11.1 **GENERAL REMARKS**

Soft computing's main characteristic is its intrinsic capability to create hybrid systems that are based on the integration of constituent technologies. This integration provides complementary reasoning and searching methods that allow combining domain knowledge and empirical data to develop flexible computing tools and solve complex problems. In this chapter, programs are developed based on GA-Fuzzy, Neuro-Fuzzy, GA-ANN and Fuzzy-GA-ANN combinations and their applications are demonstrated by solving a variety of problems. GA-Fuzzy hybridization is carried out for configuration optimization of plane truss, cost optimization of combined footing and topology optimization of plate. Next GA is employed for optimization of connections weights and topology of back propagation neural network while solving the problem of circular concrete column. Neuro-Fuzzy approach is then used for concrete mix design and finally the power of combination of all the three i.e. Fuzzy-GA-ANN is demonstrated by optimization of constituents of concrete mix for compressive strength.

11.2 **COMPUTER IMPLEMENTATION OF GA-FUZZY HYBRID APPROACH**

11.2.1 **GA-Fuzzy Formulation of Optimization Problem**

The structural optimization problems involves fuzziness and imprecision in the constraint evaluation. In the pure GA constraints are satisfied within a limit specified by a crisp or non-fuzzy number. In real sense this constraint evaluation involves many causes of approximation. Hence a design can be considered satisfactory when the constraints are satisfied within a given predetermined tolerance. When an optimization algorithm is forced to satisfy the constraints exactly it can miss the true global optimum solution which may probably exist outside the feasible region but very close to the boundary of constraint. By taking into account the fuzziness and imprecision in the constraints evaluation and employing the fuzzy set theory the objective function can further be reduced and the probability of finding actual global optimum solution can be substantially increased.

Conventional single objective multivariable optimization problem, like one which has been addressed in this section, can be stated mathematically as:
Find \( x \) which minimizes \( F(x) \), subject to:

\[
G_j(X) \leq b_j, \ j = 1 \text{ to } m,
\]

where \( x \) is the vector of design variables, \( F(x) \) is the objective function, \( G_j \) are the constraint functions, \( m \) is number of constraints and \( b_j \) represent the upper bounds for the constraints. With fuzziness considered in the constraints the design variable \( x \) corresponding to optimum solution can be obtained from a fuzzy domain \( D \) as described subsequently. The fuzzy feasible region is defined as:

\[
S = \bigcap_{j=1}^{m} G_j
\]

The degree of membership of any design vector \( x \) in the fuzzy feasible region \( S \) is given as the intersection of membership functions of various constraints which can be obtained as,

\[
\mu_S(x) = \min_{j=1}^{m} \left\{ \mu_{G_j}(g_j(x)) \right\}.
\]

The optimum solution lies in a fuzzy domain \( D \) in \( S \), which is given by

\[
D = \{ \mu_T(x) \} \cap \left\{ \bigcap_{j=1}^{m} \mu_{G_j}(g_j(x)) \right\},
\]

and its membership function is represented as,

\[
\mu_D(x) = \min \left\{ \mu_T(x), \min_{j=1}^{m} \left\{ \mu_{G_j}(g_j(x)) \right\} \right\}.
\]

Then the optimum solution \( x^* \) corresponds to a design point for which the membership function is maximum and is obtained as,

\[
\mu_D(x^*) = \max \{ \mu_D(x), x \in D \}.
\]

This max-min procedure can be solved by maximizing a scalar parameter \( \lambda \), known as the overall satisfaction parameter, to find the vector of design variable \( x^* \).

### 11.2.2 The Flowchart for GA-Fuzzy Hybridization

The hybrid approach employed here divides the fuzzy optimization problem in three non-fuzzy optimization problems. Each of these problems is then solved by pure GA. In the initial step the optimum solution is obtained by GA where constraints are considered crisp and are handled by penalty function method. In the next step constraints are relaxed by allowing...
small violation $d_j$ of constraints during the search process and the optimum solution is obtained using GA. For first two GA runs, objective function is used to find fitness value. The third step involves the calculation of membership functions for objective function and constraints. Figure 11.1 indicates the membership functions for constraints and objective function. Linear membership function has been used due to simplicity and found to give good results. An additional variable $\lambda$ is entered in the existing design variables in third GA run. It represents the membership grade and takes any value between 0 and 1. Membership grades for objective function and design constraints are used to find penalty parameters in the last GA run. In this step, $\lambda$ is directly taken as fitness function to be maximized. Optimum value of $\lambda$ and corresponding design variables give the final optimum solution. The hybrid approach used in the program is produced in the flow chart form in Fig. 11.2.

Initial population for first and second GA runs is generated randomly. The information gathered during first two runs is used in the final run. The first one third of the initial population is taken from the last generation of the first GA run and second one third is adopted from the second GA run. The remaining string length, which corresponds to the additional variable $\lambda$ is made up by generating random numbers. The last one third of the population is generated entirely using random number generation. This technique leads to faster convergence of GA.
11.3 TRUSS CONFIGURATION OPTIMIZATION THROUGH GA-FUZZY APPROACH

11.3.1 The Problem

Configuration optimization of truss structures involves simultaneously arriving at optimum values for the nodal coordinates $R$ and member cross-sectional areas $A$ that minimize the structural weight. The general form of truss configuration optimization can be described as:

Minimize, structural weight $W(x) = \rho \sum_{i=1}^{n} A_i L_i$ \hspace{1cm} \ldots (11.7)

Subject to, $g_j^l \leq g_j \leq g_j^u$, $j = 1, 2, \ldots, q$ \hspace{1cm} \ldots (11.8)

where $\rho$ is the material density, $L_i$ and $A_i$ are length and cross-sectional area of $i^{th}$ member and $g_j^l$ and $g_j^u$ are the lower and upper bounds on the inequality constrained function $g_j$. The upper and lower bounds in the constraint functions of Eq. (11.8) include the following:
11. Software Development and Application of Hybrid Tools

(i) nodal co-ordinates \((R_i^1 \leq R_i \leq R_i^a, i = 1, ..., m)\),
(ii) member cross-sectional areas \((A_i^1 \leq A_i \leq A_i^a, i = 1, ..., n)\),
(iii) member stresses \((\sigma_i^l \leq \sigma_i \leq \sigma_i^u, i = 1, ..., n)\),
(iv) nodal displacements \((\delta_i^l \leq \delta_i \leq \delta_i^u, i = 1, ..., m)\) and
(v) member buckling stresses \((\sigma_i^b \leq \sigma_i \leq 0, i = 1, ..., n)\). ...(11.9)

In this optimization problem, all the crisp constraints are converted in the fuzzy constraints and fuzzy optimization problem thus formulated is solved by converting it in to three non-fuzzy problems using traditional GA. Various aspects of GA based mathematical model for configuration optimization problem described in Eqs. (11.7) and (11.8), are outlined as under.

11.3.2 Design Variables
As mentioned earlier, \(x\) and \(y\) co-ordinates of movable nodes and member cross-sectional areas are the design variables in simultaneous size and configuration optimization. Number of design variables thus depends on number of joints and number of members.

11.3.3 Objective Function
Weight of the truss under consideration is taken as the objective function to be minimized. The objective function is calculated by adding weights of the truss members only as formulated in Eq. (11.7) without considering weight of fasteners and gusset plates.

11.3.4 Constraints and penalty functions
Three main constraints required to be imposed as per design codes are stress constraints, displacement constraints and buckling constraints. The constraint functions are evaluated based on degree of constraint violation as discussed below.

- **Stress constraint:** If \(\sigma_j\) and \(\sigma_{ja}\) are calculated and permissible stresses for \(j^{th}\) member the stress constraint is,
  \[\sigma_j \leq \sigma_{ja}\] and corresponding constraint function is,
  \[g(x) = \max (\sigma_j / \sigma_{ja} - 1, 0).\] ... (11.10)
  ... (11.11)

- **Displacement constraint:** For \(u_i\) and \(u_{ia}\) as calculated and permissible displacement of \(i^{th}\) joint respectively, the displacement constraint can be written as,
The constraint function is:

\[ u_i \leq u_{ia}, \text{ and the constraint function is,} \quad g(x) = \max \left( \frac{u_i}{u_{ia}} - 1, 0 \right). \]  \hspace{1cm} \text{(11.12)}

\[ g(x) = \max \left( \frac{u_i}{u_{ia}} - 1, 0 \right). \]  \hspace{1cm} \text{(11.13)}

- **Buckling constraint:** In this constraint the calculated stress in the \( j \)th truss member is not allowed to exceed the permissible buckling stress (\( \sigma_{jb} \)) for that member. The constraint is given by,

\[ \sigma_j \leq \sigma_{jb}, \text{ and corresponding constraint function is,} \quad g(x) = \max \left( \frac{\sigma_j}{\sigma_{jb}} - 1, 0 \right). \]  \hspace{1cm} \text{(11.14)}

\[ g(x) = \max \left( \frac{\sigma_j}{\sigma_{jb}} - 1, 0 \right). \]  \hspace{1cm} \text{(11.15)}

If constraint is violated, the constraint function takes non-zero value otherwise it takes value zero. If \( C \) is the summation of all such constraint functions for a candidate solution violating constraints, it is penalized using the penalty function given as:

\[ P(x) = (1 + K.C), \]  \hspace{1cm} \text{(11.16)}

where \( K \) is penalty parameter which is selected judiciously. In the present study \( K \) is kept low in the initial generations and is gradually increased to large values in the subsequent generations using the equation: \( K = K_{\text{initial}} \left\{ 1 + 0.2 \left( n_g - 1 \right) \right\} \), where \( n_g \) is the generation number. A value of 10 has been found suitable for \( K_{\text{initial}} \).

The penalty function value obtained in Eq. 11.16 is multiplied with objective function of Eq. 11.7 to get penalized objective function \( O_p(x) \).

**11.3.5 Fitness function**

To avoid negative fitness values following fitness function has been used in the present work:

\[ f(x) = \frac{1}{1 + O_p(x)} \]  \hspace{1cm} \text{(11.17)}

**11.3.6 GA operators**

Roulette wheel and tournament selection schemes are tried. In addition, elitism operator is also employed in the search process to ensure that optimum solution obtained in any generation is better than that obtained previously. Single point crossover is operated on every substring of the solution string.
11.3.7 Example of a 18-Bar Cantilever Plane Truss

The 18-bar cantilever plane truss, shown in Fig. 11.3, is one of the most popular truss design optimization problems. Due to its simple configuration, this structure has been used as a benchmark to verify the efficiency of various optimization methods [20, 22, 58, 59] with the following data: the material density = 27.2 kN/m³ (0.1 lb/in³), modulus of elasticity = 68947.6 MPa (10,000 ksi) and the cross-sectional areas of the members categorized into four groups: (i) \( A_{G1} = A_3 = A_7 = A_{11} = A_{15} = A_{18} \), (ii) \( A_{G2} = A_4 = A_5 = A_9 = A_{13} = A_{17} \), (iii) \( A_{G3} = A_4 = A_8 = A_{12} = A_{16} \) and (iv) \( A_{G4} = A_2 = A_6 = A_{10} = A_{14} \) with a set of vertical loads, \( P = 88.96 \) kN (20 kips), acting on the upper nodal points of the truss, as depicted in Fig. 11.3. The lower nodes 3, 5, 7 and 9 are allowed to move in any direction in the x-y plane. Thus, there are a total of 12 independent design variables that included four sizing and eight coordinate variables. Population size and maximum number of generations used for the search in the present study are 30 and 50 respectively. Cross-over and mutation probabilities considered are 0.91 and 0.05 respectively. Optimum solution is obtained after ten runs with different seed value for random number generation.

The purpose of the study is to design such a configuration for the truss that produces a minimum design weight with all the constraints satisfied to a desired level. The allowable tensile and compressive stresses are \( \pm 137.89 \) Mpa (20 ksi). The Euler buckling compressive stress limit used for the buckling constraint is computed as \(-4AE/L^2\).

The optimal configuration is displayed in Fig. 11.4. Table 11.1 lists the best solution vector and weight from hybrid GA-Fuzzy approach and also the results obtained using other optimization methods.
The optimum solution obtained in the first GA run where, crisp constraints are imposed, gave an optimum structure weight of 20.25 kN (4553.35 lb). In the second GA run, the constraints are relaxed by 15% and the optimum weight obtained is 19.09 kN (4291 lb). In the last GA run optimum value of $\lambda$ and corresponding weight of the structure obtained are 0.75 and 20.01 kN (4498.6 lb) respectively.

### Table 11.1 Optimal Results of 18-Bar Plane Truss (Comparative Study)

<table>
<thead>
<tr>
<th>Reference No.</th>
<th>[58]</th>
<th>[22]</th>
<th>[21]</th>
<th>[100]</th>
<th>Present work</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{G1}$</td>
<td>12.59</td>
<td>12.33</td>
<td>12.50</td>
<td>12.65</td>
<td>12.57</td>
</tr>
<tr>
<td>$A_{G2}$</td>
<td>17.91</td>
<td>17.97</td>
<td>16.25</td>
<td>17.22</td>
<td>17.92</td>
</tr>
<tr>
<td>$A_{G3}$</td>
<td>5.5</td>
<td>5.6</td>
<td>8.00</td>
<td>6.17</td>
<td>5.52</td>
</tr>
<tr>
<td>$A_{G4}$</td>
<td>3.55</td>
<td>3.66</td>
<td>4.00</td>
<td>3.55</td>
<td>3.25</td>
</tr>
<tr>
<td>$X_3$</td>
<td>200.9</td>
<td>202.1</td>
<td>184.4</td>
<td>195.3</td>
<td>202.7</td>
</tr>
<tr>
<td>$Y_3$</td>
<td>32.0</td>
<td>30.9</td>
<td>23.4</td>
<td>30.6</td>
<td>35.4</td>
</tr>
<tr>
<td>$X_5$</td>
<td>410.0</td>
<td>413.9</td>
<td>385.4</td>
<td>402.1</td>
<td>414.5</td>
</tr>
<tr>
<td>$Y_5$</td>
<td>97.0</td>
<td>102.0</td>
<td>72.5</td>
<td>90.5</td>
<td>102.3</td>
</tr>
<tr>
<td>$X_7$</td>
<td>640.0</td>
<td>643.3</td>
<td>610.6</td>
<td>630.3</td>
<td>640.8</td>
</tr>
<tr>
<td>$Y_7$</td>
<td>147.8</td>
<td>149.2</td>
<td>118.2</td>
<td>136.3</td>
<td>145.2</td>
</tr>
<tr>
<td>$X_9$</td>
<td>909.8</td>
<td>907.2</td>
<td>891.9</td>
<td>903.1</td>
<td>913.2</td>
</tr>
<tr>
<td>$Y_9$</td>
<td>184.5</td>
<td>184.2</td>
<td>145.3</td>
<td>174.3</td>
<td>184.6</td>
</tr>
</tbody>
</table>

### 11.3.8 Example of a 21-Bar Simply Supported Truss

Figure 11.5 shows a 21-bar simply supported truss which is subjected to concentrated load of 3000 kN acting at nodes 2, 4, 6, 8, 10. In this problem material density and modulus of elasticity are taken as 77 kN/m$^3$ and 2.1E+08 KPa respectively. Allowable compressive and tensile stresses are 104 MPa and 130 MPa respectively. Lower and upper bounds for cross sectional area are 0.01 m$^2$ and 0.1 m$^2$ respectively. Maximum displacement of any node permitted is 0.03 m. In this example also the member areas are grouped in to four groups: (i) $A_{G1} = A_2 = A_6 = A_{10} = A_{14} = A_{18} = A_{21}$, (ii) $A_{G2} = A_4 = A_8 = A_{12} = A_{16} = A_{20}$, (iii) $A_{G3} =$
A_5 = A_9 = A_{13} = A_{17} and (iv) A_{G4} = A_3 = A_7 = A_{11} = A_{15} = A_{19}. The upper nodes 3, 5, 7, 9 and 11 are allowed to move in y direction only. Thus, there are a total of 9 independent design variables that included four sizing and five co-ordinate variables. Population size and maximum number of generations used for the search are 30 and 40 respectively with crossover and mutation probabilities as 0.91 and 0.05 respectively. Optimum solution is obtained after five trials with different seed values for random number generation.

**Fig. 11.5 Example of a 21-Bar Plane Truss - Initial Configuration**

Table 11.2 lists the best solution vector form hybrid GA-Fuzzy approach and also the results obtained using pure GA [20]. The optimal configuration obtained is depicted in Fig. 11.6.

<table>
<thead>
<tr>
<th>Reference [20]</th>
<th>Optimal sectional areas A (m^2)</th>
<th>Optimal co-ordinates R (m)</th>
<th>W (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A_{G1} A_{G2} A_{G3} A_{G4} Y_3 Y_5 Y_7 Y_9 Y_{11}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Present work</td>
<td>0.082 0.022 0.046 0.028 4.31 5.73 5.87 5.73 4.31</td>
<td>382.73</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.096 0.01 0.01 0.03 3.68 5.44 5.86 5.44 3.68</td>
<td>344.08</td>
<td></td>
</tr>
</tbody>
</table>

**Fig. 11.6 21-Bar Plane Truss-Optimal Configuration**

The optimum solution obtained in the first GA run, gave an optimum structure weight of 384.89 kN. In the second GA run, the constraints are relaxed by 10 % and the optimum
weight obtained is 321.38 kN. In the last GA run, optimum value of $\lambda$ and corresponding weight of the structure obtained are 0.58 and 344.08 kN respectively.

11.4 GA-FUZZY HYBRIDIZATION FOR COMBINED FOOTING

A hybrid GA - Fuzzy approach for cost optimization of combined footing subject to various constraints involving imprecision and fuzziness is addressed in this section. Objective of this work is to improve the convergence and efficiency of GAs through the use of fuzzy set theory. The fuzzy optimization problem is tackled by dividing it in three non-fuzzy sub problems and solved individually by using GA. Fuzzy optimization problem formulation, objective function, and various constraints involved in the design of combined footing are discussed here and numerical results obtained through the program are presented at the end to confirm the efficiency and usefulness of the method.

11.4.1 Design Variables

As the cost of foundation depends on volume of concrete and weight of reinforcing bars, seven design variables considered are length, width and depth of foundation block, longitudinal reinforcement areas for negative moments under two columns and positive moment between two columns and reinforcement area parallel to width for transverse bending as depicted in Fig 11.7. Genetic search space is defined by selecting the upper and lower bound values of these variables. The developed algorithm selects these values by performing preliminary design based on data supplied by the user.

11.4.2 Constraints and Penalty Functions

All the structural engineering optimization problems are constrained optimization problems. Following constraints that govern the optimization process are imposed on a solution as per IS: 456 specifications for limit state method of design [83].

\[
\begin{align*}
(i) & \quad P_s \leq SBC & (ii) & \quad P_s \geq 0 & (iii) & \quad p_r \leq p_t \\
(iv) & \quad \tau_{avg,ss} \leq \tau_{c,ss} & (v) & \quad \tau_{avg,ds} \leq \tau_{c,ds}
\end{align*}
\]

where, $P_s$ is the soil pressure under the footing, SBC is the safe bearing capacity of soil below foundation, $p_t$ is percentage reinforcement in the solution string, $p_r$ is the reinforcement percentage required for given bending moment and dimensions of footing in the solution string selected by GA, $\tau_{avg,ss}$ and $\tau_{c,ss}$ are average and permissible shear stresses for one way shear and $\tau_{avg,ds}$ and $\tau_{c,ds}$ are for two way shear.
The penalty function modifies the objective function depending on the degree of constraint violation. The penalized objective function is then used to find the fitness function as given in Eq. (11.17).

**11.4.3 Objective Function**

Objective function, \( O(x) \) is the function of design variables which is to be minimized or maximized. Cost of the combined footing is taken as objective function and is expressed as:

\[
O(x) = V_c U_c + W_s U_s + A_f U_f 
\]  

where \( V_c \) is volume of concrete in m\(^3\), \( U_c \) is unit cost of concrete per m\(^3\), \( W_s \) is weight of steel reinforcement in kg, \( U_s \) is unit cost of steel per kg, \( A_f \) is area of formwork in m\(^2\) and \( U_f \) is unit cost of formwork per m\(^2\).

**11.4.4 Numerical Example**

Data: C/C distance between columns = 3.0 m, Size of column C1 = 0.3 × 0.3 m, Size of column C2 = 0.3 × 0.3 m, Grade of concrete = M20, Grade of steel = Fe415, DL + LL on column C1 = 480 + 170 kN, DL + LL on column C2 = 610 + 190 kN and S.B.C. of soil = 175 kN/m\(^2\).

Initial data required to carry out preliminary design of foundation is supplied through various forms developed in the software. Every solution string, which is composed of seven design variables is checked for the following three loading conditions as per IS 456 [83]:

(1) \( C_1 - DL + LL \) and \( C_2 - DL + LL \)
(2) \( C_1 - DL \) and \( C_2 - DL + LL \)
(3) \( C_1 - DL + LL \) and \( C_2 - DL \)

Maximum values of shear force and bending moments induced due to three loading conditions are calculated and the foundation is designed for the maximum values among the three cases.

Population size and maximum number of generation of 30 and 20 have been selected respectively for GA search. The probabilities of crossover and mutation adopted are 0.9 and 0.05 respectively. In the first GA run, hard constraints are imposed by using penalty function. Constraints are relaxed by 15% to improve the search near constraint boundary in the second GA run. The results obtained from the software are compared (Table 11.3).

Table 11.3 Comparison of Results

<table>
<thead>
<tr>
<th>Item</th>
<th>GA</th>
<th>FL</th>
<th>Hybrid Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of footing</td>
<td>5.59 m</td>
<td>5.1 m</td>
<td>5.4 m</td>
</tr>
<tr>
<td>Width of footing</td>
<td>1.83 m</td>
<td>2.0 m</td>
<td>1.85 m</td>
</tr>
<tr>
<td>Depth of footing</td>
<td>0.58 m</td>
<td>0.62 m</td>
<td>0.59 m</td>
</tr>
<tr>
<td>Total quantity of concrete</td>
<td>5.93 m(^3)</td>
<td>6.32 m(^3)</td>
<td>5.89 m(^3)</td>
</tr>
<tr>
<td>Total quantity of steel</td>
<td>373 kg</td>
<td>421.00 kg</td>
<td>328.00 kg</td>
</tr>
<tr>
<td>Total cost of concrete</td>
<td>Rs. 11860/-</td>
<td>Rs. 12640/-</td>
<td>Rs. 11780/-</td>
</tr>
<tr>
<td>Total cost of steel</td>
<td>Rs. 13055/-</td>
<td>Rs. 14735/-</td>
<td>Rs. 11480/-</td>
</tr>
<tr>
<td>Total Cost</td>
<td>Rs. 24915/-</td>
<td>Rs. 27375/-</td>
<td>Rs. 23260/-</td>
</tr>
</tbody>
</table>

11.5 GA-FUZZY HYBRIDIZATION FOR TOPOLOGY OPTIMIZATION OF PLATE

The problem of topology optimization of continuum plate solved earlier using pure GA is attempted here by employing GA-Fuzzy approach. The problem data is same as taken in section 8.22. The objective function, constraints handling, penalty function and fitness function used for the GA based search is also same except that in the second GA run of the hybrid approach the stress constraint is relaxed by 10%. The hybrid approach used is already discussed in detail in the preceding section. The optimum solution obtained by using pure GA for 10 x 10 is reproduced here in Fig. 11.8 whereas the optimum topology obtained through the hybrid approach is shown in Fig. 11.9. It is clear from these figures that the hybrid approach leads to more economical solution compared to pure GA.
11.6 COMPUTER IMPLEMENTATION OF GA-ANN APPROACH

Basically there are two ways of combining GA and ANN: (i) GA is employed for improvement in the learning process of ANN by optimizing various ANN parameters wherein GA is run first to get right hill and neural network may be used to climb the hill to reach its peak and (ii) Trained ANN is used to replace some of the time consuming and computationally intensive process of GA such as fitness evaluation in multivariable and/or multiobjective optimization problems such as topology optimization of continuum structures. In this work GA is employed for getting optimum parameters of ANN by developing a software named as GANN simulator.

The information (may be weight matrix, thresholds and/or network topology) about the neural network is first decoded in the genome of the GA. At the beginning, a random number of individuals are generated in the form of binary strings. These binary strings are then converted to decimal values and used along with ANN for their training. Depending on the error in the outputs, the individual’s fitness is calculated. Selection of strings and crossover and mutation takes place depending on the fitness to form new individuals. Some GANN strategies rely only on GA to find optimal network and thereby outputs of the network. In that case no further training of the network takes place. But usually GA optimized ANN is further
trained for minimization of the network error to take place. The general procedure of optimizing ANN parameters with GA is shown in Fig. 11.10.

11.7 GA-ANN Approach for Problem of Circular Concrete Column

The example of circular concrete column which was solved earlier by pure ANN in Chapter 10 (§10.3) is undertaken here and solved by GANN simulator. The use of GANN simulator in weight optimization and topology optimization of neural network is demonstrated and their
results are compared with pure BPNN. The selection of input-output variables is already outlined in Chapter 10 along with training patterns selected for training.

Training of pure BPNN with learning parameter as 0.01, error tolerance as 0.001, noise factor as 0.01, momentum factor as 0.1 and topology as 7 - 4 - 2 for 29 training patterns gives RMSE of 0.0024 in 42 minutes on Pentium IV computer system, after 500,000 cycles.

11.7.1 Weight Optimization
First GA is used to optimize the weights with population size of 10, number of generations 100, crossover probability of 0.90 and mutation rate of 0.05. It takes 7 seconds to optimize the weights. Then the network is trained using BPNN. It gives RMSE of 0.00236 with 300,000 training cycles taking training time of 29 min 16 secs. Table 11.4 gives the testing results obtained using the weights optimized with GA.

### Table 11.4 Testing Results Obtained using Optimized Weights

<table>
<thead>
<tr>
<th>INPUTS</th>
<th>OUTPUTS</th>
<th>Type of Output</th>
<th>F'ce</th>
<th>Tce</th>
</tr>
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<td>Tc</td>
<td>d</td>
<td>H</td>
<td>D</td>
<td>P</td>
</tr>
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<td>s</td>
<td>p</td>
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<td>D</td>
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<td>1500</td>
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<td>28</td>
<td>438</td>
<td>1500</td>
<td>340</td>
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<td>1500</td>
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</tr>
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<td>280</td>
<td>900</td>
<td>363</td>
<td>0.85</td>
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</table>

| 341 |
11.7.2 Topology Optimization

GA is used to optimize the topology with population size of 10, number of generations 100, crossover probability 0.90 and mutation rate of 0.05 and keeping number of hidden layer as one with 2000 ANN training cycles. It takes 1 min 31 sec to optimize the topology and after optimization it gives optimum topology of 7 - 5 - 2. With further training of ANN with this optimized topology, it gives RMSE of 2.48E-03 after 500,000 training cycles and training time of 1 hr 5 min 57 secs. Table 11.5 gives the results obtained after testing the network using the optimized topology with GA. Fig. 11.11 depicts the maximum fitness plot.

Table 11.5 Testing Results Obtained using Optimized Topology

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Outputs</th>
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<tbody>
<tr>
<td>$f_c$</td>
<td>$d$</td>
</tr>
<tr>
<td>28</td>
<td>438</td>
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<td>438</td>
</tr>
<tr>
<td>31</td>
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<tr>
<td>29.8</td>
<td>185</td>
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<td>29.8</td>
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<td>19.45</td>
<td>280</td>
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<td>19.45</td>
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</tbody>
</table>
11. Software Development and Application of Hybrid Tools

11.8 **NEURO-FUZZY APPROACH FOR CONCRETE MIX DESIGN**

11.8.1 **Sources of Randomness and Fuzziness**

Concrete is produced by mixing several discrete materials and so the number of variables governing the choice of mix proportions is necessarily large. The factors incorporating the imprecision or vagueness in concrete mix design procedure are conceptually explained below:

- **Calculation of Target strength** involves fuzziness as it is based on standard deviation which is assumed depending on quality control which is expressed in fuzzy terms such as GOOD, VERY GOOD etc.

- **Water/Cement ratio** is determined from the curves or tables which are developed on the basis of experimental works. As type of material used, method of testing etc. can vary from place to place, the experimental results used to produce curves and tables are not 100% trustable in all situations. Moreover, modified w/c ratio curves are not available for PPC therefore some imprecision is introduced when using PPC.

- **Workability** of a concrete mix is mainly determined to suit the type of construction, placing condition and the means of compaction available at site. In addition to the w/c ratio, other parameters influencing workability are the maximum size of aggregates, grading, texture and shape of aggregates which are expressed in more or less vague terms. The methods of measuring workability also involve many sources of errors causing imprecision.

- **Durability**: To take care of durability property of concrete IS code suggests minimum cement content corresponding to different exposure condition that is again expressed in fuzzy terms.

Fig. 11.11 Maximum Fitness Plot
Shape, size and grading of aggregates: For concrete mix design, the quantity of water to be added depends on shape of coarse aggregate which is expressed in fuzzy terms (i.e. angular or rounded). Grading means particle size distribution of the aggregates. The method for deciding grading also invites the randomness in mix design.

Entrapped air is the voids present in the concrete due to insufficient compaction. These voids may be of any shape and size normally embracing the contour of aggregate surfaces and they are randomly distributed throughout the concrete mass. For mix design, pre-known value of entrapped air is considered and quantity of ingredients is calculated accordingly. As the value is based on assumptions it brings in the fuzziness.

Moisture content in aggregates: In mix design calculation, the relative weight of the aggregates is based on the condition that the aggregates are saturated and surface dry. As one can not predict the site condition exactly and performs mix design merely based on certain assumptions, it leads to variation in design parameters.

The concrete mix design codes [83, 101] suggest the use of different tables and charts for estimation of certain variables involved in the mix design. Neural network is employed here to supply these table and charts to the rule based system, which will be helpful in design of forgoing rule base for the different fuzzy modules.

In the present work, mix design procedure considering compressive strength as a design parameter is modeled using five Fuzzy Rule Based modules and one Back-Propagation Neural Network (BPN) [9] module as shown in Fig. 11.12. The target strength is calculated in the fuzzy module 1, based on characteristic strength and a quality control as an input variable. Then from trained BPN network, w/c ratio is calculated and compared with maximum allowable w/c ratio corresponding to given exposure condition in the fuzzy module 2. In the fuzzy module 3, maximum quantity of water is decided and using known w/c ratio quantity of cement is calculated. The obtained quantity of cement is checked with durability criteria in fuzzy module 4. Finally, quantities of coarse and fine aggregates are calculated in fuzzy module 5.

11.8.2 Structure of Fuzzy Module
Fundamental concept for designing fuzzy module is shown in Fig. 11.13. It consists of fuzzifier, fuzzy inference engine and defuzzifier, each of which is discussed below briefly.
**Fuzzifier:** Making a fuzzy model involves analyzing a problem and setting up the fuzzy sets to define it, a process called fuzzification [72]. In which choice for shape of membership function, number of membership functions, range for universe of discourse, scaling parameters etc. are provided.

![Diagram of Neuro-Fuzzy Model for Concrete Mix Design](image-url)

**Fig. 11.12** Neuro-Fuzzy Model for Concrete Mix Design
In concrete mix design process, various parameters such as exposure condition, quality control, workability, size and shape of aggregates, target strength, characteristic strength and w/c ratio are some of the governing variables. From these variables, some are defined by crisp values and others are defined by linguistic terms which are fuzzified. In mix design procedure, exposure conditions such as mild, moderate, severe, very severe etc. can be easily described with the help of fuzzy set instead of crisp set. To design fuzzifier in present work, Triangular and S-shaped Membership Functions (MF) are used. In prepared software provision is made to select suitable membership function for the different variables in the design domain as per the choice of user.

**Fuzzy Inference Engine:** Fuzzy rule based system possesses great power in representing linguistic and structured knowledge by fuzzy sets and performing fuzzy reasoning by fuzzy logic in a qualitative manner. Fuzzy system is prepared by using available knowledge about the problem and techniques available to handle it. The basic principle of the fuzzy rule based system is that the relationships between input and output parameters are represented by simple If - Then rules. For example, “*If exposure condition is very severe Then minimum cement content is approximately 360 kg*”.

**Defuzzifier:** The output obtained from the rule base is not utilized directly to control the functioning of system. The fuzzy output is required to be defuzzified in to a crisp value. Defuzzifier converts fuzzy output into the crisp value. For that different subroutines are created considering various defuzzification methods and then based on the selected problem, defuzzification method is decided by trial and error procedure. In the present work,
defuzzification modules are based on Centroid method, Weighted average method and Mean-max method described earlier in Chapter 5.

11.8.3 Development of Neuro-Fuzzy Model

11.8.3.1 Fuzzy module for calculating target strength

The first step of concrete mix design is to calculate the target strength. Inputs required for this are cube strength and degree of quality control and output is target mean strength as shown in the Fig. 11.14. The aim of the quality control is to limit the variability as much as practicable. Fuzzy approach provides a scientific vision to the concrete designer to understand different parameters related to quality control.

As illustrated in the Fig. 11.14, core part of the model is the fuzzy screen. The construction of fuzzy screen involve the generation of fuzzy set and fuzzy membership functions, construction of fuzzy inference engine and finally the preparation of defuzzification module. The step-by-step methodology for construction of fuzzy screen to calculate the target strength is explained below.

> **Generation of fuzzy sets and fuzzy MFs:** As characteristic strength is the crisp input, only quality control and target strength are tackled by fuzzy logic concept. Quality control is given in linguistic term such as fair, good and very good. Quality control and target
strength are fuzzified through appropriate fuzzy sets. Membership functions used for quality control and target compressive strength are shown in Fig. 11.15 and 11.16 respectively.

**Antecedent fuzzy set is:**

Qua_Control = ["Very Good", "Good", "Fair"]

Equation of target strength is used for the transformation of input space to output space. Since all the fuzzy inputs are of identical membership functions, only two alpha-cuts at 0 and 1 are considered.

![Fig. 11.15 Membership Function for Quality Control](image1)

![Fig. 11.16 Membership Function for Target Compressive Strength](image2)

**Consequent fuzzy set is:**

Tar_Strength = [T25, T26, T27, T28, T29, T30,......, T75]

where T25 means target compressive strength is around 25 Mpa.
11. Software Development and Application of Hybrid Tools

- **Construction of fuzzy rule base:** Total 27 rules are developed in this module to capture all the possibilities of target strength for various types of quality control. The structure of the rules is of the form,

  1. If Char_Str is M20 and Qua_Control is Very Good Then Tar_Strength is T26.

- **Preparation of defuzzification module:** The output obtained from the rule base is not utilized directly as it is in fuzzified form hence obtained fuzzy output is required to be defuzzified in to a crisp value. Defuzzification module will convert the multiple outputs to single output, which is supplied to the next module. Here different defuzzification techniques are tried and out of which it is found that Weightage average method gives the output nearer to exact value and hence it is selected here. Next module is BPN simulator to predict w/c ratio using target strength obtained form first fuzzy module and grade of cement.

11.8.3.2 Development of BPN model for concrete mix design

In the present work, the trained 3-layer Back-Propagation Neural network is incorporated for predicting w/c ratio for 28 days target compressive strength of concrete. Here, the input layer consists of two neurons i.e. target strength obtained from fuzzy module 1 and the grade of cement. The output layer consists of one neuron corresponding to allowable w/c ratio. Various parameters used in BPN simulator are supplied to the program through form developed in the software.

Total 75 patterns for 28 days compressive strength of concrete versus w/c ratio are generated for different grades of cement by using regression equations from the accuracy point of view. From which 50 patterns are used for training and 25 patterns are taken randomly for testing of final weights. Prepared software is capable of predicting the w/c ratio for different grades of cement.

In BPN model, scaling plays a crucial role. In beginning of training phase, various types of scaling functions such as Exponential function, Sigmoid function etc. are tried, but obtained results are not satisfactory. So to take suitable scaling function Alyuda Neuro-Intelligence software is used. Among the different topologies tried, the 2-8-1 topology, proved to be the best by observing average error for the predefined number of iterations. This BPN module
after suitable training and testing is incorporated between fuzzy modules as depicted in the flow chart shown in Fig. 11.12.

The next step in the mix design is to compare the w/c ratio obtained from trained BPN model with the maximum w/c ratio suggested by the code considering the environmental exposure condition, which is explained below.

11.8.3.3 Fuzzy module for maximum water/cement ratio
Here, w/c ratio for mix design is finalized. As per the codal guidelines, the maximum w/c ratio depends on the exposure condition. Code suggests maximum w/c ratios for five-exposure conditions i.e. mild, moderate, severe, very severe and extreme. This is vague as the exposure conditions are ambiguous and overlapping and cannot be quantified by single crisp value. Here, each exposure condition is divided into sub-parameters as suggested by code, which will control the possibilities of different exposure conditions. Different shapes of MF are tried for exposure conditions from which, it is found that Triangular MF suits the best. W/C ratio is also fuzzified in to triangular fuzzy sets as shown in Fig. 11.17.

An antecedent fuzzy set is:
Exposure\_Condition = ["Mild", "Moderate", "Severe", "Very Severe", "Extreme"]

Consequent fuzzy set is:
Maximum\_W/C\_Ratio = [A0.60, A0.60, A0.50, A0.45, A0.40]
where A0.60 means maximum w/c ratio is around 0.60.
11. Software Development and Application of Hybrid Tools

- **Construction of fuzzy rule base**: This module has total five rules which will cover the different possibilities of exposure conditions versus maximum allowable w/c ratio. The prepared rules are of the form:

  *If Exposure Condition is Mild Then Maximum W/C Ratio is around 0.60.*

- **Preparation of defuzzification module**: The Different methods of defuzzification are tried and finally Weightage average method is incorporated in the defuzzification process.

11.8.3.4 Fuzzy module for quantity of water and cement

This module is used to calculate the quantity of water corresponding to maximum size of coarse aggregates. For this maximum size of the aggregate, shape of the aggregate and grade of cement are the variables in input space and allowable quantity of water is the variable in output space.

- **Generation of fuzzy sets and fuzzy MFs**: In mix design, generally size of aggregates is defined by crisp value such as 10 mm, 20 mm and 40 mm. But it does not mean that all the particles have same size, it may happen in 20 mm size some particles are of 15 mm and some are of 25 mm. In FL, one can define it with the help of fuzzy value i.e. 20 mm which represents values between 15 to 25 mm. The triangular membership function has been used for size of aggregate.

  *The antecedent fuzzy sets are:*

  - Size = [Around 10, Around 20, Around 40] and,
  - Shape = ["Angular", "Rounded"]

  *The consequent fuzzy set is:*

  - Water Content = [A208 kg/m³, A186 kg/m³, A165 kg/m³]

  where A208 kg/m³ represents approximate quantity of water.

- **Construction of fuzzy rule base**: Code has suggested the quantity of water considering maximum size of aggregate as 10 mm, 20 mm and 40 mm and shape of aggregate as angular. Considering all above conditions, rule base is designed for various combinations. Total ten rules are prepared to cover the different possibilities. The structure of the rules is:
If Size is around 10 mm And Shape of aggregates is Angular Then Water Content is around 208 kg/m³.

In this module, rule base gives the crisp answer in the output domain. Once the quantity of water is known, the amount of cement is directly calculated from the predetermined w/c ratio as obtained in module 2.

11.8.3.5 Fuzzy module for minimum cement content
The code recommends minimum cement content as crisp values corresponding to different exposure conditions considering the durability aspect of concrete structures. This module is designed considering exposure condition as input and minimum cement content as output.

★ Generation of fuzzy sets and fuzzy MFs:
The antecedent fuzzy set is:
EC = ["Mild", "Moderate", "Severe", "Very Severe", "Extreme"]
where EC - Exposure Condition.

The consequent fuzzy set is:
MCC = [A300 kg, A300 kg, A320 kg, A340 kg, A360 kg]
where MCC - Minimum Cement Content, and A300 means minimum cement content is around 300 kg.

★ Construction of fuzzy rule base: This module has rule base that works on a single input i.e. exposure condition and single output i.e. minimum cement content. Total five rules are written to capture the different possibilities and are of the form:
If EC is Mild Then MCC is around 300 kg.

Here, prepared rule base itself is capable to give the crisp value in the output domain. Considering durability of concrete, the amount of cement used should not be less than the value obtained by this module.

11.8.3.6 Fuzzy module for quantity of aggregates
In this module, percent of sand in absolute volume of total aggregate are calculated based on maximum size of coarse aggregate obtained from the module 3 and grading zone for fine aggregates. Grading zone is supplied directly or determined based on experimental results of
sieve analysis supplied by the user. The membership function for zone of sand is shown in Fig. 11.18.

![Fig. 11.18 Membership Function for Zone of Sand](image)

**Generation of fuzzy sets and fuzzy MFs:**

*The antecedent fuzzy sets are:*

- Size = [Around 10, Around 20, Around 40]
- Zone = [Zone I, Zone II, Zone III, Zone IV]

*The consequent fuzzy set is:*

- FA = [A0.30, A0.35, A0.40]

where A means Around.

**Construction of fuzzy rule base:** This module has rule base that works on a two input i.e. maximum size of coarse aggregates and grading zone of sand and single output i.e. absolute percent of sand. Total twenty rules are prepared to capture the different possibilities and are of the form:

*If Size is around 10 mm And Grading Zone for fine aggregates is I Then volume of FA is approximately 0.41 percent of absolute volume of aggregates.*

Once the percentage of sand is calculated, the next step is to determine the quantity of fine and coarse aggregates. For that value of specific gravity for different materials and assumed entrapped air content are given as input. As quantum of cement and water is already calculated in the previous module, the quantity of sand and coarse aggregates is now calculated by simple mathematical calculations. Further rule base for correction in different parameters, such as water absorbed by aggregates, extra water available in
aggregates, blending requirement etc. are also incorporated in this module. All the
mentioned corrections are carried out by the software as per the input data supplied by the
user. So at this stage all the quantities of materials are available for particular design
target strength.

11.9 CONCRETE MIX DESIGN EXAMPLES
11.9.1 Example of M20 Grade Concrete
The working of the software is checked by carrying out mix design for the M20 grade of
concrete using the following design parameters. The results obtained are compared with those
given in IS 10262 [101] and with those obtained by regression analysis [102].

(i) Grade of concrete mix = M20, (ii) Strength of cement = 53 MPa, (iii) Degree of quality
control = Good, (iv) Type of exposure condition = Mild, (v) Compacting factor = 0.90, (vi)
Type of aggregate = Angular, (vii) Maximum size of aggregate = 20 mm, (viii) Grading zone
of sand = II, (ix) Specific gravity of cement = 3.15, (x) Specific gravity of coarse aggregates
= 2.60, (xi) Specific gravity of sand = 2.60, (xii) Absorption capacity of coarse aggregates =
0.50 percent, (xiii) Absorption capacity of sand = 1.0 percent, (xiv) Moisture content in
coarse aggregates = Nil and (xv) Moisture content in sand = 2 percent.

Results obtained from Neuro-Fuzzy approach are compared with those based on IS code and
regression analysis in Table 11.6.

<table>
<thead>
<tr>
<th>Ingredients</th>
<th>Neuro-Fuzzy Approach</th>
<th>IS Code</th>
<th>Regression Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cement</td>
<td>381.00 kg</td>
<td>383.00 kg</td>
<td>396.07 kg</td>
</tr>
<tr>
<td>Water</td>
<td>192.00 lit.</td>
<td>191.61 lit.</td>
<td>192.14 lit.</td>
</tr>
<tr>
<td>Fine Aggregate</td>
<td>590.38 kg</td>
<td>546.00 kg</td>
<td>542.50 kg</td>
</tr>
<tr>
<td>Coarse Aggregate</td>
<td>1188.65 kg</td>
<td>1187.00 kg</td>
<td>1179.93 kg</td>
</tr>
<tr>
<td>C:W:FA:CA</td>
<td>1:0:0.504:1.550:3.120</td>
<td>1:0:0.5:1.426:3.100</td>
<td>1:0:0.485:1.370:2.979</td>
</tr>
</tbody>
</table>

11.9.2 Example for M30 Grade Concrete
The mix design procedure is carried out for the M30 grade of concrete using the following
design parameters: (i) Grade of concrete mix = M30, (ii) Strength of cement = 53 MPa, (iii)
Degree of quality control = Very good, (iv) Type of exposure condition = Moderate, (v) Compacting factor = 0.90, (vi) Type of aggregate = Angular, (vii) Maximum size of aggregate = 40 mm, (viii) Grading zone of sand = II, (ix) Specific gravity of cement = 3.15, (x) Specific gravity of coarse aggregates = 2.60, (xi) Specific gravity of sand = 2.70, (xii) Absorption capacity of coarse aggregates = 1.0 percent, (xiii) Absorption capacity of sand = 1.0 percent (xiv) Moisture content in coarse aggregates = Nil and (xv) Moisture content in sand = Nil

Results obtained from Neuro-Fuzzy approach are compared with those based on regression analysis in Table 11.7.

<table>
<thead>
<tr>
<th>Ingredients</th>
<th>Neuro-Fuzzy Approach</th>
<th>Regression Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cement</td>
<td>403.45 kg</td>
<td>440.30 kg</td>
</tr>
<tr>
<td>Water</td>
<td>182.90 lit.</td>
<td>204.70 lit.</td>
</tr>
<tr>
<td>Fine Aggregate</td>
<td>538.00 kg</td>
<td>590.67 kg</td>
</tr>
<tr>
<td>Coarse Aggregate</td>
<td>1284.00 kg</td>
<td>1203.83 kg</td>
</tr>
<tr>
<td>C:W:FA:CA Proportion</td>
<td>1:0:0.453:1.333:3.183</td>
<td>1.0:0.465:1.342:2.734</td>
</tr>
</tbody>
</table>

11.10 GA-NEURO-FUZZY APPROACH FOR CONCRETE MIX OPTIMIZATION
11.10.1 Concept of Multi-Objective Optimization

Many real-world problems involve simultaneous optimization of several incommensurable and often competing objectives. For example in concrete mix design, there is a trade-off between cost versus quality. Where quality itself is defined in terms of further sub-criteria and there is no definite relation between them. For such type of problems, there is usually no single optimal solution, but rather a set of alternative solutions exist. These solutions are optimal in the wider sense that no other solutions in the search space are superior to them, when all objectives are considered. They are known as Pareto-optimal solutions.

Mathematically multi-objective optimization problem can be symbolically represented as,

\[ y = f(x) = \{f_1(x), f_2(x), \ldots, f_m(x)\} \] and it is required to be maximized subject to
\[ g(x) = \{g_1(x), g_2(x), \ldots, g_J(x)\} \text{ and } h(x) = \{h_1(x), h_2(x), \ldots, h_K(x)\} \]

Considering, \( x = \{x_1, x_2, \ldots, x_N\} \in X \) and \( y = \{y_1, y_2, \ldots, y_M\} \in Y \)

where \( x \) is the vector of decision variables, \( y \) is the objective vector, \( X \) is the decision space, and \( Y \) is called as objective space.

There are several advantages to using Evolutionary Algorithms (EAs) over classical methods to solve multiple-objective problems. With a population-based search and selection based on domination, it is possible to converge to the Pareto-optimal front in a single run of the algorithm. The full tradeoff between objectives is evolved and so questions of appropriate weighting, convexity, and scaling that typically arise in classical formulations need not be asked. As the name suggests, an elite-preserving operator assures that the fitness of the best population members at any given generation will not deteriorate. The degree of elitism employed determines the balance between search and selection pressure.

A method of optimizing concrete mix proportioning based on cost and strength using Multi-Objective Evolutionary Algorithm (MOEA) is described in Fig. 11.19. The parameters for optimization problem are chosen on the basis of the IS method [101] of mix design. Here, mix design is optimized by randomly varying the controlling factors, namely fine and coarse aggregate ratio with respect to cement in such a way that the various combination of these factors would yield mix, which is low in cost and high in compressive strength.

The optimization strategy used in the present work is based on GA-Neuro-Fuzzy hybrid approach. As the use of fuzzy logic renders the mix design process more natural, flexible and humanistic and as neural networks can be used to build up the relationship from the examples presented to them, mix design procedure is formulated first by using Neuro-Fuzzy hybrid approach as described earlier. After that multi-objective optimization concept is used for concrete mix optimization through which mix design optimization problem is converted into a mathematical programming problem subjected to certain constraints like compressive strength and minimum cement content.
11.10.2 ENSGA-II Technique for Optimization

The basic idea behind None-Dominated Sorting Genetic Algorithm (NSGA) [103] is the ranking process executed before the selection operation. This process identifies non-dominated solutions in the population at each generation to form non-dominated fronts [2], based on the concept of non-dominance criterion. After this, the selection, crossover, and mutation operations are performed. The NSGA method is intended to search for non-dominated regions, and sharing helps to distribute the individuals over this region.

The NSGA implements both aspects of Goldberg’s suggestion in the better way i.e., the ranking procedure is performed according to the non-dominance definition over the population and a uniform distribution of the non-dominated is guaranteed, using a niche formation technique. Both aspects produce distinct non-dominated points to be found in the population and thus NSGA favors the schemata representing the Pareto-optimal regions.

The original NSGA had three common criticisms i.e. high computational complexity, lack of elitism, and need for specifying a sharing parameter. The Elitist Non-dominated Sorting Genetic Algorithm–II (ENSGA-II) alleviates all these difficulties and the basic concept of method is as follows.
In ENSGA-II, the offspring population $Q_i$ is first created by using the parent population $P_i$ of size $N$. However, instead of finding the non-dominated front of $Q_i$ only, first the two populations are combined together to form $R_i$ of size $2N$. Then, a non-dominated sorting is used to classify the entire population $R_i$. Although this requires more effort compared to performing a non-dominated sorting on $Q_i$ alone, it allows a global non-domination check among the offspring and parent solutions. Once the non-dominated sorting is over, the new population is filled by solutions of different non-dominated fronts, one at a time. The filling starts with the best non-dominated front and continues with solutions of the second non-dominated front, followed by the third non-dominated front, and so on. Since the overall population size of $R_i$ is $2N$, not all fronts may be accommodated in $N$ slots available in the new population. All fronts, which could not be accommodated, are simply deleted. Instead of arbitrarily discarding some members from the last front, it would be wise to use a niching strategy to choose the members of the last front, which reside in the least crowded region in that front. This scenario is illustrated in Fig. 11.20.

Fig. 11.20 Schematic Representation of ENSGA-II Technique
The considered strategy does not affect the proceedings of the algorithm much in the early stages of evolution. This is because, early on, there exist many fronts in the combined population. It is likely that solutions of many good non-dominated fronts are already included in the new population, before they add up to $N$. It then hardly matters which solution is included to fill up the population. However, during the latter stages of the simulation, it is likely that most solutions in the population lead in the best non-dominated front. It is also likely that in the combined population $R$, of size $2N$, the number of solutions in the first non-dominated front exceeds $N$. The above algorithm then ensures that niching will choose a diverse set of solutions from this set. When the entire population converges to the Pareto-optimal front, the continuation of this algorithm will ensure a better spread among the solutions.

As in the case of concrete mix design, one can take the cost and strength as objective function for the optimization. Here, aim is to maximize the strength and minimize the cost. On x-axis cost is taken and on y-axis strength is considered. If one has a number of solutions corresponding to different cost versus strength, then it can be plotted as shown in Fig. 11.21. Comparison is then carried out among each solution and different sets corresponding to non-dominated solutions are prepared. As shown in Fig. 11.21, the line in blue colour represents the example of Pareto-optimal solution.

**Fig. 11.21 Representation of Pareto-Optimal Front**
The optimization algorithm should be terminated if any one of the Pareto-optimal solutions is obtained. But in practice, since there could be a number of Pareto-optimal solutions and the suitability of any solution depends on a number of factors, including the designer’s choice and problem environment, finding the entire set of Pareto-optimal solutions may be desirable.

### 11.10.3 Working of Genetic Algorithm Module

Initially, total 15 parent populations are considered and maximum numbers of generations are taken as 10. During first run of generation, 1 population is generated from the mix proportion given by Neuro-Fuzzy model and remaining 14 populations are randomly generated as per the described range. As concrete is made of mainly four ingredients i.e. cement, water, coarse aggregates and sand, for each variable the length of string is considered as 8 digits and therefore the length of each population becomes 32 digits. Figure 11.22 represents the mathematical model for prepared GA based program.

As obtained output from Neuro-Fuzzy model is in the form of decimal value, it is required to be converted in binary form i.e. 0 and 1. For that separate module is prepared, which covers the scaling and conversion of decimal value to binary value. To generate remaining 14 populations, first of all range of cement variation is required. While varying the quantity of cement, the simultaneous correction in quantity of coarse and fine aggregates are carried out to keep the density of mix constant. Here, cement range is decided as 50 kg and based on that 14 binary strings of length 32 digits are prepared. Then each population is broken into 4 parts and corresponding to those, decimal values are calculated. So at this stage, 15 parent populations are ready.

After this, another 15 populations are produced using parent populations, which are called as child population. Then these populations are also broken up into 4 parts and corresponding decimal value is found out. Next, parent and child populations are summed up and cost of each generated population is found out considering unit cost supplied by the user. Ranking is then carried out as per the obtained cost. As the aim is to minimize the cost and maximize the strength, minimum cost will take the first position and so on. After that decimal value for all the populations are supplied to the trained BPN model and corresponding compressive strength is found out which is then used to find the cumulated weighted factor, which include cost and strength as performance parameter.
The next step is the development of non-dominated fronts where all the populations are ranked in ascending order according to their cost. To start relative comparison, first
population is directly taken in the first front because it has minimum value of cost. This value is compared with the strength of next population. If the strength is higher than all the previously selected values in that front, then only it will become the member of that particular front. The advantage of making the population in ascending order is that only one parameter is required to compare i.e. strength. So it will reduce the programming complication and run-time of the program. This procedure is repeated till all the 30 populations are compared and selected values are deleted from present list to prevent the repetitive selection. Next, the values for second front are calculated. The total number of values in all the fronts should not exceed 15. So it may happen that numbers of fronts may vary but total number of population must remain same. The development of fronts is represented in Fig. 11.23. After this weightage factor using cost and strength as objective functions is calculated as follows.

First, a local search strategy is suggested from each obtained solutions. Since a local search strategy requires a single-objective function, it is necessary to convert multi-objective into single objective function. The weighted objective function used here is:

$$ F(x) = \sum_{j=1}^{M} W_j f_j(x) $$  \hspace{1cm} \text{(11.20)}

where $W$ is weight calculated from the obtained set of solutions in a special way, $f_j$ is the objective function, and $M$ is the number of objective functions.
In this case, total two objective functions i.e. cost and strength are considered. Therefore, first of all, maximum and minimum values of both objective functions are calculated. The weighted factor for cost is calculated from the expression,

$$W^x_j = \frac{(f^{\max}_j - f^0_j(x))}{\sum_{k=1}^{M}(f^{\max}_k - f^0_k(x))}$$  \( \ldots \) (11.21)

The weighted factor for strength is calculated from the expression,

$$W^x_j = \frac{(f^0_j(x) - f^{\min}_j)}{\sum_{k=1}^{M}(f^{\max}_k - f^0_k(x))}$$  \( \ldots \) (11.22)

Here, value of k varies up to 2 i.e. cost and strength, value of j varies up to 15 as per numbers of solutions in each generation. If cost is away from maximum value, its weightage will increase and if strength is away from minimum value, its weightage will increase.

Once the cumulative weights are found for the 15 populations selected in the non-dominated front, the next step is to generate the population for next generation. From the elitist point of view, first two population of the first front are directly transferred in to the next generation. To generate rest of the populations, randomly two patterns are selected using tournament scheme and out of which pattern having maximum fitness is stored. After that mutation and crossover operations are carried out to generate the offspring population. Here constant mutation rate is considered. For crossover operation, single point crossover is used to reduce the complication. This procedure is repeated till 15 offspring populations are generated. Maximum number of generation is taken as the cutoff criteria for terminating the optimization process.

Finally, the optimum proportion is selected based on maximum fitness obtained in the last run of the program. For the checking purpose, this value is supplied to the trained BPN model, which will predict the strength of determined proportion. If obtained result doesn’t satisfy the requirement of minimum strength, then entire process is repeated again.

### 11.10.4 Role of ANN in Optimization

In concrete mix optimization problem, accurate estimation of compressive strength is an important issue. The one way to estimate the concrete strength is to follow the reverse IS method of mix design. In the reverse IS method, it may be possible that one cannot know the
various assumption that are considered for calculating the proportion of ingredients. Moreover, to formulate the reverse IS method is a very challenging job. Because of all these difficulties, in the present work artificial neural network is used for predicting the 28 days compressive strength of concrete.

Here role of ANN is to predict the strength of concrete mix based on obtained proportions. Actually ANN is used at two stages. The first is during the determination of non-dominated fronts and secondly when final optimized mix is calculated; it will justify the proportion by predicting the compressive strength of the mix without doing any experimental work.

Here, input and output layer consist of four and one neuron respectively. The training and testing of network is based on the experimental data as sited in reference [104]. Total 40 patterns are generated for various mix proportions and 28 days compressive strength of concrete. Out of which 30 patterns are used for training and 10 patterns are used for testing randomly. The number of neurons in the hidden layer is decided using a trial-and-error procedure and topology 4-4-1 proved to be the best for predicting the 28 days compressive strength.

11.10.5 Illustrative Example of Optimum Mix Design

To demonstrate the functioning of prepared software, example of M30 grade of concrete mix design is considered with following data: (i) Grade of concrete mix = M30, (ii) Strength of cement = 53 MPa, (iii) Degree of quality control = Very good, (iv) Type of exposure condition = Moderate, (v) Compacting factor = 0.90, (vi) Type of aggregate = Angular, (vii) Maximum size of aggregate = 40 mm, (viii) Grading zone of sand = II, (ix) Specific gravity of cement = 3.15, (x) Specific gravity of coarse aggregates = 2.60, (xi) Specific gravity of sand = 2.70, (xii) Absorption capacity of coarse aggregates = 1.0 percent, (xiii) Absorption capacity of sand = 1.0 percent (xiv) Moisture content in coarse aggregates = Nil and (xv) Moisture content in sand = Nil

During the execution of program, once the mix proportion for particular target strength is calculated using Neuro-Fuzzy approach, the next step is to optimize the mix. For that user has to click the optimization menu to run the module.
After that various optimization parameters used in GA i.e. string data, crossover parameters, mutation parameters and method for string selection etc. are supplied through relevant form.

Once the GA parameters are submitted to the program, immediately module based on ENSGA-II technique is executed. After the few minutes of calculations, software displays the results on the screen as shown in Fig. 11.24.

Optimized mix proportion reflects approximate saving in cement of 48 kg keeping the density of mix same. Based on the current rates, the saving in cost is indicated as Rs. 182.0 per m³ of concrete. Prepared program also displays the predicted 28 days compressive strength for the optimized mix as a final check on the obtained quantities of materials.

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Optimized Proportion

Optimized Proportion

<table>
<thead>
<tr>
<th>Proportion</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantity of Cement</td>
<td>355.00 Kg</td>
</tr>
<tr>
<td>Quantity of Coarse Aggregates</td>
<td>1344.00 Kg</td>
</tr>
<tr>
<td>Quantity of Fine Aggregates</td>
<td>559.00 Kg</td>
</tr>
<tr>
<td>Quantity of Water</td>
<td>161.00 L</td>
</tr>
</tbody>
</table>

Optimized Cost in Rs. 1743.00

Predicted Compressive Strength from Trained ANN 41.59 N/mm²

Fig. 11.24 Display of Results of Optimum Quantity of Materials
11.11 CLOSING REMARKS

The previous chapters (Chapters 8, 9 and 10) were devoted to individual soft computing techniques for optimization of various structures whereas present chapter was aimed at developing the optimization software using soft computing tools in combination. It has been observed that combination of these techniques mostly improves the performance of individual techniques. For example, (i) GA-Fuzzy approach overcomes the limitation of genetic algorithm of imposing hard constraints by relaxing them with fuzzy logic as illustrated by optimization of combined footing, configuration optimization of plane trusses and topology optimization of plate (ii) GA accelerates the speed of training of BPN network as is clear from the concrete column example, (iii) In concrete mix design procedure involving fuzziness and vagueness in several design steps, Neuro-Fuzzy hybridization not only looks realistic approach but also provides some of the parameters directly which, otherwise have to be obtained from charts and supplied to the fuzzy module and (iv) GA helps in optimization of concrete mix design carried out using Neuro-Fuzzy model.