Giant Dipole Resonance Observables

The effects of excitation on the GDR may be divided into three parts: (a) finite rotation and zero temperature, (b) the effects at finite temperature and finite rotation assuming a frozen nuclear shape (deformation), and (c) the effects of thermal fluctuations in the deformation. Various calculations indicate that the first two effects are small for moderate temperatures and spins. The most important factor, the third, is discussed in the next chapter. In this chapter the formalism for calculating GDR cross-sections and GDR $\gamma$-ray angular anisotropy parameters has been presented. To get the GDR energies of rotating nuclei an analytical method [54–57] based on rotating anisotropic harmonic oscillator potential with a separable dipole-dipole interaction, is used. In this formalism the average field of the nucleus was taken to be an oscillator potential with deformation parameter consistent with the angular momentum of the system.

The rotation-induced changes of the shape of nuclei can be simulated by the average Hamiltonian of a triaxial harmonic oscillator

$$H_{av}(\Omega) = \sum_{\nu=1}^{A} h_{\nu}(\Omega), \quad (3.1)$$

where

$$h(\Omega) = \frac{p^2}{2m} + \frac{m}{2} \left( \omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2 \right) - \Omega l_z \quad (3.2)$$
and

\[ L_z = \sum_{\nu=1}^{A} l_z(\nu) \]

is the operator of rotation about the Z axis. The eigenfunctions and the eigenvalues of the Hamiltonian (3.1) are determined from the equation

\[ [H_{av}, a^\dagger_\lambda] = \omega a^\dagger_\lambda, \quad (3.3) \]

for oscillator-quantum creation operators \( a^\dagger_\lambda \) that are linear combinations of the particle coordinates \( r_i \) and of the conjugate momenta \( p_i \). In terms of the operators \( a^\dagger_\lambda \) and \( a_\lambda \), the Hamiltonian (3.1) can be expressed as

\[
H_{av} = \sum_{\nu=1}^{A} \left\{ \omega_z \left[ (a^\dagger_\nu a_\nu) + \frac{1}{2} \right] + \omega_+ \left[ (a^\dagger_+ a_-) + \frac{1}{2} \right] \\
+ \omega_- \left[ (a^\dagger_- a_-) + \frac{1}{2} \right] \right\} \quad (3.4)
\]

The normal frequencies are then obtained as

\[
\omega_z = \omega_z, \\
\omega_\pm = \left\{ \frac{\omega^2}{2} + \Omega^2 \pm \frac{1}{2} \left[ (\omega^2 - \omega^2) + 8 \Omega^2 (\omega^2 + \omega^2) \right]^{1/2} \right\}^{1/2}. \quad (3.5)
\]

The isovector giant dipole excitation mode is then generated by adding to the Hamiltonian (3.1) the effective dipole interaction

\[
H_{int} = \eta \sum_{i=x,y,z} \frac{m \omega_i^2}{2A} \left[ \sum_{\nu=1}^{A} \tau^{(\nu)}_3 x_i(\nu) \right]^2, \quad (3.6)
\]

where \( \tau^{(\nu)}_3 \) is the third projection of the Pauli isospin matrix

\[
\tau_3 = \begin{pmatrix}
1 & 0 \\
0 & -1
\end{pmatrix}
\]
and $\eta$ is a parameter that characterizes the isovector component of the neutron or proton average field

$$V_{(n,p)}(\nu) = \frac{m}{2} \left[ 1 + \eta \frac{N - Z}{A} \right] \sum_{i=x,y,z} \omega_i^2 x_i^2(\nu). \tag{3.7}$$

The value of $\eta$ for an oscillator potential is found [54] to be 3 from experimental data on the position of the giant resonance.

The giant dipole resonance frequencies in a rotating nucleus can be obtained by diagonalizing analytically the Hamiltonian (3.1) with the effective interaction (3.6) within the framework of the standard random phase approximation (RPA) procedure by using the similarity between the linear transformation corresponding to (3.1) and the RPA transformations.

Similar to Eq.(3.3) the RPA equation for the isovector dipole modes can be written as

$$[H + H_{int}, D^\dagger] = \bar{\omega}_{\lambda} D^\dagger_{\lambda}, \tag{3.8}$$

where the dipole phonon-creation operator is given by

$$D^\dagger_{\lambda} = \frac{1}{\sqrt{A}} \sum_{\nu=1}^{A} \tau_3(\nu) \hat{a}_{\lambda}^\dagger(\nu). \tag{3.9}$$

The components of the commutator (3.8) are then obtained as

$$\sum_{i=x,y,z} \frac{m \omega_i^2}{2} \sum_{\nu=1}^{A} x_i^2(\nu) \ D^\dagger_{\lambda}$$

$$= \sum_{i=x,y,z} m \omega_i^2 \left[ x_i, a_{\lambda}^\dagger \right] \frac{1}{\sqrt{A}} \sum_{\nu=1}^{A} \tau_3(\nu) x_i(\nu) \tag{3.10a}$$

$$\frac{1}{2} \eta \left[ \sum_{i=x,y,z} \frac{m \omega_i^2}{2} \frac{1}{A} \left( \sum_{\nu=1}^{A} \tau_3(\nu) x_i(\nu) \right)^2, D^\dagger \right]$$

$$= \eta \sum_{i=x,y,z} m \omega_i^2 \left[ x_i, \hat{a}_{\lambda}^\dagger \right] \frac{1}{\sqrt{A}} \sum_{\nu=1}^{A} \tau_3(\nu) x_i(\nu) \tag{3.10b}$$
The commutator of the phonon-creation operator $D^\dagger_\lambda$ with the Hamiltonian of the interaction in (3.10b) is thus proportional to the commutator of $D^\dagger_\lambda$ with the average field operator (3.10a) and therefore we can write

$$[H + H_{int}, D^\dagger_\lambda] = [\tilde{H}_{av}, D^\dagger_\lambda],$$

(3.11)

where

$$\tilde{H}_{av} = \sum_{\nu=1}^A \frac{p^2_\nu}{2m} + (1 + \eta) \sum_{i=x,y,z} \omega_i^2 x_i^2(\nu) - \Omega L_z .$$

(3.12)

The expressions of (3.5) determine the eigenvectors and the normal frequencies of Eq.(3.11) and in this, a substitution $\omega_i = (1 + \eta)^{1/2} \omega_i$ is made. Thus, we get the giant dipole resonance frequencies in the rotating frame.

Transforming to the laboratory system, the giant dipole resonance frequencies are

$$\tilde{\omega}_z = (1 + \eta)^{1/2} \omega_z ,$$

$$\tilde{\omega}_\pm = \left\{ (1 + \eta) \frac{\omega_y^2 + \omega_x^2}{2} + \Omega^2 \right. $$

$$+ \frac{1}{2} \left[ (1 + \eta)^2 (\omega_y^2 - \omega_x^2)^2 \right.$$  

$$+ 8\Omega^2 (1 + \eta)(\omega_y^2 + \omega_x^2)]^{1/2} \right\}^{1/2} \mp \Omega $$(3.13)

If we include the splitting of these frequencies due to rotation, the final set of GDR frequencies we obtain are

$$\tilde{\omega}_z = (1 + \eta)^{1/2} \omega_z ,$$

(3.14)

$$\tilde{\omega}_2 \mp \Omega = \left\{ (1 + \eta) \frac{\omega_y^2 + \omega_x^2}{2} + \Omega^2 \right.$$  

$$+ \frac{1}{2} \left[ (1 + \eta)^2 (\omega_y^2 - \omega_x^2)^2 \right.$$  

$$+ 8\Omega^2 (1 + \eta)(\omega_y^2 + \omega_x^2)]^{1/2} \right\}^{1/2} \mp \Omega $$(3.15)
\[ \tilde{\omega}_3 \mp \Omega = \left\{ (1 + \eta) \frac{\omega_y^2 + \omega_z^2}{2} + \Omega^2 \right. \\
\left. \quad - \frac{1}{2} \left[ (1 + \eta)^2 (\omega_y^2 - \omega_z^2)^2 \right. \\
\left. \quad + 8\Omega^2 (1 + \eta)(\omega_y^2 + \omega_z^2) \right]^{\frac{1}{2}} \right\} \mp \frac{\Omega}{2} \quad (3.16) \]

Thus one get five frequencies, \( \tilde{\omega}_1, \tilde{\omega}_2 - \Omega, \tilde{\omega}_2 + \Omega, \tilde{\omega}_3 - \Omega \) and \( \tilde{\omega}_3 + \Omega \), for collectively rotating triaxial nuclei. Perhaps for all the nuclei the frequencies does split into five components. The splitting of GDR is discussed illustratively in §3.3

### 3.1. The dipole photoabsorption cross section

In the study of \( \gamma \)-ray spectra emitted from rapidly rotating hot nuclei, the dipole photoabsorption cross section as a function of angular momentum plays a vital role. Using the semi classical theory of the interaction of photons with nuclei, the shape of a fundamental resonance in the absorption cross-section is that of the Lorentz curve

\[ \sigma(E) = \frac{\sigma_m}{1 + (E^2 - E_m^2)^2 / E^2 \Gamma^2} , \quad (3.17) \]

where Lorentz parameters \( E_m, \sigma_m \) and \( \Gamma \) are the resonance energy, peak cross-section and full width at half maximum respectively. In the case of spherical nuclei, the giant dipole resonance consists of one Lorentz line. The peak cross section \( \sigma_m \) for the spherical nuclei is given by [75,76]

\[ \sigma_m = 60 \frac{2 NZ}{\pi} \frac{1}{A} \frac{1}{\Gamma_M} 0.86(1 + \alpha) , \quad (3.18) \]
where $\alpha$ is an adjustable parameter which takes care of the sum rule. For deformed spheroidal nuclei, the giant resonance consists of two such Lorentz lines corresponding to the absorption of photons which induce oscillation of the neutron and proton fluids in the nucleus against each other. In such cases,

$$\sigma(E) = \sum_{i}^{2} \frac{\sigma_{mi}}{1 + (E^2 - E_{mi}^2)^2 / E^2 \Gamma_i^2},$$

where $i = 1, 2$ correspond to the lower and higher energy lines. The lower energy line corresponds to oscillations along the longer axis and the higher energy line corresponds to oscillations along the shorter axis. For triaxial nuclei all the five GDR frequencies are distinct and results in five components of GDR. In such case the summation over $i$ should be extended to five. It is to be noted that these Lorentz lines are non-interfering, but $\Gamma_i$ is assumed to depend on energy.

### 3.2. The giant dipole resonance width

It is to be noted that this problem has not been satisfactorily solved so far even in the absence of rotation. It was observed [18] that the ground state giant dipole resonance full width at half maximum to be narrow in the region of spherical nuclei and broadened primarily by quadrupole deformation in other mass regions. Also the observed GDR full width at half maximum in excited nuclei varies more or less smoothly with mass, even though the single- and the double-Lorentzian shapes show significant differences in detail. In the same or neighbouring nuclei, the widths are generally broader than the ground state GDR widths. It was inferred [18] that the nuclear quadrupole deformation plays a fundamental role in determining the GDR shapes in excited nuclei. Nix et al.,
using a surface plus window dissipation model, could get resonance widths comparable to experimental values, not for giant dipole resonances but only for giant quadrupole and octupole resonances. The energy dependence of the GDR width can be approximated by [78]

$$\Gamma_i \approx 0.026 E_i^{1.9}$$  \hspace{1cm} (3.20)

where $E_i$ are the GDR energies. For the parameterization of the IVGDR width in a rotating nucleus, this expression is very useful, with due allowance for the corresponding changes of the energies of the resonances, and is used in the calculations.

3.3. Splitting of GDR

At zero spin in a deformed nucleus the GDR splits into three components corresponding to vibrations along each of the three principal axes, two of which are degenerate in energy if the nucleus is axially symmetric. In such cases the possible shapes of the GDR curves are illustrated in Fig. 3.1. From the figures it is clear that the shape of the GDR is not only sensitive to the deformation but also to the width. This makes the task of relating the GDR directly to the deformation parameters little difficult. For example in Fig. 3.1 case (c) and case (d) lead to similar GDR curves even though they correspond to prolate and oblate deformations respectively. This is a general case as a power law dependence of width on energy (3.20) has been established clearly [78] and hence in most cases we have $\Gamma_2 > \Gamma_1$. More such complications arise in rotating nuclei.

One have five GDR components corresponding to the frequencies, $\tilde{\omega}_1, \tilde{\omega}_2 - \Omega,$
Fig. 3.1. Variety of GDR shapes corresponding to different ratios of width for prolate and oblate shapes.
\( \tilde{\omega}_2 + \Omega, \tilde{\omega}_3 - \Omega \) and \( \tilde{\omega}_3 + \Omega \), for collectively rotating triaxial nuclei. For prolate nuclei \((\omega_x = \omega_y > \omega_z)\) rotating about an axis perpendicular to its symmetry axis, all the above five frequencies will exist. But for oblate nuclei \((\omega_x = \omega_y < \omega_z)\) rotating about its symmetry axis, as shown first by Hilton in Ref. [55], only two frequencies, namely, \( \tilde{\omega}_1 \) and \( \tilde{\omega}_2 - \Omega = \tilde{\omega}_3 + \Omega \) will exist and thus all effects due to rotation vanish and only those purely due to deformation will be left. For the spherical nuclei \((\omega_x = \omega_y = \omega_z)\), which comes under the later category, one gets only one frequency namely \( \tilde{\omega}_1 = \tilde{\omega}_2 - \Omega = \tilde{\omega}_3 + \Omega \).

A nice classical description of rotational splitting is given in ref. [40]. Fig. 3.2 shows the essential features. The GDR is considered to be the superposition of vibrations along each of the three principal axes of the nucleus. All of the angular momentum is attributed to rotation with frequency \( \Omega \) about one of the principal axes, say the \( z \) axis. The vibration along the \( z \) axis is unaffected by rotation, while the vibrations along the \( x \) and \( y \) axes are mixed by Coriolis force. In fig. 3.2, left and middle sections show the situation in the \( xy \) plane, as viewed in the rotating (internal) frame. The top two rows correspond to a prolate potential with the \( z \) axis perpendicular to the symmetry axis, and the bottom row to an oblate potential rotating about the symmetry axis. At angular momentum \( I = 0 \) (top row) the linear vibration along the \( x \) and \( y \) axes of the prolate shape are split in energy, as a result of the static deformation, as shown on the right. Each of these linear vibrations may be represented as the superposition of two counter-rotating circular orbits as shown. At finite \( I \), Coriolis forces mix the \( x \) and \( y \) vibrations and the periodic orbit or ellipses, corresponding to counter-rotating circular orbits of different amplitude. The linear vibrations along the \( x \) and \( y \) axes of the oblate shape at \( I = 0 \) become counter-rotating circular orbits.
Fig. 3.2. Classical periodic orbits in a rotating deformed harmonic oscillator potential [79]. (Top row) Prolate ($\beta = 0.25$) shape at rest. (Middle row) Prolate shape rotating along an axis perpendicular to the symmetry axis. (Left side) The periodic orbits in the rotating frame. (Center) Decomposition of the periodic orbits into circular components. (Right) The photoabsorption cross section in the rotating (internal) and in the laboratory frames, with solid bars denoting absorption associated with vibration along the axis of rotation ($\Delta I = 0$) and dashed and dot-dashed bars associated with the clockwise and counterclockwise circular components corresponding to $\Delta I = -1$ and $+1$, respectively.

for $I \neq 0$. In the rotating frame the oscillation frequencies from the classical result is the same as the eigen frequency solution (3.13) in quantum mechanics for a rotating anisotropic harmonic oscillator with a separable dipole-dipole interaction.

For the oblate case, $\omega_x = \omega_y = \omega_\perp$ and the splitting in the rotating frame is given by $\omega_{x,y} = \omega_\perp \pm \Omega$, whereas for the prolate shape illustrated in Fig. 3.2
(β ≈ 0.25) the splitting is substantially less, as given by Eq. (3.13). A further splitting or shift occurs in transforming from the rotating (internal) frame to the laboratory frame. Each counterclockwise (clockwise) orbit is shifted up (down) by frequency Ω = ω_rot (or, equivalently, by energy ħΩ). The final result for the oblate noncollective rotation in the laboratory frame is the same as for I = 0. For the prolate case, however, the spectrum is split into five pieces, with an uneven strength distribution. The relative magnitude of each of the shifted components is determined by the square of its circular orbit radius; hence the strongest components end up closest to the center of the strength distribution, which tends to minimized the splitting.

3.4. The GDR γ-ray anisotropy parameter

Fusion reactions produce highly aligned systems for which M << J, where J is the initial spin and M is the projection of J along the beam axis (M = 0 for spinless projectiles and targets). Nevertheless the angular distribution in the center of mass for statistical emission of high-energy γ rays is expected to be nearly isotropic as a result of averaging over final-state spin. An anisotropy will occur for a given J and Eγ, only if quadratic or higher order variation in the final-state level density as a function of J are important over the narrow spin range J ± 1.

Apart from this if the system possess a definite deformation, in which case one expects anisotropies that depend on the sense of the deformation (prolate vs oblate) and on the orientation of the deformed shape with respect to the rotational axis. Consider the zero-temperature limit in which the angular mo-
momentum $J$ of a nucleus is due entirely to rotation about a principal axis of the deformed shape. A dipole vibration along the axis of rotation carries no angular momentum with respect to that axis and hence in the limit $J \gg 1$ corresponds to a transition with $\Delta J = 0$. Vibration along the other two principal axes correspond in each case to $\Delta J = \pm 1$. The energy splitting of the GDR due to deformation will displace the $\Delta J = 0$ and the $\Delta J = \pm 1$ transition energies and hence give rise to an anisotropy. Anisotropy parameter in different cases are illustrated in Fig. 3.4.

Neglecting the effect of the Coriolis splitting on the GDR components, one can derive an analytical expression for the GDR $\gamma$-ray angular distribution [80] 

$$a_2(E) = -\frac{1}{2} \left[ \frac{f_x(E) + f_y(E)}{f_x(E) + f_y(E) + f_z(E)} \right] - f_z(E) \frac{3 \cos^2 \theta - 1}{2} - \frac{3}{8} \frac{f_x(E) - f_y(E)}{f_x(E) + f_y(E) + f_z(E)} \sin^2 \theta \cos 2\phi,$$  

(3.21)

where 

$$f_i(E) = \frac{\Gamma_i E^2}{(E^2 - E_i^2)^2 + \Gamma_i^2 E^2},$$  

(3.22)

and $\theta, \phi$ are the angles which specify the orientation of the nucleus with respect to its rotation axis. The anisotropy parameter is related to the cross section of the gamma ray observed at an angle $\theta$ with respect to the polarized spin direction by 

$$\sigma(E, \theta) = \sigma(E) [1 + a_2(E) P_2(\cos \theta)].$$  

(3.23)

The $a_2$ depends on $\beta$, $\gamma$ and the orientation $\Omega$ of the nucleus with respect to its rotation axis. The quantity $a_2$ is in particular sensitive to $\Omega$. This is demonstrated in fig. 3.4 where $a_2$ is shown for various orientations $(\theta, \phi)$ at three differ-
Fig. 3.3. The angular anisotropy parameter $a_2(E)$ for various orientations ($\theta, \phi$) of the nucleus with respect to its rotation axis at three different intrinsic deformations: oblate ($\gamma = -180^\circ$, left column), triaxial ($\gamma = -150^\circ$, middle column) and prolate ($\gamma = -120^\circ$, right column). Three different values of $\theta$ are shown: $0^\circ$ (top row), $45^\circ$ (middle row) and $90^\circ$ (bottom row). For each $\theta$ we show $a_2$ for three values of $\phi$: $0^\circ$ (solid line) $45^\circ$ (dashed line) and $90^\circ$ (dotted line). For each of the above shapes $\theta = 0$ is the equilibrium orientation. Notice the strong dependence of $a_2$ on orientation.

Different intrinsic deformations: oblate ($\gamma = -180^\circ$), triaxial ($\gamma = -150^\circ$) and prolate ($\gamma = -120^\circ$). In all the cases we have used $\beta = 0.4$. 