9.1 General Remarks

Many real life problem solving methodology exploits the ready made available information about problem domain, processes the information using the knowledge acquired from the subject experts which is based on some thumb rules and assumptions and makes the decision which is specific to the problem. Thus in the process of decision making one has to use thumb rules, assumptions and certain vague information and knowledge base. Fuzzy Logic assists the decision making process in such vague environment. In the fuzzy logic based optimization methodology the crisp data is converted into fuzzy data which is then processed by fuzzy arithmetic and the fuzzy output thus obtained is defuzzified to get the realistic answer. Therefore the software based on the fuzzy logic requires some specialized subroutines for fuzzification of problem data, fuzzy arithmetic and defuzzification process. The software also requires the graphical display of the fuzzified data depicting fuzzification. The software must also provide the freedom to choose various fuzzification and defuzzification methods. The programming language selected for developing such program must have all such facilities.

In traditional designs, the optimization problem is stated in precise mathematical terms. However, in many real-world problems, the design data, objective function, and constraints are stated in vague and linguistic terms. As design of RCC structures involves a large amount of uncertainty and as fuzzy logic is good in dealing with fuzzy data, the main aim of the present chapter is to explore the possibility of using the concept of fuzzy logic in one of the important areas of Structural Engineering i.e. the design of various R.C.C structures. The search for optimum solution is made without violating norms laid by IS code for limit state method of design.

This chapter is devoted to the software module developed based on fuzzy logic to deal with optimum design of R.C.C. structures. The software is developed in Visual Basic on Windows platform with menu driven input and graphical output facilities and its uses are demonstrated with the help of examples of different structures.
9.2 COMPUTER IMPLEMENTATION

The essential part of fuzzy system design is the application of fuzzy sets and fuzzy logic to a solution, or to a method of solution, provided in the conventional, nonfuzzy form. The computer implementation of the optimization algorithm by alpha-cut method is covered in this section. **Visual Basic-6.0** is selected for development of software based on fuzzy logic. Virtually all-nontrivial computer programs involve following three major tasks.

1) Entering input data (supplying information to be processed)
2) Computing the desired results (processing the input data)
3) Displaying the results (displaying the results of the computation)

Each step may be complex; its implementation may therefore require considerable time and effort. In VB, the first and last steps (data input and data output) are accomplished through the user interface. In many applications, the design of the user interface is the most complicated part of the entire program development process; though the controls built into VB simplify this process considerably. The second step (computation) is usually carried out by a series of VB instructions embedded in one or more independent event procedures. The selection and order of these VB instructions are determined by an appropriate algorithm i.e., a logical and orderly computational strategy for transferring the given data into the desired output data. These three major tasks are carried out by developing following three modules.

9.2.1 Pre-Processor Module

The pre-processor is developed to facilitate menu driven input of data. It provides graphical user interface to supply the structural data, which has following advantages over numerical data input,

- Faster than numeric input interface.
- Less chance of making mistake in supplying input data.
- The input data can be checked at a glance by getting view of drawn geometry.
- Easy editing and change in the input.

The pre-processor comprises of various forms, picture boxes, menus, toolboxes, and various command buttons. As the pre-processor module is different for each application, it is discussed in detail for each application separately. Pre-processor prompts user to supply the initial design data and dimensions of structural component based on which preliminary
design is carried out. It then asks user to supply input data related to fuzzy logic required for alpha-cut method. The data supplied by the user is then checked for its correctness.

9.2.2 Main-Processor Module
The prime requirement of the main-processor is that it should be compact and should require least computational time. The developed software comprises various subroutines for optimization of several structures such as plane frame, grid, combined footing and RCC slabs. These subroutines can be grouped into following three categories;

9.2.2.1 Analysis subroutines
- A subroutine to carry out the analysis of different types of slabs using coefficients method as per I. S. code [83].
- An analysis subroutine for combined footing calculates maximum bending moments and shear forces for various load cases.
- Analysis subroutine for Plane frame and Grid problems based on Stiffness Member Approach [92] which involves various subroutines as describe below:

- **Subroutine SDATA:** It reads the structural data, like joint co-ordinates, member indices, area of cross-section, moment of inertia, support restraints, etc.
- **Subroutine STIFF:** It constructs the member stiffness matrix and overall stiffness matrix.
- **Subroutine LOADS:** It reads joint loads, member loads and constructs overall load vector \((Ac = Aj + Ae)\)
- **Subroutine BANFAC and BANSOL:** These subroutines are developed for factorization and solution of the matrix based on Choleskey’s square root method [92] respectively.
- **Subroutine RESULTS:** This subroutine evaluates the analysis results - Joint displacements, Member forces, and Support reactions.

9.2.2.2 Fuzzy logic based subroutines
In the developed software package, following two subroutines are developed to perform the optimization based on fuzzy logic concept. These subroutines are common to all applications problems.
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- **Subroutine INPUT FUZZY SETS**: For the construction of input fuzzy set, it is necessary to compute the values of input parameter at each $\alpha$-cut depending upon their minimum and maximum values. This subroutine computes these values; stores in list box and plot the fuzzy set using graphical features of VB.

- **Subroutine FUZZY OPT**: In order to find out optimum solution, the next step is the superimposition of performance and induced fuzzy sets to get intersection point. For this the values of performance parameter and induced parameter are computed at each $\alpha$-cut and stored in the list box. These values are used for the construction of the performance and induced fuzzy sets.

The software finds out $\alpha$-cut giving minimum difference between performance parameter and induced parameter. After this the software determines the values of input parameters at this $\alpha$-cut and these input parameters are considered to give optimum solutions. The whole process of obtaining the optimum solution is very tedious and time consuming when the $\alpha$-cut is very small, but the features of VB (i.e. list box, graphics) makes it easy. The flow chart for $\alpha$-cut method is shown in Fig. 9.1.

9.2.2.3 Design subroutines

The design subroutines developed to carry out design of different R.C.C. structures using limit state method of design and Indian standard code of practice IS: 456-2000 [17] are discussed in detail for each application separately.

9.2.3 Post-Processor Module

The post-processor gives design results, reinforcement detailing and optimum cost. As the post-processor module is different for each application, it is discussed in detail for each application separately.
Decide Pre-assigned, Input, Output and Performance Parameters

Pre-assigned parameters

Carry out preliminary design

Develop Input Fuzzy Sets

Select $\alpha$ - Cut interval ($\alpha_i$)

$\alpha = 0, 1, \alpha_i$

Calculate cross over points for each input set

Generate combinations of crossover values

For each combination calculate output value

Develop Induced Fuzzy set

Superimpose Induced and Performance fuzzy sets & get match point

Find out input (optimum) values corresponding to match point

STOP

Fig. 9.1 Flowchart Showing FL Based Alpha Cut Method
9.3 **Optimum Design of Various Types of Slabs**

9.3.1 **Design Parameters**

Design parameters are classified as pre-assigned parameters, input parameters, induced fuzzy set parameters, and performance parameters. The performance parameter is subjected to some functional requirements. The term functional requirement refers to a value, range of values, or fuzzy number.

In design of slab, pre-assigned parameters are span of slab, wall thickness or beam width, loadings-live load and floor finish, grade of concrete and grade of steel. In order to design the slab, one has to decide the depth of slab. Hence the depth of slab is selected as input parameter. The design of slab is made to satisfy both serviceability and strength requirements. Generally the depth of slab is governed by deflection requirement than the requirement of strength. The serviceability requirement for deflection is controlled by shorter effective span to effective depth ratio \((l_x/d)\). The actual \((l_x/d)\) ratio should be less than permissible \((l_x/d)\) ratio. So \((l_x/d)_{\text{per}}\) is considered as performance parameter and \((l_x/d)_{\text{act}}\) is considered as induced fuzzy set parameter.

9.3.2 **Design Constraints**

- **Geometry constraints**

Geometry constraints are the restriction on thickness of the slab and percentage of reinforcement in the slab. They are as follows:

1. \(D_{\text{min}} = 80 \text{ mm}\)
2. \(D_{\text{max}} = 250 \text{ mm}\)
3. \(p_t \geq 0.12\%\) \(\ldots (9.1)\)

- **Behavior constraints**

Main behavior constraint is the deflection of slab. As discussed earlier, the deflection is controlled by \((l_x/d)\), so following constraint is imposed,

\[
(l_x/d)_{\text{act}} - (l_x/d)_{\text{per}} \leq 0
\]

\(\ldots (9.2)\)

where \(l_x = \text{Effective span in shorter direction, } D_{\text{min}} = \text{Minimum depth of slab, } D_{\text{max}} = \text{Maximum depth of slab, } p_t = \text{Percentage steel, and } d = \text{Effective depth of slab.}\)
9.3.3 Fuzzification of Input Parameters

For fuzzification of input variable first of all minimum, maximum and most acceptable values of input variable are decided based on the design experience. These values are then used to develop the fuzzy set known as input fuzzy set. The available constraints help in deciding these values of input parameter.

In design of slab, depth of slab is the main input parameter. The triangular fuzzy set \( D \) expresses different values of input parameter depending upon preferences. The constraints \( D > 80 \text{ mm} \) and \( D < 250 \text{ mm} \) decide minimum and maximum values for depth. The minimum and maximum possible values are assigned membership value equal to zero while the average value of minimum (\( D_{\text{min}} \)) and maximum (\( D_{\text{max}} \)) values is considered as most acceptable value and is assigned membership value equal to one (Refer Fig. 9.2). This gives linear triangular fuzzy set for \( D \).

Fig. 9.2 Fuzzification of Input Parameter

9.3.4 Development of Performance Fuzzy Set

As discussed earlier, the permissible effective span to effective depth ratio is considered as performance parameter. For developing performance fuzzy set it is necessary to decide the value of \( \alpha \)-cut which is the small interval at which the value of performance parameter (i.e. \( (l/d)_{\text{per}} \)) is to be found. When \( \alpha \)-cuts are applied to input fuzzy set each \( \alpha \)-cut will give two crossover points (i.e. \( D_1 \) and \( D_2 \)) from input fuzzy set. At each \( \alpha \)-cut, for each value of crossover point, \( (l/d)_{\text{per}} \) is calculated as shown below and plotted against its membership function to have graph of performance fuzzy set (Fig. 9.3.)

\[
W_u = (DL + LL + FF) \times 1.5 \quad \ldots (9.3)
\]
9.3.6 Superimposition of Performance and Induced Fuzzy Sets

After developing performance and induced fuzzy sets, next step is the superimposition of two plots so as to have match point. This point gives value of \((lx/d)\) i.e. SO which satisfies the functional requirement and at the same time has the highest performance and its appropriate membership grade \(MO\) as shown in Fig. 9.5 which finally decides the most acceptable value of depth.

\[
\begin{align*}
Mu &= (Wu \times lx \times lx) / 8 \\
Pt &= 50 \times (fck/fy) \times \left\{ 1 - (1 - (4.6 \times Mu / (fck \times b \times d^2)))^{1/2} \right\} \\
M.F. &= 1 / \left\{ 0.225 + 0.00322 \times fs - 0.625 \times \log_{10}(1/pt) \right\} \\
(lx/d)_{per} &= 20 \times M.F.
\end{align*}
\]

where \(DL\) = Dead load, \(LL\) = Live load, \(FF\) = Floor finish, \(Wu\) = Total factored design load per unit area, \(Mu\) = Maximum factored span moment, \(Pt\) = Percentage of steel, \(fck\) = Characteristic strength of concrete, \(M.F.\) = Modification factor, \(fs\) = Service stress in steel = 0.58 * \(fy\) and \(fy\) = characteristic strength in steel.

9.3.5 Development of induced fuzzy set

For developing induced fuzzy set, at each \(\alpha\)-cut the output parameter \((lx/d)\) is calculated for each value of crossover point and plotted against its membership function to have graph of induced fuzzy set, which is shown in Fig. 9.4.

\[\text{Fig. 9.3 Performance Fuzzy} \quad \text{Fig. 9.4 Induced Fuzzy Set}\]
9.3.7 Fuzzy Logic Based Processors

Three processors are developed in Visual Basic. A Pre-processor is developed to facilitate menu driven input of data, graphical display of the fuzzy set and input data editing facility before running the design program. A Main-processor is developed to find the final set of design variables according to max-min procedure of FL and to perform the design as per IS code provision [83]. A Post-processor is developed to provide reinforcement detailing. Provision is made in the software for graphical representation of induced fuzzy set and performance fuzzy set. The software is developed with menu driven input and graphical output facilities and its uses are demonstrated with the help of following four types of slab examples:

1. Simply supported one-way slab
2. Continuous one-way slab
3. Simply supported two-way slab
4. Continuous two-way slab

9.3.8 A Simply Supported One-Way Slab Example

- **Input data:** Longer span of slab = 10.00 m, Shorter span of slab = 4.00 m, Wall thickness = 0.23 m, Live load = 2.50 kN/m², Floor finish = 1.00 kN/m², Grade of concrete = M15 and Grade of steel = Fe415.

Figure 9.6 shows the main form for FL based structural design which is used to select the type of structure to be designed. The geometrical data of slab such as span of longer side, span of shorter side, wall thickness or beam width, etc. are provided using geometry form as...
it appears in Fig. 9.7(a). For inserting the material properties such as grade of concrete and grade of steel the form depicted in Fig. 9.7(b) is to be used. Figure 9.7(c) shows the form that is used to enter the live load and floor finish load. Designer can enter his preferences for values of input parameter through a form shown in Fig. 9.7(d). The software creates the geometry of slab based on supplied data, which is shown in the Fig. 9.8.

Input fuzzy set developed by main processor for depth of slab at an interval of 0.01 $\alpha$-cut is shown in the Fig. 9.9. A menu is developed to take values from the input fuzzy set menu and to calculate $(l_x/d)$ at each $\alpha$-cut. Table 9.1 shows values of induced fuzzy set and performance parameter for two crossover points at 0.20 $\alpha$-cut. These values are plotted against their relative membership function to get induced and performance fuzzy sets. The superimposition of two plots as shown in Fig. 9.9 gives a match point. The corresponding membership grade is used to find the optimum input value from input fuzzy set.

Post-processor depicts final result in a tabular form as in Fig. 9.10, in addition to reinforcement detailing of the slab as it appears in Fig. 9.11.
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GEOMETRY DATA
Cleat Next
Span of longer side 10 m
Span of shorter side 4 m
Wall thickness 0.23 m

(b) Material Form

Grade of concrete M15
Grade of steel Fe 415

(c) Loading form
Live load 2.50 kN/m²
Floor finish 1.00 kN/m²

(d) Preference form
Mini. depth of slab (Dmin) 100 mm
Max. depth of slab (Dmax) 200 mm
Acceptable depth of slab 150 mm

Fig. 9.7 Input Forms

Fig. 9.8 Geometry of Slab
Table 9.1 Intermediate Calculation Sheet

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>50.00</td>
<td>22.22</td>
<td>19.40</td>
<td>33.00</td>
</tr>
<tr>
<td>0.20</td>
<td>44.44</td>
<td>23.53</td>
<td>21.00</td>
<td>31.80</td>
</tr>
<tr>
<td>0.40</td>
<td>40.00</td>
<td>25.00</td>
<td>22.60</td>
<td>30.40</td>
</tr>
<tr>
<td>0.60</td>
<td>36.36</td>
<td>26.67</td>
<td>24.00</td>
<td>29.20</td>
</tr>
<tr>
<td>0.80</td>
<td>33.33</td>
<td>28.57</td>
<td>25.20</td>
<td>28.00</td>
</tr>
<tr>
<td>1.00</td>
<td>30.77</td>
<td>30.77</td>
<td>26.60</td>
<td>26.60</td>
</tr>
</tbody>
</table>

Fig. 9.9 Induced and Performance Fuzzy Sets

Fig. 9.10 Design Results
For validity check, the results obtained from the software are compared with those available in literature [85] and are depicted in Table 9.2.

**Table 9.2 Comparison of Results**

<table>
<thead>
<tr>
<th>DATA</th>
<th>REFERENCE [85]</th>
<th>SOFTWARE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Span of slab</td>
<td>4.00 m</td>
<td></td>
</tr>
<tr>
<td>Live load</td>
<td>2.50 kN/m2</td>
<td></td>
</tr>
<tr>
<td>Floor finish</td>
<td>1.00 kN/m2</td>
<td></td>
</tr>
<tr>
<td>Grade of concrete</td>
<td>M15</td>
<td></td>
</tr>
<tr>
<td>Grade of steel</td>
<td>Fe415</td>
<td></td>
</tr>
<tr>
<td>Cost of concrete</td>
<td>Rs. 2000/- per m3</td>
<td></td>
</tr>
<tr>
<td>Cost of steel</td>
<td>Rs. 30/- per Kg</td>
<td></td>
</tr>
<tr>
<td>RESULTS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Depth of slab</td>
<td>180 mm</td>
<td>160 mm</td>
</tr>
<tr>
<td>Quantity of concrete</td>
<td>0.761 m3</td>
<td>0.677 m3</td>
</tr>
<tr>
<td>Quantity of steel</td>
<td>24 Kg</td>
<td>22.50 Kg</td>
</tr>
<tr>
<td>Total cost (per mt. width)</td>
<td>Rs. 2240/-</td>
<td>Rs. 2040/-</td>
</tr>
<tr>
<td>% Reduction in cost</td>
<td>8.92 %</td>
<td></td>
</tr>
</tbody>
</table>
9.3.9 A Continuous One-Way Slab Example

Input data are supplied through input forms as shown in Fig. 9.12. Geometry is depicted in Fig. 9.13. The input fuzzy, induced fuzzy and performance fuzzy sets are depicted in Fig. 9.14. Provision is made in software to calculate span and support moments as per I.S. code [83] and visualize the moments. Maximum shorter span moment is considered in calculation for performance parameter.

The span and support moments are displayed in the geometry of slab as shown in Fig. 9.15. The design results are shown in Fig. 9.16 and detailing of reinforcement as carried out by post-processor is depicted in Fig. 9.17. Table 9.3 shows the comparison of results and indicates that the present software results in reduction in cost of slab.

![Fig. 9.12 Input Menus](image-url)
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**Fig. 9.13 Geometry of Slab**

**Fig. 9.14 Input, Induced, Performance Fuzzy Sets**
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Fig. 9.15 Span and Support Moments

Fig. 9.16 Design Results

**GEOMETRY DATA**
- Depth of slab: 90 mm

**REINFORCEMENT DATA**

<table>
<thead>
<tr>
<th>Location</th>
<th>Main Steel</th>
<th>Distribution Steel</th>
</tr>
</thead>
<tbody>
<tr>
<td>At near middle of end span</td>
<td>8 # @160 mm c/c</td>
<td>6 # @250 mm c/c</td>
</tr>
<tr>
<td>At middle of interior span</td>
<td>8 # @160 mm c/c</td>
<td></td>
</tr>
<tr>
<td>At the end support</td>
<td>6 # @160 mm c/c</td>
<td></td>
</tr>
<tr>
<td>At support next to end</td>
<td>10 # @160 mm c/c</td>
<td></td>
</tr>
<tr>
<td>At other interior support</td>
<td>10 # @160 mm c/c</td>
<td></td>
</tr>
</tbody>
</table>
Table 9.3 Comparison of Results

<table>
<thead>
<tr>
<th>DATA</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of spans</td>
<td>5 nos.</td>
<td></td>
</tr>
<tr>
<td>Length of longer span</td>
<td>6.50 m</td>
<td></td>
</tr>
<tr>
<td>Length of shorter span</td>
<td>2.75 m</td>
<td></td>
</tr>
<tr>
<td>Beam width</td>
<td>0.23 m</td>
<td></td>
</tr>
<tr>
<td>Live load</td>
<td>4.00 kN/m²</td>
<td></td>
</tr>
<tr>
<td>Floor finish</td>
<td>1.00 kN/m²</td>
<td></td>
</tr>
<tr>
<td>Grade of concrete</td>
<td>M15</td>
<td></td>
</tr>
<tr>
<td>Grade of steel</td>
<td>Fe415</td>
<td></td>
</tr>
<tr>
<td>Cost of concrete</td>
<td>Rs. 2000/- per m³</td>
<td></td>
</tr>
<tr>
<td>Cost of steel</td>
<td>Rs. 30/- per Kg</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>RESULTS</th>
<th>REFERENCE [85]</th>
<th>SOFTWARE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth of slab</td>
<td>120 mm</td>
<td>90 mm</td>
</tr>
<tr>
<td>Quantity of concrete</td>
<td>1.678 m³</td>
<td>1.256 m³</td>
</tr>
<tr>
<td>Quantity of steel</td>
<td>89 Kg</td>
<td>90 Kg</td>
</tr>
<tr>
<td>Total cost (per mt. width)</td>
<td>Rs. 6020/-</td>
<td>Rs. 5210/-</td>
</tr>
<tr>
<td>% Reduction in cost</td>
<td></td>
<td>13.45 %</td>
</tr>
</tbody>
</table>
9.3.10 A Simply Supported Two-Way Slab Example

- **Input data:** Effective longer span = 5.65 m, Effective shorter span = 4.15 m, Wall thickness = 0.15 m, Grade of concrete = M15, Grade of steel = Fe 415, Live load = 4.00 kN/m² and Floor finish = 1.00 kN/m².

The input menus are same as those depicted in Fig. 9.7. Also the procedure to use the software is same as outlined above. Fig.9.18 shows input fuzzy, induced and performance fuzzy sets. A reinforcement detail of slab as carried out by the software is shown in Fig. 9.19. The Comparison of results with available solution [96] is given in Table 9.4.
Fig. 9.19 Reinforcement Details of Slab

Table 9.4 Comparison of Results

<table>
<thead>
<tr>
<th>DATA</th>
<th>REFERENCE [96]</th>
<th>SOFTWARE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of longer span</td>
<td>5.65 m</td>
<td></td>
</tr>
<tr>
<td>Length of shorter span</td>
<td>4.15 m</td>
<td></td>
</tr>
<tr>
<td>Beam width</td>
<td>0.15 m</td>
<td></td>
</tr>
<tr>
<td>Live load</td>
<td>4.00 kN/m²</td>
<td></td>
</tr>
<tr>
<td>Floor finish</td>
<td>1.00 kN/m²</td>
<td></td>
</tr>
<tr>
<td>Grade of concrete</td>
<td>M15</td>
<td></td>
</tr>
<tr>
<td>Grade of steel</td>
<td>Fe415</td>
<td></td>
</tr>
<tr>
<td>Cost of concrete</td>
<td>Rs. 2000/- per m³</td>
<td></td>
</tr>
<tr>
<td>Cost of steel</td>
<td>Rs. 30/- per Kg</td>
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</tbody>
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<table>
<thead>
<tr>
<th>RESULTS</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth of slab</td>
<td>120 mm</td>
<td>170 mm</td>
</tr>
<tr>
<td>Quantity of concrete</td>
<td>3.00 m³</td>
<td>4.24 m³</td>
</tr>
<tr>
<td>Quantity of steel</td>
<td>289 Kg</td>
<td>178 Kg</td>
</tr>
<tr>
<td>Total cost</td>
<td>Rs. 14650/-</td>
<td>Rs. 13820/-</td>
</tr>
<tr>
<td>% Reduction in cost</td>
<td>5.66 %</td>
<td></td>
</tr>
</tbody>
</table>
9.3.11 A Continuous Two-Way Slab Example

❖ **Input data:** Nos. of span in longer direction = 4, Nos. of span in shorter direction = 4, 
  $L_{\text{eff}}$ of each span in longer side = 8.00 m, $L_{\text{eff}}$ of each span in shorter side = 6.00 m, Beam 
  width = 0.25 m, Grade of Concrete = M15, Grade of steel = Fe 415, Live load = 4.00 
  kN/m$^2$ and Floor finish = 1.00 kN/m$^2$.

After supplying geometry data through input form, geometry of slab is automatically created 
as shown in Fig. 9.20. Provisions are made in the software for design of continuous two-way 
slabs with various boundary conditions recognized automatically from geometry form. The 
moment coefficients for slab based on boundary conditions are shown in Fig. 9.21. The Input, 
Induced and Performance fuzzy sets are shown in Fig. 9.22. After optimum value of input 
parameter is obtained, analysis is carried out using coefficient method. The final moments 
and area of steel required are shown in tabular form in Fig. 9.23 (a) and (b) and same are 
displayed in graphical form as in Fig. 9.24 (a) and (b).

Design results for all slabs are depicted in tabular form as shown in Fig. 9.25 whereas 
reinforcement details are shown in Fig. 9.26. A comparison of results with the known 
conventional solution is presented in Table 9.5.
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Fig. 9.21 Moment Coefficients

Fig. 9.22 Input, Induced and Performance Fuzzy Sets

Fig. 9.23(a) Final Moments
### Final moments (kN.m)

### Area of steel required (Sq.mm.)

<table>
<thead>
<tr>
<th>slab</th>
<th>$\text{Ast1} x$</th>
<th>$\text{Ast1} y$</th>
<th>$\text{Ast2} x$</th>
<th>$\text{Ast2} y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>0</td>
<td>546</td>
<td>633</td>
<td>0</td>
</tr>
<tr>
<td>S2</td>
<td>0</td>
<td>480</td>
<td>558</td>
<td>464.8</td>
</tr>
<tr>
<td>S3</td>
<td>0</td>
<td>480</td>
<td>558</td>
<td>397.6</td>
</tr>
<tr>
<td>S4</td>
<td>0</td>
<td>546</td>
<td>633</td>
<td>464.8</td>
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<tr>
<td>S5</td>
<td>633</td>
<td>352.5</td>
<td>532.5</td>
<td>0</td>
</tr>
<tr>
<td>S6</td>
<td>558</td>
<td>333</td>
<td>486</td>
<td>372.4</td>
</tr>
<tr>
<td>S7</td>
<td>558</td>
<td>333</td>
<td>486</td>
<td>397.6</td>
</tr>
<tr>
<td>S8</td>
<td>633</td>
<td>352.5</td>
<td>532.5</td>
<td>372.4</td>
</tr>
</tbody>
</table>

**Fig. 9.23(b) Area of Steel Required**

### Form for Final Moments (kN.m)

**Fig. 9.24(a) Form for Final Moments (kN.m)**
Fig. 9.24(b) Form for Area of Steel (mm$^2$/m)

Fig. 9.25 Design Results
Table 9.5 Comparison of Results

<table>
<thead>
<tr>
<th>DATA</th>
<th></th>
<th>RESULTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of spans in longer direction</td>
<td>4 nos.</td>
<td>200 mm</td>
</tr>
<tr>
<td>No. of spans in shorter direction</td>
<td>4 nos.</td>
<td>170 mm</td>
</tr>
<tr>
<td>L_{eff} of each span in longer side</td>
<td>8.00 m</td>
<td>156.413 m³</td>
</tr>
<tr>
<td>L_{eff} of each span in shorter side</td>
<td>6.00 m</td>
<td>132.951 m³</td>
</tr>
<tr>
<td>Beam width</td>
<td>0.25 m</td>
<td>5646 Kg</td>
</tr>
<tr>
<td>Live load</td>
<td>4.00 kN/m²</td>
<td>6014 Kg</td>
</tr>
<tr>
<td>Floor finish</td>
<td>1.00 kN/m²</td>
<td></td>
</tr>
<tr>
<td>Grade of concrete</td>
<td>M15</td>
<td></td>
</tr>
<tr>
<td>Grade of steel</td>
<td>Fe415</td>
<td></td>
</tr>
<tr>
<td>Cost of concrete</td>
<td>Rs. 2000/- per m³</td>
<td></td>
</tr>
<tr>
<td>Cost of steel</td>
<td>Rs. 30/- per Kg</td>
<td></td>
</tr>
<tr>
<td>% Reduction in cost</td>
<td>7.45 %</td>
<td></td>
</tr>
</tbody>
</table>
9.4 Optimum Design of RCC Plane Frames

Plane frames are made of beam elements and column elements and are meant to support the slabs and transfer the load to the foundation. The developed software contains a module for optimum design of plane frame members. It analyses the frame with direct stiffness method to calculate the member forces. The optimum dimensions of the members are then determined for the calculated forces satisfying all the design constraints using fuzzy logic. The design parameters, constraints and objective function are discussed below in brief.

9.4.1 Design Parameters

The pre-assigned parameters are spans of beams, heights of columns, loading on members and joints, support conditions and grades of materials (concrete and steel). In the optimum design of frame, optimum cross sectional dimensions of the constituent members are worked out and therefore width (b) and depth (d) of the beam and column members are input parameters in this case. Total number of input parameters is twice the total number of columns and beams. For the cross sectional dimensions chosen by FL based optimization, the members are designed to satisfy strength and serviceability criteria. The actual compressive stress in the concrete is taken as the induced fuzzy set parameter and permissible stress is taken as the performance fuzzy set parameter.

9.4.2 Design Constraints

Since the design philosophy for beams and columns is different, the constraints for them are different and are given below.

❖ For Beams

1. \( A_{st} \geq 0.2 \times (b \times d) / 100 \) (geometry constraint) \( \ldots (9.8) \)
2. \( A_{st} \leq A_{st\text{max}} \) (geometry constraint) \( \ldots (9.9) \)
3. \( \tau_v \leq \tau_c\text{max} \) (behavior constraint) \( \ldots (9.10) \)

❖ For columns

1. \( A_{st} \geq 0.8 (Dxx \times Dyy)/100 \) (geometry constraint) \( \ldots (9.11) \)
2. \( A_{st} \leq 6.0 (Dxx \times Dyy)/100 \) (geometry constraint) \( \ldots (9.12) \)

where \( A_{st} = \) Area of steel reinforcement, \( A_{st\text{max}} = \) Maximum area of steel reinforcement, \( \tau_v = \) Average actual shear stress, \( \tau_c\text{max} = \) Maximum permissible shear stress of the beam section and \( Dxx \) and \( Dyy = \) Dimensions of the column member.
9.4.3 Fuzzification of input parameters

Here triangular fuzzy sets are used for simplicity. Figure 9.27 shows fuzzy sets for depth and width of one of the columns and beams. The average of maximum and minimum values supplied by the user is assigned membership grade of 1.

![Membership Function for Width of Column (mm)](image)

![Membership Function for Depth of Column (mm)](image)

![Membership Function for Width of Beam (mm)](image)

![Membership Function for Depth of Beam (mm)](image)

Fig. 9.27 Input Fuzzy Sets for Dimensions of Beam and Column

9.4.4 Development of Performance Fuzzy Set

The performance parameter for this case is compressive stress in concrete. The permissible compressive stress in concrete depends on its grade as indicated in Table 9.6. The variation of the performance parameter is considered linear between $\sigma_{oa}$ and $\sigma_{ob}$ as shown in Fig. 9.28. $\sigma_{ob}$ is maximum permissible value and $\sigma_{oa}$ is 10 percent higher value. The maximum value is assigned zero membership grade and all the values lower by 10 percent or more are assigned the membership grade of 1. The maximum permissible compressive stress depends on the grade of concrete. The performance fuzzy set is actually the set of all the feasible solutions.
### Table 9.6 Permissible Compressive Stress

<table>
<thead>
<tr>
<th>GRADE OF CONCRETE</th>
<th>PERMISSIBLE COMpressive STRESS IN CONCRETE (N/mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M15</td>
<td>6.690</td>
</tr>
<tr>
<td>M20</td>
<td>8.920</td>
</tr>
<tr>
<td>M25</td>
<td>11.150</td>
</tr>
<tr>
<td>M30</td>
<td>13.380</td>
</tr>
<tr>
<td>M35</td>
<td>15.610</td>
</tr>
<tr>
<td>M40</td>
<td>17.840</td>
</tr>
</tbody>
</table>

#### 9.4.5 Development of Induced Fuzzy Set

For development of induced fuzzy set, appropriate values of $\alpha$-cuts, preferably equispaced (0.05, 0.1, 0.15, etc) are selected and crossover points at each $\alpha$-cut are determined for every input parameter. Sets of combination of input parameters are formed at each $\alpha$-cut. $2^n$ combinations are generated where $n$ is number of input variables ($n=2$ for this case). For every set of width and depth of frame member the actual compressive is calculated and plotted against the corresponding membership grade to get induced fuzzy set for that member.

#### 9.4.6 Superimposition of Induced and Performance Fuzzy Set

The induced and performance fuzzy sets are superimposed for every member to get the membership value of the match point. This point gives value of stress ($S'$), which satisfies the functional requirement, and at the same time have the highest performance and its appropriate membership grade ($\mu'$) as shown in Fig. 9.29. The combination of the input variables corresponding to this membership value is obtained, which is the final solution. This process is explained in Fig. 9.29.

---

![Fig. 9.28 Performance Fuzzy Set](image)

![Fig. 9.29 Superimposition of the Fuzzy Sets](image)
9.4.7 Developed Processors

The preprocessor facilitates the entry of input data such as geometry data, support data, load data and material data. Provision is made in the preprocessor to display joint numbers, joint co-ordinates, member numbers, member connectivities, support conditions, joint loads, member loads, and combinations used for induced fuzzy sets.

The main processor develops various fuzzy sets, finds the optimum dimensions and carries out the design process. The various subroutines developed under this processor includes Analysis subroutines, Preliminary design subroutine, Fuzzy logic based subroutines such as Subroutine INPUT FUZZY SETS, Subroutine FUZZY OPT and Design subroutines such as Subroutine BEAM DESIGN, Subroutine SHEAR REIN DESIGN, Subroutine UNIAXIAL COL DESIGN etc.

The post processor provides the analysis results (i.e. joint displacements, member forces, support reactions) in tabular form. In addition to this it has facility to display member force diagrams. The value of these forces can also be obtained at any point within the span. The processor also provides the analysis results and design results of beam and column elements in the report form, the print out of which can be directly taken. It also provides optimum cost results and reinforcement details. It allows the designer to select the diameter of reinforcing bar for required area of steel. For selected diameter the software displays the number of bars.

9.4.8 Two - Bays, Two - Storey Frame Example

Figure 9.30 shows the geometry of a two bay, two-storey frame problem with support conditions and loadings, which is taken for size optimization. The structural (geometry) data of plane frame such as number of bays, number of stories, bay widths, storey heights, etc. are to be supplied using the developed form. The frame with joint and member number is automatically drawn for given structural data, which is shown in Fig. 9.31 along with the dialog box for assigning support conditions. Member loads like point load, UDL, and concentrated moment can be applied using form for member loads, which is shown in Fig. 9.32. Direct stiffness method based analysis of the frame requires the initial section properties, which is assigned by the section properties dialog box as shown in Fig. 9.33. Preliminary design is done based on the initial analysis results. Figure 9.34 shows the analysis results whereas Fig. 9.35 shows internal force diagram for one of the members which is produced by the software.
The software displays the preliminary design results in the tabular form on clicking the preliminary menu which are used for selecting the lower and upper bounds values for the input fuzzy sets. These values can be supplied through the preference menu as shown in Fig. 9.36, which are used to develop fuzzy sets. The developed fuzzy sets for these values are shown in Fig. 9.37 for width and depth of particular member in graphical form. On the same form fuzzy set of performance parameter is superimposed on output fuzzy set. The software finds the combination of B-D, which gives maximum stress within permissible limit out of all combination and that will be optimum width and depth for a particular member. This whole process is performed for each member. The optimum design results and reinforcement details in tabular form are depicted in Figs. 9.38 and 9.39. Figure 9.40 gives the optimum cost obtained based on fuzzy logic. Final design reports are shown in Fig. 9.41(a) and (b). The cost of structure based on FL is found as Rs. 10196/- whereas based on GA it is Rs. 9750.0/-. From the comparison it is clear that fuzzy logic gives economical design taking in to consideration fuzziness and uncertainty in the design whereas GA gives near optimal solution (more economical design) without worrying about uncertainty.
Fig. 9.31 Menu for Support Conditions

Fig. 9.32 Menu for Member Loads
9. FL Based Software Development and Applications

Fig. 9.33 Menu for Cross-section Properties

Fig. 9.34 Display of Analysis Results
Fig. 9.35 Display of Axial - Shear Force and Bending Moment Diagrams

Fig. 9.36 Preference Menu
Fig. 9.37 Input Fuzzy Sets and Superimposition of Fuzzy Sets

Fig. 9.38 Display of Design Results
### 9. FL Based Software Development and Applications

**REINFORCEMENT DATA**

**For Beam**

- **Beam Number**: 4
- **Section**: At K-end
- **Bottom steel (Ast)**: 78.54
  - Dia.: 10
  - Nos.: 2
- **Top steel (Asc)**: 223.37
  - Dia.: 12
  - Nos.: 2

<table>
<thead>
<tr>
<th>Beam No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width (mm)</td>
<td>188</td>
<td>191</td>
<td>190</td>
</tr>
<tr>
<td>Depth (mm)</td>
<td>238</td>
<td>246</td>
<td>243</td>
</tr>
<tr>
<td>Ast at End</td>
<td>2 - 10</td>
<td>2 - 10</td>
<td>2 - 10</td>
</tr>
<tr>
<td>Ast at Mid</td>
<td>2 - 12</td>
<td>2 - 12</td>
<td>2 - 12</td>
</tr>
</tbody>
</table>

### Fig. 9.39 Reinforcement Details

**Column Number**

- **Main steel**: 484.13
  - Dia.: 12
  - Nos.: 5

<table>
<thead>
<tr>
<th>Column No.</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width (mm)</td>
<td>208</td>
<td>207</td>
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<td>208</td>
</tr>
<tr>
<td>Depth (mm)</td>
<td>208</td>
<td>207</td>
<td>246</td>
<td>208</td>
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<tr>
<td>Main steel</td>
<td>4 - 12</td>
<td>4 - 12</td>
<td>5 - 12</td>
<td>4 - 12</td>
</tr>
</tbody>
</table>

### COST OPTIMIZATION

**Cost of Formwork (per sq. m.)**: 20
**Cost of Concrete (per cu. m.)**: 2000
**Cost of Steel (per kg)**: 35

**For Beams**

<table>
<thead>
<tr>
<th>Beam</th>
<th>Cost of Formwork</th>
<th>Cost of Conc</th>
<th>Cost of Steel</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>53</td>
<td>358</td>
<td>790</td>
<td>1201</td>
</tr>
<tr>
<td>2</td>
<td>55</td>
<td>376</td>
<td>682</td>
<td>1113</td>
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<td>3</td>
<td>54</td>
<td>369</td>
<td>758</td>
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<tr>
<td>4</td>
<td>60</td>
<td>440</td>
<td>661</td>
<td>1161</td>
</tr>
</tbody>
</table>

**For Columns**

<table>
<thead>
<tr>
<th>Column</th>
<th>Cost of Formwork</th>
<th>Cost of Conc</th>
<th>Cost of Steel</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>50</td>
<td>260</td>
<td>496</td>
<td>806</td>
</tr>
<tr>
<td>6</td>
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</tr>
<tr>
<td>7</td>
<td>59</td>
<td>363</td>
<td>620</td>
<td>1042</td>
</tr>
<tr>
<td>8</td>
<td>50</td>
<td>260</td>
<td>496</td>
<td>806</td>
</tr>
<tr>
<td>9</td>
<td>50</td>
<td>260</td>
<td>496</td>
<td>806</td>
</tr>
</tbody>
</table>

**Optimum Cost of Frame**: 10196

**Fig. 9.40 Calculation of Optimum Cost**
Fig. 9.41 (a) Beam Design Report

Fig. 9.41 (b) Column Design Report
9.5 Optimum Design of Grid Structures

The approach to design for torsion accompanied by bending is followed as per IS: 456 [83] for the design of grid members. In this approach bending moment and shear force are coupled with twisting moment to get equivalent moment and equivalent shear respectively. Then longitudinal reinforcement and transverse stirrups are designed respectively for equivalent bending moment and shear force.

The objective of FL based design in such problems is to minimize the cost of the grid structure subject to various design constraints such as maximum stress in the grid member. Similar to the optimum design of plane frames, in grid structures also the optimization is carried out member wise, which minimizes the cost (as objective function) of each member separately and thus finally for the complete structure.

9.5.1 Design Parameters

❖ Constant parameters

- **Geometry**: This entails the number of bays in X-direction and Z-direction, bay widths, member connectivity conditions and support conditions.

- **Loads**: These include dead load due to slabs, partition walls and nonstructural elements. It does not include the weight of beams since these are the function of design variables.

- **Material properties**: These include the grade of concrete, grade of steel, and modulus of elasticity.

❖ Input parameters

Two design variables describing the cross-sectional dimensions of rectangular beams; namely, width $B$ and depth $D$ are considered here as input parameters.

❖ Performance and induced fuzzy set parameters

Here the permissible and actual compressive stress in concrete are considered as performance and induced fuzzy set parameters respectively for beams.

9.5.2 Design Constraints

❖ For Beams

\[(1) \ A_{sl} \geq 0.2 \times (b \times d) /100 \quad (2) \ A_{sl} \leq A_{sl \text{max}} \quad (3) \ \tau_{ve} \leq \tau_{c \text{max}} \]
where \( A_{st} \) = area of reinforcement of the section, \( b \) and \( d \) = breadth and depth of a beam section respectively, \( A_{st,\text{max}} \) = maximum permissible area of tension steel, \( \tau_{ve} \) = equivalent shear stress at the section and \( \tau_{c,\text{max}} \) = maximum shear stress.

### 9.5.3 Objective Function

For the RCC structure as unit costs of both steel and concrete are different, cost is considered as the objective function here which is given by

\[
O(x) = V_c C_c + W_s C_s + A_f C_f \quad \ldots (9.13)
\]

where \( O(x) \) = objective function which is total cost of an element, \( V_c \) = volume of concrete, \( W_s \) = weight of steel, \( A_f \) = area of formwork, and \( C_c, C_s, C_f \) = unit cost of concrete, steel and formwork respectively.

This objective is achieved indirectly by maximizing the design constraint such as stress.

### 9.5.4 \( \alpha \)-Cut Procedure

Here width \( B \) and depth \( D \) of beams are considered as input parameters. These parameters are fuzzified by assigning the maximum and minimum values a membership grade 0 and the average of these a membership grade of 1. Here permissible compressive stress in concrete is considered as performance parameter. Its value depends upon the grade of concrete. The actual compressive stress in concrete is considered as induced fuzzy set parameter. In this problem 0.01 is selected as value of \( \alpha \)-cut. At each \( \alpha \)-cut, \( 2^2 \) i.e. four crossover points and total \( 2^2 \) i.e. four combinations are generated as discussed earlier. The value of induced fuzzy set parameter for each is computed by expression \( (M_{\text{max}} / Z) \). Where \( M_{\text{max}} \) is maximum moment in particular member, and \( Z \) is the modulus of section for the same member. The induced fuzzy set is superimposed on performance fuzzy set to find the match point and corresponding \( S' \) and \( \mu' \) values. The respective combination of input parameters gives the optimum solution. The grid members are then designed for these dimensions.

### 9.5.5 Seven Member Grid Example

The structural geometry, support conditions, loadings are displayed in Fig. 9.42. Concrete of grade M20 and steel of grade Fe415 are considered. Solution of the problem using the graphical user interface developed in the software is explained here in detail. The joint coordinates and member connectivity can be supplied using geometry menu of the main form of
the program, which invokes the form shown in Fig. 9.43. Upon supplying the geometry data through this form, the sketch shown in Fig. 9.44 is displayed.

![Fig. 9.42 Geometry of Grid](image)

The support conditions and loadings can be given by clicking Supports and Loading menu from the same form. Section properties can be assigned to grid members using C/S Properties menu, which is depicted in Fig. 9.44. Figure 9.45 indicates the analysis results obtained by the software and Fig. 9.46 shows internal force diagrams.

The input parameters are supplied through preference menu as shown in Fig. 9.47. The developed fuzzy sets for these values are shown in Fig. 9.48. The optimum design results and reinforcement details are depicted in Fig. 9.49 and Fig. 9.50 respectively. Figure 9.51 gives the optimum cost obtained based on fuzzy logic. The report showing design results is depicted in Fig. 9.52 whereas Table 9.7 shows the final results.

![Fig. 9.43 Joint Co-ordinates and Member Connectivity](image)
Fig. 9.44 Geometry and Section Properties

Fig. 9.45 Analysis Results
Fig. 9.46 Internal Force Diagrams

Fig. 9.47 Preference Menu
INPUT FUZZY SETS

Member Number | Width of Hor. Member (mm) | Depth of Hor. Member (mm)

- Width of Hor. Member (mm)
- Depth of Hor. Member (mm)

Compressive stress in concrete (N/mm²)

INPUT FUZZY SETS

Fig. 9.48 Fuzzy Sets

INDUCED AND PERFORMANCE FUZZY SETS

Fig. 9.49 Design Results

DESIGN RESULTS

<table>
<thead>
<tr>
<th>Beam No.</th>
<th>Length (mm)</th>
<th>Width (mm)</th>
<th>Depth (mm)</th>
<th>Ast at J-end (mm²)</th>
<th>Ast at Mid (mm²)</th>
<th>Ast at K-end (mm²)</th>
<th>Asc at J-end (mm²)</th>
<th>Asc at Mid (mm²)</th>
<th>Asc at K-end (mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2000</td>
<td>225</td>
<td>300</td>
<td>78.54</td>
<td>176.88</td>
<td>78.54</td>
<td>480.01</td>
<td>78.54</td>
<td>138.25</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>185</td>
<td>220</td>
<td>78.54</td>
<td>70.1</td>
<td>78.54</td>
<td>95.39</td>
<td>78.54</td>
<td>83.36</td>
</tr>
</tbody>
</table>

Fig. 9.49 Design Results
<table>
<thead>
<tr>
<th>Beam No.</th>
<th>Length (mm)</th>
<th>Width (mm)</th>
<th>Depth (mm)</th>
<th>Ast at J-end</th>
<th>Ast at Mid</th>
<th>Asc at J-end</th>
<th>Asc at Mid</th>
<th>Asc at K-end</th>
<th>Minimum distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5000</td>
<td>218</td>
<td>309</td>
<td>2 - 10 M</td>
<td>2 - 12 M</td>
<td>2 - 10 M</td>
<td>2 - 12 M</td>
<td>2 - 10 M</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>5000</td>
<td>182</td>
<td>225</td>
<td>2 - 10 M</td>
<td>2 - 10 M</td>
<td>2 - 10 M</td>
<td>2 - 10 M</td>
<td>2 - 10 M</td>
<td></td>
</tr>
</tbody>
</table>

**Fig. 9.50 Reinforcement Details**

**Fig. 9.51 Optimum Cost**
Table 9.7 Results Based on FL

<table>
<thead>
<tr>
<th>DATA</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
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<tbody>
<tr>
<td>Nos. of bays in X-direction</td>
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<td>Nos.</td>
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<tr>
<td>Nos. of bays in Z-direction</td>
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<td>Nos.</td>
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<td></td>
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<tr>
<td>Length of each bay in X-direction</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Length of each bay in Z-direction</td>
<td>2.00 m</td>
<td></td>
<td></td>
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<td>Grade of concrete</td>
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<td>Grade of steel</td>
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<td>Cost of formwork</td>
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<tr>
<td>Cost of steel</td>
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<table>
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<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>Width of member</td>
<td>218</td>
<td>182</td>
<td>218</td>
<td>202</td>
<td>202</td>
<td>202</td>
<td>202</td>
</tr>
<tr>
<td>Depth of member</td>
<td>309</td>
<td>225</td>
<td>309</td>
<td>272</td>
<td>272</td>
<td>272</td>
<td>272</td>
</tr>
<tr>
<td>Total cost (Rs.)</td>
<td>1924</td>
<td>2176</td>
<td>1924</td>
<td>845</td>
<td>757</td>
<td>757</td>
<td>673</td>
</tr>
<tr>
<td>Overall cost (Rs.)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>9056</td>
</tr>
</tbody>
</table>
9.6 **Optimum Design of Retaining Wall**

In case of cantilever retaining wall both stem and base slab act as individual cantilever slabs. So for optimum design of retaining wall it is necessary to design each component individually. The software finds out the optimum dimensions and reinforcement areas using FL based optimization module taking in to account the latest IS code provisions. The results obtained for stem and base dimensions along with the cost of retaining wall are compared with those based on conventional design.

9.6.1 **Design Parameters**

In cantilever retaining wall design, pre-assigned parameters are height of backfill, density and angle of repose of backfill, co-efficient of friction between wall and material, grade of concrete and grade of steel. When top surface of backfill is not horizontal then angle of repose is also to be provided in advance. When backfill is subjected to extra surcharge then intensity of surcharge is also considered as pre assigned parameter. In design of base slab, thickness of base and percentage steel required in base are input parameters. In design of stem, thickness of stem and percentage steel required are main input parameters. The tensile stress in steel should be less than the permissible tensile stress in steel, so actual tensile stress in steel is considered as output parameter and permissible tensile stress in steel is taken as performance parameter.

9.6.2 **Constraints**

❖ **Geometry Constraints**

Geometry constraints are the restriction imposed over the geometry of the structure.

1. $0.15 \, \text{m} \leq d_b \leq 0.5 \, \text{m}$
2. $0.20 \, \text{m} \leq d_s \leq d_p + 0.25 \, \text{m}$
3. $0.12\% \leq p_{tb} \leq 1\%$
4. $0.12\% \leq p_{ts} \leq 1\%$ ... (9.14)

❖ **Behavior Constraints**

Behavior constraints are the restriction imposed over the performance of the structure.

1. (F.O.S) sliding $\geq 1.55$
2. (F.O.S) overturning $\geq 1.55$
3. $P_{\text{max}} = \frac{\Sigma W + \Sigma M}{A} \leq SBC$
4. $P_{\text{min}} = \frac{\Sigma W - \Sigma M}{A} \geq 0$ ... (9.15)

where $d_b =$ Thickness of base, $d_s =$ Thickness of stem, $d_p =$ Preliminary available depth of stem, $p_{tb} =$ Percentage reinforcement in base, $p_{ts} =$ Percentage reinforcement in stem, $P_{\text{max}} =$ Maximum base pressure on the soil, $P_{\text{min}} =$ Minimum base pressure on soil, $\Sigma \, W =$ Total
vertical force to which the base is subjected, $\Sigma M = $ Moment about the center line of the base, $A = $ Area of base, $Z = $ Section modulus and $SBC = $ Soil bearing capacity.

### 9.6.3 Fuzzification of Input Parameters

In optimization of retaining wall, optimum RC section of base slab and stem are determined. In each case thickness and percentage steel are input parameters and corresponding input fuzzy sets are shown in Fig. 9.53 (a) and (b).

#### 9.6.4 Development of Performance Fuzzy Set

Here the performance parameter is tensile stress in steel, which depends upon grade of steel. The membership function for performance parameter is considered to be of trapezoidal shape as shown in Fig. 9.54. The same performance fuzzy set is used for both design of stem and base.

![Fig. 9.53 Input parameters](image)

![Fig. 9.54 Performance fuzzy set](image)

![Fig. 9.55 Induced fuzzy set](image)
9.6.5 Development of Induced Fuzzy Set
For developing induced fuzzy set it is necessary to decide the value of $\alpha$-cut. Here 0.01 is selected as value of $\alpha$-cut. At each $\alpha$-cut there will be $2 \times 2$ i.e. four crossover points $d_b$, $d_s$, $p_b$ and $p_s$. At each $\alpha$-cut, output values are calculated for all four combinations and plotted against their membership value to have induced fuzzy set graph. Figures 9.55 shows induced fuzzy set for base and stem for only two combinations.

- **For Base**: For optimum design of base, first of all it optimizes the heel and finds the most acceptable combination of depth of base and percentage steel. For the same depth, the required percentage of steel for toe portion is calculated. After having net pressures on the heel, it is possible to calculate total shear and moment acting on the heel. If heel is subjected to shear force $v_a$ and bending moment $m_{ub}$ then with combination of $d_{xb}$-$p_{xb}$ at any $\alpha$-cut the actual tensile stress is given by

$$\sigma_{t,act, b} = \frac{(m_{ub} \times 1000000)}{(a_{stb} \times j \times d_{xb})} \quad \text{... (9.16)}$$

where $\sigma_{t,act, b}$ = actual tensile stress in steel in base, $m_{ub}$ = factored bending moment for the heel and $a_{stb} = p_{xb} \times 1000 \times d_{xb}/100$.

- **For Stem**: Stem is subjected to moment due to backfill. Maximum moment occurs at the base of stem. If $m_{us}$ is moment at the base of stem then for any combination of $d_{xs}$-$p_{xs}$,

$$\sigma_{t,act, s} = \frac{(m_{us} \times 1000000)}{(a_{sts} \times j \times d_{xs})} \quad \text{... (9.17)}$$

where $\sigma_{t,act, s}$ = actual tensile stress in steel in stem, $m_{us}$ = factored moment acting at base of stem, and $a_{sts} = p_{xs} \times 1000 \times d_{xs}/100$.

9.6.6 Superimposition of Induced and Performance Fuzzy Sets
On superimposition it is possible to have intersecting point. This point gives value of stress ($S_o$), which satisfies the functional requirement at the same time has the highest performance and its appropriate membership grade ($M_o$) as shown in Fig. 9.56.
9.6.7 FL Based Processors

Preprocessor: The form developed under this processor accepts the data such as height of backfill, density and angle of internal friction of material, angle of repose, intensity of surcharge, the grade of concrete and grade of steel.

Main Processor: This module gives optimum thickness and steel for base as well as for stem. After fuzzification of input parameters, at each \( \alpha \)-cut four stress values are obtained corresponding to four combinations and graphs are displayed on the screen corresponding to maximum and minimum stress. On the same graph fuzzy set of performance parameter is superimposed. Then the software finds the optimum combination of \( d \)-pt, which gives maximum stress within permissible limit out of all combinations which corresponds to the optimum thickness and percentage steel for base. Similarly the software finds the optimum thickness of stem at base and percentage steel required in stem.

Post Processor: The post processor shows the detailing of all components. Using the optimum percentage of steel and the depth, the total area of steel for toe, heel and stem are calculated and are displayed on the screen.

9.6.8 Example of a Cantilever Retaining Wall

Following data [85] is considered for the design of retaining wall: Height of retain = 4 m, Density of backfill and foundation material = 17 kN / m\(^3\), Angle of internal friction for backfill and foundation material = 30 degree, Co-efficient of friction between base and material = 0.55, Intensity of surcharge = 0, Angle of repose = 30, Grade of concrete = M15 and Grade of steel = Fe 415.
The data is supplied through the form depicted in Fig. 9.57. The lower and upper bound values of the input fuzzy sets are supplied through one of the forms developed under this software. Using these fuzzy sets, induced fuzzy sets are developed and are superimposed on performance fuzzy set to get the final results. Figure 9.58 shows the screen shot of superimposed fuzzy sets for stem and base. The reinforcement detail for the same is displayed in Fig. 9.59.

<table>
<thead>
<tr>
<th>Backfill Material Properties</th>
<th>Foundation Material Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (kN/m³)</td>
<td>Density (kN/m³)</td>
</tr>
<tr>
<td>Angle of Internal Friction</td>
<td>Angle of Internal Friction (degree)</td>
</tr>
<tr>
<td>Inclination of Top Surface</td>
<td>Soil Bearing Capacity</td>
</tr>
<tr>
<td>Intensity of Uniform Surcharge</td>
<td>Co-efficient of Friction Between Wall and Base</td>
</tr>
<tr>
<td>17</td>
<td>17</td>
</tr>
<tr>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

- Concrete Grade: M15
- Reinforcement Grade: Fe415

**Fig. 9.57 Form for Supply of Input**

**Fig. 9.58 Superimposed Fuzzy Sets for Stem and Base**
Comparison of results with the available solution (Table 9.8) indicates reduction in cost by about 8%.

Table 9.8 Comparison of Results

<table>
<thead>
<tr>
<th>RESULTS</th>
<th>REFERENCE[85]</th>
<th>SOFTWARE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width of Base</td>
<td>2.7 m</td>
<td>2.5 m</td>
</tr>
<tr>
<td>Thickness of Base</td>
<td>350 mm</td>
<td>290 mm</td>
</tr>
<tr>
<td>Thickness of Stem at Base</td>
<td>350 mm</td>
<td>260 mm</td>
</tr>
<tr>
<td>Cost</td>
<td>59970/-</td>
<td>55195/-</td>
</tr>
</tbody>
</table>

9.7 **COST OPTIMIZATION OF ISOLATED FOOTING**

9.7.1 **Design Parameters**

In the case of isolated footing the pre-assigned parameters are soil bearing capacity, size of column, total axial load, total moments in both direction, horizontal force acting in x and y direction, grade of concrete for column, grade of concrete for footing and grade of steel. In the design of isolated footing, width of footing, length of footing and depth of footing are
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main input parameters. As the design is mostly governed by two-way shear, it is considered as the output parameter and the performance parameter is two-way shear strength of concrete, which depends on the grade of concrete.

9.7.2 Design Constraints

❖ Geometry constraints:
1. \( b_{\text{min}} \leq b \leq b_{\text{max}} \)
2. \( L_{\text{min}} \leq L \leq L_{\text{max}} \)
3. \( (b - b_{c}) = (L - L_{c}) \)
4. \( 0.4 \text{m} < d < 1.5 \text{m} \)
5. \( 0.2\% < p_t \) ... (9.18)

where, \( L \) = length of footing, \( b \) = width of footing, \( L_{c} \) = length of column, \( b_{c} \) = width of column, \( L_{\text{min}} \) = lower bound value of length, \( L_{\text{max}} \) = upper bound value of length, \( b_{\text{min}} \) = lower bound value of width, \( b_{\text{max}} \) = upper bound value of width and \( d \) = depth of footing at the face of column.

❖ Behavior Constraints:
1. \( p_{\text{max}} = \frac{P}{A} + \frac{M}{Z} < S.B.C. \)
2. \( p_{\text{min}} = \frac{P}{A} - \frac{M}{Z} > 0 \)
3. \( \tau_{v} \leq \tau_{c} \) for one-way shear
4. \( \tau_{v} \leq K_s \cdot \tau_{c} \) for two-way shear ... (9.19)

where \( p_{\text{max}} \) = maximum pressure developed below footing, \( p_{\text{min}} \) = minimum pressure developed below footing, \( P \) = total axial load on the column, \( M \) = total moment on the column, \( A \) = area of footing, \( Z \) = section modulus of the footing area, \( \tau_{v} \) = shear stress developed for one way and two way shear check at a suitable section in both cases, \( \tau_{c} \) = shear strength of concrete which depends upon the grade of concrete, \( K_s = (0.5 + \beta_{c}) \) but not greater than 1 with \( \beta_{c} \) being the ratio of short side to long side of the column.

9.7.3 \( a \)-Cut Procedure

In footing design problem, width of footing and depth of footing at the face of column are the main input parameters. The triangular fuzzy set for each input parameter is taken in which lower and upper bound values are assigned membership grade of 0, and average of them takes the membership grade of 1. The performance parameter is two way shear strength of concrete, which depends upon the grade of concrete. The critical section for two-way shear is at \( d/2 \) from the face of the column where \( d \) is effective depth at the face of column. The design shear stress is taken as \( k_s \cdot \tau_{c} \) where \( k_s \) and \( \tau_{c} \) are as defined above. The values \( k_s \cdot \tau_{c} \) and lower are assigned membership grade 1 and 10% higher value is assigned membership grade 0. Thus performance fuzzy set is considered to be of trapezoidal shape.
At each \( \alpha \)-cut (\( \alpha = 0.01 \)) four crossover points \( b_1, b_2, d_1 \) and \( d_2 \) are available with which total four combinations (\( b_1-d_1, b_1-d_2, b_2-d_1, b_2-d_2 \)) are generated. For each combination the value of output parameter is calculated as follows. If \( V_p \) is maximum shear force for two way shear stress at a distance \( d/2 \) from column then

\[
V_u = 1.5 * V_p \quad \text{and} \quad \tau_v = V_u / bd.
\]  

(9.20)

where \( V_p \) = maximum shear force, \( V_u \) = maximum factored shear force, \( \tau_v \) = actual shear stress, \( b \) = width of footing and \( d \) = depth of footing at the face of column. If all four combinations are generated at all \( \alpha \)-cut and plotted against their membership grade then the available graph is the output fuzzy set.

Superimposition of two plots gives an intersection point having value of stress \( (S_0) \), which satisfies the functional requirement and at the same time has the highest performance and its appropriate membership grade \( (M_0) \) as shown in Fig. 9.60. This membership grade will give all corresponding crossover points and selected stress will decide most acceptable combination.

![Fig. 9.60 Superimposition of fuzzy sets](image)

**9.7.4 Developed Processors**

Preprocessor facilitates interactive data entry of soil data, column and load data and material data. The load data supplied is displayed in a graphical form to confirm the data entry. The software also provides the form for entering the lower and upper bound values for the input parameters. The main processor starts with the display of the form showing the triangular input fuzzy sets developed based on the values supplied by the user. The processor then takes the input values from the input fuzzy set and calculates two-way shear stress at each \( \alpha \)-cut.
It generates four combinations and all corresponding output values are plotted against their membership grade to get output fuzzy sets. This fuzzy set is then superimposed on performance fuzzy set to get the optimum solution. After having the optimum width and depth of the foundation, the software calculates pressure distribution below the foundation, calculates the moments at the face of column and designs the foundation. The post processor shows the superimposition of two fuzzy plots. It also provides the graphical display of the pressure diagram and final output in tabular and graphical form.

9.7.5 Example of Isolated Footing

To facilitate comparison with the available solution, following data is considered.

1. Soil Bearing Capacity = 220 kN / m²
2. Size of Column = 230 x 600 mm
3. Moment about its major axis = 100 kNm
4. Axial Load = 600 kN
5. Grade of concrete for column = M30
6. Depth of Footing = 1.6 m
7. Grade of concrete for footing = M15
8. Grade of Steel = Fe 415.

Figures 9.61 to 9.63 show the forms for supplying respectively soil data, column data and lower and upper bounds for footing. The graphical display of the column data is produced by the program to confirm the correct data entry and is reproduced in Fig. 9.64. The input fuzzy sets produced depending on the data supplied is indicated in Fig. 9.65. Figure 9.66 depicts the superimposition plot produced by the program. Pressure diagram plotted by the postprocessor for optimum dimensions is displayed in Fig. 9.67.

![Fig. 9.61 Soil Data Form](image-url)
COLUMN DATA

* Type of column
* Size of column
** Col Length (X-Dir) L (mm)
** Col Width (Y-Dir) B (mm)

Fig. 9.62 Column Data Form

Geometry Of Footing

* Min Length in X-Dir (Lmin) 0.5 M
* Max Length in X-Dir (Lmax) 4 M
* Min Width in Y-Dir (Bmin) 0.5 M
* Max Width in Y-Dir (Bmax) 4 M
* Depth of Footing below G.L. 1.6 M

Fig. 9.63 Form for Supplying Lower and Upper Bound Values

Load and Moment on the Column

Fig. 9.64 Display of Loading on Column
The final output results are shown in tabular and graphical forms as in Figures 9.68 and 9.69. A comparison of results, given in Table 9.9 indicates that the design based on the fuzzy logic leads to an economical design. A reduction of 22% in the cost of foundation is found for the example undertaken in the present work.
### Table 9.9 Comparison of Results

<table>
<thead>
<tr>
<th>DESIGN PARAMETERS</th>
<th>REFERENCE [85]</th>
<th>FL BASED SOFTWARE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of footing</td>
<td>2.7 m</td>
<td>2.328 m</td>
</tr>
<tr>
<td>Width of footing</td>
<td>1.7 m</td>
<td>1.957 m</td>
</tr>
<tr>
<td>Depth of footing at face of column</td>
<td>520 mm</td>
<td>574 mm</td>
</tr>
<tr>
<td>Depth of footing at edge</td>
<td>520 mm</td>
<td>150 mm</td>
</tr>
<tr>
<td>Steel parallel to longer side</td>
<td>1670 mm²</td>
<td>2150 mm²</td>
</tr>
<tr>
<td>Steel parallel to shorter side</td>
<td>2580 mm²</td>
<td>2490 mm²</td>
</tr>
<tr>
<td>Cost</td>
<td>5865/-</td>
<td>4530/-</td>
</tr>
</tbody>
</table>
9.8 Cost Optimization of Combined Footing

Here optimum design of combined footing, and strap footing for given geometry restriction of plan dimensions, soil data, and loads based on fuzzy logic concept is considered. The optimum design of combined footing involves optimization of thickness and that of strap footing involves optimization of plan dimensions and thickness and depth of strap beam.

9.8.1 Design Parameters

Pre-assigned parameters are, (i) Geometry data which includes the distance of columns from restraint edge, c/c distance between the columns, size of columns, and length or width of footing, (ii) Soil data which includes type of soil, soil bearing capacity, unit weight of soil and angle of repose, (iii) Load data which entails total axial load, total moments in both directions, horizontal force acting in X and Y direction and (iv) Material data which includes grade of concrete for column, grade of concrete for footing and grade of steel.

Here, design of rectangular combined footing is considered as constant thickness pad footing. So thickness of footing is considered as main input parameter. The design of strap footing involves design of footing and strap beam. For design of footing, width and thickness of footing are considered as input parameters. In design of strap beam, the depth of beam is considered as input parameter.

In design of rectangular combined footing, thickness of footing is determined considering that the shear is resisted without shear reinforcement. Hence, the shear strength of concrete is considered as performance parameter and shear stress developed for one-way shear is considered as induced fuzzy set parameter. In case of strap footing, the performance and induced fuzzy set parameters are same as discussed above for design of footing. For design of strap beam the permissible and actual compressive stress in concrete is considered as performance and induced fuzzy set parameters respectively.

9.8.2 Design Constraints

❖ Geometry constraints

Following are the main geometry constraints.

1. $0.20 \text{ m} \leq d \leq 2.00 \text{ m}$
2. $pt \geq 0.20 \%$.

... (9.21)

where $d = \text{depth of footing}$ and $pt = \text{percentage of steel}$. 

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❖ Behavior constraints

1. \( p_{\text{max}} = \frac{(P/A)}{Z} + \frac{(M/Z)}{S.B.C.} \)
2. \( p_{\text{min}} = \frac{(P/A)}{Z} - \frac{(M/Z)}{\geq 0} \)
3. \( \tau v \leq \tau c \) for one-way shear
4. \( \tau v \leq ks \times \tau c \) for two-way shear

\[ \text{(9.22)} \]

where \( p_{\text{max}} \) = maximum pressure developed below footing, \( p_{\text{min}} \) = minimum pressure developed below footing, \( P \) = Total axial load on the column, \( M \) = Total moment on the column, \( A \) = Area of footing, \( Z \) = Section modulus of the footing area, \( \tau v \) = Shear stress induced at a critical section due to one-way and two-way shear, \( \tau c \) = Shear strength of concrete which depends upon the grade of concrete and \( ks = (0.5 + \beta c) \) but not greater than 1, \( \beta c \) being the ratio of short side to long side of the column.

9.8.3 Fuzzification of Input Parameter

For rectangular combined footing, thickness of footing is considered as input parameter. Given input parameter \( df \) take values in a set \( Df \) that is usually an interval of real numbers. The triangular fuzzy set \( F Df \) express different values of input parameter depend on the preferences. The minimum \( (Df)_{\text{min}} \) and maximum \( (Df)_{\text{max}} \) values are assigned membership value equal to zero. The average value \( (Df)_{\text{avg}} \) of minimum and maximum values may be considered as most acceptable value and is assigned membership value equal to one.

The strap footing involves design of footings and design of strap beam. In design of footings, width and thickness of footing are considered as input parameters. The depth of strap beam is considered as input parameter in case of design of strap beam. The procedure to develop fuzzy sets is same as outlined above.

9.8.4 Development of Performance Fuzzy Set

In case of rectangular combined footing, the shear strength of concrete is taken as performance parameter which depends upon the grade of concrete and minimum percentage steel. This value is the most acceptable value and is assigned membership grade value equal to one. The value 10 % larger than the permissible value takes membership grade value equal to zero with trapezoidal variation of performance parameter with its membership grade. Table 9.10 shows the values of shear strength for different grade of concrete at 0.25 % steel. It also shows the permissible stress values for different grades of concrete.
In case of strap footing, the shear strength of concrete is considered as performance parameter in design of footings. The procedure to develop performance fuzzy set is same as discussed above.

In design of strap beam, the permissible compressive stress in concrete is considered as the performance parameter. Its value depends upon the grade of concrete. Here also preferences for different values are expressed by the linear variation having the variation of performance parameter trapezoidal with its membership grade.

### 9.8.5 Induced Fuzzy Sets

In rectangular combined footing, 0.01 is selected as value of $\alpha$-cut. At each $\alpha$-cut there will be two crossover points $d_{f1}$ and $d_{f2}$. Thus there will be two values of induced fuzzy set parameter at each $\alpha$-cut. If at all $\alpha$-cuts $\tau_v$ values are plotted against their membership value then available graph is considered as induced fuzzy sets. If $V$ is maximum shear force for one-way shear stress at a distance $d$ from column face then

\[
\tau_v = \frac{1.5 \times V}{b \times de}
\]  

... (9.23)

where $\tau_v$ = Actual shear stress, $V$ = Maximum shear force, $b$ = Width of footing and $de$ = Effective depth of footing at the face of column.

In optimum design of strap footing, shear stress for one-way shear for footing and actual compressive stress in concrete for strap beam is considered as induced fuzzy set parameters.

In case of footing, at each $\alpha$-cut there will be 2*2 i.e. four crossover points, $b_1$, $b_2$, $d_{f1}$, and $d_{f2}$. For each $\alpha$-cut the value of induced fuzzy set parameter is calculated as outlined above and plotted against respective membership value.
In case of strap beam, the actual compressive stress in concrete is considered as induced fuzzy set parameter. At each \( \alpha \)-cut there will be two crossover points, \( db_1 \) and \( db_2 \). For each crossover point the value of induced fuzzy set parameter is computed by expression \( \frac{M_{max}}{Z} \), where \( M_{max} \) is maximum moment in the strap beam and \( Z \) is the modulus of section of strap beam.

**9.8.6 Superimposition of Two Plots**

The two fuzzy sets are superimposed to obtain the match point which gives value of stress \((S')\) and its membership grade \((\mu')\) as shown in Fig. 9.70. The input parameters combination corresponding to \( \mu' \) is considered as the most acceptable combination.

**Fig. 9.70 Superimposition of Performance and Induced Fuzzy Sets**

![Fig. 9.70 Superimposition of Performance and Induced Fuzzy Sets](image)

**9.8.7 Processors Developed**

- **Pre-processor**: Various menus developed in the pre-processor for the supply of input are:
  1. Geometry menu, 
  2. Soil data menu, 
  3. Material menu and 
  4. Load data menu. On clicking these menus respective forms will open through which the related data can be supplied. The software provides data entry for strap footing as well as combined footing.

- **Main Processor**: The analysis is carried out for following three loading conditions:
  1. \( C1 - DL + LL \) and \( C2 - DL + LL \)
  2. \( C1 - DL \) and \( C2 - DL + LL \)
  3. \( C1 - DL + LL \) and \( C2 - DL \)
The maximum values of axial loads, moments and horizontal shear obtained from various combinations are considered for design. The software shows the analysis results also. FL based optimum design is carried out by clicking on optimization menu which calculates the optimum depth of footing and then design of the footing is carried as per codal provisions [83].

The optimum design of strap footing involves optimization of footing dimensions and optimization of depth of strap beam. For the optimization of footing, the width and thickness of footing are considered as input parameters. The software develops the induced and performance fuzzy sets and superimposes them. Finally it gives the optimum value of width of footing and thickness of footing. After having the dimensions of footing, the analysis is carried out.

In case of strap beam, the depth of beam is considered as input parameter. The performance fuzzy set is developed by the program for the permissible compressive stress values which depends on the grade of concrete. The maximum moment is considered for computing induced fuzzy set parameter which is actual compressive stress in concrete. Upon superimposing the two sets the software gives the geometry of strap footing with optimum dimensions. The software then performs the design as per the code [83].

❖ Post-processor: The post-processor is developed to give the design results in the tabular form and to provide reinforcement detailing of footing in graphical form.

9.8.8 A Rectangular Combined Footing Example

Data: C/C distance between columns = 3.00 m, Size of column C1 = 0.30 × 0.30 m, Size of column C2 = 0.30 × 0.30 m, Grade of concrete for columns = M20, Grade of steel for columns = Fe 415, Dead load on column C1 = 480 kN, Dead load on column C2 = 610 kN, Live load on column C1 = 170 kN, Live load on column C2 = 190 kN, Restricted width of footing = 2.00 m, Grade of concrete for footing = M15, Grade of steel for footing = Fe415 and S.B.C. of soil = 175 kN/m².

The restraint condition of geometry is given by clicking restraint conditions menu. It shows form for geometry data of selected restraint conditions (Fig. 9.71). The data such as c/c distance between columns, size of columns, distance of columns from restraint edges and
length or width of footing is supplied through this form. The soil data are supplied using the soil data form shown in Fig. 9.72. This form provides different type of soil options with their approximate bearing capacity as per IS: 1904-1978 [98]. The material data are supplied through the form as depicted in Fig. 9.73. The load data such as axial loads, moments and horizontal shear on columns C1 and C2 are given using the form of Fig. 9.74. Figure 9.75 shows analysis results for given geometry and loading in graphical form.

Fig. 9.71 Geometry Data

Fig. 9.72 Soil Data Form

Fig. 9.73 Material Data Form
Fig. 9.74 Load Data

Fig. 9.75 Analysis Results
The lower and upper bound values are given through the form of Fig. 9.76. The input fuzzy sets for this values and induced fuzzy sets for all the combinations are generated and superimposed on performance fuzzy set as shown in Fig. 9.77. This form also shows the optimum depth obtained through this process. Figures 9.78 and 9.79 show the final dimensions and reinforcement details in tabular and graphical form.

The results obtained from the software are compared with the available literature [85] and depicted in Table 9.11. It indicates about 8.1 % cost optimization.
**DESIGN RESULTS**

**GEOMETRY OUTPUT**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth of foundation</td>
<td>1.00 m</td>
</tr>
<tr>
<td>Width of footing</td>
<td>2.00 m</td>
</tr>
<tr>
<td>Length of footing</td>
<td>5.10 m</td>
</tr>
<tr>
<td>Thickness of footing</td>
<td>0.62 m</td>
</tr>
</tbody>
</table>

**REINFORCEMENT SUMMARY**

<table>
<thead>
<tr>
<th>(As)reqd.</th>
<th>Dia.</th>
<th>Nos.</th>
<th>Spacing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal steel for span moment</td>
<td>2296 sq mm</td>
<td>16</td>
<td>12</td>
</tr>
<tr>
<td>Longitudinal steel for support moment</td>
<td>2296 sq mm</td>
<td>16</td>
<td>12</td>
</tr>
<tr>
<td>Transverse steel (Top Bottom) per mt.length</td>
<td>1127.50 sq mm</td>
<td>16</td>
<td>29</td>
</tr>
</tbody>
</table>

---

**Fig. 9.78 Design Results**

**Fig. 9.79 Reinforcement Detailing**
### Table 9.11 Comparison of results

<table>
<thead>
<tr>
<th>COLUMN DATA</th>
<th>Cl</th>
<th>C2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column number</td>
<td>Cl</td>
<td>C2</td>
</tr>
<tr>
<td>C/C distance between columns</td>
<td>3.00 m</td>
<td></td>
</tr>
<tr>
<td>Size of column</td>
<td>0.30 x 0.30 m</td>
<td>0.30 x 0.30 m</td>
</tr>
<tr>
<td>Grade of concrete</td>
<td>M20</td>
<td>M20</td>
</tr>
<tr>
<td>Grade of steel</td>
<td>Fe415</td>
<td>Fe415</td>
</tr>
<tr>
<td>Dead load</td>
<td>480 kN</td>
<td>610 kN</td>
</tr>
<tr>
<td>Live load</td>
<td>170 kN</td>
<td>190 kN</td>
</tr>
</tbody>
</table>

| FOOTING DATA         |     |     |
| Restricted width of footing | 2.00 m |     |
| Grade of concrete    | M15 |     |
| Grade of steel       | Fe415 |     |

| OTHER DATA           |     |     |
| S.B.C. of soil       | 175 kN / m2 |     |
| Cost of concrete     | 2000 Rs. / m3 |     |
| Cost of steel        | 35 Rs. / Kg |     |

<table>
<thead>
<tr>
<th>RESULTS</th>
<th>REFERENCE [85]</th>
<th>SOFTWARE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of footing</td>
<td>5.10 m</td>
<td>5.10 m</td>
</tr>
<tr>
<td>Thickness of footing</td>
<td>0.67 m</td>
<td>0.62 m</td>
</tr>
<tr>
<td>Total quantity of concrete</td>
<td>6.83 m3</td>
<td>6.32 m3</td>
</tr>
<tr>
<td>Total quantity of steel</td>
<td>460.4 Kg</td>
<td>421.0 Kg</td>
</tr>
<tr>
<td>Total cost of concrete</td>
<td>Rs. 13660 /-</td>
<td>Rs. 12640 /-</td>
</tr>
<tr>
<td>Total cost of steel</td>
<td>Rs. 16114 /-</td>
<td>Rs. 14735 /-</td>
</tr>
<tr>
<td>Total Cost</td>
<td>Rs. 29774 /-</td>
<td>Rs. 27375 /-</td>
</tr>
<tr>
<td>% Reduction in cost</td>
<td>8.10%</td>
<td></td>
</tr>
</tbody>
</table>

#### 9.8.9 A Strap Footing Example

**Data:** C/C distance between columns = 5.70 m, Size of column C1 = 0.23 x 0.50 m, Size of column C2 = 0.23 x 0.50 m, Grade of concrete for columns = M20, Grade of steel for columns = Fe415, Characteristic load on column C1 = 500 kN, Characteristic load on column C2 = 610 kN, Width of strap beam = 0.60 m, Grade of concrete for footing = M15, Grade of steel for footing = Fe415 and S.B.C. of soil = 200 kN/m².

Figures 9.80 to 9.86 explain the complete procedure whereas Table 9.12 provides the comparison of result.
Fig. 9.80 Geometry Data

Fig. 9.81 Form for Load Data
Fig. 9.82 Fuzzy Sets for Footing

Fig. 9.83 Fuzzy Sets for Strap Beam
**Fig. 9.84 Optimum Results**

**Fig. 9.85 Design Results**

**Fig. 9.86 Reinforcement Detailing**
Table 9.12 Comparison of Results

<table>
<thead>
<tr>
<th>COLUMN DATA</th>
<th>C1</th>
<th>C2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column number</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C/C distance between columns</td>
<td>5.70 m</td>
<td></td>
</tr>
<tr>
<td>Size of column</td>
<td>0.23 × 0.50 m</td>
<td>0.23 × 0.50 m</td>
</tr>
<tr>
<td>Grade of concrete</td>
<td>M20</td>
<td>M20</td>
</tr>
<tr>
<td>Grade of steel</td>
<td>Fe415</td>
<td>Fe415</td>
</tr>
<tr>
<td>Characteristic load</td>
<td>500 kN</td>
<td>600 kN</td>
</tr>
</tbody>
</table>

| FOOTING DATA         |                         |                         |
| Width of strap beam  | 0.60 m                  |                         |
| Grade of concrete    | M15                     |                         |
| Grade of steel       | Fe415                   |                         |

| OTHER DATA           |                         |                         |
| S.B.C. of soil       | 200 kN / m²              |                         |
| Cost of concrete     | 2000 Rs. / m³            |                         |
| Cost of steel        | 35 Rs. / Kg              |                         |

| RESULTS              | REFERENCE [85]           | SOFTWARE                |
| Width of footing     | 2.00 m                   | 1.96 m                  |
| Length of footing under col. C1 | 1.26 m | 1.27 m |
| Length of footing under col. C2 | 1.82 m | 1.87 m |
| Thickness of footing | 0.40 m                   | 0.31 m                  |
| Depth of strap beam  | 1.00 m                   | 1.055 m                 |
| Total quantity of concrete | 5.31 m³ | 5.12 m³ |
| Total quantity of steel | 262.05 Kg | 249.02 Kg |
| Total cost of concrete | Rs. 10620/- | Rs. 10240/- |
| Total cost of steel  | Rs. 9172/-               | Rs. 8716/-              |
| Total Cost           | Rs. 19792/-              | Rs. 18956/-             |
| % Reduction in cost  |                         | 4.22%                   |
9.9 OPTIMUM DESIGN OF SILOS

9.9.1 Design Parameters

In silo design pre-assigned parameters are density of filling material, angle of repose of filling material, coefficient of friction between wall and material, grade of concrete, grade of steel and weight of material for which silo is to be designed. In this problem diameter of silo and thickness of side wall are input parameters. The actual tensile stress in concrete is considered as the output (induced) parameter and allowable tensile stress in steel is performance parameter.

9.9.2 Constraints

The problem is subjected to following constraints.

1. \(3 \text{m} \leq d \leq 10\text{m}\)
2. \(1\text{m} \leq h \leq 30\text{m}\)
3. \(p_t \geq 0.2\%\)
4. \(h / d \geq 1.7\)
5. \(100\text{ mm} \leq Sp \leq 300\text{ mm}\)
6. \(\theta = \Theta + 15\)
7. \(100\text{ mm} \leq t \leq 300\text{ mm}\) \hspace{1cm} \ldots (9.24)

where \(d = \) diameter of silo, \(h = \) height of silo, \(p_t = \% \) of main steel required in side wall, \(\theta = \) angle of hopper bottom made with horizontal, \(\Theta = \) Angle of repose, \(Sp = \) spacing of reinforcement and \(t = \) thickness of sidewall.

9.9.3 Fuzzification of Input Parameters

In this process input variables are expressed by appropriate fuzzy sets known as input fuzzy sets. Triangular input fuzzy sets for \(d\) and \(t\) are shown in Fig. 9.87 (a) and (b) respectively.

- **Fig. 9.87 Fuzzy sets**

\[\text{Membership Function}\]

\[\text{Diameter of Silo (m)} \quad \text{Thickness of wall (mm)} \quad \text{Tensile Stress} \quad \text{Tensile Stress}\]

\[\text{(a) Input} \quad \text{(b) Input} \quad \text{(c) Performance} \quad \text{(d) Output}\]

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9.9.4 Development of Performance Fuzzy Set

Depending upon functional requirement, the performance parameter takes the values. In this problem performance parameter is tensile stress in concrete, which depends upon grade of concrete. Here also preferences for different values are expressed by the linear variation as shown in Fig. 9.87(c).

9.9.5 Development of Induced Fuzzy Sets

For developing output fuzzy set the value of $\alpha$-cut for this problem is selected as 0.01. For two variables there will be total four combinations ($d_1$-$t_1$, $d_1$-$t_2$, $d_2$-$t_1$, $d_2$-$t_2$) at each $\alpha$-cut. Figure 9.87(d) shows the induced fuzzy set. Horizontal pressure ($p_h$) is calculated by Janssen's theory [87].

\[
x = (\mu n h / R), p_h = W R (1 - \exp(-x)) / \mu, F_t = 0.5 * p_h * d \quad \ldots (9.25)
\]

Tensile stress in concrete = \( (F_t * 1000) / (A_c + m A_{st}) \) \ldots (9.26)

where, $p_h$ = horizontal pressure acting on the side wall due to material in kN/ m\(^2\), $F_t$ = hoop tension in cylindrical wall per meter height in kN, $\mu$ = co-efficient of friction between wall and material = tan ($\Theta$), $n$ = ratio of horizontal to vertical pressure intensity = (1-sin ($\Theta$)) / (1+sin ($\Theta$)), $\Theta$ = angle of repose, $h$ = height of wall in m, $R$ = hydraulic mean radius = $d / 4$ in m, $d$ = diameter of silo in m, $w$ = density of filling material in kN/m\(^3\), $A_c$ = area of concrete in mm\(^2\) = 1000 * $t$, $t$ = thickness of side wall in mm, $m$ = modular ratio and $A_{st}$ = Area of steel provided in side wall in mm\(^2\).

9.9.6 Superimposition of Two Plots

As mentioned earlier, superimposition of two plots gives match point and corresponding stress value ($s'$) and membership grade ($\mu'$) (Fig. 9.88). This membership grade gives all corresponding crossover points and selected stress decides most acceptable combination.

![Fig. 9.88 Superimposition of Two Plots](image)
9.9.7 Processors Developed

The software developed provides the design of cylindrical silos, which can store cement up to 40,000 kN.

❖ Preprocessor: Provision is made in the software to design the silo with large variety of storing materials like coal, cement, food grain, ash, ore etc. With the type of material their density and angle of internal friction are also provided in software. The other inputs like weight of material for which silo is to be designed, grade of concrete and grade of steel are also to be entered before starting optimization.

❖ Main processor: In this processor complete optimization of given problem is carried out to get the optimum height, diameter and thickness of silo. For this initially it generates triangular fuzzy sets of diameter and thickness. At each $\alpha$-cut it calculates four crossover points and four combinations. These four combinations give four stress values which are used to plot induced fuzzy sets. On these fuzzy sets performance fuzzy is superimposed to get the member ship grade at the match point. The combination of $d$ and $t$ w.r.t this match point is considered to be optimum diameter of silo and thickness of side wall. For these dimensions design is carried out by Janssen’s Theory. At three or four different levels, the hoop tension is calculated and corresponding steel is calculated.

❖ Postprocessor: The post processor gives the optimum geometry and reinforcement detail in tabular and graphical form.

9.9.8 Example of Cylindrical Silo

Data: (i) Volume of material to be stored = 620 m$^3$, (ii) Weight of material = 5000 kN, (iii) Co-efficient of friction = 0.444, (iv) Density of wheat = 8kN/m$^3$, (v) Ratio of horizontal to vertical pressure = 0.4, (vi) Angle of repose = 25°, (vii) Grade of concrete = M15 and (viii) Grade of steel = Fe 415.

The above input data are supplied through the form shown in Fig. 9.89. Based on the range of input parameters suggested by the user, the software develops fuzzy sets and displays on the form as shown in Fig. 9.90. Figure 9.91 shows the superimposition process as displayed by the software. The software also gives the final sketch of silo along with the optimum dimensions as in Fig. 9.92. The final results after complete design process are displayed on the form shown in Fig. 9.93. The form is also developed to show the cross section of the silo.
wall indicating the reinforcement detail (Fig. 9.94). The comparison of results is given in Table 9.13.

<table>
<thead>
<tr>
<th>General data</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight of Material (kN)</td>
<td>35000</td>
</tr>
<tr>
<td>Grade of Concrete</td>
<td>M20</td>
</tr>
<tr>
<td>Grade of Steel</td>
<td>Fe415</td>
</tr>
</tbody>
</table>

Fig. 9.89 Input Data Form for Silo

![Input Data Form for Silo](image)

<table>
<thead>
<tr>
<th>Membership Function</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter of Silo (m)</td>
<td>0 1 2 3 4 5 6 7 8 9 10 11 12</td>
</tr>
<tr>
<td>Thickness of Wall (mm)</td>
<td>0 20 40 60 80 100 120 140 160 180 200 220 240 260 280 300</td>
</tr>
</tbody>
</table>

Fig. 9.90 Input Fuzzy Set for Silo

![Input Fuzzy Set for Silo](image)
9. FL Based Software Development and Applications

Fig. 9.91 Superimposition of Induced and Performance Fuzzy Sets for Silo

![Superimposition of Induced and Performance Fuzzy Sets for Silo](image)

Fig. 9.92 Optimum Geometry of Silo

![Optimum Geometry of Silo](image)

<table>
<thead>
<tr>
<th>Number</th>
<th>Diameter (m)</th>
<th>Height (m)</th>
<th>Dia of Outlet (m)</th>
<th>Height of hopper bottom (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.025</td>
<td>27.387</td>
<td>1.64</td>
<td>3.342</td>
</tr>
</tbody>
</table>

**REINFORCEMENT OUTPUT**

(1). In main cylinder

<table>
<thead>
<tr>
<th>Height (m)</th>
<th>7</th>
<th>14</th>
<th>21</th>
<th>27</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter (mm)</td>
<td>10</td>
<td>12</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>Number</td>
<td>63</td>
<td>63</td>
<td>77</td>
<td>86</td>
</tr>
<tr>
<td>Spacing (mm)</td>
<td>110</td>
<td>110</td>
<td>90</td>
<td>80</td>
</tr>
</tbody>
</table>

(2). In hopper bottom

| Diameter (mm) | 8   | 8   |
| Number        | 13  | 53  |
| Spacing (mm)  | 250 | 111 |

Fig. 9.93 Result Form
9.9.9 Design graphs

Following are some of the graphs (Figures 9.95 to 9.97) which are depicted on screen by the developed software. They are prepared with weight of material as a main function. With different values of weight of cement to be stored, their corresponding optimum diameter, height, thickness and optimum cost are calculated by this software and plotted. As results are obtained by fuzzy logic there are chances of changing the values of variables from point to point. Thus graph cannot give exact optimum solution at each point but it can explain variation in value of different variables with increase in the size, their contribution to overall result, their effect on value of other variables etc.

Fig. 9.94 Reinforcement Detail for Silo
Fig. 9.95 Diameter and Height of Silo

Fig: 9.96 Thickness of Side Wall

Fig. 9.97 Optimum Cost of Silo
Table 9.13 Comparison of Results

<table>
<thead>
<tr>
<th>RESULT</th>
<th>REFERENCE [87]</th>
<th>SOFTWARE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter of silo</td>
<td>6 m</td>
<td>7.375 m</td>
</tr>
<tr>
<td>Depth of cylindrical portion</td>
<td>20 m</td>
<td>12.936 m</td>
</tr>
<tr>
<td>Thickness of side wall</td>
<td>150 mm</td>
<td>175 mm</td>
</tr>
<tr>
<td>Depth of hopper bottom</td>
<td>2.5 m</td>
<td>2.724 m</td>
</tr>
<tr>
<td>Diameter of opening at hopper bottom</td>
<td>1 m</td>
<td>1.23 m</td>
</tr>
<tr>
<td>Cost of silo</td>
<td>121740/-</td>
<td>115426/-</td>
</tr>
</tbody>
</table>

Figures 9.95 and 9.96 show that up to 22000 kN capacity the diameter and height both increase and then the diameter becomes steady i.e. 10 m which is maximum limit with minimum thickness of 100 mm up to 32000 kN. Then diameter reduces because of increasing thickness. Beyond 37000 kN capacity there is tremendous increase in thickness.

9.10 OPTIMUM DESIGN OF FOLDED PLATES

9.10.1 Design parameters

In the design of folded plates, main pre-assigned parameter that governs the design is area covered by roof. The other pre-assigned parameters are number of plates to be provided, the value of live load, grade of concrete and grade of steel to be used. In order to fix the geometry of the V-shaped folded plates fully, one has to decide about the thickness of plates and inclination of plates. Hence these two are selected as input parameters. The compressive stress at any point should be less than the permissible compressive stress in concrete so the permissible stress in concrete is considered as performance parameter. Since the compressive stress is maximum at mid span, it is considered as output parameter.

9.10.2 Constraints

- Geometry constraints
  1. $60 \text{mm} \leq t \leq 110 \text{ mm}$
  2. $30 \leq x \leq 80$
  3. $b_{\text{max}} = \frac{L}{3}$

where $t =$ thickness of folded plate, $x =$ angle of inclination, $b_{\text{max}} =$ maximum width of an individual plate, $L =$ span of the roof. Constraint $b_{\text{max}} = L/3$ indicates that the maximum width of an individual plate should be one third of the span in order to eliminate two way behavior and deep beam action and thus minimum three number of plates are necessary.
❖ **Behavior constraint**

Main behavior constraint is the compressive stress in concrete. The compressive stress at any point should be less than the permissible compressive stress in concrete. Since the stress is maximum at mid span the constraint imposed is \( \sigma - \sigma_{cb} \leq 0 \) where \( \sigma \) is maximum compressive stress at mid span and \( \sigma_{cb} \) is permissible compressive stress in concrete.

9.10.3 **\( \alpha \)-Cut Procedure**

Thickness of plate and inclination of plate are the main input parameters. The geometry constraints help in developing fuzzy sets for these inputs. These constraints indicate that input values should be within the minimum and maximum possible values depending upon provision of IS: 2210 code [88]. Adopted linear triangular fuzzy sets \( T \) for thickness and \( X \) for angle are shown in Figs. 9.98(a) and (b) respectively.

The permissible compressive stress in concrete which depends on grade of concrete is considered as performance parameter (Fig. 9.99). Figure 9.100 shows the induced fuzzy set in which output parameters for only two combinations are plotted. For each combination the value of output parameter is calculated.

Superimposition of two plots gives the match point. This point gives value of stress \( (S') \), which satisfies the functional requirement and at the same time has the highest performance and its appropriate membership grade \( (\mu') \) as shown in Fig. 9.101. This will give all corresponding crossover points and selected stress will decide most acceptable combination.

![Membership Function](a) Input Fuzzy Set \( T \)

![Membership Function](b) Input Fuzzy Set \( X \)

Fig. 9.98 Input Fuzzy Sets
9.10.4 Fuzzy Logic Based Processors

A **Pre-processor** is developed to facilitate menu driven input of data. Various forms are developed under this processor to supply geometry data, loading data, and material data. A **Main-processor** is developed to find optimum solution based on fuzzy logic. Under this processor input fuzzy sets are developed for thickness of plate and inclination of plate. Output parameter is then calculated for all the combinations at every \( \mu \)-cut. These values are plotted against their relative membership function to get induced fuzzy sets. The program provides the graphical display of the induced fuzzy set developed. When this induced fuzzy set is superimposed on the trapezoidal performance fuzzy set it gives a match point (intersection point) for optimum solution. Multiple match points may be some times obtained when number of combinations is more than two. The match point corresponding to maximum membership is considered to give optimum results. The input parameter values corresponding to this match point are obtained and the design of the plate is carried out as per the standard IS 2210 [88]. The final design results are then supplied to the **Post-processor** to depict results in tabular form and reinforcement detailing. It also provides graphical representation of induced fuzzy set, and superimposition of fuzzy sets. A separate form is developed to depict superimposition of performance fuzzy set and induced fuzzy sets.
9.10.5 Folded Plate Example

Data: Span = 8 m, Length = 20 m, Number of Plates = 4, Live Load = 0.6 kN/m², Grade of concrete = M15 and Grade of Steel = Fe 415.

The geometry detail of folded plates such as length and width of the area to be covered and number of plates required are provided using geometry form whereas to enter the live load for which plates are to be designed and material details such as grade of concrete and grade of steel the appropriate menus are to be used. The superimposition of the fuzzy sets carried out by the program is displayed as shown in Fig. 9.102. Provision is made in the software to design the folded plate and to depict complete reinforcement detailing as shown in Fig. 9.103. Form is also developed to provide result in tabular form as shown in Fig. 9.104.

![Fig. 9.102 Superimposition of Induced and Performance Fuzzy Sets](image1)

![Fig. 9.103 Form for Detailing of Folded Plate](image2)
For this problem, a comparison of results given in Table 9.14 indicates that the design based on the fuzzy logic leads to an economic design.

Table 9.14 Comparison of Results

<table>
<thead>
<tr>
<th>RESULTS</th>
<th>REFERENCE [87]</th>
<th>FL SOFTWARE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of Plate</td>
<td>2.82 m</td>
<td>3.109 m</td>
</tr>
<tr>
<td>Height of plate</td>
<td>2 m</td>
<td>2.38 m</td>
</tr>
<tr>
<td>Angle of Inclination</td>
<td>45</td>
<td>50</td>
</tr>
<tr>
<td>Thickness of Plate</td>
<td>100</td>
<td>80</td>
</tr>
<tr>
<td>Cost</td>
<td>94000/-</td>
<td>76000/-</td>
</tr>
</tbody>
</table>

9.11 OPTIMUM DESIGN OF HYPER SHELL

Hyperbolic paraboloid generally known as hyper shell is a doubly curved anticlastic shell of translation. The hyper shells are of two types: Inverted Umbrella Type Shell and Tilted Umbrella Type shell. Inverted umbrella type of roof consists of four hyper units joined together at the center where the main supporting column is provided. The structural components of a typical inverted type umbrella roof are as follows: Central supporting column, Edge beam, Sloping ribs connecting the column, and Shell surface. Tilted inverted umbrella has same structure features as inverted umbrella. The main difference is that in inverted umbrella all four edges are at same level whereas in tilted umbrella type roof all four edges are at different levels. In both the cases edge members are in tension and interior members are in compression.
Inverted umbrella type shell is addressed here for optimum design, taking into account latest IS code provisions. Pre- and post-processors are developed to facilitate the input and output and are explained briefly with suitable application. The results obtained for dimensions of shell surface, tie member and compression member and for the cost of shell are compared with those based on conventional design.

In design of shell surface, the main variable is the thickness and percentage steel required in surface. The overall thickness of a reinforced concrete shell should not be less than 50 mm for singly curved shell and 40 mm for doubly curved shell [87]. Generally a minimum reinforcement of 0.15 percent of the gross cross section area in the principle direction is recommended for thin shell structures. Shell is subjected to small loading and in shell surface all stresses are within the permissible limits even with minimum depth and minimum percentage of reinforcement. So for shell surface there is no need to apply fuzzy logic and it can be designed with minimum criteria. Main constraint is shear stress which should be within the permissible range. If shear stress is large then depth is to be increased.

9.11.1 Design Parameters
In design of hyper shell, main pre-assigned parameter is area covered by roof. The other pre-assigned parameters are the value of live load, value of weight of waterproofing, grade of concrete and grade of steel. In case of Tilted Umbrella type hyper shell maximum and minimum height of the two opposite ends are also to be provided before starting design.

In case of design of interior beams, the main input parameters are width of member, depth of member and percentage of steel to be provided in the member. As the member is subjected to compressive force, at any point the direct compressive stress should be less than the permissible compressive stress in concrete. So value of permissible direct compressive stress is considered as performance parameter. The actual compressive stress in concrete is output parameter. In case of design of edge beams, the main input parameters are width of member and depth of member. As the member is subjected to tensile force, the tensile stress in concrete should be less than the permissible tensile stress in concrete at any point. So value of permissible tensile stress is considered as performance parameter. The actual tensile stress in concrete is output parameter.
9.11.2 Constraints
The problem is subjected to following constraints for rib and edge beam:

❖ Compression rib
   1. $100 \leq b_c \leq 1000 \text{ mm}$
   2. $dm \leq d_c \leq 400 \text{ mm}$
   3. $0.2\% \leq p_t \leq 3\%$ ...(9.27)

❖ Edge beam
   1. $150 \leq b_e \leq 800 \text{ mm}$
   2. $150 \text{ mm} \leq d_e \leq 400 \text{ mm}$

where $b_c$ = width of compression rib, $d_c$ = depth of compression rib, $dm = d_s + 130$, $d_s$ = thickness of shell surface, $p_t$ = percentage of steel provided in compression rib, $b_e$ = width of edge beam, and $d_e$ = depth of edge beam.

9.11.3 Fuzzification of Input Parameters
In compression rib design, the fuzzy sets for the input parameters $b_c$, $d_c$ and $p_t$, will take the triangular shape as in Fig. 9.105 Similarly, for edge beam the triangular fuzzy sets for width $b_e$ and $D_e$ are shown in Fig. 9.106

![Fig. 9.105 Input Fuzzy Sets for Compression Rib](image)

![Fig. 9.106 Input Fuzzy Sets for Edge Beam](image)
9.11.4 Performance Fuzzy Sets

In case of design of compression member, the permissible direct compressive stress in concrete is considered as performance parameter. In case of design of edge beam, the permissible tensile stress in concrete is considered as performance parameter. The performance fuzzy sets for these two parameters are shown in Fig. 9.107.

\[ \sigma_{cc} = (P * 1000 - \sigma_{sc} * A_{sc}) / A_c \]
where \( \sigma_{cc} \) = direct compressive stress in concrete, \( P \) = maximum compressive force in rib, \( \sigma_{sc} \) = compressive stress in steel, \( A_{sc} \) = area of steel in compression and \( A_c \) = area of concrete.

9.11.5 Induced Fuzzy Sets

❖ **Interior beams:** In case of compression rib, 0.01 is selected as value of \( \alpha \)-cut. At each \( \alpha \)-cut there will be 2*3 i.e. six crossover points which will lead to total eight combinations of b, d and pt. For each combination the value of output parameter is calculated by, \( \sigma_{ct} = P1 * 1000 / (A_c + (m-1) * A_{st}) \)
where \( \sigma_{ct} \) = tensile stress in concrete, \( P1 \) = maximum tensile force in tie member, \( A_c \) = area of concrete, \( m \) = modular ratio and \( A_{st} \) = area of steel.

❖ **Edge beams:** In case of edge beam also, 0.01 is selected as value of \( \alpha \)-cut. At each \( \alpha \)-cut there will be 2*2 crossover points with total four combinations of b and d. For each combination the value of output parameter is calculated by \( \sigma_{ct} = P1 * 1000 / (A_c + (m-1) * A_m) \)
where \( \sigma_{ct} \) = tensile stress in concrete, \( P1 \) = maximum tensile force in tie member, \( A_c \) = area of concrete, \( m \) = modular ratio and \( A_m \) = area of steel.

9.11.6 Superimposition of Two Plots

After having both performance and induced fuzzy sets, the next step is superimposition of two plots which gives an intersection point. This point gives value of stress, which satisfies the functional requirement and at the same time has the highest performance and its corresponding membership gives optimum combination of design variables.
9.11.7 Developed Processors
Pre-processor is developed to facilitate menu driven input of data such as area to be covered, water proofing load, live load and grade of materials. Main-processor is developed to find optimum solution using $\alpha$-cut method. The software also displays the input, output and performance fuzzy sets and superimposition of fuzzy sets. Post-processor is developed to provide final optimum solution and reinforcement detailing.

9.11.8 Inverted Umbrella Type Hyper Shell Example
**Data:** Roof area = 12 x 12 m, Water proofing load = 0.22 kN/m$^2$, Live load = 0.5 kN/m$^2$, Grade of Concrete = M 20 and Grade of Steel = Fe 415.

Input data is supplied through various forms. FL based optimization is carried out for compression rib and edge beam separately through the main processor which finds the match point after superimposition of fuzzy sets as shown in Fig. 9.108 and Fig. 9.109. The final results obtained are displayed along with the reinforcement detail. Figure 9.110 shows the reinforcement detail of the slab. Section through rib is displayed as in Fig. 9.111. The detail of section crossing slab and tie beam is given in Fig. 9.112. Section cutting rib and slab is depicted in Fig. 9.113.
Reinforcement Detail in Hyper Shell

Fig. 9.110 Detailing of Hyper Shell

Section Through Compression Rib (Section — AA)

C.I. Drain
Column bar

Fig. 9.111 Detail at Section A-A

Fig. 9.112 Detail at Section B-B
9.11.9 Design graphs

Design graphs are prepared with different area to be covered as a main function and are plotted for a fix assigned parameter using the developed software as shown in Figures 9.114 to 9.117. If problem consists of fix input parameter with only area as a variable then these graphs can be calibrated. The graphs can be used for finding optimum values of variable without using the software. As results are obtained by fuzzy logic, there are chances of changing the values of variables from point to point. Thus final result will depend upon the combination selected. Although these graphs cannot give exact optimum solution at each point, they can explain variation in value of different variables with increase in size, their contribution to overall result, their effect on value of other variables etc.

![Graph of Percentage Steel vs Size of Hyper Shell](image1)

**Fig. 9.114 Variation of Percentage Steel with Size of Hyper Shell**
The comparison of the obtained results with those available is shown in Table 9.15 which indicates that the design based on the fuzzy logic leads to an economic design.
### Table 9.15 Comparison of Results

<table>
<thead>
<tr>
<th>PARAMETERS</th>
<th>REFERENCE [87]</th>
<th>SOFTWARE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height</td>
<td>1.2 m</td>
<td>1.2 m</td>
</tr>
<tr>
<td>Thickness of Slab</td>
<td>70 mm</td>
<td>70 mm</td>
</tr>
<tr>
<td>Width of Compression Rib</td>
<td>600 mm</td>
<td>240 mm</td>
</tr>
<tr>
<td>Depth of Compression Rib</td>
<td>200 mm</td>
<td>380 mm</td>
</tr>
<tr>
<td>Percentage Steel in Compression Rib</td>
<td>1 %</td>
<td>0.606 %</td>
</tr>
<tr>
<td>Width of Tension Member</td>
<td>310 mm</td>
<td>320 mm</td>
</tr>
<tr>
<td>Depth of Tension Member</td>
<td>200 mm</td>
<td>220 mm</td>
</tr>
<tr>
<td>Cost</td>
<td>46698/-</td>
<td>41223/-</td>
</tr>
</tbody>
</table>

### 9.12 Cost Optimization of Machine Foundation

Optimum dimensions of block foundation (i.e. Length, Width and Depth) are found using amplitude as a performance parameter. The design giving maximum amplitude from the domain of feasible designs is considered to be the optimum due to the fact that the design giving amplitude nearer to the permissible amplitude gives minimum possible dimensions satisfying all the design constraints.

#### 9.12.1 Design Parameters

The pre-assigned input data are machine data, data related to spring absorber, preliminary dimensions of the machine foundation supplied by machine manufacturer, soil data and material data. Length, width and depth of foundation are selected as the main input parameters. As the design of machine foundation is governed by the amplitude the actual value of the amplitude is taken as output parameter and permissible amplitude is considered to be the performance parameter. As optimization of machine foundation is constraint optimization problem, following two types of constraints are imposed during optimization.

#### 9.12.2 Design Constraints

- **Geometrical constraints**
  
  (a) \( L_{\text{min}} \leq L \leq L_{\text{max}} \)  
  (b) \( B_{\text{min}} \leq B \leq B_{\text{max}} \)  
  (c) \( D_{\text{min}} \leq D \leq D_{\text{max}} \)  

  where, \( L_{\text{min}} \) and \( L_{\text{max}} \) are lower and upper bound values of length, \( B_{\text{min}} \) and \( B_{\text{max}} \) are lower and upper bound values of width and \( D_{\text{min}} \) and \( D_{\text{max}} \) are lower and upper bound values of
depth. Designer is free to choose any value for upper and lower limit based on dimensions outlined by the machine manufacturer.

**Behaviour constraints**

(a) \( A_z, A_x, A_{top}, A_{rot} \leq \) Permissible amplitude

(b) \( 1.4f_m < f_{int}, f_{n1}, f_{n2} < 0.5f_m \) (constraint preventing resonance) \[89\] \(...(9.30)\)

where \( A_z \) is vertical amplitude, \( A_x \) is horizontal amplitude, \( A_{top} \) is net amplitude at the top of foundation, \( A_{rot} \) is rotational amplitude, \( f_m \) is vertical natural frequency, \( f_{n1} \) is first coupled natural frequency, \( f_{n2} \) is second coupled natural frequency and \( f_m \) is operating frequency of machine.

### 9.12.3 Fuzzification of Input Parameters

In the design of machine foundation length, width and depth of foundation are main input parameters. Degree of inclusion of any given value of \( L, B \) or \( D \) can be expressed by a membership function. The values supplied by machine manufacturer are assigned a membership value of 1. Lower limit and upper limit of values are assigned a membership value of 0. Linear membership function is selected for all the variables in the present work. Figure 9.118 shows fuzzy sets for \( L, B, \) and \( D \). \( L_{mm}, B_{mm} \) and \( D_{mm} \) are dimensions provided by machine manufacturer. In Fig. 9.118, \( L_1-L_2, B_1-B_2 \) and \( D_1-D_2 \) are cross over points for length, breadth and depth respectively.

![Fig. 9.118 Input Fuzzy Sets for (a) Length (b) Width (c) Depth](image)

### 9.12.4 Performance Fuzzy Set

Depending on the functional requirement performance fuzzy set is decided. A permissible value is considered to be the most favorable value which is assigned a membership grade of 1. In FL based design a little constraint violation is tolerable to achieve economical design. Normally the tolerance of 10 to 20% may be considered for structural design. In the present
work the 10 % tolerance is chosen keeping in mind dynamic loading. Figure 9.119 shows performance fuzzy set with $A_{mf}$ = most favorable amplitude and $A_{mt}$ = maximum tolerable amplitude

9.12.5 Development of Output Fuzzy Set

For getting output fuzzy set, 0.01 is selected as value of $\alpha$-cut. At each $\alpha$-cut there will be 2*n i.e. six crossover points $L_1$, $L_2$, $B_1$, $B_2$, $D_1$ and $D_2$. After having cross over points, total eight ($2^n$) combinations, ($L_1$-$B_1$-$D_1$, $L_1$-$B_1$-$D_2$, $L_1$-$B_2$-$D_1$, $L_1$-$B_2$-$D_2$, $L_2$-$B_1$-$D_1$, $L_2$-$B_1$-$D_2$, $L_2$-$B_2$-$D_1$, $L_2$-$B_2$-$D_2$) are generated. For each combination the value of output parameter is calculated as follows.

$$a_z = \frac{P_z}{m(\omega_{nz}^2 - \omega_m^2)}$$  \hspace{1cm} (9.31)

where $a_z$ = vertical amplitude, $P_z$ = vertical exciting force, $m$ = mass of the machine foundation system, $\omega_{nz}$ = vertical natural frequency and $\omega_m$ = operating frequency of a machine.

All the above parameters are dependent on $L$, $B$ and $D$ of foundation. Figure 9.120 shows the plot of output values versus membership function for any two combinations.

9.12.6 Superimposition of Two Plots

To get optimum result, performance and induced fuzzy sets are superimposed as shown in Fig. 9.121. Intersecting point gives value of amplitude ($A'$), which satisfies the functional requirement at the same time has the highest performance. The corresponding membership grade ($M'$) is also shown in the figure. The difference between induced and performance fuzzy set at each $\alpha$-cut value is calculated. The values of $L$, $B$ and $D$ at $M'$ are obtained from input fuzzy sets. These dimensions are considered to be optimum.
9.12.7 The Developed Processors

❖ **Pre-processor**: The interactive data entry of various input values such as machine data, spring data, material data and soil data is facilitated through various forms developed under this processor. It provides interactive entry of limiting values of input parameters and gives immediate response to this data entry by displaying the fuzzy sets graphically on the same form. The data supplied through the preprocessor are supplied to the main processor.

❖ **Main processor**: Various subroutines and functions are developed under this processor. It calculates the crossover points for all input fuzzy sets and generates all the possible combinations of input parameters at each \( \alpha \)-cut value which is taken as 0.01. Subroutine **InputFuzz** is used for this purpose. Subroutine **Str_Dynamics** analyses the machine foundation system for every combination of input values and calculates output value (i.e. amplitude) to get induced fuzzy set. Using the given permissible value of amplitude a performance fuzzy set is obtained with the help of **Performance** subroutine. Subroutine **Match** finds the match point by superimposing induced and performance fuzzy sets and gets the \( \alpha \)-cut (\( M' \)) value giving maximum performance. Subroutine **Opt** returns the dimensions of foundation corresponding to this \( M' \) that are considered as optimum dimensions. The processor then calculates the bending moment at critical sections and finds out the reinforcement.

❖ **Post processor**: Under this processor various forms are developed to display the output results. These forms are invoked by clicking respective menus on the main form of this processor named OUTPUT form. It includes the form for displaying superimposition of fuzzy
sets and match point. One of these forms shows induced fuzzy set values for maximum and minimum amplitudes and corresponding performance fuzzy set values and input values for all $\alpha$-cut. It also highlights the values corresponding to match point indicating optimum dimensions. The software displays the reinforcement detail for the optimum dimensions upon clicking reinforcement menu of the main form.

9.12.8 Example With Illustrated Output

Here an example of a block foundation for a Diesel Engine is solved and results are compared with the solution available in the literature [89]. The data considered is as follows:

(i) Total weight of machine : 6 t
(ii) Operating speed of machine ($f_m$) : 120 rpm
(iii) Vertical exciting force ($P_z$) : $\pm 1.5$ t
(iv) Permissible amplitude at floor level : 0.04 mm
(v) Nature of soil : stiff clay
(vi) Dimensions of Foundation (L X B X D) : 5.5 X 3.0 X 1.5 m
(vii) Centre of gravity of machine parts:

<table>
<thead>
<tr>
<th>Part</th>
<th>$W_i$ (t)</th>
<th>$X_i$ (m)</th>
<th>$Z_i$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engine</td>
<td>4.0</td>
<td>3.5</td>
<td>2.55</td>
</tr>
<tr>
<td>Motor</td>
<td>2.0</td>
<td>1.5</td>
<td>2.25</td>
</tr>
</tbody>
</table>

Figure 9.122 shows the forms developed for data entry related to machine, soil, vibration absorber and dimensions suggested by the machine manufacturer. The input fuzzy sets considered are as shown in Fig. 9.123. The graph showing superimposition of output fuzzy set and performance fuzzy set is given in Fig. 9.124. Finally, Figs. 9.125 and 9.126 show the forms giving optimum dimensions and reinforcement details of machine foundation respectively. It should be noted that dimensions of foundation are so large that the maximum bending moment is very less and therefore nominal reinforcement has been provided as per the specifications given in the literature [89]. The dimensions suggested by machine manufacturer are 5.5m $\times$ 3.0m $\times$ 1.5m, whereas fuzzy logic based software gives the dimensions of machine foundation as 4.8m $\times$ 2.3m $\times$ 0.8m.
Fig. 9.122 Input Data

Fig. 9.123 Input Fuzzy Sets for Length, Width and Depth
Fig. 9.124 Superimposition of Performance and Output Fuzzy Sets

Fig. 9.125 Amplitudes and Optimum Dimensions of Foundation

Fig. 9.126 Structural Details of Block Foundation
9.13 CLOSING REMARKS

In this chapter a software package was developed to facilitate optimum design of various R.C.C. structures namely different types of slabs, plane frames, grids, isolated footings, combined footings, retaining walls, silos, folded plates, hyper shells subjected to static loading and machine foundation subjected to dynamic loading based on fuzzy logic concept. The graphical features of VB were effectively used to facilitate menu driven input and to produce graphical output of the analysis and design results. Various subroutines such as analysis subroutines, fuzzy logic based subroutines and design subroutines were developed in the software. The analysis of plane frame and grid problems was carried out using stiffness matrix method. The software develops the input fuzzy sets, performance fuzzy sets and induced fuzzy sets and performs optimization using fuzzy logic based subroutines. The design of various structures was carried out as per IS code provisions for limit state method. The results obtained were compared, wherever possible, with the results available in the literature to validate the working of developed software.

In design formulation based on FL, the selection of performance and induced fuzzy set parameters is very important and requires thorough knowledge of the problem in hand. Selection of range of input parameters for developing input fuzzy sets is also very important because the optimum input parameters will take the values form this range. Also optimum solution is likely to differ by changing this range. Though the min-max methodology adopted here is simple it gives the competitive results when compared to conventional design procedure.