7. Conclusion

In Chapter 2, an implicit difference approximation scheme (IDAS) for fractional sub-diffusion equation with Neumann boundary conditions is constructed. The unconditional stability and convergence are proved by the energy method. The convergence order in a discrete 2 norm is $O(\tau + h^{1.5})$. Some numerical examples show that the implicit difference approximation scheme (IDAS) has convergence order of $O(\tau + h^2)$. It is an open problem to prove that the difference scheme has convergence order of $O(\tau + h^2)$. This open problem is first proposed by Sun [54]. In Chapter 3 and Chapter 4, a combination of compact finite difference scheme in space and modified trapezoidal rule in time is proposed for the fractional sub-diffusion equation with Dirichlet and Neumann boundary conditions respectively. Numerical solutions of the fractional heat equation with Dirichlet and Neumann boundary conditions are compared with $L1C$ scheme [16] and Box scheme [61] respectively. As can be realized from Tables 4.2-4.7, the numerical scheme has convergence of order $O(\tau^2 + h^4)$. This shows that the method is more accurate. In Chapter 5 and Chapter 6, a second order discretization for the spatial derivative is proposed for one and two dimensional space and time fractional heat equation with variable coefficients respectively. Numerical solutions of the space and time fractional heat equation are compared with $SIL1$ scheme in [6]. As can be realized from the Tables 5.7-5.8, the numerical scheme has convergence of order two in both time and space. This shows that the method is more accurate.