CHAPTER III

Economic Design of Control Charts for Variables with Known and Unknown Sigma

3.1 In this chapter the economic design of control charts for variables is developed for a process subject to a single assignable cause. The cost model used is an adaptation of the cost model developed for np-control chart in chapter II.

Duncan (1956, 1971) and Knappengerger and Grandage (1969) developed the economic design of $\bar{x}$-control charts under the assumption that the process standard deviation $\sigma$ is known. However, if $\sigma$ is unknown, then the case is treated differently. In this situation one may use $T^2$-control chart to monitor the production process.

In Section 3.2 the economic design of $\bar{x}$-control charts is developed under the assumption that $\sigma$ is known. In section 3.3 economic design of $T^2$-control chart is developed under the assumption that $\sigma$ is unknown. Of course, the normality of the variable under study is assumed through out.

3.2 Economic Design of $\bar{x}$-Control Charts under $\sigma$ Known

3.2.1 The Production Process and the Sampling Scheme

The production process starts in an in-control state in which the process mean is $\mu_0$. A single assignable cause produces a shift of the process mean from $\mu_0$ to $\mu_0 + \delta\sigma$. Thus there is only one out-of-control state in which the process mean is $\mu_0 + \delta\sigma$. 
The assignable cause is assumed to occur according to a Poisson process with an intensity of \( \lambda \) occurrences per operating hour. Hence the time until the process remains in the in-control state is an exponential random variable with mean \( 1/\lambda \) operating hours. Once the process is in the out-of-control state it stays there until the shift in the process is detected by the control chart. The process parameters \( \mu_0, \delta \) and \( \sigma \) are assumed to be known.

This process is monitored by an \( \bar{x} \)-control chart with central line \( \mu_0 \) and the upper and lower control limits \( \mu_0 \pm L \sigma / \sqrt{n} \). After every production of \( k \) units, \( n \) units are sampled and inspected. The sample mean \( \bar{x} \) is calculated. If the value of \( \bar{x} \) falls within the control limits, the process is declared to be in control and the production continues. If the value of \( \bar{x} \) falls outside the control limits, the process is declared to be out of control. The production at this stage may or may not be stopped and a search for the assignable cause is undertaken.

The design variables \( n, k, L \) are to be determined such that the expected cost per unit of the product during the production cycle is minimized.

### 3.2.2 The Probability of Type I Error and the Power of \( \bar{x} \)-Control Chart

When the assignable cause occurs, the probability that it will be detected on any subsequent sample is

\[
q_1 = P( \bar{x} < \mu_0 - L \sigma / \sqrt{n} \mid \mu = \mu_0 + \delta \sigma ) + P( \bar{x} > \mu_0 + L \sigma / \sqrt{n} \mid \mu = \mu_0 + \delta \sigma )
\]

\[
= \Phi(-L-\delta \sqrt{n}) + 1 - \Phi(L-\delta \sqrt{n}) \quad \ldots (3.2.1)
\]
where \( \Phi(y) \) is the distribution function of the standard normal variate \( Y \),

\[
\Phi(y) = \int_{-\infty}^{y} \frac{1}{\sqrt{2\pi}} \exp(-z^2/2) \, dz.
\]

The quantity \( q_1 \) is the power of the control chart.

Next, the probability of a false alarm is

\[
q_0 = P(\overline{X} < \mu_0 - L\sigma/\sqrt{n} \mid \mu = \mu_0) + P(\overline{X} > \mu_0 + L\sigma/\sqrt{n} \mid \mu = \mu_0)
\]

\[
= \Phi(-L) + 1 - \Phi(L)
\]

\[
= 2[1 - \Phi(L)]
\]

\[ \ldots \text{(3.2.2)} \]

The quantity \( q_0 \) is the probability of type I error.

### 3.2.3 The Proportions of Nonconforming Units

A unit is considered to be nonconforming if its measurement falls outside the specification limits \((u_1, u_2)\). Let \( p_i (i=0,1) \) be the proportion of nonconforming units when the process is in state \( \mu_i (i=0,1) \).

Then,

\[
p_i = P(X < u_1 \mid \mu = \mu_i) + P(X > u_2 \mid \mu = \mu_i)
\]

Thus,

\[
p_0 = 1 - \Phi \left[ \frac{u_2 - \mu_0}{\sigma} \right] + \Phi \left[ \frac{u_1 - \mu_0}{\sigma} \right] \ldots \text{(3.2.3a)}
\]

\[
p_1 = 1 - \Phi \left[ \frac{u_2 - \mu_0}{\sigma} - \delta \right] + \Phi \left[ \frac{u_1 - \mu_0}{\sigma} - \delta \right] \ldots \text{(3.2.3b)}
\]

It may be noted that both the proportions \( p_0 \) and \( p_1 \) are known constants since they are functions of known constants.
3.2.4 The Expected Cost Model.

We compute the expected cost per unit of controlling the process during the production cycle. The cost model used is an adaptation of the cost model for np-control chart developed in chapter II.

Recall the definitions of the following terms well explained there

(i) $C_1, C_2, C_3$
(ii) $N, N(0), B_0, \theta$
(iii) $D, S, \Delta$

Then using the derivations of that section the expression for the expected total cost per unit (ECPU) of controlling the process is

$$\frac{(a_1 + a_2 n) [\theta/(1-\theta) + 1/q_1] + a_3, 1, q_0 \theta/(1-\theta) + a_3, 2 + a_4, 1 S + a_4, 2 (D-S)}{[\theta/(1-\theta) + 1/q_1] k}$$

...(3.2.4)

For ready reference and continuity the expressions for $D, S, \Delta$ and $\theta$ are reproduced here.

$$D = [k \theta/(1-\theta) + \Delta k] \rho_0 + [k/q_1 - \Delta k] \rho_1$$

...(3.2.5)

$$S = np_0 \theta/(1-\theta) + np_1/q_1$$

...(3.2.6)

$$\Delta = \frac{1-(1+\lambda k/R) \theta}{(1-\theta) \lambda k/R}$$

$$\theta = \exp(-\lambda k/R)$$

It should be noted that the expression (2.3.21), obtained after substitutions, for ECPU for np-control chart and the expression (3.2.4) given above look alike but are different in
the sense that the expressions required for $q_0$ and $q_1$ are different. In the evaluation of (3.2.4) one has to use (3.2.2) and (3.2.1) for $q_0$ and $q_1$ respectively, whereas while evaluating (2.3.21) one has to use (2.3.10) and (2.3.4) for $q_0$ and $q_1$ respectively. Minimization of the objective function $E_{CPU}$ with appropriate substitution of $q_0$ and $q_1$ gives the optimal values of the design variables of $\bar{x}$-control chart or np-control chart as the case be. The reason for mentioning this point elaborately is explained in the next few lines.

The practitioners of the control charts quite often switch over from the control charts for the proportion of nonconforming units to the control charts for variables since the general feeling is that one requires a smaller number of units for inspection for the control chart for variables. However, one should not forget the point that the cost of inspection for variables is comparatively higher than the cost of merely classifying a unit as conforming-nonconforming unit. Hence it is thought to compare the performance of the two types of charts from the cost point of view. The similarity and the difference in the cost structure of these charts is useful to have a comparative study of these charts.

In the next section a numerical comparison is made between the performance of $\bar{x}$-control chart and np-control chart from the cost point of view.
3.2.5 Numerical Example

Let $\mu_0=0$, $\sigma=1$, $u_1=-2.58$, $u_2=2.58$, $\delta=1.3$.
Thereby $p_0=0.01 (=1\%)$ and $p_1=0.10 (=10\%)$.
Thus the problem of controlling the process average at zero (i.e. $\mu=\mu_0=0$) and detecting the shift of 1.3 on the process average (i.e. $\mu=\mu_1=\mu_0+\delta \sigma=1.3$) by $\bar{X}$-chart is comparable with controlling the process by np-chart for in-control state $p_0$ at 1% and for out-of-control state $p_1$ at 10%. We consider the same set of cost coefficients and the set of systems parameters of the numerical example of np-control chat of Section 2.3.5 of Chapter II, except one change for the cost of sampling and inspection per unit. Four different values for $a_2$ are considered in addition to $a_2=1$. The maximum cost of $a_2$ considered for inspection by variables is three times the cost of inspection by attributes.
Thus taking

$a_1 = $10, $a_2 = ($1, $1.5, $2, $2.5, $3 one at a time),
$a_3,1 = $100, $a_3,2 = $100, $a_4,1 = $10, $a_4,2 = $15,$
$\lambda = 1$, $R = 1000$.

the objective function ECPU given by (3.2.4) is minimized using the direct search technique explained in Section 2.3.4 of Chapter II. Since in this case only two design variables $n$ and $k$ are discrete the precaution for the proper step size and for the reduction factor is required only for two variables. The listing of the FORTRAN program developed for calculating the objective function ECPU using (3.2.2) and (3.2.1) for $q_0$ and $q_1$ is given at the end of this chapter.
The optimal values of the design variables \( n, k, L \) along with the other findings are listed in the Table 3.1. The last row of the table gives the reproduction of the optimal values along with the other findings of the np-control chart from the numerical example of the Section 2.3.5 of Chapter II.

The following points are revealed from the Table 3.1.

(I) Comparison of row(5) and row(6).

Though the cost of sampling and inspection per unit for \( \bar{x} \)-control chart is 3 times that for np-control chart, it is seen that \( \bar{x} \)-control chart leads to smaller ECPU as compared to that for np-control chart.

(II) Comparison of rows as listed (1) through (5).

As the cost of sampling and inspection per unit for \( \bar{x} \)-control chart increases, the optimal value of \( n \) decreases. However the total expected cost per unit (ECPU), as one expects, increases.

### 3.3 Economic Design under Unknown \( \sigma \)

We shall assume that the process standard deviation \( \sigma \) is unknown. Though unknown it is assumed to be attaining some constant value throughout the production cycle. All the other assumptions of the model and the system discussed in Section 3.2 are continued to be true throughout the present section.

### 3.3.1 \( T^2 \)-Control Chart

It is proposed that the production process be monitored by \( T^2 \)-control chart.
### Table 3.1

Findings for $\bar{x}$-control chart

<table>
<thead>
<tr>
<th>Row No.</th>
<th>$a_2$</th>
<th>Optimal Values</th>
<th>ECP</th>
<th>$E(C_1)$</th>
<th>$E(C_2)$</th>
<th>$E(C_3)$</th>
<th>N(0)</th>
<th>N</th>
<th>$q_0$</th>
<th>$q_1$</th>
<th>D</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>1.0</td>
<td>11 252 2.30</td>
<td>0.4448</td>
<td>0.0833</td>
<td>0.0853</td>
<td>0.2761</td>
<td>4</td>
<td>5</td>
<td>0.0214</td>
<td>0.9778</td>
<td>23.70</td>
<td>1.51</td>
</tr>
<tr>
<td>(2)</td>
<td>1.5</td>
<td>9 254 2.20</td>
<td>0.4623</td>
<td>0.0925</td>
<td>0.0863</td>
<td>0.2835</td>
<td>4</td>
<td>5</td>
<td>0.0278</td>
<td>0.9554</td>
<td>24.42</td>
<td>1.25</td>
</tr>
<tr>
<td>(3)</td>
<td>2.0</td>
<td>9 254 2.20</td>
<td>0.4801</td>
<td>0.1102</td>
<td>0.0863</td>
<td>0.2835</td>
<td>4</td>
<td>5</td>
<td>0.0278</td>
<td>0.9554</td>
<td>24.42</td>
<td>1.25</td>
</tr>
<tr>
<td>(4)</td>
<td>2.5</td>
<td>7 257 2.05</td>
<td>0.4917</td>
<td>0.1070</td>
<td>0.0885</td>
<td>0.2962</td>
<td>4</td>
<td>5</td>
<td>0.0403</td>
<td>0.9176</td>
<td>25.71</td>
<td>1.00</td>
</tr>
<tr>
<td>(5)</td>
<td>3.0</td>
<td>7 257 2.05</td>
<td>0.5053</td>
<td>0.1206</td>
<td>0.0885</td>
<td>0.2962</td>
<td>4</td>
<td>5</td>
<td>0.0403</td>
<td>0.9176</td>
<td>25.71</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Findings for np-control chart

<table>
<thead>
<tr>
<th>Row No.</th>
<th>$a_2$</th>
<th>Optimal Values</th>
<th>ECP</th>
<th>$E(C_1)$</th>
<th>$E(C_2)$</th>
<th>$E(C_3)$</th>
<th>N(0)</th>
<th>N</th>
<th>$q_0$</th>
<th>$q_1$</th>
<th>D</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>(6)</td>
<td>1.0</td>
<td>37 350 2.00</td>
<td>0.5457</td>
<td>0.1343</td>
<td>0.0804</td>
<td>0.3310</td>
<td>3</td>
<td>4</td>
<td>0.0528</td>
<td>0.8964</td>
<td>33.00</td>
<td>5.00</td>
</tr>
</tbody>
</table>
Here the statistic $T^2$ based on $n$ observations $x_i (i=1,2,...n)$ of the sample of size $n$ is defined as

$$T^2 = n(\bar{x} - \mu_0)^2 / s_{n-1}^2$$

where

$$s_{n-1}^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

and

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

...(3.3.1)

...(3.3.2)

...(3.3.3)

It may be noted that $T^2$ has $F$ distribution with 1 and $(n-1)$ d.f.

A typical $T^2$-control chart is given below.

$$\begin{array}{c|c|c|c|c|c|c|c}
| \alpha | F_{\alpha,1,n-1} ( upper (100\alpha)\% point of F distribution ) |
\end{array}$$

$$\begin{array}{c|c|c|c|c|c|c|c}
| T^2 |
\end{array}$$

$$\begin{array}{c|c|c|c|c|c|c|c}
| 0 | 1 | 2 | 3 | 4 | 5 | 6 |
\end{array}$$

Sample Number

The sampling scheme and the control procedure are as follows. After the production of every $k$ units, $n$ units are sampled and examined. For each sample, sample mean $\bar{x}$, sample variance $s_{n-1}^2$ and $T^2$ are calculated. If $T^2 \leq F_{\alpha,1,n-1}$ then the process is declared to be in control and the production is continued. If $T^2 > F_{\alpha,1,n-1}$ the process is declared to be out of control and a search for the assignable cause is undertaken. Here $F_{\alpha,1,n-1}$ is the upper $(100\alpha)\%$ point of $F$ distribution such that

$$P(F > F_{\alpha,1,n-1}) = \alpha$$

...(3.3.4)

The design variables $n$, $k$, $F_{\alpha,1,n-1}$ are to be determined such that the expected total cost per unit of the product is minimum.
One may use T-control chart in place of $T^2$-control chart to monitor the production process. If T-control chart is used, one has to use $t$ and noncentral $t$-distributions to calculate the probability of type I error ($q_0$) and the power of the test ($q_1$). (The expressions for $q_0$ and $q_1$ are derived in the next section). Whereas if $T^2$-control chart is used, one needs central and noncentral $F$-distributions for the calculation of $q_0$ and $q_1$. The subroutines developed for central and noncentral $F$-integrals can be used further for multivariate $T^2$-control chart also. Hence with a view to extending the present model for multivariate $T^2$-control chart, we prefer $T^2$-control chart rather than T-control chart in univariate case also.

3.3.2 The probability of Type - I Error and the Power of $T^2$-Control Chart

We recall that $T^2$-control chart is proposed to find whether the process is in the in-control state $\mu_0$ or whether the process is in the out-of-control state $\mu_0 + \delta \sigma$ due to assignable cause.

Hence when the assignable cause occurs, the probability that it will be detected by any subsequent sample is

$$q_1 = P(T^2 > F_{\alpha,1,n-1} \mid \mu_0 + \delta \sigma) \ldots (3.3.5)$$

where $T^2$ has noncentral $F$ distribution with 1 and $(n-1)$ d.f. and the noncentrality parameter $n \delta^2$. The probability $q_1$ is known as the power of the $T^2$-control chart. It may be noted that $q_1$ is independent of $\sigma$. 

65
The probability of a false alarm is

\[ q_0 = P \left[ T^2 > F_{\alpha,1,n-1} \mid \mu_0 \right] \quad \text{(3.3.6)} \]

where \( T^2 \) has the F-distribution with 1 and \((n-1)\) d.f.. The probability \( q_0 \) is known as the probability of type-I error.

### 3.3.3 The Proportions of Nonconforming Units

The definitions of \( p_0 \) and \( p_1 \) remain the same as given in Section 3.2.3 and are to be obtained by the expressions (3.2.3a) and (3.2.3b) given there. Immediately one can see that one has to face the problem of unknown \( \sigma \) in evaluating these expressions. This problem can be solved in the following way. One may calculate the sample variance on the basis of a preliminary sample of some suitable size taken when the process is in control. The square root of this sample variance will give a quick estimator of \( \sigma \). Using this estimator in place of \( \sigma \) in the expressions (3.2.3a) and (3.2.3b) one gets the approximate values of \( p_0 \) and \( p_1 \).

However, if one wants to maintain some stipulated value of \( p_0 \) when the process is in the in-control state \( \mu_0 \), then the value of \( p_1 \) can be obtained as follows. Making an appropriate break-up of \( p_0 \) and referring to standard normal tables one can obtain the values for \((u_1-\mu_0)/\sigma\) and \((u_2-\mu_0)/\sigma\) using (3.2.3a). Substituting these values in (3.2.3b) one can find \( p_1 \) for known \( \delta \). In many cases \( u_1 \) and \( u_2 \) are equally spaced from \( \mu_0 \) on either side. In these situations \((u_1-\mu_0)/\sigma\) and \((u_2-\mu_0)/\sigma\) are numerically equal but opposite in signs so that \( p_0 = 2\Phi((u_1-\mu_0)/\sigma) \). For instance, if stipulated value of \( p_0 \) is 1 % then \((u_2-\mu_0)/\sigma = 2.58 \) and...
\(\frac{u_1 - \mu_0}{\sigma} = -2.58\). It is easy to see that in the above method the knowledge of \(\sigma\) is not required.

### 3.3.4 The Expected Cost Model

Since we have assumed that all the assumptions of the model of the case of known \(\sigma\) prevail here also, the expression for the total expected cost per unit (ECPU) is the same as given by (3.2.4). While evaluating this expression, one has to use (3.3.6) and (3.3.5) for \(q_0\) and \(q_1\) respectively. Substitution for \(p_0\) and \(p_1\) is just discussed in Section 3.3.3.

### 3.3.5 Solution Method and Numerical Example

**(A) Program**

A computer program on FORTRAN is developed to calculate the expected total cost per unit of the product for the given values of \(n, k, F_n, i, n-1\). This program computes the probabilities \(q_0\) and \(q_1\) using central and noncentral F-distributions. A subroutine is developed for the calculation of central F-distribution using Trapezoid Rule. The noncentral F-integrals are calculated from central F-integrals using the following results given in a book by Abramowitz and Stegun (1972) pp. 946 - 947.

These results are as follows.

1. **Distribution Function of central F**

   \[ P(F|\nu_1,\nu_2) = I_x \left( \frac{\nu_1}{2}, \frac{\nu_2}{2} \right) \]  

   \(\ldots(3.3.7)\)
where $I_x \left( \frac{V_1}{2}, \frac{V_2}{2} \right)$ is incomplete beta integral with $x = \frac{V_1 F}{V_2 + V_1 F}$.

(2) Distribution Function of noncentral $F$

\[
P \left( F | V_1, V_2, \lambda \right) = \sum_{j=0}^{\infty} \frac{\exp(-\lambda/2) (\lambda/2)^j}{j!} \cdot I_x \left( \frac{V_1}{2} + j, \frac{V_2}{2} \right)
\]

where $x = \frac{V_1 F}{V_2 + V_1 F}$ and $\lambda$ is noncentrality parameter.

The proportions $p_0$ and $p_1$ are supplied externally.

This program is linked to Hooke-Jeeves search technique to find the optimal values of $n$, $k$, $F_{a_1,n-1}$ which minimize ECPU. For the objective function understudy, two design variables $n$ and $k$ are discrete and the third design variable $F_{a_1,n-1}$ is continuous. However, by giving the suitable initial values for $(n, k, F_{a_1,n-1})$ and by choosing the proper step size and reduction factor, Hooke-Jeeves' procedure works successfully and gives the optimal solution. The listing associated with all the programs is given at the end of this chapter.

(B) Numerical Example

Let $a_1 = 10.0, a_2 = 1.0, a_3,1 = 100, a_3,2 = 100, a_4,1 = 10, a_4,2 = 15$.

Let $\lambda = 1, R = 1000, u_1 = -2.58, u_2 = 2.58$.

Let $\mu_0 = 0, \delta = 1.3$.

We take assessment of $\sigma$ to be 1 to calculate $p_0$ and $p_1$.

For these values of the cost coefficients and systems
parameters the search technique yielded the following optimal procedure.

\[ n = 11, \ k = 253, \ F = 5.3 \text{ with minimum ECPU } = \$ 0.4560. \]

For these optimal design variables we give the values of some intermediate terms required in the calculation of ECPU.

\[ N(0) = 4, \ N = 5, \ q_0 = 0.0522, \ q_1 = 0.9653 \]

\[ D = 24.10, \ S = 1.52 \]

\[
\frac{E(C_1)}{N_k} = \$ 0.0830, \quad \frac{E(C_2)}{N_k} = \$ 0.0934, \quad \frac{E(C_3)}{N_k} = \$ 0.2796
\]

Comparing the ECPU derived in this example with the corresponding example given by row (1) of Table 3.1 (when \( \sigma \) is known), one can see that the lack of knowledge of \( \sigma \) leads to cost penalty of \( (0.4560 - 0.4448 =) \$ 0.0112 \) per unit of the product.
SUBROUTINE XBC(RK, NSTAGE, SUM, A1, A2, A3, A3P, A4, A4P, ALEMDA, RATE, CPN, FNOT, FONE)

C FILE NAME IS VCC
C PROGRAM FOR ECPU OF XBAR-CHART

DIMENSION RK(IO)
WRITE(*,2) A1, A2, A3, A3P, A4, A4P
2 FORMAT(IX,'A1=','F10.4,'A2=','F10.4,'A3=','F10.4','A3P=','F10.4,
1 'A4=','F10.4','A4P=','F10.4)
WRITE(*,4) ALEMDA, RATE, CPN
4 FORMAT(IX, 'ALEMDA=','F10.4,'RATE=','F10.4,'CPN=','F10.4)
N=RK(1)
K=RK(2)
CL=RK(3)
WRITE(*,6) N, K, CL
6 FORMAT(IX,'N=','I5,'K=','I5,'CL=','F10.4)
WRITE(X,8) FNOT, FONE
8 FORMAT(IX, 'FNOT=','F10.6,'FONE=','F10.6)

PQWER=ALEMDA#K/RATE
PPQWER=—POWER
THEETA=EXP(PPOWER)
WRITE(*,9) THEETA
9 FORMAT(IX,'THEETA=','F10.6)

X=CL
CALL NDTR(X, P, D)
QNOT=2*(1.0-P)
WRITE(*,10) QNOT
10 FORMAT(IX,'QNOT=','F10.6)

AN=N
X=CL—SORT(AN)*CPN
CALL NDTR(X, P, D)
QONE=1-P
WRITE(*,40) QONE
40 FORMAT(IX,'QONE=','F10.6)

TNOS=THEETA/(1-THEETA)+1/QONE
NOS=TNOS+0.5
WRITE(*,50) NOS
50 FORMAT(IX,'NOS=','I5)

EC1=(A1+A2*N)*NOS
BNOT=QNOT*THEETA/(1-THEETA)
EC2=A3*BNOT+A3P
TAW=(1-(1+POWER)*THEETA)/(1-THEETA)
WRITE(*,55) TAW
55 FORMAT(IX,'TAW=','F10.4)

H=K/RATE
D=(RATE*FNOT/ALEMDA)+(H/QONE-TAW)*RATE*FONE
S=THEETA*N*FNOT/(1-THEETA)+N*FONE/QONE
WRITE(*,60) D, S
60 FORMAT(IX,'D=','F12.4,'S=','F12.4)

EC3=A4*S+A4P*(D-S)
EC=EC1+EC2+EC3
ECPU=EC/(NOS*K)
WRITE(*,65) EC1, EC2, EC3, EC, ECPU
65 FORMAT(IX,'EC1=','F12.4,'EC2=','F12.4,'EC3=','F12.4,'EC=', 'F12.4,
1 'ECPU=','F12.4)

SUM=ECPU
RETURN
END

L3.1
SUBROUTINE NDTR(X,P,D)
    AX=ABS(X)
    T=1.0/(1.0+.2316419*AX)
    D=0.39B9423*EXP(-X*X/2.0)
    P=1.0-D*(((1.330274ST-1.821256)*T+1.781478)*T-0.3565638)*T
    IF(X) 1,2,2
1    P=1.0-P
2    RETURN
END

SUBROUTINE XBC(RK,NSTAGE,SUM,A1,A2,A3,A3P,A4,A4P,ALEMDA, RATE,CPN,FNOT,FONE)
C FILE NAME IS XBAR
C COST MODEL FOR TSQR CHART FOR UNKNOWN VARIANCE
DIMENSION P(IOO),Q(IOO),R(IOO),RK(IO)
WRITE(*,2) A1,A2,A3,A3P,A4,A4P,ALEMDA, RATE,CPN,FNOT,FONE
2 FORMAT(1X,'Al=',F10.4,'A2=',F10.4,'A3=',F10.4,'A3P=',F10.4,
1 'A4=',F10.4,'A4P=',F10.4)
WRITE(St, 4) ALEMDA,RATE,CPN
4 FORMAT(1X, 'ALEMDA=',F10.4,'RATE=',F10.4,'CPN=',F10.4)
N=RK(1)
K=RK(2)
F=RK(3)
WRITE(*,6) N,K,F
6 FORMAT(1X,'N=',I5,'K=',I5,'F=',F10.4)
WRITE(*,8) FNOT,FONE
8 FORMAT(1X,'FNOT=',F10.6,'FONE=',F10.6)
CPN1=NSCFN
POWER=ALEMDA*K/RATE
PPOWER=-POWER
THEETA=EXP(PPOWER)
WRITE(*,9) THEETA
9 FORMAT(iX,'THEETA=',F10.6)
Y1=0
Y2=F/((N-I)+F)
A=0.5
B=(N-l)/2
H=0.01
CALL QR(Y1,Y2,A,B,H,BI)
QNOT=1-BI
WRITE(*,10) QNOT
10 FORMAT(1X,'QNOT=',F10.6)
DO 15 J=1,90
   Y1=0
   Y2=F/((N-I)+F)
   A=0.5+J
   B=(N-l)/2
   H=0.01
   CALL QR(Y1,Y2,A,B,H,BI)
   P(J)=BI
   IF(P(J).LT.0.00001)GO TO 21
15 CONTINUE
21 ISTOP=J-1

L3.2
WRITE(*,32) (P(J),J=1,ISTOP)
P0W=CPN1/2.0
PPQW=POW
R(I)=EXP(PPW)*POW*P(I)
IST=ISTOP-1
DO 30 J=1,IST
30 R(J+1)=POW#P(J+1)*R(J)/(P(J)*(J+1))
WRITE(*,32) (R(J),J=1,ISTOP)
FORMAT(1X,7F10.6)
Q0NE=EXP(PPW)*(1-Q0W)
TEM=Q0NE
DO 35 J=1,ISTOP
TEM=TEM+R(J)
35 CONTINUE
Q0NE=1-TEM
WRITE(*,40) Q0NE
FORMAT(1X,'QONE=',F10.6)
TN0S=THEETA/(1-THEETA)+1/Q0NE
N0S=TN0S+0.5
WRITE(*,50) N0S
FORMAT(1X,'NOS=',153
EC1=(A1+A2*N3#N0S
BN0T=Q0NE*THEETA/(1-THEETA)
EC2=A3*BN0T+A3P
TAW=(1-(1+POWER#THEETA)/(1-THEETA)
WRITE(*,55) TAW
FORMAT(1X,'TAW=',F10.4)
H=K/RATE
D=(RATE#BN0T/ALEM0A)+(H/QONE-TAW)*RATE#FONE
S=THEETA*N#FONE/(1-THEETA)+N#FONE/QONE
WRITE(*,60) D,S
FORMAT(1X,'D=',F12.4,'S=',F12.4)
EC3=A4#S+A4P#(D-S)
EC=EC1+EC2+EC3
ECPU=EC/(N0S#K)
WRITE(*,65)EC1,EC2,EC3,EC,ECPU
FORMAT(1X,'EC1=',F12.4,'EC2=',F12.4,'EC3=',F12.4,'EC=',F12.4,
1 'ECPU=',F12.4)
SUM=ECPU
RETURN
END

SUBROUTINE QR(Y1,Y2,A,B,H,BI)
C FILE NAME IS XBAR1
Y1=Y1
Y2=Y2
A=A
B=B
H=H
CALL BITA(Y1,Y2,A,B,H,PROB2)
BIN=PROB2
Y1=PROB2
Y1=Y1
Y2=1.0
A=A
B=B
H=H

L3.3
SUBROUTINE BITA(Y1,Y2,A,B,H,PROB2)
C COMPUTATION OF INCOMPLETE BETA INTEGRAL BY TRAPEZOID RULE
C FILE NAME IS TRALS
NOY1=(Y2-Y1)/H
NOY=NOY1+1
IF((NOY1 H) .EQ. (Y2-Y1)) GO TO 80
GO TO 85
80 WRITE(*,20)
20 FORMAT(IX,'NOY1 AND NOY ARE REALLY INTEGER AND NOT BY COM
1 TECH BOTH PROB AND PROB1 SAME')
GO TO 90
85 WRITE(*,22)
22 FORMAT(IX,'NOY AND NOY1 ARE NOT REALLY INTEGER AND ARE MADE
1 INTEGER , PROB AND PROB1 NOT EXPECTED SAME')
90 WRITE(*,24) NOY
24 FORMAT(IX,'NOY='18)
S1=0
DO 100 M=1,NOY
RM=M
YSUB=Y1+(RM-1.0)*H
IF(YSUB.EQ.0.) GO TO 200
GO TO 210
200 F=0.
GO TO 212
210 F=(YSUB**(A-1.0))**((1-YSUB)**(B-1.0))
212 IF(M.EQ.1) GO TO 120
IF((M .LT. M) .AND. (M .LT. NOY)) GO TO 125
IF (M .EQ. NOY) GO TO 130
120 F=F/2.
S1=S1+F
GO TO 100
125 S1=S1+F
GO TO 100
130 F=F/2.
S1=S1+F
FY2=(Y2**(A-1.0))**((1-Y2)**(B-1.0))
T=(F+FY2)**((Y2-Y1)-(RM-1.)*H)
S2=S1+T
100 CONTINUE
WRITE(*,26)FY2,T
26 FORMAT(IX,'FY2='18.10,'T='18.10)
PROB1=S1*H
PROB2=PROB1+T
WRITE(*,28) PROB1,PROB2
28 FORMAT(IX,'PROB1='18.10,'PROB2='18.10)
RETURN
END