A large number of ultrasonic probes suitable for various applications have been
developed. Generally single probe transducers are being used for inspection and
detection of underwater pipeline defects, but a substantial time saving in the inspection
can be achieved by using suitable arrays. Fabrication of ultrasonic arrays is feasible
using integrated circuit and hybrid construction techniques. The formulation and
computation results of array gain and beam pattern of an annular ring array and
annular cylindrical array suited for pipeline inspection are highlighted in this chapter.
The array gain and beam pattern are computed for a selectively energised section of
an annular ring and annular cylindrical array. The computations are carried out for different configurations and array radius of the annular ring and annular cylindrical arrays. The sidelobe levels and 3dB beamwidths are determined for different configurations of the array.

4.1 INSPECTION SYSTEM

The transducer array designed for the inspection of underwater pipelines comprises transducer elements arranged in an annular ring or annular cylindrical fashion as in figures 4.1 and 4.2.

*Fig.4.1 Configuration and orientation of an annular ring array around the pipeline*

The configuration of the annular cylindrical array can be pictured as staves of elements arranged in such a way that they form a cylindrical structure concentric to the test pipeline as in Figure 4.2. Each stave consists of *n* elements arranged as a linear array. A consecutive set of staves focus the ultrasound at a point on the surface of
The focused ultrasound has to be moved around the contour of the pipeline. For this, the next consecutive set of staves are energised to focus the beam at a point farther on the pipeline and so on until the whole contour of the pipeline is inspected. An $m \times n$ element annular cylindrical array is constituted by $m$ staves of $n$ elements each. For an annular ring array $n=1$. A similar set of elements form the receiving part of the array. Manual rotation of the array is not necessary due to electronic switching and there is a substantial reduction in inspection time compared to single probe system.

The transducer elements are excited by a sine wave of frequency between 1 to 10 MHz, in the continuous wave excitation. In the pulsed mode a gated sine wave is used. The assumptions made in the computation of the annular ring and annular cylindrical array are
The transducer elements are considered to be point source elements with half wavelength spacing between elements. Half wavelength spacing ensures, the elimination of grating lobes and minimum interelement interaction.

Since point source of elements are considered, the amount of energy reradiated and captured by the sources can be considered to be negligible.

Surface of the pipeline is assumed to be smooth so that scattering effect may be neglected.

However when a practical array is realised, the finite size of the elements are to be taken into account and the reradiation effects and interelement interaction are of considerable significance.

4.2 CROSSCORRELATION COEFFICIENTS

The cross correlation coefficients of signal $\rho_s$ and that of noise $\rho_n$ for an annular ring array or annular cylindrical array insonified by a single frequency sinusoidal wave with time delay $\tau_e$ is,

$$\rho_s = \cos \omega (\tau_w + \tau_e)$$  \hspace{1cm} (4.1)

and

$$\rho_n = \frac{\sin(\omega d/c)}{\omega d/c} \cos \omega \tau_e$$  \hspace{1cm} (4.2)
where

\[ w = 2 \pi f, \quad f \text{ being the frequency of the acoustic wave incident on the array, } r_w \text{ is the} \]

transit time of the signal between array elements, \( r_e \) is the electrical time delay, \( c \) is
the acoustic velocity and \( d \) is the spacing between the elements.

### 4.2.1 Annular ring array

The crosscorrelation coefficient of signal between \( i^{th} \) and \( j^{th} \) element \((\rho_s)_{ij}\) for a single frequency zero time delay acoustic signal incident at an angle \( \theta \) on a reference element of the annular ring array is

\[ (\rho_s)_{ij} = \cos(\omega r_{ij}) \] (4.3)

where

\[ r_{ij} = \frac{d_{ij}}{c} \cos \theta \] (4.4)

and

\[ d_{ij} = 2R \sin \left( \frac{|i-j| d'}{2R} \right) \] (4.5)

\( R \) being the radius of the array and \( d \) the interelement spacing. The crosscorrelation coefficient of noise \((\rho_n)_{ij}\) is

\[ (\rho_n)_{ij} = \sin \left( \frac{d_{ij}}{\omega \frac{c}{c}} \right) \] (4.6)

### 4.2.2 Annular cylindrical array

The crosscorrelation coefficient of signal for the annular cylindrical array
between \((ij)_{th}\) element and \((kl)_{th}\) element, where \(i\) and \(k\) represent linear positions and \(j\) and \(l\) represent positions along the annular ring is, 

\[
(\rho_s)_{ijkl} = \cos(\omega \tau_{ijkl}) \tag{4.7}
\]

where

\[
\tau_{ijkl} = \frac{d_{ijkl}}{c} \cos \theta \tag{4.8}
\]

and

\[
d_{ijkl} = \sqrt{2R \sin \left[ \frac{|i-k|}{2R} \right]^2 + (|j-l|d)^2} \tag{4.9}
\]

where \(R\) is the radius of the array and \(d\) the interelement spacing. The crosscorrelation coefficient of noise \((\rho_n)_{ijkl}\) is

\[
(\rho_n)_{ijkl} = \frac{\sin \left( \omega \frac{d_{ijkl}}{c} \right)}{\omega \frac{d_{ijkl}}{c}} \tag{4.10}
\]

### 4.3 ARRAY GAIN

The array gain of the annular ring and annular cylindrical arrays are computed from their respective crosscorrelation coefficients as

\[
AG = 10 \log \left( \sum_i \sum_j \frac{(\rho_s)_{ij}}{\sum_i \sum_j (\rho_n)_{ij}} \right) \tag{4.11}
\]
The array gain computed for different configurations of an annular ring array are shown in Table 4.2. The array gain for the annular cylindrical array is

\[
AG = 10 \log \frac{\sum \sum \sum \sum (\rho_s)_{ijkl}}{\sum \sum \sum \sum (\rho_n)_{ijkl}}
\]  

(4.12)

The array gain computed for different configurations of the annular cylindrical array are shown in Table 4.4.

4.4 BEAM PATTERN

The beam pattern of a transducer array represents graphically its directional response to sound waves incident in a specified plane and at a specified frequency. The beam patterns of annular ring and annular cylindrical arrays are presented below.

4.4.1 Annular ring array

A plane sinusoidal wave is incident on an annular ring array. \( \theta \) and \( \phi \) are the incident angle and azimuth angle with which the wavefront is incident at the first element. The path difference of the wavefront incident on the two adjacent elements A and B is computed from the geometry of the array as

\[
U = R \left[ \sin \left( \theta^* + \frac{d^*}{R} \right) - \sin \theta^* \right] \sin \phi
\]

(4.13)
Where $R$ is the radius of the annular ring, $d$ is the interelement spacing and $d'$ is the arclength between A and B.

Fig. 4.3 Geometrical variables used for the computation

The path difference of the wavefront for the $m^{th}$ element relative to the first element is

$$U_{(m-1)} = R \left[ \sin \left( \theta^* + (m-1) \frac{d'}{R} \right) - \sin \theta^* \right] \sin \phi$$

(4.14)

The beam pattern is computed as
\[ b(\theta) = \left( \frac{V}{M} \right)^2 \]  \tag{4.15} 

Where \( V \) is the total output voltage from \( M \) element section of the receiving array.

Total output voltage is computed as

\[ V = e^{ikU_0} + e^{ikU_1} + \ldots + e^{ikU_{(M-1)}} \]  \tag{4.16} 

\[ V = \sum_{m=0}^{M-1} \cos \left[ kR \left[ \sin(\theta' + \frac{md}{R}) - \sin\theta' \right] \sin\phi \right] \]
\[ + j \sum_{m=0}^{M-1} \sin \left[ kR \left[ \sin(\theta' + \frac{md}{R}) - \sin\theta' \right] \sin\phi \right] \]  \tag{4.17} 

\( k=2\pi/\lambda \) the wave vector, \( \lambda \) being the acoustic wavelength

\[ b(\theta) = \frac{1}{M^2} \left[ \sum_{m=0}^{M-1} \cos \left[ kR \left[ \sin(\theta' + \frac{md}{R}) - \sin\theta' \right] \sin\phi \right] \right]^2 \]
\[ + \frac{1}{M^2} \left[ \sum_{m=0}^{M-1} \sin \left[ kR \left[ \sin(\theta' + \frac{md}{R}) - \sin\theta' \right] \sin\phi \right] \right]^2 \]  \tag{4.18} 

\( b(\theta) \) is plotted against \( \sin(\theta) \) to obtain the beam pattern. Beam pattern for different configurations of an annular ring array with array radius 6cm and element spacing \( \lambda/2 \), operating at 1MHz are shown in Figures 4.4 and 4.5.

The array parameters like radius of the array, number of elements in the selected section of the array etc are varied and the beam pattern corresponding to these variations are plotted. Effect of array radius on the beam pattern is shown in
Fig. 4.4 Beam pattern of an annular ring array for different configurations
Fig. 4.5 Beam pattern of an annular ring array for different configurations
Fig. 4.6 Effect of array radius on the beam pattern
Figure 4.6. The beam characteristics of a 5 element section of an annular ring array operating at 5 MHz for different array radii are shown in Table 4.1. Table 4.2 shows the beam characteristics for different configurations of an annular ring array.

**Table 4.1**

*Beam characteristics of a 5 element annular ring array*

<table>
<thead>
<tr>
<th>Radius (A)</th>
<th>Most Intense sidelobe level (dB)</th>
<th>Sidelobe level at 90° (dB)</th>
<th>3 dB Beamwidth (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>-22.802</td>
<td>-28.107</td>
<td>22</td>
</tr>
<tr>
<td>20</td>
<td>-23.928</td>
<td>-28.623</td>
<td>20</td>
</tr>
<tr>
<td>30</td>
<td>-24.294</td>
<td>-28.719</td>
<td>18</td>
</tr>
<tr>
<td>40</td>
<td>-24.457</td>
<td>-28.753</td>
<td>17</td>
</tr>
<tr>
<td>50</td>
<td>-24.408</td>
<td>-28.768</td>
<td>17</td>
</tr>
<tr>
<td>60</td>
<td>-24.375</td>
<td>-28.777</td>
<td>17</td>
</tr>
<tr>
<td>70</td>
<td>-24.352</td>
<td>-28.782</td>
<td>16</td>
</tr>
<tr>
<td>80</td>
<td>-24.335</td>
<td>-28.785</td>
<td>16</td>
</tr>
<tr>
<td>90</td>
<td>-24.322</td>
<td>-28.788</td>
<td>16</td>
</tr>
<tr>
<td>100</td>
<td>-24.311</td>
<td>-28.789</td>
<td>16</td>
</tr>
</tbody>
</table>

Frequency=5MHz, \( d = \lambda/2 \), wavelength \( \lambda = 0.03\text{cm} \), \( m = 5 \)
Table 4.2

Beam characteristics for different configurations of annular ring array

<table>
<thead>
<tr>
<th>No. of elements</th>
<th>Most Intense sidelobe level (dB)</th>
<th>Corresponding $\theta$ in degrees</th>
<th>Sidelobe level at $90^\circ$</th>
<th>3 dB Beamwidth (degrees)</th>
<th>Array Gain in dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>-22.609</td>
<td>50.40</td>
<td>-48.209</td>
<td>20.0</td>
<td>5.768</td>
</tr>
<tr>
<td>5</td>
<td>-24.071</td>
<td>36.00</td>
<td>-28.789</td>
<td>16.0</td>
<td>6.647</td>
</tr>
<tr>
<td>6</td>
<td>-24.816</td>
<td>30.60</td>
<td>-48.397</td>
<td>14.0</td>
<td>7.347</td>
</tr>
<tr>
<td>7</td>
<td>-25.232</td>
<td>26.10</td>
<td>-35.465</td>
<td>12.0</td>
<td>7.922</td>
</tr>
<tr>
<td>8</td>
<td>-25.490</td>
<td>22.50</td>
<td>-48.618</td>
<td>11.0</td>
<td>8.404</td>
</tr>
<tr>
<td>9</td>
<td>-25.569</td>
<td>20.25</td>
<td>-40.942</td>
<td>10.8</td>
<td>8.816</td>
</tr>
<tr>
<td>11</td>
<td>-25.492</td>
<td>17.10</td>
<td>-45.826</td>
<td>9.0</td>
<td>9.481</td>
</tr>
</tbody>
</table>

Frequency=5MHz, Radius=100$\lambda$, d=$\lambda$/2, $\Lambda=0.03$cm

From Table 4.1, the 3dB beamwidths and sidelobe levels are seen to remain constant when array radius is $\geq 40\lambda$. 

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4.4.2 Annular cylindrical array

The annular cylindrical array can be pictured as staves of elements arranged in a cylindrical manner and concentric to the pipeline. The section of the annular cylindrical array comprising $m$ elements along the curvature and $n$ elements linear is energised. For a linear array constituted by $n$ elements and element spacing $d$, the beam pattern is

$$b'(\theta) = \left[ \frac{\sin \left( \frac{nm}{A} d \sin \theta \sin \phi \right)}{n \sin \left( \frac{m}{A} d \sin \theta \sin \phi \right)} \right]^2$$

(4.19)

The beam pattern of an annular ring array constituted by $m$ elements and element spacing $d$ is

$$b(\theta) = \frac{1}{M^2} \left[ \sum_{m=0}^{M-1} \cos \left( kR \left[ \sin \left( \theta' + \frac{md'}{R} \right) - \sin \theta' \right] \sin \phi \right) \right]^2$$

$$+ \frac{1}{M^2} \left[ \sum_{m=0}^{M-1} \sin \left( kR \left[ \sin \left( \theta' + \frac{md'}{R} \right) - \sin \theta' \right] \sin \phi \right) \right]^2$$

(4.20)

The beam pattern $B(\theta)$ of the portion of the annular cylindrical array, can be expressed to a good approximation as [11],

$$B(\theta) = b(\theta) b'(\theta)$$

(4.21)

Where $b(\theta)$ is the beam pattern of an annular ring array of $m$ elements and $b'(\theta)$
Fig. 4.7 Beam pattern of an annular cylindrical array compared with an annular ring array
is the beam pattern of a linear array of \( n \) elements. Figure 4.7 shows the beam pattern of a 6x10 element annular cylindrical array compared with a 6 element annular ring array. The parameters like, the radius of the array and number of array elements selected are varied and the beam pattern is plotted. The results from these plots are tabulated in Tables 4.3 and 4.4.

Table 4.3

*Beam characteristics of an annular cylindrical array*

<table>
<thead>
<tr>
<th>Radius (( A ))</th>
<th>Most Intense sidelobe level (dB)</th>
<th>Sidelobe level at 90° (dB)</th>
<th>3 dB Beamwidth (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>-15.14</td>
<td>-42.25</td>
<td>14.2</td>
</tr>
<tr>
<td>20</td>
<td>-15.29</td>
<td>-42.19</td>
<td>14.1</td>
</tr>
<tr>
<td>30</td>
<td>-15.34</td>
<td>-42.18</td>
<td>14.1</td>
</tr>
<tr>
<td>40</td>
<td>-15.36</td>
<td>-42.17</td>
<td>14.0</td>
</tr>
<tr>
<td>50</td>
<td>-15.38</td>
<td>-42.17</td>
<td>14.0</td>
</tr>
<tr>
<td>60</td>
<td>-15.39</td>
<td>-42.17</td>
<td>14.0</td>
</tr>
<tr>
<td>100</td>
<td>-15.40</td>
<td>-42.17</td>
<td>14.0</td>
</tr>
</tbody>
</table>

\( m=3, \ n=10, \ Frequency=1MHz, \ d=\lambda/2 \)
Table 4.4

Beam characteristics for different configurations of annular cylindrical array

<table>
<thead>
<tr>
<th>No. of elements m</th>
<th>Most Intense sidelobe level (dB)</th>
<th>Sidelobe level at 90°  (dB)</th>
<th>3 dB Beamwidth (degrees)</th>
<th>Array Gain in dB ( \Theta=n/6, \phi=n/2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-13.90</td>
<td>-26.17</td>
<td>14.2</td>
<td>11.69</td>
</tr>
<tr>
<td>3</td>
<td>-15.40</td>
<td>-42.17</td>
<td>14.0</td>
<td>12.92</td>
</tr>
<tr>
<td>4</td>
<td>-17.61</td>
<td>-30.53</td>
<td>13.6</td>
<td>13.69</td>
</tr>
<tr>
<td>5</td>
<td>-20.50</td>
<td>-35.67</td>
<td>13.2</td>
<td>14.23</td>
</tr>
</tbody>
</table>

Frequency=1MHz, \( d=\lambda/2 \), Radius \( R=100\lambda \), Number of linear elements \( n=10 \)