IST for the months January, April, July and October of years 1995-2001. Since the total reflection height is only around 98 km in the years 1995 and 1996, there is no group retardation upto the height 92 km. As the ray approaches total reflection height, there is a group retardation of around 2-7 km at heights 94-96 km. At 98 km, it becomes greater than 20 km. Since total reflection height is much lower than 98 km in the years 1997-2000, group retardation can be seen even from the height 92 km. It can be inferred that because of this group retardation, the signal appears to arrive from much higher heights than the sampled height.

The above observations of the group retardation has provided a prediction of the height up to which the data can be used without any correction due to the group retardation. For the years 1995-98, the wind data are valid up to 92 km without any correction, but for the years 1998-2001, the maximum height up to which the data can be used without correction falls to 90 km. Hence in the present study, the vertical wavelengths of tides and planetary waves are estimated from their phase gradient with heights 80-90 or 92 km only.

2.7 Mathematical techniques

2.7.1 Estimation of tidal amplitudes and phases

The MLT wind data acquired by the MF radar are harmonically analyzed using the series constructed from hourly averaged values. The time series for the zonal and meridional wind components are assumed to consist of prevailing and 24-, 12- and 8-hour harmonic components. Hence each wind component is represented as a function of time by the expression,

\[ F(t) = A_0/2 + \sum_{j=1}^{3} \left( A_j \cos(2\pi t/T_j) + B_j \sin(2\pi t/T_j) \right), \]  

(2.7)
where $A_0$ is the prevailing component and $A_j$ and $B_j$ are the amplitude and phase of the $j$th harmonic component with a period of $T_j$ and are given by

$$A_j = 2/n \sum_{t=0}^{n-1} F(t) \cos(2\pi t/T_j) \quad \text{and} \quad B_j = 2/n \sum_{t=0}^{n-1} F(t) \sin(2\pi t/T_j).$$  \hfill (2.8)

The amplitudes ($R_j$) and the phases ($\phi_j$) of the harmonics are determined as

$$R_j = (A_j^2 + B_j^2)^{1/2} \quad \text{and} \quad \phi_j = \tan^{-1}(B_j/A_j)$$ \hfill (2.9)

### 2.7.2 Spectral analysis

A physical process can be described either in the time domain or in the frequency domain. One representation can be converted into other by means of Fourier transform.

The Fourier transform may be written in discrete form as

$$C_k = \sum c_j \exp(2\pi ijk/N), \quad k = 0 \text{ to } N-1. \hfill (2.12)$$

The discrete Fourier transform maps $N$ complex numbers (the $c_j$'s) into $N$ complex numbers (the $C_k$'s). The computation of DFT described above involves $N^2$ complex computations. It can be computed in $N \log_2 N$, which is much less than $N^2$ operations with an algorithm called the fast Fourier transform [Cooley and Tukey, 1960].

The Fourier coefficients obtained by using FFT/DFT are then used to estimate the power spectrum, which is the energy content of the signal at particular frequency. The power spectrum curve shows how the variance of the process is distributed with frequency. The variance contributed by frequencies in the range $f$ to $f+\delta f$ is given by the area under the power spectrum curve between the two ordinates $f$ and $f+\delta f$. The periodogram estimate of the power spectrum is given by

$$P(f_k) = 1/N^2 \{C_k^2 + C_{N-k}^2\}, \quad k = 1, 2, \ldots, (N/2-1),$$ \hfill (2.13)
The power spectral density (PSD) is given by

\[ P(k) = N\Delta P(k) = (\Delta N)\{C_k^2 + C_{N,k}^2\} \]

(2.14)

Amplitude \( (R_k) = \sqrt{P(k)} = \sqrt{(\Delta N)\{C_k^2 + C_{N,k}^2\}} \)

(2.15)

To test of significance the amplitude \( (R_k) \), Nowroozi [1967] has given a method in which the probability \( P \) that the ratio \( R_k^2/\Sigma R_k^2 \) (summation runs from \( k=1 \) to \( m \)) exceeds a parameter '\( g \)' is given by

\[ P = m(1-g)^{m-1} \cdot m(m-1)(1-2g)^{m-1}/2^{m+1} + \ldots + (-1)^m m!(1-Lg)^{m-1}/(L!(m-L)!) \]  \[ (2.16) \]

It can be shown that \( \Sigma R_k^2 = 2/2m+1 \{ \Sigma (X_i-x_{\text{mean}})^2 \} \).

It is, therefore, not necessary to calculate all the harmonics. The error introduced in neglecting the higher order term is only 0.1% for \( p=0.05 \) (95% confidence level). Therefore, the parameter \( g \) can be calculated only from \( P = m(1-g)^{m-1} \). The value of \( g \) for different values of \( p \) and \( m \) is determined. The parameter \( g_k \) is given by

\[ g_k = R_k^2/(2N)\Sigma (X_i-x_{\text{mean}})^2 \]  \[ (2.17) \]

If \( g_k > g_{p=0.05} \) (for 95% confidence level), the amplitude is 95% significant.

2.7.3 Digital filtering

To extract the signal lying only in a certain frequency band, the band-pass filter is used. Filtering is more convenient in the frequency domain. The whole data record is subjected to FFT. The FFT output is multiplied by a filter function \( H(f) \). Inverse FFT is taken to get the filtered data set in time domain.
The nonrecursive or finite impulse response (FIR) filter function is given by

\[ H(f) = \sum_{k=0}^{M} c_k \exp(-2\pi i k f). \] (2.18)

Here, the filter response function is just a discrete Fourier transform. The transform is easily invertible, giving the desired small number of \( c_k \) coefficients in terms of the same small number of values of \( H(f) \) at some discrete frequencies \( f \). However, this fact is not very useful, as \( H(f) \) will tend to oscillate wildly between the discrete frequencies where it is pinned down to specific values. Hence, the following procedure of Press et al. [1993] is adopted.

i. A relatively large value of \( M \) is chosen.

ii. The \( M \) coefficients \( c_k, k=0, \ldots, M-1 \) can be found by an FFT.

iii. Most of the \( c_k \)'s are set to zero except only the first \( k \) \( (c_0, c_1, \ldots, c_{k-1}) \) and last \( K-1 \) \( (c_{M-K}, \ldots, c_{M-1}) \).

iv. As the last few \( c_k \)'s are filter coefficients at negative lag, because of the wrap-around property of the FFT, the array of \( c_k \)'s are cyclically shifted to bring everything to positive lag. The coefficients will be in the following order

\[(c_{M-K+1}, \ldots, c_{M-1}, c_0, c_1, \ldots, c_{K-1}, 0, 0, \ldots, 0)\] (2.19)

v. The FFT of the array will give an approximation to the original \( H(f) \). If the new filter function is acceptable, then we will have a set of \( 2K-1 \) filter coefficients. If it is not acceptable, either \( K \) is increased and the same procedure is repeated or the magnitudes of the unacceptable \( H(f) \) are modified to bring it more in line with the original \( H(f) \), and then FFT is taken to get new \( c_k \)'s.

vi. Now, all coefficients, except the first \( 2K-1 \) values are set to zero. Inverse transform is the taken to get a new \( H(f) \).
Estimation of the characteristics of planetary-scale waves

Unlike atmospheric tides, planetary-scale waves do not have fixed period. They exhibit a range of periods and these periods may vary with height as well as time. To compute the characteristics of these waves (the amplitude, phase and the period), the following procedure is adopted. The hourly averaged wind data are subjected to the time-domain filtering with finite impulse response (FIR) filters [Press et al. 1992] to get filtered data consisting of the particular planetary wave periods only. The filtered data are then subjected to harmonic analysis with period varied in the range of expected periods of concerned planetary wave. The wave parameters are determined in least squares sense.

2.7.4 Bispectral analysis

Power spectrum gives only the power of each spectral component and the information about the phase relation between different spectral components is suppressed. But, the bispectrum, which is a higher order spectrum, is an ensemble average of the product of three spectral components and can be used to determine whether there is any phase relation between the three spectral components. If each spectral component is independent of other components, then the amplitude and phase of the component will be different from those of other components and the bispectrum will give a nearly zero value. If the phases associated with the three components sum to the same constant each time they occur, then the bispectrum will give a non-zero value [Nikias and Raghuveer, 1987].

In the present analysis, the monthly data (720 points) are divided into ‘K’ segments containing ‘M’ samples (256 data points). 75% overlap between the segments is ensured to get a large value for K. The mean of each segment is calculated and removed. The DFT of each segment is computed. The bispectrum of $i^{th}$ segment is given by

$$B(f_1, f_2) = X_i(f_1) X_i(f_2) X_i(f_1 + f_2),$$  \hspace{1cm} (2.20)
where $X_i$ is the Fourier Transform of the $i^{th}$ segment.

The bispectral estimates are averaged across all segments. Then, they are normalized by dividing each value by the maximum value, resulting in a relative amplitude ranging from zero to one.

The bispectrum graphs are generally plotted with smaller frequencies along y-axis and next higher frequencies along x-axis. It will only be non-zero at locations $(f_1, f_2)$ where $f_3 = f_1 + f_2$ and $\phi_3 = \phi_1 + \phi_2$, where $\phi_i$ denotes the phase of the $i^{th}$ component. Non-linear interactions not only yield sum and difference frequencies, but also phase relations, which are of the same form as frequency relations. Such a phase relation is termed as quadratic phase coupling. The non-zero value of bispectrum is the result of quadratic phase coupling indicating non-linear interaction. If $\phi_3$ is random and independent of $\phi_1$ and $\phi_2$, then the bispectrum will be zero revealing that the phases are not related.

The interpretation of peaks in bispectral plots is given in Clark and Bergin [1997]. As a simple case, the presence of two frequencies and their sum denotes existence of quadratic phase coupling (i.e., non-linear mixing). If the bispectrum shows large value (peak) at any x-y point, it can be interpreted as a response to quadratic phase coupling between x-frequency, y-frequency and the sum frequency. The x- and y-coordinates (bifrequencies) account for two of the frequencies and the sum frequency is represented by a diagonal through the peak, which intersects x- and y-axes at the same value. The initial two mixing components could be at any two of the three frequencies.

In the present work, bispectral analysis is used to determine the occurrence of non-linear interaction between tides and planetary waves.

2.7.5 Singular spectral analysis

The power spectrum obtained using FFT gives only an average contribution from a specific oscillation to the total variance and the phase information is lost in deriving the power spectral density. Singular spectral analysis extracts principal components of the variability even when the system is non-stationary. This method generates data adaptive
filters, whose transfer functions highlight regions where sharp spectral peaks occur and thus helps reconstruction of the original time series with just a few principal components close to the spectral peaks. In contrast to the Fourier components, the principal components need not be sinusoidal in nature. Briefly SSA decomposes the original time series into its significant signal components with least noise [Rangarajan and Araki, 1997].

Assume a finite time series \( y(t) \) of length \( N \).

\[
y(t) = y(K \cdot t_0), \quad K=1,2,3,\ldots, N
\]  
and \( t_0 \) is sampling interval. The series is normalized using the mean \((\bar{y})\) and standard deviation \((\sigma_y)\). The new series will be

\[
x(t) = \frac{y(t) - \bar{y}}{\sigma_y}, \quad t=1,2,3,\ldots, N
\]  
The sampled time series is then embedded in an \( M \)-dimensional space, the consecutive sequences of \( X(t) \)

\[Z = \begin{bmatrix} X_1 & X_2 & \ldots & X_M \\ X_2 & X_3 & \ldots & X_{M+1} \\ \vdots & \vdots & \ddots & \vdots \\ X_{N-M+1} & \ldots & \ldots & X_N \end{bmatrix}
\]  

The matrix so derived is called the Trajectory matrix. For different choices of \( M \), different trajectory matrices can be obtained. However, \( M \) should be larger than the autocorrelation time (the lag at which the first zero occurs). The eigen values of this matrix are then evaluated in descending order of magnitude together with the corresponding eigen vectors. As the matrix is positive symmetric Toeplitz (whose all diagonal elements are equal), the eigen values will always be positive. Ideally, the number of non-zero eigen values will correspond to the number of independent variables in the system. When quasi-periodic fluctuations are present in the time series, the eigen vectors appear as even/odd pair in phase quadrature, with corresponding eigen values nearly equal in magnitude. As the PCs are filtered versions of the original series with the \( M \) elements of the eigen vectors serving as appropriate filter weights, the resulting series would be of length \((N-M+1)\). Vautard et al. [1992] has given a method to extract a series...
of length \( N \) corresponding to a given set of eigen elements which have been called the reconstructed components (RC). The formulae for the \( k \)-th component

\[
R(X_{ij})^k = \frac{1}{i} \sum_{j=1}^{M} E_{ij}^k, \quad \text{for } 1 \leq i \leq (M-1) \text{ and } j=1 \text{ to } M
\]

\[
= \frac{1}{M} \sum_{i=1}^{M} E_{ij}^k, \quad \text{for } M \leq i \leq (N-M+1) \text{ and } j=1 \text{ to } M
\]

\[
= \frac{1}{(N+1)} \sum_{i=1}^{N} E_{ij}^k, \quad \text{for } (N-M+2) \leq i \leq N \text{ and } j=1 \text{ to } M
\]

where \( E_{ij} \) are the \( M \) eigen elements of the \( k \)-th component and

\[
A_{ik} = \sum_{j=1}^{M} E_{ij}^k, \quad 1 \leq i \leq (N-M) \text{ and } j=1 \text{ to } M.
\]

The percentage of the total variance accounted by each of the reconstructed component can be computed from the ratio of the individual eigen value to the sum of all the \( M \) eigen values which then gives rise to an immediate idea of the relative importance of a particular component to the time series. In the present work, this technique is used to extract the principal long period components of the zonal wind.

### 2.7.6 Auto- and Cross-correlation functions

The correlation between two continuous functions \( g(t) \) and \( h(t) \), which is denoted by \( \text{corr}(g,h) \), and is a function of lag \( t \). The correlation will be large at some value of \( t \) if the first function \( g(t) \) is a close copy of the second \( h(t) \) but lags in time by \( t \), i.e., the first function is shifted to the right of the second. Likewise, the correlation will be large for some negative value of the first function leads the second, i.e., is shifted to the left of the second. The relation that holds when the two functions are interchanged is

\[
\text{Corr}(g,h)(t) = \text{Corr}(h,g)(-t)
\]

The discrete correlation of two sampled functions \( g_k \) and \( h_k \), each periodic with time \( N \), is defined by

\[
\text{Corr}(g,h)_k = \sum g_{nk} h_k
\]
According to the discrete correlation theorem, this discrete correlation of two real functions $g$ and $h$ is one member of the discrete Fourier transform pair

$$\text{Corr}(g,h) \Leftrightarrow G_k H_k^*, \quad (2.28)$$

where $G_k$ and $H_k$ are the discrete Fourier transforms of $g_j$ and $h_j$, and the asterisk denotes complex conjugate.

The practical procedure adopted to find the correlation of two data sets using FFT is given below.

i. FFT is taken for the two data sets

ii. One resulting transform is multiplied by the complex conjugate of the other.

iii. Inverse transform is taken for the product

If the input data are real, the result ($r_k$) will normally be a real vector of length $N$. The components $r_k$ are the correlation at different lags (both positive and negative). The correlation at zero lag is in $r_0$, the first component; the correlation at lag 1 is in $r_1$, the second component; the correlation at lag $-1$ is in $r_{N-1}$, the last component; etc. In the present work, auto- and cross-correlations are used to study the correlation between tidal parameters.