CHAPTER 4

JACOBI TRANSITION AND GIANT DIPOLE RESONANCE

4.1 ROTATION INDUCED JACOBI TRANSITION

The aim of this chapter is to study the giant dipole resonance as a sign-
nature to the Jacobi transition in the fp shell region. Since such Jacobi
transition occurs as a spin driven phenomenon we first restrict our calcu-
lations to $T=0.0$ MeV for obtaining the potential energy surfaces for the
considered nuclei. The deformation energy is now taken as

$$ E_T = E_{RLDM} + (E_{sp} - E_{sp}) $$

where

$$ E_{RLDM} = E_{LDM} - \frac{1}{2} \Omega_{rig} \omega^2 + \hbar \omega \tilde{I} $$

the second term on the right hand side being the rotational energy. Here
the liquid drop energy $E_{LDM}$ is given by the sum of Coulomb and surface
energies and $\Omega_{rig}$, the rigid body moment of inertia defined by $\beta$ and $\gamma$
including the surface diffuseness correction.

It is to be noted however that in the above expression the constant
$\omega$ rotation energy is Legendre transformed to constant spin rotation energy.
This is to ensure that our calculations may not get involved into the problem
faced by Alhassid and Whelan [1], namely, the rotating liquid drop at con-
stant $\omega$ may become unstable at the critical angular
velocity. However this problem was not encountered
by us when we have used the finite tempera-
ture cranked Strutinsky calculations in chapter 3. The one drawback that we faced in such calculations is that at very high spins the results showed having a slight deviation compared to the results obtained by other similar methods. For example, in the case $^{48}\text{Cr}$ we were unable to get the hyper deformation at the spin of $32\hbar$ which was reported by Betts [2]. The present formalism has completely overcome the difficulty as far as the fp shell region is concerned. We show in figures 1(a) to 1(c) the zero temperature potential energy surfaces for the nucleus $^{44}\text{Ti}$ at spin $26\hbar$, $28\hbar$ and $34\hbar$. It is very gratifying to note that the Jacobi transition takes place in this nucleus at the spin $28\hbar$ where the non-collective oblate shape with $\beta = 0.5$ has a co-existing hyper deformed minimum at the exact collective prolate line as seen in Fig 1(b). This is an improvement to our result obtained in chapter 3 for this nucleus where we could get the hyper deformation with triaxiality only. It is also seen from Fig 1(c) that at spin $34\hbar$, the shape of $^{44}\text{Ti}$ at $T = 0.0\text{MeV}$ has already changed to a hyper deformed prolate shape. Fig 2(a) to 2(c) show similar results for the case of $^{48}\text{Cr}$. A nice feature of these figures is that the potential energy surface obtained by us at the $32\hbar$ exactly coincides with that of Betts [2].

4.2 GIANT DIPOLE RESONANCE AS A SIGNATURE OF JACOBI TRANSITION

Giant resonances are small amplitude, high frequency, simple, collective
modes of excitations of nuclei. The study of giant resonances has been and still is a major topic of research in nuclear physics. As early as 1947, it was Baldwin and Klaiber [3] who discovered that absorption of high-energy photons showed a resonance-like behaviour and that the photo absorption cross-section exhibited a strong resonance (the giant resonance) at energies of the order of 15 to 20 MeV. This resonance was soon found in other photon-induced reactions such as \((\gamma, n), (\gamma, p)\) etc., and this photo nuclear giant resonances turned out to be a general feature of all nuclei.

Many of the features of nuclei indicate that nuclear motion does not consist only of simple single-particle excitations as might be suggested by the shell model. But there are different typical effects which favour a collective motion, that is a motion where many nucleons move coherently, with well defined phases. One such collective behavior involving the nuclear interior is established in the photo nuclear giant resonance motion where the electric field \(E\) of the photon acts on protons only and as the centre of mass has to be at rest, the neutrons have to move in a direction opposite to that of the motion of protons.

The photo-emission of nucleons is the most dominant photo nuclear reaction. The threshold for proton or neutron emission is about 8 MeV in medium and heavy nuclei. The schematic photo absorption spectrum shown in [4] can be divided into three parts. At low energies only a few discrete levels approaches the particle emission threshold. Beyond the threshold, the
nucleons can escape and hence the levels have widths which also increase(II). The most prominent feature of the photon absorption cross-section is found around 12 to 20 MeV where we have a strong resonance - the giant resonance of width $\Gamma = 3-4$ MeV(III). As the states in the continuum (II and III) decay predominantly by particle emission rather than by $\gamma$-ray emission, one finds that the widths in the region II and III are very much larger than in region I. The structure in regions I and II depends on the shell structure of the nucleus and hence varies considerably from nucleus to nucleus.

The giant resonances are characterized by the following features.

1. All nuclei exhibit this photo absorption peak.

2. The resonance energy ($E_m$) varies with $A$, i.e., $E_m \propto A^{-\frac{1}{3}}$ for medium and heavy nuclei.

3. The full width at half maximum $\Gamma_m$ varies from 4 MeV (in $^{208}$Pb) to 7 MeV (in $^{65}$Cu). The width is small for closed shell nuclei and large for deformed nuclei.

The mechanism of giant resonances can be easily understood from the following.

1. When a photon is absorbed into the nucleus, the electric field $E$ of the photon wave induces a coherent motion of the protons in a direction. The electric field $E$ of the photon pushes the protons upwards. Due to the centre-of-mass conservation, the neutrons move in the opposite direction. As the wavelength $\lambda \gg R$ where $R$ is the nuclear radius, the electric field $E(t)$
is almost homogeneous over the nucleus and it is time-dependent.

2. As the nucleons scatter each other, the energy of the coherent state is distributed among all nucleons. Friction between the proton and the neutron fluids of the nucleus easily explains the damping mechanism. Then a thermalisation of the energy occurs and it leads to the disappearance of the coherent mode.

3. From the complex giant resonance configuration, neutrons and protons are evaporated. The nucleus cools off. The evaporated nucleons have, due to their origin, a statistical energy distribution. When the photon energy is increased, it is observed in the energy distribution for the nucleons.

For more than thirty years experimental and theoretical efforts have been made to the study of giant resonances in nuclei. The best-known and the most pronounced of those resonances is the giant dipole resonance (GDR) which is observed in nearly all nuclei at a resonance energy of approximately $78\, A^{-\frac{1}{3}}$ and it corresponds to an isovector mode, where the protons and neutrons move out of phase. When the protons and neutrons move in phase, it corresponds to the isoscalar mode. Proton scattering [5] experiments revealed the existence of the isoscalar giant quadrupole (GQR) resonances. It occurs at an excitation energy of approximately $65\, A^{-\frac{1}{3}} MeV$ and the isovector component at $130\, A^{-\frac{1}{3}} MeV$. Another resonance to be located was the isoscalar giant monopole resonance (GMR) [6, 7]. This is located in heavy nuclei at an excitation energy of approximately $80\, A^{-\frac{1}{3}} MeV$. This resonance
is of special importance since it can be related to the incompressibility of nuclei and of nuclear matter that matters much in the field of astrophysics.

The isovector giant dipole resonance (IVGDR) is thus an out-of-phase, small amplitude collective oscillation of protons against neutrons. Heavy ion collisions lead to the formation of compound nuclear systems with large angular momenta and internal excitation and the behaviour of such systems has been attracting much attention in recent years. Such excitation may not only alter the shape of the nuclei but also the excitation spectra built on them. But, until recently, only the giant dipole resonances built on the ground state of a nucleus were investigated. Morinaga[8] and Brink [9] have already pointed out that there should be giant dipole resonances based on each nuclear state, i.e., not only the ground state but also the excited states should have giant dipole resonances built on them.

On the experimental side, Newton et al., [10] first observed giant dipole resonances built on highly excited rotating compound states, which are built after heavy-ion fusion reactions. This was found in heavier nuclei. In light nuclei, these have been studied in detail for A=28 systems by Dowell et al. [11] and for nuclei near A = 40 by the Seattle group [12]. Experimental information on the shapes of highly rotating nuclei is now thus becoming available through studies of the spectra of giant dipole resonances built on nuclear excited states. The study of such resonances in hot rotating systems is interesting because it gives us two additional degrees of freedom, namely,
rotation and temperature which can provide us with new information on the nuclear structure. Such studies may lead to a quite exciting new spectroscopy in which one can study the dynamical structure of excited states just by looking at the properties of the dipole resonances built on them. It thus becomes necessary to theoretically study the changes that take place in the shapes of nuclei with spin and temperature and the properties of dipole excitations built on them. Many macroscopic and microscopic studies concentrate on the first of these problems while the problem of dipole vibrations in the hot rotating nuclei has gained attention only recently. Eventhough much theoretical effort has been made to explain the giant dipole resonances in excited heavier nuclei, not very much attention has been paid in the case of light and fp shell nuclei. In such nuclei, as first suggested by Hilton [13] one can expect a much stronger influence of nuclear rotation on the giant dipole resonances since the corresponding angular velocities in this region are greater. The measurements of the giant dipole resonance properties in fp shell nuclei is thus seen a worthwhile investigation which should manifest such effects most clearly. With this view, in this thesis, attention is focussed on the study of isovector giant dipole resonance (IVGDR) built on excited states of nuclei in the fp shell region. The purpose of this work is two-fold: (1) to examine the giant dipole resonance systematic in excited fp nuclei by isolating their spin dependence and temperature dependence and (ii) to consider the role of thermal shape fluctuations around mean field values which are not ordinarily
negligible in finite nuclear systems.

The isovector giant dipole resonances (IVGDR's) are described as out-of-phase small-amplitude collective oscillations of neutron distribution against proton distribution. Recent investigations of such resonances built on high spin states, have opened up the possibility of studying high spin states by just looking at the properties of giant resonances built on them. The fact that emerges out of the recent experiments on IVGDR's at high spin [14] is that there is the broadening of the overall widths at high angular momentum. It was first found that the splitting due to nuclear rotation was small for the rare earth nuclei. Since the splittings induced in heavy nuclei by nuclear rotation are small, they are difficult to observe. But, in light and fp shell nuclei, as first suggested by Hilton, one can expect a much stronger influence of nuclear rotation on the IVGDR's since the corresponding angular velocities are greater in this region. It is to be noted that the region of light and fp shell nuclei have not been studied with that much attention as the heavier nuclei. With this in view, attention has been focussed on the study of Jacobi transitions and IVGDR's in certain nuclei in the fp shell region wherein the bulk of the angular momentum is of an aligned nature. Use is made of a simple analytical microscopic model [15, 13, 16] for studying the effect of rotation on the IVGDR's in $^{44}Ti$ and $^{48}Cr$.

The equilibrium deformations of the nuclei mentioned above, were first determined by the use of the microscopic cranked-Nilsson-Strutinsky method
for cold nucleus [17] considered in the previous chapter. The next step in the calculations is to get the IVGDR energies of the rotating nuclei considered. For this, an analytical method [15, 13] is used. In this method, the average field of the nucleus was taken to be an oscillator potential with deformation parameter consistent with the angular momentum of the system.

The rotation-induced changes of the shape of nuclei can be stimulated by the average Hamiltonian of a triaxial harmonic oscillator

\[
H_{av}(\Omega) = \sum_{\nu=1}^{A} h_{\nu}(\Omega)
\]

where

\[
h(\Omega) = \frac{p^2}{2m} + \frac{m}{2} \left( \omega_z^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2 \right) - \Omega_z
\]

and \( L_z = \sum_{\nu=1}^{A} l_z(\nu) \) is the operator of rotation about the z-axis. The eigen functions and the eigen values of the Hamiltonian \( H_{av} \) are determined from the equation

\[
[H_{av}, a^\dagger_\lambda] = \omega_\lambda a^\dagger_\lambda
\]

for oscillator-quantum creation operators \( a^\dagger_\lambda \) that are linear combinations of the particle coordinates \( r_i \) and of the conjugate momenta \( p_i \). In terms of the operators \( a^\dagger_\lambda \) and \( a_\lambda \), the Hamiltonian \( H_{av} \) can be expressed as

\[
H_{av} = \sum_{\nu=1}^{A} \left\{ \omega_z \left[ \left( a^\dagger_\nu a_\nu \right) + \frac{1}{2} \right] + \omega_+ \left[ \left( a^\dagger_+ a_- \right) + \frac{1}{2} \right] + \omega_- \left[ \left( a^\dagger_- a_+ \right) + \frac{1}{2} \right] \right\}
\]
The normal frequencies are then obtained as

\[
\omega_{\pm} = \omega_{z} \pm \sqrt{\frac{\omega_y^2 + \omega_z^2}{2} + \Omega^2 \pm \frac{1}{2} \left[ 8 \Omega^2 \left( \omega_y^2 + \omega_z^2 \right) \right]}
\] (4.3)

The isovector giant dipole excitation mode is then generated by adding to the Hamiltonian \( H_{av} \) the effective dipole interaction

\[
H_{int} = \eta \sum_{i=x,y,z} \frac{m \omega_i^2}{2A} \left[ \sum_{\nu=1}^{A} \Gamma_3 (\nu) x_i (\nu) \right]^2
\]

where \( \Gamma_3 (\nu) \) is the third projection of the Pauli isospin matrix

\[
\Gamma_3 = \begin{bmatrix}
1 & 0 \\
0 & -1 
\end{bmatrix}
\]

and \( \eta \) is a parameter that characterizes the isovector component of the neutron or proton average field

\[
V_{(n)} (\nu) = m 2 \left[ 1 + \eta \frac{N - Z}{A} \right] \sum_{i=x,y,z} \omega_i^2 x_i^2 (\nu)
\]

The value of \( \eta \) for an oscillator potential is found [15] to be 3 from experimental data on the position of the giant resonance.
The giant dipole resonance frequencies in a rotating nucleus can be obtained by diagonalising analytically the Hamiltonian $H_{av}$ with the effective interaction $H_{int}$ within the framework of the standard random phase approximation (RPA) procedure by using the similarity between the linear transformation corresponding to $H_{av}$ and the RPA transformations to get the giant dipole resonance frequencies in the rotating frame.

Transforming to the laboratory system, the giant dipole resonance frequencies become

$$\tilde{\omega}_z = (1 + \eta)^{\frac{1}{2}} \omega_z$$

$$\tilde{\omega}_\pm \mp \Omega = \left\{ \left( 1 + \eta \right) \left( \frac{\omega_y^2 + \omega_z^2}{2} \right) \right\}^{\frac{1}{2}}$$

$$+ \Omega^2 \pm \frac{1}{2} \sqrt{(1 + \eta)^2 \left[ \left( \omega_y^2 - \omega_z^2 \right)^2 + 8\Omega^2 (1 + \eta) \left( \omega_y^2 + \omega_z^2 \right) \right]} \mp \Omega$$

The expression for $\tilde{\omega}_\pm$ may be written as

$$\tilde{\omega}_+ = \sqrt{(1 + \eta) \frac{\omega_y^2 + \omega_z^2}{2} + \Omega \frac{1}{2} \sqrt{(1 + \eta)^2 \left[ \left( \omega_y^2 - \omega_z^2 \right)^2 + 8\Omega^2 (1 + \eta) \left( \omega_y^2 + \omega_z^2 \right) \right]}} - \Omega$$

$$\tilde{\omega}_- = \sqrt{(1 + \eta) \frac{\omega_y^2 + \omega_z^2}{2} + \Omega \frac{1}{2} \sqrt{(1 + \eta)^2 \left[ \left( \omega_y^2 - \omega_z^2 \right)^2 + 8\Omega^2 (1 + \eta) \left( \omega_y^2 + \omega_z^2 \right) \right]}} + \Omega$$

Thus one gets five frequencies, $\tilde{\omega}_z$, $\tilde{\omega}_+ - \Omega$, $\tilde{\omega}_+ + \Omega$, $\tilde{\omega}_- - \Omega$ and $\tilde{\omega}_- + \Omega$ for the collectively rotating triaxial nuclei. For prolate nuclei, $(\omega_z = \omega_y \neq \omega_x)$ rotating about an axis perpendicular to its symmetry axis, all the above
five frequencies will exist. But for oblate nuclei, \( \omega_x = \omega_y \neq \omega_z \) rotating about its symmetry axis, as first shown by Hilton in Ref. 2, only two frequencies, namely, \( \omega_1 \) and \( \omega_1 - \Omega = \omega_- + \Omega \) will exist and thus all effects due to rotation vanish and only those purely due to deformation will be left. For the spherical nuclei \( \omega_x = \omega_y = \omega_z \), which comes under the latter category, one gets only one frequency, namely \( \omega_1 = \omega_+ - \Omega = \omega_+ + \Omega \).

In the study of \( \gamma \)-ray spectra emitted from rapidly rotating nuclei, the dipole photo absorption cross section as a function of angular momentum plays a vital role. Using the semi-classical theory of the interaction of photons with nuclei, the shape of a fundamental resonance in the absorption cross section is that of the Lorentz curve

\[
\sigma(E) = \frac{\sigma_m}{1 + \left( \frac{E^2 - E_m^2}{E^2 \Gamma^2} \right)^2}
\]

where the Lorentz parameters \( E_m, \sigma_m \) and \( \Gamma \) are the resonance energy, peak cross section, and full width at half maximum, respectively.

In the case of a spherical nuclei, the giant dipole resonance consists of one Lorentz line. The peak cross section \( \sigma_m \) for the spherical nuclei is given by [18],

\[
\sigma_m = 60 \frac{2}{\pi} \frac{NZ}{A} \frac{1}{\Gamma_m} 0.86(1 + \alpha)
\]

where \( \Gamma_m \) is the width at half maximum and \( \alpha \) is an adjustable parame-
ter. For deformed spherical nuclei, the giant resonance consists of two such Lorentz lines corresponding to the absorption of photons which induce oscillations of the neutron and proton fluids in the nucleus against each other. In such cases,

$$\sigma(E) = \sum_{i=1}^{2} \frac{\sigma_{mi}}{1 + \left( \frac{(E^2 - E_m^2)^2}{E^2 \Gamma^2} \right)}$$

(4.14)

where $i = 1,2$ correspond to the lower and higher energy lines. The lower energy line corresponds to oscillations along the longer axis and the higher energy line corresponds to oscillations along the shorter axis. It is to be noted that these Lorentz lines are non interfering, but $\Gamma_m$ is assumed to depend on energy.

It is to be noted that this problem has not been solved satisfactorily so far even in the absence of rotation. It was observed [19] that the ground state giant dipole resonance (GDR) full width at half maximum is found to be narrow in the region of spherical nuclei and broadened primarily by quadrupole deformation in other mess regions. Also the observed GDR full width at half maximum in excited nuclei varies more or less smoothly with mass, even though the single-and the double-Lorentzian shapes show significant differences in detail. In the same or neighbouring nuclei, the widths are generally broader than the ground state GDR widths. It was inferred [19] that the nuclear quadrupole deformation plays a fundamental role in
determining the GDR shapes in excited nuclei. Recently, Nix et al. [20] using a surface plus windows dissipation model, could get resonance widths comparable to experimental values, not for giant dipole resonance but only for giant quadrupole and octupole resonances. It was shown in [21] that the energy dependence on the GDR width can be approximated by the relation

\[ \Gamma_{\lambda} \approx 0.026 \ E_{\lambda}^{1.9} \] (4.5)

For the parameterization of the IVGDR width in a rotating nucleus, this expression is very useful, with due allowance for the corresponding changes of the energies of the resonances, and is used in the calculations.

The magnitude of the angular velocity of nuclear rotation \( \Omega_{n} \), can be simply estimated by considering the nuclear rotation as that of a rigid body. As is well known, this assumption seems to provide a reasonable picture of nuclear rotation. Thus we have the relation

\[ \Omega = \frac{\sqrt{I(I+1)}}{\mathcal{I}_{\text{rig}}} = \frac{I}{\mathcal{I}_{\text{rig}}} \] (4.6)

which relates the angular velocity \( \Omega \), angular momentum \( I \), and the nuclear moment of inertia \( \mathcal{I}_{\text{rig}} \). Here \( \mathcal{I}_{\text{rig}} \) denotes a rigid body moment of inertia for a nucleus of a given shape which may itself depend on angular momentum.
I. Looking for the lowest rotational states, one should employ the largest possible moment of inertia,

\[ \frac{\Omega}{\hbar^2} = \frac{A^{3/2}}{72} \left[ 1 - \sqrt{\frac{5}{4\pi}} \epsilon \cos \left( \gamma - \frac{2\pi}{3} \right) \right] \text{MeV}^{-1} \tag{4.7} \]

In order to obtain the 1VGDR energies of the rotating nucleus, the average field of the nucleus has been taken to be an oscillator potential with deformation parameters consistent with the angular momentum of the system.

Using the above formalism the GDR cross-sections have been obtained for \(^{44}\text{Ti}\) at spins \(26\hbar, 28\hbar\) and \(34\hbar\) and for \(^{48}\text{Cr}\) at spins \(30\hbar, 32\hbar, 44\hbar\). Fig 3(a) gives the GDR corresponding to the non-collective oblate shape before Jacobi transition in \(^{44}\text{Ti}\). When the spin is raised, this nucleus undergoes a Jacobi transition which is seen from Figure 3(b). Figure 3(c) describes the situation well after the Jacobi transition has occurred. Similar results can be read from figures 4(a) to 4(c). A nice point to note in figure 4(c) is the occurrence of super hyper deformation and the related soft GDR.
REFERENCES


14. J.E. Draper, J.O. Newton, L.G. Sabotka, H. Lindenberger, 
G.J. Wozniak, L.G. Moretto, F.S. Stephens, R.M. Diamond 


17. G. Shanmugam and V. Devanathan, Phys. Scr. 24, 17 (1981); 
25, 607 (1982); G. Shanmugam and V. Devanathan, in 
Proc. of Nat. Symp. on Medium energy 
Physics and Nuclear Structure, Madras, India (1982). 


FIG URE CAPTIONS

FIG 1(a) - 1(c) : Potential energy surfaces for $^{44}$Ti at temperature $T = 0.0$ MeV and spins $I = 26\hbar, 28\hbar$ and $34\hbar$.

FIG 2(a) - 2(c) : Potential energy surfaces for $^{48}$Cr at temperature $T = 0.0$ MeV and spins $I = 30\hbar, 32\hbar$ and $44\hbar$.

FIG 3(a) - 3(c) : GDR cross sections for $^{44}$Ti at temperature $T = 0.0$ MeV and spins $I = 26\hbar, 28\hbar$ and $34\hbar$.

FIG 4(a) - 4(c) : GDR cross sections for $^{48}$Cr at temperature $T = 0.0$ MeV and spins $I = 30\hbar, 32\hbar$ and $44\hbar$. 

162
$^{44}$Ti: $T = 0.0$ MeV
Spin: 26 $\hbar$
$^{44}$Ti: $T=0.0$ MeV  \hspace{1cm} \text{Spin: } 28\hbar

\[ \beta \cos \gamma \]

\[ \gamma = -120^\circ \]

\[ \beta \sin \gamma \]

\[ \gamma = -180^\circ \]

FIG 1(b)

165
$^{48}$Cr (T=0.0 MeV)

Spin: 32 $\hbar$

FIG 2(b)
FIG 3(a)

44Ti : 26 h

σ (mb)

Energy (MeV)

FIG 3(a)
FIG 3(b)
\[ \sigma \text{(mb)} \]

Energy (MeV)

FIG 3(c)

\[ ^{44}\text{Ti} : 34 \text{ h} \]
FIG 4(a)
FIG 4(b)
$^{48}\text{Cr} : 44\, \text{h}$