CHAPTER 2

ROTATING LIQUID DROP MODEL

AND JACOBI TRANSITION

The atomic nucleus consists of protons and neutrons (collectively termed as nucleons) held together by a strong interacting force - the nuclear force. Various nuclear models have been put forward in order to account for the properties of nuclei both qualitatively and quantitatively. The two main models are

i) The liquid drop model and

ii) The shell model

The liquid drop model of the nucleus draws an analogy between the nucleus and a charged liquid drop. It emphasises the strong coupling between the motion of the individual nucleons in the nucleus. The fission process discovered in 1939 could also be qualitatively understood from this simple model. However, subsequent measurement of nuclear binding energies showed a variation from nucleus to nucleus that could not be explained by the liquid drop model. With the shell model, it was possible to explain such variations and also to get a general understanding of ground state spins and parities for nuclei with an odd number of particles.

In this model, each nucleon is assumed to be moving in an independent orbital in a field created by the rest of the nucleons. One then had a nuclear
model for single-particle degrees of freedom (the shell model) and another for collective degrees of freedom (the liquid drop model). The shell model, in spite of its great successes, failed to predict the large values of the observed quadrupole moments of nuclei with many particles outside the closed shell. J. Rainwater in 1950, suggested that this discrepancy might be overcome in odd-\(A\) nuclei by considering the polarization of the even-even core by the motion of the odd nucleons. Thus the nuclear core would have a spheroidal rather than a spherical shape. This distortion would make an additional contribution to the quadrupole moment. A unified description of single particle motion and collective excitations of the surface as vibrations and rotations, was developed by Bohr and Mottelson. The recognition of rotational bands in deformed nuclei gave experimental support to this model. In its simplest case, such a band obeys the rotational - energy formula.

\[ E^* = \frac{\hbar^2 I(I + 1)}{2\mathcal{I}} \]  

(2.1)

where \(I\) is the spin and \(\mathcal{I}\) the moment of inertia of the nucleus. The strength of the unified model was subsequently demonstrated by the development of a model for the motion of independent particles in the intrinsic deformed field. It now becomes possible to interpret intrinsic excitations and to calculate equilibrium shapes.

It was however found that the moments of inertia evaluated from rotational spectra were not in conformity with the deformations extracted from measurements of quadrupole moment. The former were found to be much
smaller than that of a rigid body, expected for a nucleus consisting of independent particles moving in an average nuclear field. This discrepancy is due to the pairing interaction which was also evidenced by other experimental data.

The simple rotation energy formula mentioned above might be considerably changed, as the rotation of the nucleus disturbs the individual particles in it. This disturbance is, in classical mechanics, realised by the Coriolis and centrifugal force acting in the intrinsic reference frame of the nucleus. The Coriolis force strives to get a moving particle to align its spin vector along the axis of rotation. The above mentioned pairing force couples the nucleons pair-wise together to spin zero and thus counteracts such an alignment.

However, in an odd-A nucleus, where for axially symmetric shape and no rotation, the spin vector of the odd particles is quantised along the symmetry axis. The Coriolis force will especially act on the odd particle, trying to decouple its spin vector from the symmetry axis of the nucleus and instead align it along the rotation axis. Decoupled rotational spectra formed in this way have been observed in several odd nuclei. The rotation of the nucleus can affect the paired nucleons also. If the Coriolis force is strong enough, it can break up a pair of nucleons and align their spins along the rotation axis. If this alignment occurs suddenly the total angular momentum will increase fairly much for a small change in rotational frequency.

At very high spins, there is competition between the single particle motion
and the collective rotation to carry angular momentum most efficiently. This leads to a compromise between the two limiting situations. At lower spins, in well deformed nuclei, relatively pure collective rotation that follows the geometrical relationship very well is found to predominate. In nuclei near closed shells the shell model does an excellent job in explaining states up to several MeV in excitation and carrying tens of units of angular momentum. But at very high spins, nuclei seem to have some characteristics from each of these limiting cases[1] and it is of interest to understand this compromise.

The first systematic investigation trying to understand what happens to a nucleus with very high spin, was undertaken in 1974 by Cohen, Plasil and Swiatecki[2]. They considered the nucleus to be structureless, charged liquid drop subject to Coulomb and surface forces. The drop was assumed to be rotating with the rigid body moment of inertia. The shape transitions were then extracted by minimising the deformation energy as a function of deformation and non-axiality at each spin.

2.1 THEORETICAL FRAMEWORK OF THE ROTATING LIQUID DROP MODEL

The theoretical framework is formulated on the basis of the liquid drop model for the nucleus [3] as follows. \(E_t^o = a, A^{2/3}\) (2.2)

and
\[ E_0^e = a_c \frac{Z^2}{A^{1/3}} \]  \hspace{1cm} (2.3)

where \( a_s \) and \( a_c \) are surface and Coulomb constants, \( Z \) and \( A \) the atomic and the mass number of the nucleus.

For a deformed nucleus the surface and Coulomb energies are given by

\[ E_S^e = a_s A^{2/3} f(shape) \]  \hspace{1cm} (2.4)

\[ E_C^e = a_c \frac{Z^2}{A^{1/3}} g(shape) \]  \hspace{1cm} (2.5)

The function \( f(shape) \) gives the dependence of the surface energy on shape and is equal to the dimensionless ratio of the surface area of the shape in question to the area of the original sphere. The function \( g(shape) \) is the dimensionless ratio of the electrostatic energy of a distorted shape distribution to that of the sphere.

In the case of an ellipsoidal shape described by deformation parameter \( \beta \) and shape parameter \( \gamma \), the semi axes \( R_x, R_y, R_z \) are given by

\[ R_x = R_0 \exp \left[ \sqrt{\frac{5}{4\pi}} \beta \cos \left( \gamma - \frac{2\pi}{3} \right) \right] \]  \hspace{1cm} (2.6)

\[ R_y = R_0 \exp \left[ \sqrt{\frac{5}{4\pi}} \beta \cos \left( \gamma - \frac{4\pi}{3} \right) \right] \]  \hspace{1cm} (2.7)

and

\[ R_z = R_0 \exp \left[ \sqrt{\frac{5}{4\pi}} \beta \cos \gamma \right] \]  \hspace{1cm} (2.8)

By volume conservation we have

\[ R_x R_y R_z = R_0^3 \]  \hspace{1cm} (2.9)

where \( R_0 \) is the radius of the spherical nucleus.

The moment of inertia is given by
\[ \frac{\Sigma_{\text{rig}}(\beta, \gamma)}{\hbar^2} = \frac{1}{5} \frac{\text{AM}(R_x^2 + R_y^2)}{\hbar} \]  

(2.10)

Z-axis being the rotation axis and

\[ R_0^\beta = r_0 A^{\frac{5}{3}} \quad (r_0 = 1.16 \text{ fm}) \]  

(2.11)

The rotational energy of the nucleus now reads as

\[ \frac{\hbar^2 I^2}{2 \Sigma_{\text{rig}}(\beta, \gamma)} \]  

where

I is the spin of the nucleus.

Now the total energy of the rotating nucleus is

\[ E_{\text{RLDM}}(\beta, \gamma, I) = E_{\text{LDM}}(\beta, \gamma) + \frac{\hbar^2 I^2}{2 \Sigma_{\text{rig}}(\beta, \gamma)} \]  

(2.12)

where the non-rotating liquid drop energy is

\[ E_{\text{LDM}}(\beta, \gamma) = B_s a_s A^{\frac{2}{3}} + B_c a_c Z^2 / A^{\frac{1}{3}} \]  

(2.13)

Here \( B_s \) and \( B_c \) are the relative surface and Coulomb energies of the nucleus. Both \( B_s \) and \( B_c \) are elliptic integrals which depend on the semi-axes lengths.

2.2 RESULTS AND DISCUSSION

The first systematic investigation trying to understand what happens to a nucleus with very high spin, was undertaken in 1974 by Cohen, Plasil and Swiatecki [2]. They considered the nucleus to be structureless, charged liquid drop subject to Coulomb and surface forces. The drop was assumed to be rotating with the rigid body moment of inertia. One interesting result of
their investigation concerns the shape changes that the nucleus undergoes with increasing spin. At rest the liquid drop has a spherical shape. When the nucleus begins to rotate, the larger moment of inertia associated with oblate shapes, minimises the total energy of the system in the same way that the rotation of the Earth gives rise to such a shape. For very high angular momenta the stability associated axial symmetry is lost leading to an equilibrium shape with a triaxial form and ultimately to the disappearance of fission barrier[4].

The deformation energies obtained by us as a function of $\beta$ and $\gamma$ by using the rotating liquid drop model are diagramatically represented by the potential energy surfaces in the case of $^{45}$Sc, $^{44}$Ti, $^{48}$Cr, $^{52}$Fe and $^{56}$Ni in figures 1 to 5. These potential energy surfaces are drawn taking $\beta \sin \gamma$ along the X-axis and $\beta \cos \gamma$ along the Y-axis. The $\gamma = -180^\circ$ line denotes the non-collective oblate shape whereas $\gamma = -120^\circ$ line the collective prolate shape. The $\gamma$-space is restricted to $-180^\circ$ to $-120^\circ$ because we are mainly interested in the Jacobian transition which is defined as a drastic shape transition from non-collective oblate to a collective prolate or near prolate shape. This is also the reason for using Hill-Wheeler parameterization for the semi-axes of the ellipsoidal shape so as to cater to very large deformation. Figure 1(a) gives the ground state shape and deformation of $^{45}$Sc at spin $7/2 \hbar$ which is non-collective oblate with $\beta = 0.2$. When the spin is raised to $55/2 \hbar$ the shape moves to nearly prolate triaxial shape with a deformation around $\beta = 0.4$. 
At spin $63/2 \hbar$, $^{45}\text{Sc}$ almost reaches a prolate shape with a super deformation $\beta = 0.6$. Thus this nucleus shows the tendency of Jacobi transition even in the RLDM limit. Similar potential energy surfaces for $^{44}\text{Ti}$ at spins $0\hbar, 32\hbar$ and $44\hbar$ are given in figures 2(a) to 2(c). The Jacobi transition at $32\hbar$ as well as the formation of hyper deformation at $44\hbar$ in this nucleus are clearly seen from these figures. $^{48}\text{Cr}$ which is prolate at ground state with $\beta = 0.2$ moves towards triaxiality at $30\hbar$. The hyper deformation at $44\hbar$ is clearly visible from figure 3(c). In the case of $^{52}\text{Fe}$, as we go from $34\hbar$ to $44\hbar$, Jacobi transition is clearly noticed in figures 4(b) and 4(c). Finally, in the case of $^{56}\text{Ni}$, co-existing minima at the non-collective oblate axis and the collective prolate axis are seen even at $0\hbar$ spin. At the spin of $24\hbar$, $^{56}\text{Ni}$ undergoes Jacobi transition as shown in figure 5(b). The occurrence of super-hyper deformation in $^{56}\text{Ni}$ at the spin of $40\hbar$ is a nice feature to be noted in figure 5(c).

Thus the rotating liquid drop model clearly indicates that Jacobi transition should be possible at very high spins not only in $^{45}\text{Sc}$ but also in the entire $N = Z$ nuclei in the fp shell region. This clearly demonstrates the fact that the Jacobi transition is mainly a rotation induced effect.
REFERENCES:


FIGURE CAPTIONS

FIG 1(a) - 1(c) : Potential energy surfaces for $^{45}$ Sc at temperature $T = 0.0$ MeV and spins $I = 3.5h$, $27.5h$ and $31.5h$.

FIG 2(a) - 2(c) : Potential energy surfaces for $^{44}$ Ti at temperature $T = 0.0$ MeV and spins $I = 0h$, $32h$ and $44h$.

FIG 3(a) - 3(c) : Potential energy surfaces for $^{48}$ Cr at temperature $T = 0.0$ MeV and spins $I = 0h$, $30h$ and $44h$.

FIG 4(a) - 4(c) : Potential energy surfaces for $^{52}$ Fe at temperature $T = 0.0$ MeV and spins $I = 0h$, $34h$ and $44h$.

FIG 5(a) - 5(c) : Potential energy surfaces for $^{56}$ Ni at temperature $T = 0.5$ MeV and spins $I = 0h$, $24h$ and $40h$. 

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FIG 1(a)
FIG 1(b)

45Sc : Spin 55/2 h

\begin{align*}
\beta \cos \gamma \\
\gamma = -120^\circ \\
\gamma = -180^\circ
\end{align*}

\begin{align*}
\beta \sin \gamma
\end{align*}
$^{44}\text{Ti} : \text{Spin } 32 \hbar$

FIG 2(b)
FIG 2(c)
FIG 3(a)
FIG 3(b)
$^{48}\text{Cr} : \text{Spin } 44\hbar$

\[ \beta \cos \gamma \]

\[ \beta \sin \gamma \]

\[ \gamma = -120^\circ \]

\[ \gamma = -180^\circ \]

\[ \text{FIG 3(c)} \]
FIG 4(a)
FIG 4(c)
$^{56}$Ni: $T=0.0$ MeV
Spin $0^+$

$\beta \cos \gamma$

$\gamma = -120^\circ$

$\beta \sin \gamma$

$\gamma = -180^\circ$

FIG 5(a)
FIG 5(b)