In the standard theory the V-A character of weak interactions is obtained as the low energy limit of spontaneously broken $SU(2)_L \times U(1)_Y$ gauge interactions. By allowing only the left-handed chiral fermions to transform nontrivially under $SU(2)_L \times U(1)_Y$, the V-A structure is built into the theory in a certain sense. On the other hand, it is natural to conjecture that both V-A and V+A type of charged and neutral currents are present, with equal strengths at a larger scale $M_R > > M_W$, so that as we come down to the W-boson mass and lower energies the V+A structure is damped out. The weak CP violation manifested in $K^0$ decay does not have a spontaneous origin in the standard model. On the other hand, the model which embodies both V-A and V+A structures can ascribe the weak CP violation to have a spontaneous origin. The neutrino in the standard model is massless. At present there are reasons to believe, both experimentally and cosmologically, that the neutrino might have a small mass. In the left-right symmetric models, since both left and right-handed helicities of the neutrino are included, the neutrino can naturally have a Dirac mass. On the other hand, if neutrinos are Majorana particles, there could be suitable Higgs representations which can also generate
Majorana neutrino masses. The gauge models based upon the electroweak gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, or $SU(2)_L \times SU(2)_R \times SU(4)_C$, proposed by Mahapatra and Pati, and Pati and Salam, do possess these desirable and attractive features. We concentrate upon the left-right model, based upon the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ in this chapter. In Sec. 4.1, introduction of the model is provided.

In sec 4.2, we mention fermion representation in L-R symmetric model. In Sec 4.3, Higgs sectors without generating neutrino masses are discussed. The gauge boson masses in LR model are derived in sec 4.4. The constraints arising from minimization of potential are discussed in Sec. 4.5. In section 4.6, we review how neutrino masses are generated in L-R models. The see-saw mechanism which explains about the small neutrino masses is also discussed. We discuss in sec. 4.7 about the neutrinoless double-$\beta$ decay which is the most interesting prediction of the model. A brief summary of the chapter is given in sec 4.9.

4.2 Fermion representation in L-R models

Considering the gauge theory based on the group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ under which the fermions transform in the left-right symmetric manner. We denote the $SU(2)_L$, $SU(2)_R$, and $U(1)_{B-L}$ coupling constants by $g_L$, $g_R$ and $g'$. 
respectively. The immediate consequence of the left-right
symmetry which we will impose on the Lagrangian is the left
and right gauge couplings of fermions with gauge bosons
are the same i.e. $g_L = g_R = g$.

The quarks and lepton of three generations are placed
in $SU(2)_L$ and $SU(2)_R$ doublets as given below.

\begin{align}
Q_{1L} &= \begin{pmatrix} u \\ d \end{pmatrix}^{r, y, b} , \quad Q_{2L} = \begin{pmatrix} c \\ s \end{pmatrix}^{r, y, b} , \quad Q_{3L} = \begin{pmatrix} t \\ b \end{pmatrix}^{r, y, b} \\
Q_{1R} &= \begin{pmatrix} u \\ d \end{pmatrix}^{r, y, b} , \quad Q_{2R} = \begin{pmatrix} c \\ s \end{pmatrix}^{r, y, b} , \quad Q_{3R} = \begin{pmatrix} t \\ b \end{pmatrix}^{r, y, b}
\end{align}

(4.2.1)

\begin{align}
\psi_{1L} &= \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L , \quad \psi_{2L} = \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L , \quad \psi_{3L} = \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \\
\psi_{1R} &= \begin{pmatrix} \nu_e \\ e \end{pmatrix}_R , \quad \psi_{2R} = \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_R , \quad \psi_{3R} = \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_R
\end{align}

(4.2.2)

The representation content of fermionic multiplet is

\begin{align}
Q_{iL} &= (\frac{1}{2}, 0, \frac{1}{3}) , \quad Q_{iR} = (0, \frac{1}{2}, \frac{1}{3}) \\
\psi_{iL} &= (\frac{1}{2}, 0, -1) , \quad \psi_{iR} = (0, \frac{1}{2}, -1)
\end{align}

(4.2.3)
where \( i = 1,2,3 \) and the symbol \( \frac{1}{2}(0) \) implies a doublet (singlet) under the gauge group.

The electric charge operator is defined as

\[
Q_{el} = T_{3L} + T_{3R} + \frac{B - L}{2}
\]  
\[
(4.2.4)
\]

Where \( T_{L}, T_{R} \) and \( B-L \) are the generators of the \( SU(2)_L \), \( SU(2)_R \) and \( U(1)_{B-L} \) respectively.

In the standard model, fermions appeared as left-handed doublets and right handed singlets. But in L-R symmetric models, they appear in both as doublets.

In the standard model, \( \nu_{eR}, \nu_{\mu R}, \nu_{\tau R} \) are absent which causes any neutrino to be massless. But in the LR models, the presence of these right-handed neutrinos allows Dirac mass of the neutrino to exist like quark and charged lepton masses.

4.3 Description of Higgs sector in left-right models

The original idea of Pati and Mahapatra is based upon left right symmetric (LRS) gauge theory. In this case both \( SU(2)_L \) and \( SU(2)_R \) possess the same coupling constant \( (g_L = g_R = g) \) with fermions at a mass scale when \( SU(2)_L \times SU(2)_R \times U(1)_{B-L} \) is a good symmetry. There could be left-right gauge groups with unequal gauge coupling constants \( (g_L \neq g_R) \) which are known as left-right asymmetric (LRA) models. In this case the gauge
group is also based upon $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. In the left-right symmetric or asymmetric models based upon the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, the symmetry is broken spontaneously to the Weinberg-Salam model, $SU(2)_L \times U(1)_Y$ by a Higgs field with and without generating neutrino masses. The sets of Higgs fields in the two cases are different as we describe them below.

Higgs sector without generating Majorana neutrino masses

In this case there is a left(right) handed doublets, $\chi_L(\chi_R)$ carrying $B-L=1$ which are needed to break $SU(2)_R \times U(1)_{B-L} \rightarrow U(1)_Y$. Simultaneously maintaining L-R symmetry. Besides, the Weinberg-Salam doublet is contained in the scalar field $\phi$ which transform as a doublet both under $SU(2)_L$ and $SU(2)_R$.

$$\chi_L = \begin{pmatrix} \chi_L^+ \\ \chi_L^- \end{pmatrix} = (\tfrac{1}{2}, 0, 1), \quad \chi_R = \begin{pmatrix} \chi_R^+ \\ \chi_R^- \end{pmatrix} = (0, \tfrac{1}{2}, 1)$$

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^- \end{pmatrix} = (\tfrac{1}{2}, \tfrac{1}{2}, 0)$$

$$\tilde{\phi} = \tau_2 \phi^* \tau_2 = \begin{pmatrix} \phi^+_2 - \phi^-_2 \\ -\phi^-_2 + \phi^+_2 \end{pmatrix}$$

(4.3.1)

The symmetry breaking is realized by giving vacuum expectation
value (VEV)

\[ \langle x_L \rangle = \begin{pmatrix} 0 \\ v_L \end{pmatrix}, \quad \langle x_R \rangle = \begin{pmatrix} 0 \\ v_R \end{pmatrix} \]

and

\[ \langle \phi_1 \rangle = \langle \phi \rangle = \begin{pmatrix} k \\ 0 \\ 0 \end{pmatrix}, \quad \langle \phi_2 \rangle = \langle \bar{\phi} \rangle = \begin{pmatrix} k' \\ 0 \\ 0 \end{pmatrix}, \quad \langle \bar{\phi} \rangle = \begin{pmatrix} k' \\ 0 \\ 0 \end{pmatrix} \]

where \( k' \) could be zero.

4.4 Gauge boson masses in the left-right models

In this section we discuss derivation of gauge boson masses in the left-right asymmetric (LRA) model, where the SU(2)_L and SU(2)_R have unequal coupling constants (\( g_L \neq g_R \)). We show how the formulas reduce to the LRS case when we use \( g_L = g_R = g \). We will discuss the cases of Higgs sectors without and with Majorana neutrino masses in each case.

(a) LRA model without Majorana neutrino masses

In this case the Higgs sectors are represented by

\[ x_L = \begin{pmatrix} x_L^+ \\ x_L^0 \end{pmatrix}, \quad x_R = \begin{pmatrix} x_R^+ \\ x_R^0 \end{pmatrix}, \quad \phi = \begin{pmatrix} \phi_1^0 \\ \phi_1^+ \\ \phi_2^- \\ \phi_2^0 \end{pmatrix} \]

and

\[ \langle x_L \rangle = \begin{pmatrix} 0 \\ v_L \end{pmatrix}, \quad \langle x_R \rangle = \begin{pmatrix} 0 \\ v_R \end{pmatrix}, \quad \langle \phi \rangle = \begin{pmatrix} k \\ 0 \\ 0 \\ k' \end{pmatrix} \]

with \( u_R > k > k', \ v_L \)
The gauge boson masses are obtained from the scalar contribution to the Lagrangian

\[ \alpha_{\text{scalar}} = (D_\mu \chi_L)^+ (D_\mu \chi_L) + (D_\mu \chi_R)^+ (D_\mu \chi_R) \]

\[ + \text{Tr}(D_\mu \phi^+ (D_\mu \phi) - V(\chi_L, \chi_R, \phi) \] (4.4.2)

Where,

\[ D^\mu \chi_L = \partial^\mu \chi_L - \frac{1}{2} i g_L \gamma^\mu \chi_L^+ \]

\[ D^\mu \chi_R = \partial^\mu \chi_R - \frac{1}{2} i g_R \gamma^\mu \chi_R^+ \]

\[ D^\mu \phi = \partial^\mu \phi - \frac{1}{2} i g_L \gamma^\mu \phi^+ \]

Using the vacuum expectation values from (4.4.1) in (4.4.2)

then gives charged gauge boson mass matrix

\[ M = \begin{pmatrix} W_L^2 & -g_L g_R k k' \\ -g_L g_R k k' & \frac{1}{2} g_R^2 (k^2 + k'^2 + u_R^2) \end{pmatrix} \] (4.4.4)

Defining the mass eigen states as

\[ W_1^+ = W_L^+ \cos \xi + W_R^+ \sin \xi \]

\[ W_2^+ = -W_L^+ \sin \xi + W_R^+ \cos \xi \] (4.4.5)

The eigen values of (4.4.4) are given by,

\[ M_{w}^{\pm 2} = \frac{1}{4} \left( g_L^2 (v_L^2 + k^2 + k'^2) + g_R^2 (v_R^2 + k^2 + k'^2) \right) \pm \left\{ \left[ g_L^2 (k^2 + k'^2 + v_L^2) - g_R^2 (v_R^2 + k^2 + k'^2) \right]^2 + g_L^2 g_R^2 k^2 k'^2 \right\}^{1/2} \] (4.4.6)

Where, the left-right mixing angles \( \xi \), is given by the relation

\[ \tan 2\xi = \frac{4 g_L g_R k k'}{g_R^2 (v_R^2 + k^2 + k'^2) - g_L^2 (v_L^2 + k^2 + k'^2)} \] (4.4.7)
It is noted that second term inside the radical in (4.4.6) is proportional to \( \tan^2 \theta \) and vanishes in the limit \( \theta \to 0 \). In the limit of vanishing L-R mixing, the left-handed \((W_L^\pm)\) and right-handed \((W_R^\pm)\) charged gauge boson masses computed using minus and plus signs, respectively in (4.4.6)

\[
M^2_{W_L^\pm} = \frac{1}{2} g_L^2 (v_L^2 + k^2 + k'^2) \tag{4.4.8}
\]

\[
M^2_{W_R^\pm} = \frac{1}{2} g_R^2 (v_R^2 + k^2 + k'^2) \tag{4.4.9}
\]

This can be directly verified from (4.4.4) neglecting off-diagonal elements. If we use the conditions \( v_R^2 \gg k^2 \gg k'^2, v_L^2 \)

eqs (4.4.8) and (4.4.9) yield

\[
M^2_{W_R} = \frac{1}{2} g_R^2 v_R^2 \tag{4.4.10}
\]

\[
M^2_{W_L} = \frac{1}{2} g_L^2 \phi^2
\]

satisfying \( M_{W_R} \gg M_{W_L} \). In such a limit the L-R mixing angle is

\[
\tan 2\theta = \frac{4g_L k k'}{g_R v_R^2} \tag{4.4.11}
\]

\( W_L^\pm \) are charged W-bosons of the \( SU(2)_L \times U(1)_Y \) model, after the symmetry is spontaneously broken down to \( U(1)_{\text{em}} \). These have predominantly V-A structure of weak currents. \( W_R^\pm \) are the charged and heavy right-handed bosons of the \( SU(2)_R \times U(1)_{B-L} \), after the symmetry is broken down to \( U(1)_Y \). They have predominantly V+A couplings with quarks and leptons.
Neutral gauge boson masses - The neutral gauge boson mass matrix for the spontaneous breaking,

\[
\begin{align*}
\text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_{B-L} \xrightarrow{\langle \chi_R \rangle \neq 0} \text{SU}(2)_L \times \text{U}(1)_{Y} \xrightarrow{\langle \phi \rangle \neq 0} \text{U}(1)_{\text{em}}
\end{align*}
\]

can be similarly written as

\[
M_Z^2 = \begin{bmatrix}
W_L^3 & W_R^3 & B \\
W_L^3 & \frac{1}{2}g_L^2(v_L^2+k^2+k'^2) & -\frac{1}{2}g_L^2g_R^2(k^2+k'^2) & -g'g_Lv_L^2 \\
W_R^3 & \frac{1}{2}g_Lg_R^2(k^2+k'^2) & \frac{1}{2}g_R^2(v_R^2+k^2+k'^2) & -g'g_Rv_R^2 \\
B & -g'g_Lv_L^2 & -g'g_Rv_R^2 & g'^2(v_L^2+v_R^2)
\end{bmatrix}
\]

(4.4.12)

Where \( W_L^3, W_R^3 \) and \( B \) are the neutral components of gauge bosons in \( \text{SU}(2)_L, \text{SU}(2)_R \) and \( \text{U}(1)_{B-L} \), respectively.

Using the definitions

\[
\begin{align*}
\chi_\omega &= \sin^2\theta_W = \frac{e^2(M_W)}{g_L^2(M_W)} \\
\frac{1}{R_1} &= \frac{1}{g_R^2(M_R)} + \frac{1}{g_L^2(M_R)} + \frac{1}{g'^2(M_R)} \\
R^2 &= g_R^2/g_L^2 \\
\eta_R &= \frac{k^2+k'^2}{v_R^2} \\
\eta_L &= \frac{v_L^2}{k^2+k'^2} \\
Z &= \frac{2k'^2}{k^2+k'^2}
\end{align*}
\]

(4.4.13)

The mass matrix can be written as

\[
M_Z^2 = \frac{1}{2}g_L^2v_R^2 \begin{bmatrix}
\eta_R(1+\eta_L) & -R\eta_L & -2\epsilon\eta_R\eta_L \\
-R\eta_R & R^2(2+\eta_R) & -2R\epsilon \\
2\epsilon\eta_R\eta_L & -2R\epsilon & 2\epsilon^2(1+\eta_R\eta_L)
\end{bmatrix}
\]

(4.4.14)
This yields for the left and right handed neutral gauge bosons \((Z_1, Z_2)\)

\[
M_{Z_{1,2}}^2 = \frac{1}{2} g_L^2 v_R^2 \left( \frac{(R^2 - x_w)}{R^2 - (R^2 + 1)x_w} \right) \left[ 1 + \frac{(R^2 - (R^2 + 1)x_w)(R^2 - 1 + n_R^n L)}{(R^2 - x_w)(1 + n_R^n L)} \right]
\]

\[
\pm \left\{ 1 + \frac{R^2 - (R^2 + 1)x_w}{(R^2 - x_w)^2 (1 + n_R^n L)^2} \left[ (R^2 - (R^2 + 1)x_w)(R^2 - 1 + n_R^n L) \frac{(R^2 + 1)}{2} \right]^2 \right\}
\]

\[-2x_w (1 + n_R^n L) (R^2 - 1 + n_R^n L) \frac{(R^2 + 1)}{2} \]

\[-4R^4 n_R^n L + 2R^2 (1 + n_R^n L) (R^2 - 1 + n_R^n L) \frac{(1 - R^2)}{2} \right) \right\}^{1/2} \]

In (4.4.15), the lighter (heavier) of the two masses corresponds to \(Z_1(Z_2)\)-neutral boson of the \(SU(2)_L \times U(1)_Y\) theory.

(b) Gauge boson masses in the LRS model

In this case the \(SU(2)_L\) and \(SU(2)_R\) gauge groups have the same coupling constants at \(\mu = M_{W_L^\pm} = M_{W_R^\pm}\), i.e. \(g_L(M_R) = g_R(M_R) = g\).

Then in (4.5.15), \(R^2 = 1\),

\[
\frac{1}{e^2(M_R)} = \frac{2}{g^2} + \frac{1}{g'^2}
\]

(4.4.16)

The left-right \(W_L^\pm - W_R^\pm\) mixing angle for \(v_R^2 \gg k^2 >> k'^2\), \(v_L^2\) is given by

\[
\tan 2\xi = \frac{4kk'}{v_L^2} \]

(4.4.17)

In the vanishing mixing limit \(\xi \rightarrow 0\) \((k' \rightarrow 0, Z \rightarrow 0)\), the charged
and neutral gauge boson masses are given by,

\[ M_{W_L}^2 = \frac{1}{2} g^2 (k^2 + k'^2 + \nu_L^2) \]
\[ M_{W_R}^2 = \frac{1}{2} g^2 (k^2 + k'^2 + \nu_R^2) \]
\[ M_{Z_1}^2 = \frac{1}{2} g^2 \left( \frac{g^2 + 2g'^2}{g^2 + g'^2} \right) (k^2 + k'^2 + \nu_L^2) \]
\[ M_{Z_2}^2 = \frac{1}{2} g^2 \left( \frac{g^2 + g'^2}{g'^2} \right) (\nu_R^2 + k^2 + k'^2) \]

(c) Gauge boson masses in the LRA and LRS models with Higgs sector generating Majorana neutrino masses

The Higgs doublets \( \chi_L \) and \( \chi_R \) used in sec (4.4a) and (4.4b) do not possess \( B-L = 2 \). The LRS doublet \( \phi(\frac{1}{2}, \frac{1}{2}, 0) \) has vanishing \( B-L \) charge. As we have seen in previous sections, if neutrino is a Majorana particle a coupling of the type \( \Delta_L \overline{\nu} \nu \) or \( \Delta_R \overline{\nu} \nu \) is possible if the Higgs particles \( \Delta_L \) and \( \Delta_R \) carry \( 6-L=2 \). As described earlier, the choice of the Higgs sector for generating Majorana neutrino masses consists of the following:

\[ \phi \equiv (\frac{1}{2}, \frac{1}{2}, 0), \quad \phi = \tau_2 \phi^* \tau_2 \equiv (\frac{1}{2}, \frac{1}{2}, 0) \]
\[ \Delta_L \equiv (1, 0, 2), \quad \Delta_R \equiv (0, 1, 2) \]

(4.4.20)

Where \( \Delta_L \) (\( \Delta_R \)) is the left(right) handed triplet carrying \( B-L = 2 \). The neutral components of \( \Delta_L \) and \( \Delta_R \) are assigned VEV's of the most general form.
with these VEV's the charged $\mathbb{W}_L^\pm$ and $\mathbb{W}_R^\pm$ gauge boson-masses and also the neutral gauge boson masses assume the same form as expressed in eqs (4.4.4) - (4.4.19).

Before closing this section we write down the structure of the neutral gauge bosons including the massless photon ($A_\mu$) in terms of $\mathbb{W}_L^3$, $\mathbb{W}_R^3$ and $B_\mu$ and the electroweak mixing angle $\theta_w$.

\[
\begin{align*}
A_\mu &= \sin \theta_w (\mathbb{W}_L^3 + \mathbb{W}_R^3) + (\cos 2\theta_w)^{1/2} B_\mu \\
Z_{1\mu} &= Z_{L\mu} = \cos \theta_w \mathbb{W}_L^3 - \sin \theta_w \tan \theta_w \mathbb{W}_R^3 - \tan \theta_w (\cos 2\theta_w)^{1/2} B_\mu (4.4.22) \\
Z_{2\mu} &= Z_{R\mu} = (\cos 2\theta_w)^{1/2} \mathbb{W}_R^3 - \tan \theta_w B_\mu
\end{align*}
\]

where

\[
\tan \theta_w = \frac{g'}{(g^2 + g'^2)^{1/2}} \quad (4.4.23)
\]

In the LRS model,

\[
M_{ZL} = \frac{M_{WL}}{\cos \theta_w} \quad (4.4.24)
\]

as in the standard model, but $M_{ZR}$ is related to the corresponding
charged gauge boson mass by a different relation

\[ M_{ZR} = \frac{M_R \cos \theta_W}{\sqrt{\cos 2\theta_W}} \quad (4.4.25) \]

These relations can be verified from eqs (4.4.18) and (4.4.19).

4.5 Minimisation of Higgs potential in LRS model

Higgs potential can be written as

\[
V = - \sum_{i,j=1}^{2} \mu_{ij} \text{tr} \phi_i^+ \phi_j + \sum_{i,j,k,l} \lambda_{ijkl} \text{tr} \phi_i^+ \phi_j \phi_k^+ \phi_l - \mu^2 (\chi_{L}^+ \chi_{L}^+ + \chi_{R}^+ \chi_{R}^+) \\
+ \rho_1 [(\chi_{L}^+ \chi_{L}^+)^2 + (\chi_{R}^+ \chi_{R}^+)^2] + \rho_2 (\chi_{L}^+ \chi_{L}^+ \chi_{L}^+ \chi_{L}^+ + \chi_{R}^+ \chi_{R}^+ \chi_{R}^+ \chi_{R}^+) \\
+ \rho_3 (\chi_{L}^+ \chi_{R}^+ \chi_{L}^+ \chi_{R}^+ + \sum_{i,j=1} \alpha_{ij} \text{tr} \phi_i^+ \phi_j (\chi_{L}^+ \chi_{L}^+ + \chi_{R}^+ \chi_{R}^+) \\
+ \sum_{i,j=1} \beta_{ij} (\text{tr} \chi_{L}^+ \chi_{L}^+ \phi_i^+ \phi_j + \text{tr} \chi_{R}^+ \chi_{R}^+ \phi_i^+ \phi_j) \\
+ \sum_{i,j=1} \gamma_{ij} \text{tr} \chi_{L}^+ \phi_i^+ \chi_{R}^+ \phi_j^+) \quad (4.5.1) \]

Left-right symmetry under which the fields \( \phi_1 \) and \( \phi_2 \) transform as \( \phi_i \leftrightarrow \phi_i^+ \) \((i = 1,2)\) dictates \( \mu_{ij} = \mu_{ji}, \lambda_{1212} = \lambda_{2121}, \)

\[
\lambda_{iijk} = \lambda_{iikj}, \lambda_{ijkk} = \lambda_{jikk}, \lambda_{ijkl} = \lambda_{klij} = \lambda_{jklj} = \lambda_{jikl} \quad (4.5.2) \]

and the potential becomes
\[ v(x_L, x_R, k, k') \]

\[ = \left[ -\left( \mu_1^2 + \mu_2^2 \right) + \left( \lambda_1 1111 + \lambda_1 1222 + \lambda_2 2221 + \lambda_2 2222 \right) \right] \left( k^2 + k'^2 \right) + \left( \lambda_1 1111 + \lambda_1 2222 \right) \left( \lambda_1 1222 + \lambda_1 2112 \right) k^2 k'^2 \]

\[ + \left( \lambda_2 1221 + \lambda_2 2212 \right) \left( k^2 + k'^2 \right) \]

\[ + 4 \left( \lambda_1 1212 + \lambda_1 2212 \right) k^2 k'^2 \]

\[ - \mu^2 \left( \frac{v_L^2 + v_R^2}{2} \right) + \rho_1 \left( v_L^4 + v_R^4 \right) + \rho_2 \left( \frac{v_L^4}{2} + v_R^4 \right) \]

\[ + \rho_3 \left( v_L^2 v_R^2 \right) \left( (\alpha_1 11 + \alpha_2 22 + \beta_1 11) k^2 \right) \]

\[ + \left( \alpha_1 11 + \alpha_2 22 + \beta_2 22 \right) k'^2 + \left( 4\alpha_1 12 + 2\beta_1 12 \right) k' \]

\[ + 2v_L v_R \left( (\gamma_1 11 + \gamma_2 22) k k' + \gamma_1 12 (k^2 + k'^2) \right) \] \hfill (4.5.3)

From the extremizing conditions:

\[ \frac{\partial v}{\partial u_L} = 0 = \frac{\partial v}{\partial u_R} \] \hfill (4.6.4)

We obtain:

\[ -2\mu^2 u_L + 4 \rho_1 u_L^3 + 4 \rho_2 u_L^3 + 2 \rho_3 u_L u_R^2 \]

\[ + 2u_L (\alpha_1 11 + \alpha_2 22 + \beta_1 11) k^2 + 2u_L (\alpha_1 11 + \alpha_2 22 + \beta_2 22) k'^2 \]

\[ + 2u_R (\gamma_1 11 + \gamma_2 22) k k' + 2u_R (\gamma_1 12 + \gamma_2 22) (k^2 + k'^2) = 0 \] \hfill (4.5.5)

or

\[ \mu^2 u_L = 2u_L^3 (\rho_1 + \rho_2) + \rho_3 u_L u_R^2 + u_L (\alpha_1 11 + \alpha_2 22 + \beta_1 11) k^2 \]

\[ + u_L (\alpha_1 11 + \alpha_2 22 + \beta_2 22) k'^2 + u_R (\gamma_1 11 + \gamma_2 22) k k' + u_R (\gamma_1 12 + \gamma_2 22) (k^2 + k'^2) \] \hfill (4.5.6)
\[ \mu^2 v_R = 2v_R^3(\rho_1 + \rho_2) + 3v_L v_R \]
\[ + v_R (\alpha_{11} + \alpha_{22} + \beta_{11}) k^2 + v_R (\alpha_{11} + \alpha_{22} + \beta_{22}) k'^2 \]
\[ + v_L (\gamma_{11} + \gamma_{22}) k k' + v_L \gamma_{12} (k^2 + k'^2) \]  
(4.5.7)

Multiplying equation (4.5.6) by \( v_R \) and equation (4.5.7) by \( v_L \) and subtracting one obtains
\[
\begin{align*}
&[v_L v_R \{ 2(\rho_1 + \rho_2) - \rho_3 \} - \gamma_{12} (k^2 + k'^2) ] \\
&- (\gamma_{11} + \gamma_{22}) k k' \right) (v_L^2 - v_R^2) = 0
\end{align*}
(4.5.8)

The possible solutions are

(i) \( v_L^2 = v_R^2 \), in this case, LR symmetry is not broken, 

(ii) \( v_L v_R = \frac{\gamma_{12} (k^2 + k'^2) + (\gamma_{11} + \gamma_{22}) k k'}{2(\rho_1 + \rho_2) - \rho_3} \)  
(4.5.9)

for \( v_L^2 \neq v_R^2 \)

In the approximation \( k' \ll k \) and \( v_L = 0 \) from eq (4.5.3)
\[
V(x_R, k) = \left\{ - (\mu_1^2 + \mu_2^2) + (\lambda_{1111} + \lambda_{1122} + \lambda_{2211} + \lambda_{2222}) k^2 \right\} k^2 \\
+ (\lambda'_{1111} + \lambda'_{2222}) k^4 \\
- \mu^2 v_R^2 + (\rho_1 + \rho_2) v_R^4 + v_R^2 (\alpha_{11} + \alpha_{22} + \beta_{11}) k^2
\]  
(4.5.10)

Again from the extremizing condition \( \frac{\partial V}{\partial v_R} = 0 \) we obtain
\[
-2\mu^2 v_R + 4(\rho_1 + \rho_2) v_R^3 + 2v_R (\alpha_{11} + \alpha_{22} + \beta_{11}) k^2 = 0
\]
or \(- \mu^2 + 2(\rho_1 + \rho_2) v_R^2 + 2(\alpha_{11} + \alpha_{22} + \beta_{11}) k^2 = 0 \)
or \( v_R^2 = \frac{\mu^2 - 2(\alpha_{11} + \alpha_{22} + \beta_{11}) k^2}{2(\rho_1 + \rho_2)} \)  
(4.5.11)
which is the minimum solution and the parity is spontaneously broken. From eq.(4.5.9)

\[ \nu_L \nu_R = \frac{\beta}{\rho - \rho'}, \quad k^2 \]

in the limit \( k >> k' \) and \( \beta = \gamma_{12} \)

\[ \rho - \rho' = 2(\rho_1 \Phi \rho_2) - \rho_3 \]

\[ \nu_L = \frac{\gamma k^2}{\nu_R} \quad \text{with} \quad \gamma = \frac{\rho}{\rho - \rho'} \]

The fermion mass in the present case is generated by the Lagrangian

\[ \alpha_{\text{mass}} = h_1 \bar{\psi}_L \phi \psi_R + h_2 \bar{\psi}_L \bar{\phi} \psi_R + h_3 \bar{\psi}_L \phi \psi_R + h_4 \bar{\psi}_L \bar{\phi} \psi_R \quad (4.5.12) \]

There is no coupling of the fermions to the \( \chi' \) fields, and the neutrino gets the Dirac mass in a manner similar to quarks and leptons due to the VEV's of \( \phi \) and \( \bar{\phi} \)

\[ m_{\nu e} = h_1 k + h_2 k' \]

\[ m_e = h_1 k' + h_2 k \quad (4.5.13) \]

\[ m_u = h_3 k + h_4 k' \]

\[ m_d = h_3 k' + h_4 k \]

But such large neutrino masses are not favoured by experiments.

4.6 Generation of neutrino masses by Higgs triplets:

In this case left-handed and right handed triplets
of Higgs scalars $\Delta_L$ and $\Delta_R$ are used to break left-right symmetry. Assuming neutrino to be a majorana particle, due to the coupling of $\Delta'$ fields to the farmions, neutrino acquires a majorana mass.

The symmetry breaking can be realized by two L-R conjugate triplets $\Delta_L(1,0,2), \Delta_R(0.1,2)$ and

$$\phi (1/2, 1/2^*, 0), \bar{\phi} \equiv \tau_2 \phi^* \tau_2 (1/2, 1/2^*, 0)$$

With the numbers in the brackets denoting $SU(2)_L, SU(2)_R$ and $U(1)_{B-L}$ quantum numbers respectively. Writing in the matrix form,

$$\Delta_{L,R} = \begin{pmatrix}
\frac{1}{\sqrt{2}} & \delta^{+} \\
\delta & -\frac{1}{\sqrt{2}}
\end{pmatrix}$$

The most general form of the vacuum expectation value (VEV) consistent with $U(1)_{em}$ is

$$\langle \phi \rangle = \begin{pmatrix} k & 0 \\ 0 & k' \end{pmatrix}, \langle \phi_2 \rangle = \langle \bar{\phi} \rangle = \begin{pmatrix} k' & 0 \\ 0 & k \end{pmatrix}$$

and

$$\langle \Delta_{L,R} \rangle = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}_{L,R}$$

and the Higgs potential satisfying gauge, left-right
and discrete symmetries \((\Delta_L \rightarrow \Delta_L', \Delta_R \rightarrow \Delta_R', \phi \rightarrow e^{i\pi/2}\phi)\)
can be written as

\[
V = -\sum_{i,j=1}^{2} \mu_{ij} \text{tr} \phi_i^+ \phi_j^+ + \sum_{i,j,k,l=1}^{2} \lambda_{ijkl} \text{tr}(\phi_i^+ \phi_j) \text{tr}(\phi_k^+ \phi_l) \\
+ \sum_{i,j,k,l=1}^{2} \lambda_{ij} \text{tr} \phi_i^+ \phi_j^+ \phi_k^+ \phi_l^+ - \mu^2 \text{tr} (\Delta_L^+ \Delta_L + \Delta_R^+ \Delta_R) \\
+ \rho_1 [(\text{tr} \Delta_L^+ \Delta_L)^2 + (\text{tr} \Delta_R^+ \Delta_R)^2] \\
+ \rho_2 (\text{tr} \Delta_L^+ \Delta_L \Delta_L^+ \Delta_L + \text{tr} \Delta_R^+ \Delta_R \Delta_R^+ \Delta_R) + \rho_3 \text{tr} \Delta_L^+ \Delta_L \Delta_R^+ \Delta_R \\
+ \sum_{i,j=1}^{2} \alpha_{ij} \text{tr} \phi_i^+ \phi_j^+ \text{tr} (\Delta_L^+ \Delta_L + \text{tr} \Delta_R^+ \Delta_R) \\
+ \sum_{i,j=1}^{2} \delta_{ij} (\text{tr} \Delta_L^+ \Delta_L \phi_i^+ + \text{tr} \Delta_R^+ \Delta_R \phi_j^+) \\
+ \sum_{i,j=1}^{2} \gamma_{ij} \text{tr} \Delta_L^+ \Delta_R \phi_i^+ \phi_j^+ \quad (4.6.4)
\]

which gives

\[
V(\Delta_L, \Delta_R, k_1, k_2) = -\mu^2 (V_L^2 + V_R^2) + \frac{\rho}{4} (V_L^4 + V_R^4) \\
+ \frac{\rho'}{2} V_L^2 V_R^2 + (V_L^2 + V_R^2) [(\alpha_{11} + \alpha_{22} + \beta_{11}) k^2 \\
+ (\alpha_{11} + \alpha_{22} + \beta_{22}) k'^2 + (4\alpha_{12} + 2\beta_{12}) kk'] \\
+ 2V_L V_R [(\gamma_{11} + \gamma_{22}) kk' + \gamma_{12}(k^2 + k'^2)] \quad (4.6.5) \\
+ \text{terms which depend on } k, k' \text{ only.}
\]

where \(\rho = 4(\rho_1 + \rho_2)\), \(\rho' = 2\rho_3\). \(4.6.6\)
as previously, in the approximation

\[ k' \ll k, \]

\[ V(\Delta_L, \Delta_R, k) = -\mu^2(V^2_L + V^2_R) + \frac{\rho}{4} (V^2_L + V^2_R) \]

\[ + \frac{\rho'}{2} V^2_{L,R} + \frac{\alpha}{2} (V^2_L + V^2_R)k^2 + \beta V_L V_R k^2 \quad (4.6.7) \]

with \( \alpha = 2(\alpha_{11} + \alpha_{22} + \beta_{11}) \)

\[ \beta = 2\gamma_{12} \]

Again from the extremizing condition

\[ \frac{\partial V}{\partial V_L} = \frac{\partial V}{\partial V_R} = 0, \]

we obtain

\[ \mu^2 V_L = \rho V^3_L + \rho' V_L V^2_R + \alpha k^2 V_L + \beta k^2 V_R \quad (4.6.9) \]

\[ \mu^2 V_R = \rho V^3_R + \rho' V_R V^2_L + \alpha k^2 V_R + \beta k^2 V_L \quad (4.6.10) \]

and

\[ [(\rho - \rho') V_L V_R - \beta' k^2] (V^2_L - V^2_R) = 0 \quad (4.6.11) \]

The possible solutions are

(a) \( V^2_L = V^2_R \)

for which \( L - R \) symmetry is not broken.

(b) \( V_L \neq V_R \) in which case,

\[ V_L V_R = \frac{\beta}{\rho - \rho'} k^2 \quad (4.6.12) \]

writing \( \gamma = \frac{\beta}{\rho - \rho'} \),

\[ V_L = \frac{\gamma k^2}{V_R} \quad (4.6.13) \]

Unlike the \( \chi' \) fields, if neutrino is a majorana particle, there is a coupling with the \( \Delta' \) fields which generates majorana mass
terms for the neutrino. The fermion mass is generated by the Lagrangian,

\[ \alpha = h_1 \bar{\psi}_L \psi_R + h_2 \bar{\psi}_L \phi \psi_R + h_3 \bar{\phi}_L \phi \bar{Q}_R + h_4 \bar{Q}_L \phi \bar{Q}_R + \left[ i h_5 (\bar{\psi}_L^T \tau_2 \Delta_L \psi_R + \bar{\psi}_R^T \tau_2 \Delta_R \psi_R) + H.C. \right] \]  \tag{4.6.14}

where \( \phi \equiv \tau_2 \phi^\tau_2 \) and \( C \) is the Dirac change-conjugation matrix. Considering only one generation of fermions, the masses for charged fermions are

\[ m_e = h_1 k' + h_2 k \]
\[ m_u = h_3 k + h_4 k' \]  \tag{4.6.15}
\[ m_d = h_3 k' + h_4 k \]

where \( m_e, m_u, m_d \) are the masses of electron, up and down quark respectively. The Lagrangian for \( \nu_L, \nu_R \) sector becomes

\[ \alpha_{\text{mass}} = h_5 [\nu_L^T (\nu_L \nu_L + \nu_R \nu_R^* \bar{\nu}_L^* \nu_R^*) + \nu_R (\nu_R \nu_L + \nu_R^* \nu_L^*)] + (h_1 k + h_3 k') (\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L) \]  \tag{4.6.16}

which is a mixture of Majorana and Dirac mass terms. This Lagrangian can also be written as

\[ \alpha_{\text{mass}} = h_5 (\nu_L^T \nu_L + \nu_R^T \nu_R^*) - \nu_R N^T C N + H.C. \]  \tag{4.6.17}

where \( \nu \equiv \nu_L \) and \( N \equiv C (\bar{\nu}_R)^T \)

and using the properties of charge conjugation matrix

\[ C^T = -C, \quad C^2 = -1 \quad \text{and} \quad C \gamma_\mu C^T = -\gamma_\mu^T \]
\[ \nu_R^T C \nu_R^* = -N^T C N \quad \text{and} \quad \bar{\nu}_R \nu_L = N^T C \nu = \nu^T C N. \]

In the matrix form, the above Lagrangian takes the form

\[ \alpha_{\text{mass}} = (\nu^T N^T) MC (\nu_N) + H.C. \]  \tag{4.6.18}

where \( M = \begin{pmatrix} a & c \\ c & b \end{pmatrix} \)  \tag{4.6.19}
a, b denote the Majorana mass and C denotes the Dirac mass term and

\[ a = h_5 V_L, \quad b = -h_5 V_R, \quad C = \frac{1}{2} (h_1 k + h_2 k') \quad (4.6.20) \]

The eigen states of this mass matrix is given by,

\[ \nu = \nu_e \cos \zeta + N_e \sin \zeta \]
\[ N = -\nu_e \sin \zeta + N_e \cos \zeta \quad (4.6.21) \]

with \( \tan 2 \zeta = \frac{2c}{b-a} = \frac{2c}{b} \quad (4.6.22) \)

Studying the eigen values of (4.6.20) assuming \( k' \ll k \), Mahapatra and Senjanović. obtain light and heavy Majorana neutrino masses as

\[ m_{\nu e} = a \frac{c^2}{b} \quad (4.6.23) \]
\[ m_N = b. \]

Using eqs. (4.6.13) and (4.6.20), masses can be written as,

\[ m_{\nu e} = (h_5 Y + \frac{1}{4} \frac{h_1^2}{h_5^2} ) \frac{k^2}{V_R} \quad (4.6.24) \]
\[ m_N = -h_5 V_R \]

For simplicity we put \( k' = 0 \). If all Yukawa couplings are equal, \( h_1 = h_2 = h_5 = h \), then it is easy to see that,

\[ m_N = \frac{h}{g} m_{WR} \quad (4.6.25) \]

and

\[ m_{\nu e} = \frac{g}{h} \frac{m_e^2}{m_{WR}} \]

Where \( g \) is the gauge coupling constant in the LRS model, and, as will be demonstrated subsequently in this dissertation.

\[ M_{WR} \sim g V_R, \quad \text{where} \quad M_{WR} \quad \text{stands for the} \quad W_R^+ \quad \text{gauge boson mass.} \]
If \( h/g = 1 \),
\[
m_{\nu e} = \frac{m_e^2}{m_{WR}} \tag{4.6.26}
\]

Thus, as \( m_{WR} \to \infty \), \( m_{\nu e} \to 0 \).
i.e. in the V-A limit of charged and neutral currents, the
neutrino has zero mass as in the standard model. But for
any finite value of \( m_{WR} \), the neutrino acquires a mass much
smaller compared to the Dirac mass of the quark or charged
lepton in the same family. The formula (4.6.26) holds for
every generation and can be generalised as
\[
m_{\nu \alpha} = \frac{m^2_\alpha}{m_{WR}} \tag{4.6.27}
\]
\( \alpha = e, \mu, \tau \).

We calculate the neutrino masses for assumed value of \( m_{WR} \)
in the range \( 1 \, \text{TeV} - 10^{12} \, \text{GeV} \). as presented in Table 4.1.

### Table 4.1 Neutrino masses for three generations as a function of \( W_R^\pm \) boson masses - using see-saw mechanism and left-right symmetric model.

<table>
<thead>
<tr>
<th>( m_{WR} ) (GeV)</th>
<th>( m_{\nu e} ) (eV)</th>
<th>( m_{\nu\mu} ) (keV)</th>
<th>( m_{\nu\tau} ) (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^3 )</td>
<td>0.261</td>
<td>11.2</td>
<td>3.5</td>
</tr>
<tr>
<td>( 10^4 )</td>
<td>0.026</td>
<td>1.12</td>
<td>0.35</td>
</tr>
<tr>
<td>( 10^5 )</td>
<td>( 2.6 \times 10^{-3} )</td>
<td>0.11</td>
<td>0.04</td>
</tr>
<tr>
<td>( 10^6 )</td>
<td>( 2.6 \times 10^{-10} )</td>
<td>( 1.12 \times 10^{-8} )</td>
<td>( 3.5 \times 10^{-9} )</td>
</tr>
</tbody>
</table>
4.7 Neutrinoless double \( \beta \)-decay

In order to confirm the finite neutrino mass, there are various experiments such as the \( \beta \)-decays of \( ^3\text{He} \), \( ^{35}\text{S} \), and \( ^{63}\text{Ni} \), the neutrino oscillation, the electron capture in \( ^{163}\text{Ho} \) and so on. Concerning the V+A charged current, the precise measurements on the \( \beta \)-and \( \mu \)-decays have been performed. In addition to these problems, the question whether neutrinos are Dirac or Majorana particle can be tested the neutrinoless double \( \beta \)-decay directly.

The process can take place in the second order in Fermi coupling if neutrinos are Majorana particles (shown in the fig. 4.1)

\[ n \xrightarrow{W_L(W_R)} e \]

\[ \nu(N) \xrightarrow{W_L(W_R)} e \]

\[ n \xrightarrow{W_L(W_R)} p \]

Fig.4.1. The diagram showing neutrino less double \( \beta \)-decay through exchange of \( W_L \) and \( W_R \) or \( N_i \) and \( \nu_i \).

The neutrinoless double \( \beta \)-decay \((\beta\beta)_0\) has been analysed in terms of the amplitude \( \eta \), the lepton number nonconserving parameter or the admixture of left and right handed neutrino electron currents, where \( \eta \) appears in the leptonic current as
through the finite mass of the electron neutrino. The value obtained for the contribution involving $W_L-W_R$ and $\nu-N$ mixing is

$$\eta \leq 10^{-2} \frac{m_e}{m_N} \leq 10^{-7} \quad (4.7.2)$$

for $m_N \geq 100$ GeV. The values of $\eta$ obtained for $m_N = 10^2-10^4$ are listed in the table below.

Table 4.2 The values of $\eta$ for $m_N = 10^2-10^4$ GeV are obtained from the relation (4.7.2)

<table>
<thead>
<tr>
<th>$m_N$ (GeV)</th>
<th>$10^2$</th>
<th>$10^3$</th>
<th>$10^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta$</td>
<td>5x10^{-8}</td>
<td>5x10^{-9}</td>
<td>5x10^{-10}</td>
</tr>
</tbody>
</table>

We see the relation (4.7.2) is satisfied for the values of

$$m_N = 10^2 - 10^4$$
i.e. $m_N \geq 10^2$.

The limit on $\eta$ consistent with $(\beta\beta)_0$ decay rates of $^{48}\text{Ca} \rightarrow ^{48}\text{Ti}$, $^{76}\text{Ge} \rightarrow ^{76}\text{Se}$, and $^{82}\text{Se} \rightarrow ^{82}\text{Kr}$ is,

$$| \eta | \leq 5x10^{-4}$$

The value of $\eta$ corresponds to $m_{\nu_e} \leq 1$ KeV for the light neutrino and implies a half-life for $(\beta\beta)_0$ decay of $^{82}\text{Se}$, eg,
\[ T(\beta\beta)_o \approx \frac{1.1 \times 10^{14} \pm 2}{\eta^2} \text{ Yr} \quad (4.7.3) \]

\[ \geq 40 \times 20^{20} \pm 2 \text{ Yr} \]

Again due to the tiny neutrino masses, the exchange of heavy right handed leptons \( N_i (i = e, \mu, \tau) \) will obviously dominate and \( \eta \) is given by

\[ \eta \leq \left( \frac{m_{WL}}{m_{WR}} \right)^4 \frac{1}{m_N} f_{\text{nuc}}. \quad (4.7.4) \]

Where \( f_{\text{nuc}} \) is the nuclear structure estimated by Halprin et al to be about 0.35 GeV.

For \( m_{Ne} \geq 100 \text{ GeV} \), and \( \left( \frac{m_{WL}}{m_{WR}} \right)^2 \leq \frac{1}{10} \)

\( \eta \) becomes \( \leq 3.5 \times 10^{-5} \)

For the values of \( \eta \) in eq (4.7.4) \((\beta\beta)_o\) would require a half-life measurement of order \( 8 \times 10^{22} \pm 2 \text{ yr} \). The values of \( \eta \) are found out for \( m_N = m_{WR} = 10^2 - 10^4 \text{ GeV} \).

**Table 4.3** The values of \( \eta \) obtained for \( m_N = m_{WR} = 10^2 - 10^4 \text{ GeV} \) from the relation (4.7.4)

<table>
<thead>
<tr>
<th>( m_N ) (GeV)</th>
<th>( 10^2 )</th>
<th>( 10^3 )</th>
<th>( 10^4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta )</td>
<td>3.5 \times 10^{-5}</td>
<td>3.5 \times 10^{-6}</td>
<td>3.5 \times 10^{-7}</td>
</tr>
</tbody>
</table>

A measurement of a half-life for \((\beta\beta)_o\) decay in the range \( 10^{20} \) to
$10^{24}$ Yr would place a limit on $m_N$ for the left-right symmetric electroweak model if the limit on $\left(\frac{m_{WL}}{m_{WR}}\right)^2$ is known from other considerations. Neutrinoless double $\beta$-decay is a lepton number violating process which provides constraints on neutrino masses and couplings. The existence of Majorana neutrinos is one of the simplest ways to account for masses much smaller than those of charged leptons (or of quarks). The effective neutrino mass $<m_\nu>$ can be derived from the life time and from the knowledge of the nuclear matrix element. Unfortunately, strong discrepancies still exist between different theoretical estimates of this last quantity, as an example, upper limit on $<m_\nu>$ derived from the UCS-LBL result on $^{76}$Ge, are given by 1eV, 2eV and 19eV.

We now briefly comment on some other muon and electron number changing processes. The interesting possible decay such as $\mu \rightarrow e\gamma$ and $\mu \rightarrow 3e$ may sometimes impose severe constraints on models with neutrino masses. The processes $\mu \rightarrow e\gamma$ receives its two dominant contributions from the $U_W^H W_R^H$ and $U_L^H W_L^H$ triangle loop diagrams (Fig.4.2). These contributions to the branching ratio may be estimated respectively.

$$\mathcal{B}_L = \frac{\Gamma(\mu \rightarrow e + \gamma)}{\Gamma_\mu \text{ total}} \left[ \frac{3}{32\pi} \sin \theta \cos \theta \frac{m_{u_2}^2 - m_{u_1}^2}{m_{W_L}^2} \right]$$  \hspace{1cm} (4.7.5)$$

$$\mathcal{B}_R = \frac{3\alpha}{32\pi} \left( \frac{m_{W_L}^2}{m_{W_R}^2} \right)^2 \left( \frac{\sin \theta \cos \theta}{\sin \theta' \cos \theta'} \frac{m_{W_2}^2 - m_{N_1}^2}{m_{W_L}^2} \right)$$  \hspace{1cm} (4.7.6)$$
For $m_W \sim 80$ GeV, and the limiting values $m_{\nu e} \leq 30$ ev, $m_{\nu \mu} \sim 0.6$ MeV, $B_L < 4.5 \times 10^{-26}$ which is too small. Eq. (4.7.6) holds for L-R symmetric model where neutrinos are predominantly left-handed and light while the N's are predominantly right-handed and heavy, but the masses smaller than but of the same order as $m_{WR}$. In this case, eq (4.7.6) for $\frac{m_{WL}}{2} \sim \frac{1}{10}$, $m_{L2} - m_{N1} \sim 10^4$ GeV.

\[ B_R \approx 4 \times 10^{-8} (\sin \theta' \cos \theta')^2 \]  

(4.7.7)

From cosmological limits on the $\nu_e, \nu_\mu$ masses, $\theta' \leq \left( \frac{m_e}{m_\mu} \right)^{1/2}$

and $B_R \leq 2 \times 10^{-10}$  

(4.7.8)

When compared with $\mu \rightarrow e\gamma$ process, the branching ratio becomes

\[ B(\mu \rightarrow eee) = \Gamma(\mu \rightarrow eee) / \Gamma(\mu \rightarrow e \nu \bar{\nu} e) \approx \frac{\alpha}{\sin^2 \theta_W} B(\mu \rightarrow e\gamma) \]  

For $\sin^2 \theta_W = 0.22$, 

\[ B(\mu \rightarrow eee) / B(\mu \rightarrow e\gamma) \approx (1-10)\% \]  

(4.7.9)

Other possible muon and lepton number changing process are $e \bar{\nu} \rightarrow \mu \bar{\nu}$,

\[ \mu^- + A(Z) \rightarrow e^- + A(Z), \]  

(4.7.10)

\[ \mu^- + A(Z) \rightarrow e^+ + A(Z-2), \text{ etc.} \]  

(4.7.11)

For the process (4.7.10)

\[ B \leq 1.3 \times 10^{-10} \sin^2 \theta' \cos^2 \theta' \]  

(4.7.12)
Choosing $\theta' \sim \left( \frac{m_e}{m_\mu} \right)^{1/2}$

$B \leq 6 \times 10^{-13}$ \hspace{1cm} (4.7.13)

and for the process (4.7.11)

$B \leq 4 \times 10^{-10} \sin^2 \theta' \cos^2 \theta' \left( \frac{m_2 - m_1}{m_A} \right)^2$ \hspace{1cm} (4.7.14)

Where $m_1$, $m_2$ are the masses of Majorana neutrinos and $m_A$ is the target mass.

Detection of both conversion processes with comparable branching ratios provides clear cut evidence for the existence of heavy Majorana neutrinos and for the L-R symmetric electroweak model.

4.8 Summary

Unified electroweak gauge theories based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ in which breakdown of parity invariance is spontaneous, lead most naturally to a massive neutrino. Dirac mass is generated by Yukawa interaction of fermions with Higgs doublets carrying $B-L = 0$ and $Y = +1$. But when Higgs triplets with $B-L = \pm 2$ are used, neutrino also gets Majorana mass. The small mass of the physical neutrino is then generated by a see-saw mechanism. Mahapatra and Senjanovic have shown that smallness of neutrino mass can be understood as a result of the observed maximality of parity violation in low energy weak interactions. In particular, in the limit $m_{\nu_R} \rightarrow \infty$, $m_{\nu_e} \rightarrow 0$, the weak interaction becomes
pure V-A type. In left-right symmetric model, the V-A limit of charged and neutral currents corresponds to the vanishing of neutrino mass. The neutrinoless double β-decay is expected to provide strong evidence for a heavy neutrino in the mass range 100 GeV - few TeV.

In the next section we discuss gauge model based upon $SU(2)_L \times U(1)_R \times U(1)_{B-L}$ where it is possible to generate nonvanishing Majorana neutrino mass even if $M_R \to \infty$. Here it has been found that vanishing neutrino mass is a consequence of V-A limit of neutral currents only. Neutrino masses from various experiments are also summarised in the next chapter.