CHAPTER 3

MODELLING AND CONTROL OF A FIVE-PHASE VOLTAGE SOURCE INVERTER

3.1 INTRODUCTION

This chapter is devoted to the development of a comprehensive model of a Five-phase voltage source inverter based on space vector theory. Proper modelling of voltage source inverters is important in devising appropriate control algorithm. The complete model is broadly classified into two group namely square wave and PWM mode based on the operation of the inverter. The leg voltages and line voltages along with phase voltages are illustrated. The Fourier analysis of output phase-to-neutral voltages and non-adjacent voltages is performed for square wave mode. The output phase-to-neutral voltage is shown to be essentially identical to those obtainable with a three-phase voltage source inverter. At each step simulation results are provided to support the analytical approach used. The relationship between the phase and line voltages for a Five-phase system is also established.

For high power application stepped operation of inverter is preferred over PWM mode to avoid switching losses. Square wave mode of operation is elaborate for various conduction modes such as 180°, 144° and 108° and the performance is evaluated in terms of harmonic content of phase-to-neutral voltages. The Total harmonic distortion is evaluated for each case and a comparison is provided. Complete experimental set-up is provided for a prototype Five-phase inverter developed in the laboratory. Experimental results are also provided in the chapter for stepped operation a Five-phase voltage source inverter. Experimental and analytical results are shown to verify each other.
3.2 MODELLING OF A FIVE-PHASE VSI

Power circuit topology of a Five-phase VSI is shown in Fig. 3.1. Each switch in the circuit consists of two power semiconductor devices, connected in anti-parallel. One of these is a fully controllable semiconductor, such as a bipolar transistor or IGBT, while the second one is a diode. The input of the inverter is a dc voltage, which is regarded further on as being constant. The inverter outputs are denoted in Fig. 3.1 with lower case symbols \(a, b, c, d, e\), while the points of connection of the outputs to inverter legs have symbols in capital letters \(A, B, C, D, E\). The basic operating principles of the Five-phase VSI are developed in what follows assuming the ideal commutation and zero forward voltage drop.

Discrete switching of power switches in an inverter leads to stepped wave output termed as square wave operation of the inverter. Conventionally 180° conduction mode is considered leading to ten-step output phase voltages from the inverter. Two more conduction angles 144° and 108° are also taken up in this section.

3.2.1 SQUARE WAVE MODE OF OPERATION

Discrete switching of power switches in an inverter leads to stepped wave output termed as square wave operation of the inverter. Conventionally 180° conduction mode is considered leading to ten-step output phase voltages from the inverter. Two more conduction angles 144° and 108° are also taken up in this section.

3.2.1A 180° CONDUCTION MODE

Each switch is assumed to conduct for 180° and the phase delay between firing of two switches in any subsequent two phases is equal to \(360°/5 = 72°\). The driving control gate/base signals
for the ten switches of the inverter in Fig. 3.1 are illustrated in Fig. 3.2. One complete cycle of operation of the inverter can be divided into ten distinct modes indicated in Fig. 3.2 and summarised in Table 3.1. It follows from Fig. 3.2 and Table 3.1 that at any instant in time there are five switches that are ‘ON’ and five switches that are ‘OFF’. In the ten-step mode of operation there are two conducting switches from the upper five and three from the lower five, or vice versa.

Table 3.1. Modes of operation of the five-phase voltage source inverter (ten-step operation).

<table>
<thead>
<tr>
<th>Mode</th>
<th>Switches ON</th>
<th>Terminal polarity</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>1,7,8,9,10</td>
<td>A'B'C'D'E'</td>
</tr>
<tr>
<td>10</td>
<td>8,9,10,1,2</td>
<td>A'B'C'D'E'</td>
</tr>
<tr>
<td>1</td>
<td>9,10,1,2,3</td>
<td>A'B'C'D'E'</td>
</tr>
<tr>
<td>2</td>
<td>10,1,2,3,4</td>
<td>A'B'C'D'E'</td>
</tr>
<tr>
<td>3</td>
<td>1,2,3,4,5</td>
<td>A'B'C'D'E'</td>
</tr>
<tr>
<td>4</td>
<td>2,3,4,5,6</td>
<td>A'B'C'D'E'</td>
</tr>
<tr>
<td>5</td>
<td>3,4,5,6,7</td>
<td>A'B'C'D'E'</td>
</tr>
<tr>
<td>6</td>
<td>4,5,6,7,8</td>
<td>A'B'C'D'E'</td>
</tr>
<tr>
<td>7</td>
<td>5,6,7,8,9</td>
<td>A'B'C'D'E'</td>
</tr>
<tr>
<td>8</td>
<td>6,7,8,9,10</td>
<td>A'B'C'D'E'</td>
</tr>
</tbody>
</table>

Space vector of phase voltages in stationary reference frame is defined, using power variant transformation,

$$\mathbf{v} = \frac{2}{5}(\mathbf{v}_a + \mathbf{a} \mathbf{v}_b + \mathbf{a}^2 \mathbf{v}_c + \mathbf{a}^* \mathbf{v}_d + \mathbf{a}^* \mathbf{v}_e)$$

(3.1)

where $a = \exp(j2\pi/5)$, $a^2 = \exp(j4\pi/5)$, $a^* = \exp(-j2\pi/5)$, $a^{*2} = \exp(-j4\pi/5)$ and $*$ stands for a complex conjugate.

Leg voltages (i.e. voltages between points $A, B, C, D, E$ and the negative rail of the dc bus $N$ in Fig. 3.1) are considered first. The leg voltages from Fig. 3.2 are substituted in expression (3.1) to obtain their corresponding space vectors given as in equation (3.2)
\[ v_{1\text{leg}} \]
\[ v_{2\text{leg}} \]
\[ v_{3\text{leg}} \]
\[ v_{4\text{leg}} \]
\[ v_{5\text{leg}} \]
\[ v_{6\text{leg}} \]
\[ v_{7\text{leg}} \]
\[ v_{8\text{leg}} \]
\[ v_{9\text{leg}} \]
\[ v_{10\text{leg}} \]

\[ = \frac{2}{5} V_{DC} \left( 2 \cos \frac{\pi}{5} \right) \]

(3.2)

Fig. 3.2 Driving switch signals of a five-phase voltage source inverter in the ten-step mode
It is seen that the leg voltages have magnitude of $\frac{2}{5}V_{dc}2\cos(\pi/5)$ and are 36° apart forming a decagon.

Phase-to-neutral voltages are investigated next. Phase-to-neutral voltages of the star connected load are most easily found by defining a voltage difference between the star point $n$ of the load and the negative rail of the dc bus $N$. The following correlation then holds true:

\[ V_B = v_a + v_{nN} \]
\[ V_C = v_b + v_{nN} \]
\[ V_D = v_c + v_{nN} \]
\[ V_E = v_d + v_{nN} \]  

(3.3)

Since the phase voltages in a star connected load sum to zero, summation of the equations (3.3) yields

\[ v_{nN} = \left(\frac{1}{5}\right)(v_A + v_B + v_C + v_D + v_E) \]  

(3.4)

Substitution of (3) into (2) yields phase-to-neutral voltages of the load in the following form:

\[ v_a = \left(\frac{4}{5}\right)v_A - \left(\frac{1}{5}\right)(v_B + v_C + v_D + v_E) \]
\[ v_b = \left(\frac{4}{5}\right)v_B - \left(\frac{1}{5}\right)(v_A + v_C + v_D + v_E) \]
\[ v_c = \left(\frac{4}{5}\right)v_C - \left(\frac{1}{5}\right)(v_A + v_B + v_D + v_E) \]
\[ v_d = \left(\frac{4}{5}\right)v_D - \left(\frac{1}{5}\right)(v_A + v_B + v_C + v_E) \]
\[ v_e = \left(\frac{4}{5}\right)v_E - \left(\frac{1}{5}\right)(v_A + v_B + v_C + v_D) \]  

(3.5)

The phase voltages in different modes are obtained by substituting leg voltages into equation (3.5) and their space vectors are determined using equation (3.1). The space vectors of phase-to-neutral voltage are identical to the leg voltage space vectors. The phase-to-neutral voltages for various modes are given in Fig. 3.3.

In order to relate the input dc link voltage of the inverter with the output phase-to-neutral, Fourier analysis of the voltage waveforms is undertaken.

Using definition of the Fourier series for a periodic waveform
\( v(t) = V_o + \sum_{n=1}^{\infty} (A_n \cos n\omega t + B_n \sin n\omega t) \) 

where the coefficients of the Fourier series are given with

\[
V_o = \frac{1}{T} \int_0^T v(t) dt = \frac{1}{2\pi} \int_0^{2\pi} v(\theta) d\theta
\]

\[
A_n = \frac{2}{T} \int_0^T v(t) \cos n\omega t dt = \frac{1}{\pi} \int_0^{2\pi} v(\theta) \cos n\theta d\theta
\]

\[
B_n = \frac{2}{T} \int_0^T v(t) \sin n\omega t dt = \frac{1}{\pi} \int_0^{2\pi} v(\theta) \sin n\theta d\theta
\]

Fig. 3.3. Phase-to-neutral voltages of the five-phase VSI in the ten-step mode of operation.
\[ v(t) = V_o + \sum_{n=1}^{\infty} (A_n \cos n \omega t + B_n \sin n \omega t) \]  \hfill (3.6)

where the coefficients of the Fourier series are given with

\[ V_o = \frac{1}{T} \int_0^T v(t) dt = \frac{1}{2\pi} \int_0^{2\pi} v(\theta) d\theta \]

\[ A_n = \frac{2}{T} \int_0^T v(t) \cos n \omega t dt = \frac{1}{\pi} \int_0^{2\pi} v(\theta) \cos n \theta d\theta \]  \hfill (3.7)

\[ B_n = \frac{2}{T} \int_0^T v(t) \sin n \omega t dt = \frac{1}{\pi} \int_0^{2\pi} v(\theta) \sin n \theta d\theta \]

Fig. 3.3. Phase-to-neutral voltages of the five-phase VSI in the ten-step mode of operation.
and observing that the waveforms possess quarter-wave symmetry and can be conveniently taken as odd functions, one can represent phase-to-neutral voltages with the following expressions:

\[
v(t) = \sum_{n=1}^{\infty} B_{2n-1} \sin(2n-1)\omega t; \text{ where } B_{2n-1} = \frac{4}{\pi} \int_{0}^{\frac{\pi}{2}} v(\theta) \sin(2n-1)\theta d\theta; \text{ and } n = 1, 2, 3, \ldots (3.8)
\]

In the case of the phase-to-neutral voltage \( v_n \), shown in Fig. 3.3, one further has for the coefficients of the Fourier series

\[
B_{2n-1} = \frac{8V_{DC}}{5\pi(2n-1)} \left[ 1 + \cos(2n-1)\frac{3\pi}{5} \cos(2n-1)\frac{4\pi}{5} \right]; \text{ where } k = 1, 2, 3, \ldots (3.9)
\]

The expression in brackets of equation (3.9) equals zero for all the harmonics whose order is divisible by five. For all the other harmonics it equals 2.5. Hence one can write the Fourier series of the phase-to-neutral voltage as

\[
v(t) = \frac{2}{\pi} V_{DC} \left[ \sin \omega t + \frac{1}{3} \sin 3\omega t + \frac{1}{7} \sin 7\omega t + \frac{1}{9} \sin 9\omega t + \ldots \right] \]

(3.10)

From (3.10) it follows that the fundamental component of the output phase-to-neutral voltage has an RMS value equal to

\[
V_1 = \frac{\sqrt{2}}{\pi} V_{DC} = 0.45V_{DC} (3.11)
\]

From Fig. 3.3, mean square value is determined as:

\[
\text{Mean Square Value} = \frac{1}{\pi} \left[ \left( \frac{2}{5} V_{DC} \right)^2 \times \frac{3\pi}{5} + \left( \frac{3}{5} V_{DC} \right)^2 \times \frac{2\pi}{5} \right] = \frac{6}{25} V_{DC}^2 (3.12)
\]

\[
V_{rms} = \frac{\sqrt{6}}{5} V_{DC} (3.13)
\]

Total harmonic r.m.s. voltage (p.u.) is given by

\[
V_{Hrms} = \sqrt{(V_{rms})^2 - (V_1)^2} = \sqrt{\left( \frac{\sqrt{6}}{5} \right)^2 - \left( \frac{\sqrt{2}}{\pi} \right)^2} = 0.193281227 (3.14)
\]

Hence total harmonic distortion is
\[ THD = \frac{r.m.s. \text{ total harmonic voltage}}{r.m.s. \text{ total voltage}} \]
\[ = \frac{0.193281227}{\sqrt{6/5}} = 0.3945336525 \text{ or } 39.45\% \]  \hfill (3.15)

This is the same voltage as obtainable with a three-phase VSI operating in six-step mode. It is important to note at this stage that the space vectors described by (3.1) provides mapping of inverter voltages into a two-dimensional space. However, since five-phase inverter essentially requires description in a five-dimensional space not all the harmonics contained in (3.10) will be encompassed by the space vector of (3.1). In particular, space vectors calculated using (3.1) will only represent harmonics of the order \(10k \pm 1, k = 0, 1, 2, 3, \ldots\), that is, the first, the ninth, the eleventh, and so on. Harmonics of the order \(5k, k = 1, 2, 3, \ldots\) cannot appear due to the isolated neutral point. However, harmonics of the order \(5k \pm 2, k = 1, 3, 5, \ldots\) are present in (3.10) but are not encompassed by the space vector definition of (3.1). These harmonics in essence appear in the second two-dimensional space, which requires introduction of the second space vector for the five-phase system.

Simulation is performed to obtain the harmonic spectrum of inverter phase voltage in ten-step mode of operation, shown in Fig. 3.4. The dc voltage is kept at 1 p.u. The harmonic spectrum is in compliance with the expression (3.10) and (3.11).

![Fig. 3.4. Inverter phase 'a' voltage time domain waveform and its harmonic spectrum in frequency domain (Fundamental frequency is 50 Hz, Fundamental voltage 0.4504 (p.u.)).](image-url)
The fundamental component is equal to 0.4504 p.u., which is same as what is obtainable with a three-phase VSI. The Sub harmonic components are 3rd and 7th in Fig. 3.4 and their magnitudes are 33.33% and 14.3%, respectively. These sub harmonics will appear in the x-y plane and will cause distortion in the stator currents and consequently increase the losses in the machine. The lowest harmonic appearing on d-q plane are 9th and 11th with their magnitude as 11.1% and 9.1%, respectively. These harmonic will further add to the losses in addition to tenth harmonic pulsating torque under steady state conditions.

The Line-to-line voltages are elaborated next. There are two system of line-to-line voltage; adjacent and non-adjacent, in contrast to a three-phase system where only one line-to-line voltage is defined. The adjacent line-to-line voltages at the output of the five-phase inverter are defined in Fig. 3.5, for a fictitious load. Since each line-to-line voltage is a difference of corresponding two leg voltages, the values of nonadjacent line-to-line voltages will yield higher magnitude compared to the adjacent line voltages, hence only former case is taken up in the thesis and later is omitted from further consideration.

![Fig. 3.5. Adjacent line-to-line voltages of a five-phase star-connected load.](image)

There are two sets of non-adjacent line-to-line voltages. Due to symmetry, these two sets lead to the same values of the line-to-line voltage space vectors, with a different phase order. Only the set \( v_{ac}, v_{ad}, v_{ce}, v_{de}, v_{eb} \) is analysed for this reason. Table 3.2 lists the states and the values for these line-to-line voltages.
Table 3.2. Non-adjacent line-to-line voltages for 180° conduction mode.

<table>
<thead>
<tr>
<th>Switching state</th>
<th>Switches ON</th>
<th>Space vector</th>
<th>( V_{dc} )</th>
<th>( V_{bd} )</th>
<th>( V_{ae} )</th>
<th>( V_{de} )</th>
<th>( V_{eb} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9,10,1,2,3</td>
<td>( y_{1u} )</td>
<td>( V_{DC} )</td>
<td>( V_{DC} )</td>
<td>( -V_{DC} )</td>
<td>( -V_{DC} )</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>10,1,2,3,4</td>
<td>( y_{2u} )</td>
<td>( V_{DC} )</td>
<td>( V_{DC} )</td>
<td>0</td>
<td>( -V_{DC} )</td>
<td>( -V_{DC} )</td>
</tr>
<tr>
<td>3</td>
<td>1,2,3,4,5</td>
<td>( y_{3u} )</td>
<td>0</td>
<td>( V_{DC} )</td>
<td>( V_{DC} )</td>
<td>( -V_{DC} )</td>
<td>( -V_{DC} )</td>
</tr>
<tr>
<td>4</td>
<td>2,3,4,5,6</td>
<td>( y_{4u} )</td>
<td>( -V_{DC} )</td>
<td>( V_{DC} )</td>
<td>( V_{DC} )</td>
<td>0</td>
<td>( -V_{DC} )</td>
</tr>
<tr>
<td>5</td>
<td>3,4,5,6,7</td>
<td>( y_{5u} )</td>
<td>( -V_{DC} )</td>
<td>0</td>
<td>( V_{DC} )</td>
<td>( V_{DC} )</td>
<td>( -V_{DC} )</td>
</tr>
<tr>
<td>6</td>
<td>4,5,6,7,8</td>
<td>( y_{6u} )</td>
<td>( -V_{DC} )</td>
<td>( -V_{DC} )</td>
<td>( V_{DC} )</td>
<td>( V_{DC} )</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>5,6,7,8,9</td>
<td>( y_{7u} )</td>
<td>( -V_{DC} )</td>
<td>( -V_{DC} )</td>
<td>0</td>
<td>( V_{DC} )</td>
<td>( V_{DC} )</td>
</tr>
<tr>
<td>8</td>
<td>6,7,8,9,10</td>
<td>( y_{8u} )</td>
<td>0</td>
<td>( -V_{DC} )</td>
<td>( -V_{DC} )</td>
<td>( V_{DC} )</td>
<td>( V_{DC} )</td>
</tr>
<tr>
<td>9</td>
<td>7,8,9,10,1</td>
<td>( y_{9u} )</td>
<td>( -V_{DC} )</td>
<td>( -V_{DC} )</td>
<td>0</td>
<td>( V_{DC} )</td>
<td>( V_{DC} )</td>
</tr>
<tr>
<td>10</td>
<td>8,9,10,1,2</td>
<td>( y_{10u} )</td>
<td>( V_{DC} )</td>
<td>0</td>
<td>( -V_{DC} )</td>
<td>( -V_{DC} )</td>
<td>( V_{DC} )</td>
</tr>
</tbody>
</table>

Space vectors of non-adjacent line-to-line voltages are determined once more using the defining expression (3.1) and are summarised as

\[
\begin{bmatrix}
V_{1u} \\
V_{2u} \\
V_{3u} \\
V_{4u} \\
V_{5u} \\
V_{6u} \\
V_{7u} \\
V_{8u} \\
V_{9u} \\
V_{10u}
\end{bmatrix} = \frac{8}{5} V_{dc} \cos \left( \frac{\pi}{5} \right) \cos \left( \frac{\pi}{10} \right) \\
\begin{bmatrix}
e^{j\pi/10} \\
e^{j3\pi/10} \\
e^{j\pi/2} \\
e^{j9\pi/10} \\
e^{j13\pi/10} \\
e^{j15\pi/10} \\
e^{j17\pi/10} \\
e^{j19\pi/10}
\end{bmatrix}
\]

Time-domain waveforms of non-adjacent line-to-line voltages are illustrated in Fig. 3.6.
The Fourier analysis is further carried out for non-adjacent line-to-line voltage following the same procedure outlined in conjunction with Fourier analysis of phase voltages. The non-adjacent line voltages waveform possess quarter wave odd symmetry hence the Fourier coefficient can be evaluated as

\[ B_{2n-1} = \frac{4V_{DC}}{(2n-1)\pi} \left( \cos(2n-1)\frac{\pi}{10} \right) \text{ Where } n = 1, 2, 3, \ldots \text{ and } \]

the Fourier series of nonadjacent line-to-line voltage can be written as

\[ v(t) = \frac{4}{\pi} V_{DC} \left[ \cos \left( \frac{\pi}{10} \right) \sin(\omega t) + \frac{1}{3} \cos \left( \frac{3\pi}{10} \right) \sin(3\omega t) + \frac{1}{7} \cos \left( \frac{7\pi}{10} \right) \sin(7\omega t) + \ldots \right] \]  

Thus the peak of the fundamental is

\[ V_u = \frac{4}{\pi} V_{DC} \cos \left( \frac{\pi}{10} \right) = 1.211V_{DC} \]  

From Fig. 3.6, mean square value is determined as:

\[ \text{Mean Square Value} = \frac{1}{\pi} \left( V_{DC} \right)^2 \times \frac{4\pi}{5} = \frac{4}{5} V_{DC}^2 \]  

\[ V_{rms} = \frac{2}{\sqrt{5}} V_{DC} = 0.89442719 V_{DC} \]  

Total harmonic r.m.s. voltage (p.u.) is given by
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\[ V_{H_{rms}} = \sqrt{(I_{r_{rms}})^2 - (I_1')^2} = \sqrt{\left(\frac{2}{\sqrt{5}}\right)^2 - \left(\frac{2\sqrt{2}}{\pi} \cos\left(\frac{\pi}{10}\right)\right)^2} = 0.2585208456 \]  

(3.21)

Hence total harmonic distortion is

\[ THD = \frac{r.m.s. \text{ total harmonic voltage}}{r.m.s. \text{ total voltage}} \]

\[ = \frac{0.2585208456}{2/\sqrt{5}} = 0.2890350922 \text{ or } 28.9\% \]  

(3.22)

A simulation study is performed using Matlab/Simulink to determine the non-adjacent line-to-line voltages spectrum and is shown in Fig. 3.7. The harmonic spectrum is in compliance with the expressions (3.17) and (3.18).

![Fig. 3.7. Inverter non-adjacent line voltage time domain waveform and its harmonic spectrum in frequency domain (Fundamental frequency is 50 Hz, Fundamental voltage 0.8562 (p.u)).](image)

3.2.1B 144° CONDUCTION MODE

In this mode each switch conducts for 144°. The gating signals are shown in Fig. 3.8 and the corresponding switches being on are listed in Table 3.3.

It is seen from Fig. 3.8 that a dead band of 36° is available providing inherent protection of the power switches of the same leg from short circuit. The phase-to-neutral voltages are found using equation (3.5) and corresponding Leg voltage values from Table 3.4 and the resulting values are
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\[ V_{H_{rms}} = \sqrt{\left(\frac{2}{\sqrt{5}}\right)^2 - \left(\frac{2\sqrt{2}}{\pi} \cos \left(\frac{\pi}{10}\right)\right)^2} = 0.2585208456 \]  

(3.21)

Hence total harmonic distortion is

\[ THD = \frac{r.m.s.\ total\ harmonic\ voltage}{r.m.s.\ total\ voltage} \]

\[ = \frac{0.2585208456}{2/\sqrt{5}} = 0.2890350922 \text{ or } 28.9\% \]  

(3.22)

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![Inverter non-adjacent line voltage time domain waveform and its harmonic spectrum in frequency domain (Fundamental frequency is 50 Hz, Fundamental voltage 0.8562 (p.u)).](image)

**Fig. 3.7.** Inverter non-adjacent line voltage time domain waveform and its harmonic spectrum in frequency domain (Fundamental frequency is 50 Hz, Fundamental voltage 0.8562 (p.u)).

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listed in Table 3.5. The waveform of phase voltages is shown in Fig. 3.9. $[0X]$ indicates floating nature of the leg polarity.

<table>
<thead>
<tr>
<th>Modes</th>
<th>Duration</th>
<th>Switches ON</th>
<th>Polarity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0-36°</td>
<td>8,9,10,1</td>
<td>+ - 0 +</td>
</tr>
<tr>
<td>2</td>
<td>36°-72°</td>
<td>9,10,1,2</td>
<td>+ 0 - +</td>
</tr>
<tr>
<td>3</td>
<td>72°-108°</td>
<td>10,1,2,3</td>
<td>+ + - 0</td>
</tr>
<tr>
<td>4</td>
<td>108°-144°</td>
<td>1,2,3,4</td>
<td>+ + 0 -</td>
</tr>
<tr>
<td>5</td>
<td>144°-180°</td>
<td>2,3,4,5</td>
<td>0 + + -</td>
</tr>
<tr>
<td>6</td>
<td>180°-216°</td>
<td>3,4,5,6</td>
<td>- + + 0</td>
</tr>
<tr>
<td>7</td>
<td>216°-252°</td>
<td>4,5,6,7</td>
<td>- 0 + +</td>
</tr>
<tr>
<td>8</td>
<td>252°-288°</td>
<td>5,6,7,8</td>
<td>- - + 0</td>
</tr>
<tr>
<td>9</td>
<td>288°-324°</td>
<td>6,7,8,9</td>
<td>- - 0 +</td>
</tr>
<tr>
<td>10</td>
<td>324°-360°</td>
<td>7,8,9,10</td>
<td>0 - - +</td>
</tr>
</tbody>
</table>

Table 3.3. Switches position in each mode for 144° conduction mode.

Fig. 3.8. Gating signals for 144° conduction mode.
### Table 3.4. Leg Voltages for 144° conduction mode

<table>
<thead>
<tr>
<th>Modes</th>
<th>Duration</th>
<th>( V_A )</th>
<th>( V_B )</th>
<th>( V_C )</th>
<th>( V_D )</th>
<th>( V_E )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0-36°</td>
<td>( V_{DC} )</td>
<td>0</td>
<td>0</td>
<td>( \frac{1}{2} V_{DC} )</td>
<td>( V_{DC} )</td>
</tr>
<tr>
<td>2</td>
<td>36°-72°</td>
<td>( V_{DC} )</td>
<td>( \frac{1}{2} V_{DC} )</td>
<td>0</td>
<td>0</td>
<td>( V_{DC} )</td>
</tr>
<tr>
<td>3</td>
<td>72°-108°</td>
<td>( V_{DC} )</td>
<td>( V_{DC} )</td>
<td>0</td>
<td>0</td>
<td>( \frac{1}{2} V_{DC} )</td>
</tr>
<tr>
<td>4</td>
<td>108°-144°</td>
<td>( V_{DC} )</td>
<td>( V_{DC} )</td>
<td>( \frac{1}{2} V_{DC} )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>144°-180°</td>
<td>( \frac{1}{2} V_{DC} )</td>
<td>( V_{DC} )</td>
<td>( V_{DC} )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>180°-216°</td>
<td>0</td>
<td>( V_{DC} )</td>
<td>( V_{DC} )</td>
<td>( \frac{1}{2} V_{DC} )</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>216°-252°</td>
<td>0</td>
<td>( \frac{1}{2} V_{DC} )</td>
<td>( V_{DC} )</td>
<td>( V_{DC} )</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>252°-288°</td>
<td>0</td>
<td>0</td>
<td>( V_{DC} )</td>
<td>( V_{DC} )</td>
<td>( \frac{1}{2} V_{DC} )</td>
</tr>
<tr>
<td>9</td>
<td>288°-324°</td>
<td>0</td>
<td>0</td>
<td>( \frac{1}{2} V_{DC} )</td>
<td>( V_{DC} )</td>
<td>( V_{DC} )</td>
</tr>
<tr>
<td>10</td>
<td>324°-360°</td>
<td>( \frac{1}{2} V_{DC} )</td>
<td>0</td>
<td>0</td>
<td>( V_{DC} )</td>
<td>( V_{DC} )</td>
</tr>
</tbody>
</table>

### Table 3.5. Phase-to-neutral voltages for 144° conduction mode.

<table>
<thead>
<tr>
<th>Modes</th>
<th>Duration</th>
<th>( V_a )</th>
<th>( V_b )</th>
<th>( V_c )</th>
<th>( V_d )</th>
<th>( V_e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0-36°</td>
<td>( \frac{1}{2} V_{DC} )</td>
<td>-( \frac{1}{2} V_{DC} )</td>
<td>-( \frac{1}{2} V_{DC} )</td>
<td>0</td>
<td>( \frac{1}{2} V_{DC} )</td>
</tr>
<tr>
<td>2</td>
<td>36°-72°</td>
<td>( \frac{1}{2} V_{DC} )</td>
<td>0</td>
<td>-( \frac{1}{2} V_{DC} )</td>
<td>-( \frac{1}{2} V_{DC} )</td>
<td>( \frac{1}{2} V_{DC} )</td>
</tr>
<tr>
<td>3</td>
<td>72°-108°</td>
<td>( \frac{1}{2} V_{DC} )</td>
<td>( \frac{1}{2} V_{DC} )</td>
<td>-( \frac{1}{2} V_{DC} )</td>
<td>-( \frac{1}{2} V_{DC} )</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>108°-144°</td>
<td>( \frac{1}{2} V_{DC} )</td>
<td>( \frac{1}{2} V_{DC} )</td>
<td>0</td>
<td>-( \frac{1}{2} V_{DC} )</td>
<td>-( \frac{1}{2} V_{DC} )</td>
</tr>
<tr>
<td>5</td>
<td>144°-180°</td>
<td>0</td>
<td>( \frac{1}{2} V_{DC} )</td>
<td>( \frac{1}{2} V_{DC} )</td>
<td>-( \frac{1}{2} V_{DC} )</td>
<td>-( \frac{1}{2} V_{DC} )</td>
</tr>
<tr>
<td>6</td>
<td>180°-216°</td>
<td>-( \frac{1}{2} V_{DC} )</td>
<td>( \frac{1}{2} V_{DC} )</td>
<td>( \frac{1}{2} V_{DC} )</td>
<td>0</td>
<td>-( \frac{1}{2} V_{DC} )</td>
</tr>
<tr>
<td>7</td>
<td>216°-252°</td>
<td>-( \frac{1}{2} V_{DC} )</td>
<td>0</td>
<td>( \frac{1}{2} V_{DC} )</td>
<td>( \frac{1}{2} V_{DC} )</td>
<td>-( \frac{1}{2} V_{DC} )</td>
</tr>
<tr>
<td>8</td>
<td>252°-288°</td>
<td>-( \frac{1}{2} V_{DC} )</td>
<td>-( \frac{1}{2} V_{DC} )</td>
<td>( \frac{1}{2} V_{DC} )</td>
<td>( \frac{1}{2} V_{DC} )</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>288°-324°</td>
<td>-( \frac{1}{2} V_{DC} )</td>
<td>-( \frac{1}{2} V_{DC} )</td>
<td>0</td>
<td>( \frac{1}{2} V_{DC} )</td>
<td>( \frac{1}{2} V_{DC} )</td>
</tr>
<tr>
<td>10</td>
<td>324°-360°</td>
<td>0</td>
<td>-( \frac{1}{2} V_{DC} )</td>
<td>-( \frac{1}{2} V_{DC} )</td>
<td>( \frac{1}{2} V_{DC} )</td>
<td>( \frac{1}{2} V_{DC} )</td>
</tr>
</tbody>
</table>
In order to relate the input dc link voltage of the inverter with the output phase-to-neutral, Fourier analysis of the voltage waveforms is undertaken. Observing that the waveforms of the phase to neutral voltage possess odd quarter-wave symmetry. In case of phase-to-neutral voltage $V_a$, shown in Fig. 3.9, the coefficients of the Fourier series are:

$$B_{2n-1} = \sum_{n=1}^{\infty} \frac{2V_{dc}}{(2n-1)\pi} \cos(2n-1) \frac{\pi}{10}$$ \hspace{1cm} (3.23)

The expression in equation (3.23) equals to zero for all the harmonics whose order is divisible by 5. Hence one can write the phase-to-neutral voltages sinusoidal series as:

$$V(t) = \sum_{n=1}^{\infty} \frac{2V_{dc}}{(2n-1)\pi} \frac{\pi}{10} \sin(2n-1)\omega t$$ \hspace{1cm} (3.24)

From (3.24) it follows that the fundamental sinusoidal component of the output phase-to-neutral voltages has an RMS value equal to

$$V_a = 0.428126V_{dc}$$ \hspace{1cm} (3.25)

From Fig. 3.9, mean square value is determined as:

$$Mean\ Square\ Value = \frac{1}{\pi} \left[ \left( \frac{1}{2} V_{dc} \right)^2 \times \frac{4\pi}{5} \right] = \frac{1}{5} V_{dc}^2$$ \hspace{1cm} (3.26)

$$V_{rms} = \frac{\sqrt{5}}{5} V_{dc}$$ \hspace{1cm} (3.27)
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In order to relate the input dc link voltage of the inverter with the output phase-to-neutral, Fourier analysis of the voltage waveforms is undertaken. Observing that the waveforms of the phase to neutral voltage possess odd quarter-wave symmetry. In case of phase-to-neutral voltage $V_a$, shown in Fig. 3.9, the coefficients of the Fourier series are:

$$B_{2a-1} = \sum_{n=1}^{\infty} \frac{2V_{dc}}{(2n-1)\pi} \cos\left(\frac{2n-1}{10}\pi\right)$$  \hspace{1cm} (3.23)

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$$V(t) = \sum_{n=1}^{\infty} \frac{2V_{dc}}{(2n-1)\pi} \cos\left(\frac{2n-1}{10}\pi\right) \sin(2n-1)\omega t$$  \hspace{1cm} (3.24)

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$$V_{rms} = \frac{\sqrt{5}}{5}V_{dc}$$  \hspace{1cm} (3.27)
Total harmonic rms voltage (p.u.) is given by

\[
V_{Hrms} = \sqrt{(V_{rms})^2 - (V_1')^2} = \sqrt{\left(\frac{1}{\sqrt{5}}\right)^2 - \left(\frac{\sqrt{2}}{\pi} \cos\left(\frac{\pi}{10}\right)\right)^2} = 0.1292604228
\]  

(3.28)

Hence total harmonic distortion is

\[
THD = \frac{\text{rms total harmonic voltage}}{\text{rms total voltage}}
\]

\[
= \frac{0.12926}{1/\sqrt{5}} = 0.2890350922 \text{ or } 28.9\%
\]  

(3.29)

The theoretical value of THD in this conduction mode is significantly reduced compared to conventional 180° conduction mode. This is due to the symmetrical switching pattern in this conduction mode.

FFT analysis of inverter phase ‘V_a’ voltage is performed using Matlab/Simulink code and is shown in Fig. 3.10 and this validates equation (3.24).

![Phase Voltage 'V_a' time domain waveform and its harmonic spectrum in frequency domain](image)

Fig 3.10 Inverter phase ‘V_a’ voltage time domain waveform and its harmonic spectrum in frequency domain (Fundamental frequency is 50 Hz)

The spectrum shows the fundamental equal to 0.428 p.u. and the lowest order harmonic as 3\textsuperscript{rd} and 7\textsuperscript{th} equal to 20.6% and 8.8%, respectively. Once again a significant reduction in these harmonics are achieved. Thus the losses and hence the efficiency of the motor drive using this
conduction mode will improve. The most dominant harmonics of d-q plane are 9th and 11th as 11.1% and 9.1%, respectively. It is important to note that these harmonic do not change and hence the tenth harmonic pulsating torque will remain unaltered with this conduction mode.

Non-adjacent line-to-line voltages are determined next. Table 3.6 summarizes the values of the first non-adjacent line-to-line voltages in the ten 36° of equal intervals. Wave shapes are shown in Fig. 3.11, resulting in further higher fundamental voltages due to increased conduction period (180°).

It is observed that the output of non-adjacent line-to-line voltages takes on values of
$$\pm \frac{1}{2} V_{DC}, \pm V_{DC} \text{ and } 0.$$ The on duration is 180°.

The coefficients of the Fourier series for non-adjacent line-to-line voltages ($V_{ac}$) as they possess odd quarter-wave symmetry, are:

$$B_{2n-1} = \sum_{n=1}^{\infty} \frac{4V_{DC}}{(2n - 1)\pi} \cos^2 \left( \frac{2n - 1}{10} \right)$$

And hence, the series is:

$$V(t) = \sum_{n=1}^{\infty} \frac{4V_{DC}}{(2n - 1)\pi} \cos^2 \left( \frac{2n - 1}{10} \right) \sin \left( 2n - 1 \right) \omega t$$

From (3.31) it follows that the fundamental sinusoidal component of the output first non-adjacent line-to-line voltages has an RMS value equal to

$$V_{\infty} = 0.8143476V_{DC}$$

From Fig. 3.9, mean square value is determined as:

$$MeanSquareValue = \frac{1}{\pi} \left[ \left( \frac{1}{2} V_{DC} \right)^2 \times \frac{\pi}{5} + \left( V_{DC} \right)^2 \times \frac{3\pi}{5} + \left( \frac{1}{2} V_{DC} \right)^2 \times \frac{\pi}{5} \right] = \frac{7}{10} V_{DC}^2$$

$$V_{rms} = \sqrt{\frac{7}{10}} V_{DC}$$

Total harmonic r.m.s. voltage (p.u.) is given by

$$V_{Hrms} = \sqrt{\left( V_{rms} \right)^2 - \left( V_1 \right)^2}$$

$$= \sqrt{\left( \sqrt{\frac{7}{10}} \right)^2 - \left( \frac{2\sqrt{2}}{\pi} \cos \left( \frac{\pi}{10} \right) \right)^2} = 0.1919485542$$

Hence total harmonic distortion is
\[ THD = \frac{r.m.s. \text{ total harmonic voltage}}{r.m.s. \text{ total voltage}} \]
\[ = \frac{0.1919485442}{\sqrt{\frac{7}{10}}} = 0.2294223915 \text{ or } 22.94\% \] 

It is clearly observed that the THD improves in this conduction mode compared to the conventional 180° conduction mode. The reduction in THD is almost 6%.

<table>
<thead>
<tr>
<th>Modes</th>
<th>Duration</th>
<th>( V_{ac} )</th>
<th>( V_{bd} )</th>
<th>( V_{dc} )</th>
<th>( V_{da} )</th>
<th>( V_{eb} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0-36°</td>
<td>( V_{dc} )</td>
<td>(-\frac{1}{2} V_{dc})</td>
<td>(-V_{dc})</td>
<td>(-\frac{1}{2} V_{dc})</td>
<td>( V_{dc} )</td>
</tr>
<tr>
<td>2</td>
<td>36°-72°</td>
<td>( V_{dc} )</td>
<td>( \frac{1}{2} V_{dc} )</td>
<td>(-V_{dc})</td>
<td>(-V_{dc})</td>
<td>( \frac{1}{2} V_{dc} )</td>
</tr>
<tr>
<td>3</td>
<td>72°-108°</td>
<td>( V_{dc} )</td>
<td>( V_{dc} )</td>
<td>(-\frac{1}{2} V_{dc})</td>
<td>(-V_{dc})</td>
<td>(-\frac{1}{2} V_{dc})</td>
</tr>
<tr>
<td>4</td>
<td>108°-144°</td>
<td>( \frac{1}{2} V_{dc} )</td>
<td>( V_{dc} )</td>
<td>( \frac{1}{2} V_{dc} )</td>
<td>(-V_{dc})</td>
<td>(-V_{dc})</td>
</tr>
<tr>
<td>5</td>
<td>144°-180°</td>
<td>(-\frac{1}{2} V_{dc})</td>
<td>( V_{dc} )</td>
<td>( V_{dc} )</td>
<td>(-\frac{1}{2} V_{dc})</td>
<td>(-V_{dc})</td>
</tr>
<tr>
<td>6</td>
<td>180°-216°</td>
<td>(-V_{dc})</td>
<td>( \frac{1}{2} V_{dc} )</td>
<td>( V_{dc} )</td>
<td>( \frac{1}{2} V_{dc} )</td>
<td>(-V_{dc})</td>
</tr>
<tr>
<td>7</td>
<td>216°-252°</td>
<td>(-V_{dc})</td>
<td>(-\frac{1}{2} V_{dc})</td>
<td>( V_{dc} )</td>
<td>( V_{dc} )</td>
<td>(-\frac{1}{2} V_{dc})</td>
</tr>
<tr>
<td>8</td>
<td>252°-288°</td>
<td>(-V_{dc})</td>
<td>(-V_{dc})</td>
<td>( \frac{1}{2} V_{dc} )</td>
<td>( V_{dc} )</td>
<td>( \frac{1}{2} V_{dc} )</td>
</tr>
<tr>
<td>9</td>
<td>288°-324°</td>
<td>(-\frac{1}{2} V_{dc})</td>
<td>(-V_{dc})</td>
<td>(-\frac{1}{2} V_{dc})</td>
<td>( V_{dc} )</td>
<td>( V_{dc} )</td>
</tr>
<tr>
<td>10</td>
<td>324°-360°</td>
<td>( \frac{1}{2} V_{dc} )</td>
<td>(-V_{dc})</td>
<td>(-V_{dc})</td>
<td>(-\frac{1}{2} V_{dc})</td>
<td>( V_{dc} )</td>
</tr>
</tbody>
</table>
Fig. 3.11. Non-adjacent line-to-line voltages symmetrical for 144° conduction mode.

FFT analysis of inverter first non-adjacent line-to-line ‘Vac’ voltage is performed using Matlab/Simulink code and is shown in Fig. 3.12 and this validates equation (3.31).

Fig. 3.12. Inverter Non-adjacent line-to-line ‘Vac’ voltage time domain waveform and its harmonic spectrum in frequency domain(Fundamental frequency is 50 Fundamental : 0.814344 ; Highest harmonics : Orders=[3 9] Values=[12.7322% 11.1111%].
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\[ V_{ac} \]
\[ V_{bd} \]
\[ V_{ce} \]
\[ V_{da} \]
\[ V_{eb} \]

Fig. 3.11 Non-adjacent line-to-line voltages symmetrical for 144° conduction mode.

FFT analysis of inverter first non-adjacent line-to-line 'Vac' voltage is performed using Matlab/Simulink code and is shown in Fig. 3.12 and this validates equation (3.31)

![Non-adjacent line-line voltage 'Vac']

Fig. 3.12. Inverter Non-adjacent line-to-line 'Vac' voltage time domain waveform and its harmonic spectrum in frequency domain (Fundamental frequency is 50 Fundamental: 0.814344, Highest harmonics Orders=[3 9] Values=[12.7322% 11.1111%].

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3.2.1C 108° CONDUCTION MODE

In this conduction mode each power switch is remain on for 108°. The gating signals are shown in Fig. 3.13 and switches being ‘ON’ are listed in Table 3.7. It is seen that once again two legs are kept idle for every 72° interval.

The phase-to-neutral voltages are found once again using equation (3.5) and Leg Voltage Table 3.8. The resulting values for phase-to-neutral are tabulated in Table 3.9 and its waveform is shown in Fig. 3.14.

Table 3.7. Switches position in each mode for 108° conduction mode.

<table>
<thead>
<tr>
<th>Modes</th>
<th>Duration</th>
<th>Switches ON</th>
<th>Polarity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0-36°</td>
<td>9,10,1</td>
<td>+0−0−0−0+ABCDE</td>
</tr>
<tr>
<td>2</td>
<td>36°-72°</td>
<td>10,1,2</td>
<td>+0−−0−0−0ABCDE</td>
</tr>
<tr>
<td>3</td>
<td>72°-108°</td>
<td>1,2,3</td>
<td>+0−0−0−0−0ABCDE</td>
</tr>
<tr>
<td>4</td>
<td>108°-144°</td>
<td>2,3,4</td>
<td>0+0−−0−0−0ABCDE</td>
</tr>
<tr>
<td>5</td>
<td>144°-180°</td>
<td>3,4,5</td>
<td>0+0+0−−0−0ABCDE</td>
</tr>
<tr>
<td>6</td>
<td>180°-216°</td>
<td>4,5,6</td>
<td>−0−0−0−0−0ABCDE</td>
</tr>
<tr>
<td>7</td>
<td>216°-252°</td>
<td>5,6,7</td>
<td>−0−0+0−0−0ABCDE</td>
</tr>
<tr>
<td>8</td>
<td>252°-288°</td>
<td>6,7,8</td>
<td>−0+0+0−0−0ABCDE</td>
</tr>
<tr>
<td>9</td>
<td>288°-324°</td>
<td>7,8,9</td>
<td>0−0+0−0−0ABCDE</td>
</tr>
<tr>
<td>10</td>
<td>324°-360°</td>
<td>8,9,10</td>
<td>0−0−0−0−0−0ABCDE</td>
</tr>
</tbody>
</table>

Table 3.8 Leg Voltages (108° conduction mode)

<table>
<thead>
<tr>
<th>Modes</th>
<th>Duration</th>
<th>( V_A )</th>
<th>( V_B )</th>
<th>( V_C )</th>
<th>( V_D )</th>
<th>( V_E )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0-36°</td>
<td>( V_{DC} )</td>
<td>( (2/3)V_{DC} )</td>
<td>0</td>
<td>( (2/3)V_{DC} )</td>
<td>( V_{DC} )</td>
</tr>
<tr>
<td>2</td>
<td>36°-72°</td>
<td>( V_{DC} )</td>
<td>( (1/3)V_{DC} )</td>
<td>0</td>
<td>0</td>
<td>( (1/3)V_{DC} )</td>
</tr>
<tr>
<td>3</td>
<td>72°-108°</td>
<td>( V_{DC} )</td>
<td>( V_{DC} )</td>
<td>( (2/3)V_{DC} )</td>
<td>0</td>
<td>( (2/3)V_{DC} )</td>
</tr>
<tr>
<td>4</td>
<td>108°-144°</td>
<td>( (1/3)V_{DC} )</td>
<td>( V_{DC} )</td>
<td>( (1/3)V_{DC} )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>144°-180°</td>
<td>( (2/3)V_{DC} )</td>
<td>( V_{DC} )</td>
<td>( V_{DC} )</td>
<td>( (2/3)V_{DC} )</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>180°-216°</td>
<td>0</td>
<td>( (1/3)V_{DC} )</td>
<td>( V_{DC} )</td>
<td>( (1/3)V_{DC} )</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>216°-252°</td>
<td>( (2/3)V_{DC} )</td>
<td>( V_{DC} )</td>
<td>( V_{DC} )</td>
<td>( (2/3)V_{DC} )</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>252°-288°</td>
<td>0</td>
<td>( (1/3)V_{DC} )</td>
<td>( V_{DC} )</td>
<td>( (1/3)V_{DC} )</td>
<td>( V_{DC} )</td>
</tr>
<tr>
<td>9</td>
<td>288°-324°</td>
<td>( (2/3)V_{DC} )</td>
<td>0</td>
<td>( (2/3)V_{DC} )</td>
<td>( V_{DC} )</td>
<td>( V_{DC} )</td>
</tr>
<tr>
<td>10</td>
<td>324°-360°</td>
<td>( (1/3)V_{DC} )</td>
<td>0</td>
<td>0</td>
<td>( (1/3)V_{DC} )</td>
<td>( V_{DC} )</td>
</tr>
</tbody>
</table>
Chapter 3: Modelling and Control of a Five-phase voltage source inverter

Fig. 3.13. Gating signals for 108° conduction mode.

Table 3.9 Phase Voltages (108° conduction mode).

<table>
<thead>
<tr>
<th>Modes</th>
<th>Duration</th>
<th>$V_a$</th>
<th>$V_b$</th>
<th>$V_c$</th>
<th>$V_d$</th>
<th>$V_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0-36°</td>
<td>(1/3)V_{DC}</td>
<td>0</td>
<td>(-2/3)V_{DC}</td>
<td>0</td>
<td>(1/3)V_{DC}</td>
</tr>
<tr>
<td>2</td>
<td>36°-72°</td>
<td>(2/3)V_{DC}</td>
<td>0</td>
<td>(-1/3)V_{DC}</td>
<td>(-1/3)V_{DC}</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>72°-108°</td>
<td>(1/3)V_{DC}</td>
<td>(1/3)V_{DC}</td>
<td>0</td>
<td>(-2/3)V_{DC}</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>108°-144°</td>
<td>0</td>
<td>(2/3)V_{DC}</td>
<td>0</td>
<td>(-1/3)V_{DC}</td>
<td>(-1/3)V_{DC}</td>
</tr>
<tr>
<td>5</td>
<td>144°-180°</td>
<td>(1/3)V_{DC}</td>
<td>(1/3)V_{DC}</td>
<td>0</td>
<td>(2/3)V_{DC}</td>
<td>(-1/3)V_{DC}</td>
</tr>
<tr>
<td>6</td>
<td>180°-216°</td>
<td>(1/3)V_{DC}</td>
<td>(1/3)V_{DC}</td>
<td>0</td>
<td>(2/3)V_{DC}</td>
<td>(-1/3)V_{DC}</td>
</tr>
<tr>
<td>7</td>
<td>216°-252°</td>
<td>(-2/3)V_{DC}</td>
<td>0</td>
<td>(1/3)V_{DC}</td>
<td>(1/3)V_{DC}</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>252°-288°</td>
<td>(-1/3)V_{DC}</td>
<td>(-1/3)V_{DC}</td>
<td>0</td>
<td>(2/3)V_{DC}</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>288°-324°</td>
<td>0</td>
<td>(-2/3)V_{DC}</td>
<td>0</td>
<td>(1/3)V_{DC}</td>
<td>(1/3)V_{DC}</td>
</tr>
<tr>
<td>10</td>
<td>324°-360°</td>
<td>(-1/3)V_{DC}</td>
<td>(-1/3)V_{DC}</td>
<td>0</td>
<td>(2/3)V_{DC}</td>
<td>0</td>
</tr>
</tbody>
</table>
Fig. 3.13. Gating signals for 108° conduction mode.

Table 3.9 Phase Voltages (108° conduction mode)

<table>
<thead>
<tr>
<th>Modes</th>
<th>Duration</th>
<th>$V_a$</th>
<th>$V_b$</th>
<th>$V_c$</th>
<th>$V_d$</th>
<th>$V_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0-36°</td>
<td>$(1/3)V_{DC}$</td>
<td>0</td>
<td>$(2/3)V_{DC}$</td>
<td>0</td>
<td>$(1/3)V_{DC}$</td>
</tr>
<tr>
<td>2</td>
<td>36°-72°</td>
<td>$(2/3)V_{DC}$</td>
<td>0</td>
<td>$(1/3)V_{DC}$</td>
<td>$(1/3)V_{DC}$</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>72°-108°</td>
<td>$(1/3)V_{DC}$</td>
<td>$(1/3)V_{DC}$</td>
<td>0</td>
<td>$(2/3)V_{DC}$</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>108°-144°</td>
<td>0</td>
<td>$(2/3)V_{DC}$</td>
<td>0</td>
<td>$(1/3)V_{DC}$</td>
<td>$(1/3)V_{DC}$</td>
</tr>
<tr>
<td>5</td>
<td>144°-180°</td>
<td>0</td>
<td>$(1/3)V_{DC}$</td>
<td>$(1/3)V_{DC}$</td>
<td>0</td>
<td>$(2/3)V_{DC}$</td>
</tr>
<tr>
<td>6</td>
<td>180°-216°</td>
<td>$(1/3)V_{DC}$</td>
<td>0</td>
<td>$(2/3)V_{DC}$</td>
<td>0</td>
<td>$(-1/3)V_{DC}$</td>
</tr>
<tr>
<td>7</td>
<td>216°-252°</td>
<td>$(2/3)V_{DC}$</td>
<td>0</td>
<td>$(1/3)V_{DC}$</td>
<td>$(1/3)V_{DC}$</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>252°-288°</td>
<td>$(1/3)V_{DC}$</td>
<td>$(1/3)V_{DC}$</td>
<td>0</td>
<td>$(2/3)V_{DC}$</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>288°-324°</td>
<td>$(2/3)V_{DC}$</td>
<td>0</td>
<td>$(1/3)V_{DC}$</td>
<td>$(1/3)V_{DC}$</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>324°-360°</td>
<td>$(1/3)V_{DC}$</td>
<td>$(1/3)V_{DC}$</td>
<td>0</td>
<td>$(2/3)V_{DC}$</td>
<td>0</td>
</tr>
</tbody>
</table>
Fourier analysis of the voltage waveforms is undertaken using the same approach as that of previous sections. Observing that the waveforms of the phase to neutral voltage possess odd quarter-wave symmetry. In case of phase-to-neutral voltage $V_a$, shown in Fig. 3.15, the coefficients of the Fourier series are:

$$B_{2n-1} = \sum_{n=1}^{\infty} \frac{8V_{dc}}{3\pi(2n-1)} \cos(2n-1) \frac{3\pi}{10} \cos(2n-1) \frac{\pi}{10}$$

The expression in equation (3.37) equals to zero for all the harmonics whose order is divisible by 5. Hence one can write the phase-to-neutral voltages sinusoidal series as:

$$V(t) = \sum_{n=1}^{\infty} \frac{8V_{dc}}{3\pi(2n-1)} \cos(2n-1) \frac{3\pi}{10} \cos(2n-1) \frac{\pi}{10} \sin(2n-1) \omega t$$

From (3.38) it follows that the fundamental sinusoidal component of the output phase-to-neutral voltages has an RMS value equal to
Fourier analysis of the voltage waveforms is undertaken using the same approach as that of previous sections. Observing that the waveforms of the phase to neutral voltage possess odd quarter-wave symmetry. In case of phase-to-neutral voltage \( V_n \) shown in Fig. 3.15, the coefficients of the Fourier series are:

\[
B_{2n-1} = \sum_{n=1}^{\infty} \frac{8V_{DC}}{3\pi(2n-1)} \cos(2n-1) \frac{3\pi}{10} \cos(2n-1) \frac{\pi}{10} 
\]

The expression in equation (3.37) equals to zero for all the harmonics whose order is divisible by 5. Hence one can write the phase-to-neutral voltages sinusoidal series as:

\[
V(t) = \sum_{n=1}^{\infty} \frac{8V_{DC}}{3\pi(2n-1)} \cos(2n-1) \frac{3\pi}{10} \cos(2n-1) \frac{\pi}{10} \sin(2n-1) \omega t 
\]

From (3.38) it follows that the fundamental sinusoidal component of the output phase-to-neutral voltages has an RMS value equal to
\[ V_a = 0.335528V_{DC} \tag{3.39} \]

From Fig. 3.14, mean square value is determined as:

\[ \text{MeanSquareValue} = \frac{1}{\pi} \left[ \left( \frac{1}{3} V_{DC} \right)^2 \times \frac{\pi}{5} + \left( \frac{2}{3} V_{DC} \right)^2 \times \frac{\pi}{5} + \left( \frac{1}{3} V_{DC} \right)^2 \times \frac{\pi}{5} \right] = \frac{1}{5} V_{DC}^2 \tag{3.40} \]

\[ V_{rms} = \frac{1}{\sqrt{5}} V_{DC} \tag{3.41} \]

Total harmonic r.m.s. voltage (p.u.) is given by

\[ V_{Hrms} = \sqrt{(I_{rms})^2 - (I_1)^2} \]

\[ = \sqrt{\left( \frac{1}{\sqrt{5}} \right)^2 - \left( 4\sqrt{2} \cos \left( \frac{\pi}{10} \right) \cos \left( \frac{3\pi}{10} \right) \right)^2} = 0.2956702675 \tag{3.42} \]

Hence total harmonic distortion is

\[ \text{THD} = \frac{\text{r.m.s. total harmonic voltage}}{\text{r.m.s. total voltage}} \]

\[ = \frac{0.2956702675}{\sqrt{1/5}} = 0.661138817 \text{ or } 66.11\% \tag{3.43} \]

FFT analysis of inverter phase ‘V_a’ voltage is performed using Matlab/Simulink code and is shown in Fig. 3.15 and this validates equation (3.38).

![Phase Voltage 'Va'](image)

![Voltage spectrum RMS (p.u.)](image)

Fig. 3.15. Inverter phase ‘V_a’ voltage time domain waveform and its harmonic spectrum in frequency domain (Fundamental frequency is 50 Hz, Fundamental : 0.335528 ; Highest harmonics : Orders=[3 7] Values=[33.3333%, 14.2857%].

46
\[ V_a = 0.335528V_{DC} \] (3.39)

From Fig. 3.14, mean square value is determined as:

\[
\text{MeanSquareValue} = \frac{1}{\pi} \left[ \left( \frac{1}{3} V_{DC} \right)^2 \times \frac{\pi}{5} + \left( \frac{2}{3} V_{DC} \right)^2 \times \frac{\pi}{5} + \left( \frac{1}{3} V_{DC} \right)^2 \times \frac{\pi}{5} \right] = \frac{1}{5} V_{DC}^2 \] (3.40)

\[ V_{rms} = \frac{1}{\sqrt{5}} V_{DC} \] (3.41)

Total harmonic r.m.s. voltage (p.u.) is given by

\[
V_{thrms} = \sqrt{\left( V_{rms} \right)^2 - \left( V_1 \right)^2}
\]

\[
= \sqrt{V_{rms}^2 - \left( \frac{4\sqrt{2}}{3\pi} \cos\left( \frac{\pi}{10} \right) \cos\left( \frac{3\pi}{10} \right) \right)^2} = 0.2956702675
\] (3.42)

Hence total harmonic distortion is

\[
THD = \frac{\text{r.m.s. total harmonic voltage}}{\text{r.m.s. total voltage}}
\]

\[
= \frac{0.2956702675}{\sqrt{1/5}} = 0.661138817 \text{ or } 66.11\%
\] (3.43)

FFT analysis of inverter phase 'V_a' voltage is performed using Matlab/Simulink code and is shown in Fig. 3.15 and this validates equation (3.38).

---

**Fig. 3.15.** Inverter phase 'V_a' voltage time domain waveform and its harmonic spectrum in frequency domain (Fundamental frequency is 50 Hz, Fundamental: 0.335528; Highest harmonics Orders=[3 7] Values=[33.3333%, 14.2857%].
It is observed that the fundamental is reduced by almost 25% compared to the ten-step mode of operation (180° conduction mode) but the harmonic contents remain the same. Thus this conduction mode is of no practical significance.

Non-adjacent line-to-line voltages are determined next. Table 3.10 summarizes the values of the non-adjacent line-to-line voltages in the ten 36° of equal intervals. Wave shapes are shown in Fig. 3.16, resulting in further higher fundamental voltages due to increased conduction period (144°).

<table>
<thead>
<tr>
<th>Modes</th>
<th>Duration</th>
<th>V_ac</th>
<th>V_bd</th>
<th>V_cd</th>
<th>V_da</th>
<th>V_ch</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0-36°</td>
<td>V_DC</td>
<td>0</td>
<td>V_DC</td>
<td>(-1/3)V_DC</td>
<td>(1/3)V_DC</td>
</tr>
<tr>
<td>2</td>
<td>36°-72°</td>
<td>(1/3)V_DC</td>
<td>0</td>
<td>V_DC</td>
<td>-V_DC</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>72°-108°</td>
<td>V_DC</td>
<td>V_DC</td>
<td>0</td>
<td>-V_DC</td>
<td>(-1/3)V_DC</td>
</tr>
<tr>
<td>4</td>
<td>108°-144°</td>
<td>V_DC</td>
<td>(1/3)V_DC</td>
<td>V_DC</td>
<td>(-1/3)V_DC</td>
<td>-V_DC</td>
</tr>
<tr>
<td>5</td>
<td>144°-180°</td>
<td>V_DC</td>
<td>(1/3)V_DC</td>
<td>V_DC</td>
<td>0</td>
<td>-V_DC</td>
</tr>
<tr>
<td>6</td>
<td>180°-216°</td>
<td>V_DC</td>
<td>0</td>
<td>V_DC</td>
<td>(1/3)V_DC</td>
<td>(-1/3)V_DC</td>
</tr>
<tr>
<td>7</td>
<td>216°-252°</td>
<td>V_DC</td>
<td>(-1/3)V_DC</td>
<td>(1/3)V_DC</td>
<td>V_DC</td>
<td>V_DC</td>
</tr>
<tr>
<td>8</td>
<td>252°-288°</td>
<td>V_DC</td>
<td>0</td>
<td>V_DC</td>
<td>(1/3)V_DC</td>
<td>V_DC</td>
</tr>
<tr>
<td>9</td>
<td>288°-324°</td>
<td>V_DC</td>
<td>(-1/3)V_DC</td>
<td>(1/3)V_DC</td>
<td>V_DC</td>
<td>V_DC</td>
</tr>
<tr>
<td>10</td>
<td>324°-360°</td>
<td>V_DC</td>
<td>(1/3)V_DC</td>
<td>V_DC</td>
<td>-V_DC</td>
<td>0</td>
</tr>
</tbody>
</table>

It is observed that the output of non-adjacent line-to-line voltages takes on values of ± V_{DC}, ± V_{DC} and 0. The on duration is 144°.

The coefficients of the Fourier series for non-adjacent line-to-line voltages (V_ac) as they possess odd quarter-wave symmetry, are:

\[
B_{2n-1} = \sum_{n=1}^{\infty} \frac{4V_{DC}}{3\pi(2n-1)} \left[ \cos(2n-1) \frac{\pi}{10} + 2 \cos(2n-1) \frac{3}{10} \right] \tag{3.44}
\]

And hence, the series is:

\[
V(t) = \sum_{n=1}^{\infty} \frac{4V_{DC}}{3\pi(2n-1)} \left[ \cos(2n-1) \frac{\pi}{10} + 2 \cos(2n-1) \frac{3}{10} \right] \sin(2n-1)\omega t \tag{3.45}
\]

From (3.45) it follows that the fundamental sinusoidal component of the output non-adjacent line-to-line voltages has an RMS value equal to

\[
V_{ac} = 0.6382123V_{DC} \tag{3.46}
\]
From Fig. 3.14, mean square value is determined as:

\[
\text{Mean Square Value} = \frac{1}{\pi} \left( \left( \frac{1}{3} V_{DC} \right)^2 \times \frac{\pi}{5} + (V_{DC})^2 \times \frac{2\pi}{5} + \left( \frac{1}{3} V_{DC} \right)^2 \times \frac{\pi}{10} \right) = \frac{4}{9} V_{DC}^2 \quad (3.47)
\]

\[V_{\text{rms}} = \frac{2}{3} V_{DC} \quad (3.47)\]

Total harmonic r.m.s. voltage (p.u.) is given by

\[
V_{H\text{rms}} = \sqrt{(V_{\text{rms}})^2 - (V_1)^2} = \sqrt{\left( \frac{2}{3} \right)^2 - (0.63821233)^2} = 0.1926900783 \quad (3.48)
\]

Hence total harmonic distortion is

\[
THD = \frac{\text{r.m.s. total harmonic voltage}}{\text{r.m.s. total voltage}} = \frac{0.1926900783}{2/3} = 0.2890351175 \text{ or } 28.9\% \quad (3.49)
\]

![Fig. 3.16. Non-adjacent line-to-line voltages for 108° conduction mode.](image)
From Fig. 3.14, mean square value is determined as:

\[
\text{Mean Square Value} = \frac{1}{\pi} \left( \left( \frac{1}{3} V_{DC} \right)^2 \times \frac{\pi}{5} + \left( V_{DC} \right)^2 \times \frac{2\pi}{5} + \left( \frac{1}{3} V_{DC} \right)^2 \times \frac{\pi}{10} \right) = \frac{4}{9} V_{DC}^2 \quad (3.47)
\]

\[
V_{rms} = \frac{2}{3} V_{DC} \quad (3.47)
\]

Total harmonic r.m.s. voltage (p.u.) is given by

\[
V_{Hrms} = \sqrt{\left( l_{rms}' \right)^2 - \left( l_1 \right)^2} = \sqrt{\left( \frac{2}{3} \right)^2 - \left( 0.63821233 \right)^2} = 0.1926900783 \quad (3.48)
\]

Hence total harmonic distortion is

\[
THD = \frac{\text{r.m.s. total harmonic voltage}}{\text{r.m.s. total voltage}} = \frac{0.1926900783}{2/3} = 0.2890351175 \text{ or } 28.9\% \quad (3.49)
\]

Fig. 3.16. Non-adjacent line-to-line voltages for 108° conduction mode.
THD also remains same as that of the conventional 180° conduction mode.

FFT analysis of inverter first non-adjacent line-to-line ‘Vac’ voltage is performed using Matlab/Simulink code and is shown in Fig. 3.17 and this validates equation (3.45).

![Non-Adjacent Line-Line Voltage "Vac"](image)

Fig 3.17 Inverter first non-adjacent line-to-line ‘Vac’ voltage time domain waveform and its harmonic spectrum in frequency domain (Fundamental frequency is 50 Fundamental 0.638228, Highest harmonics Orders=[3 9] Values=[20 601% 11 1084%] )

3.2.1D PERFORMANCE COMPARISON OF VARIOUS CONDUCTION MODES

A comparative study of the quality of output phase-to-neutral voltages, adjacent line voltages and non-adjacent line voltages are carried out using the simulation results and are listed in Table 3.11, Table 3.12 and Table 3.13, respectively.

Table 3.11 Comparison of fundamental phase voltages and its harmonic content

<table>
<thead>
<tr>
<th>Conduction Mode</th>
<th>Fundamental rms (p.u.)</th>
<th>3(^{rd}) %</th>
<th>7(^{th}) %</th>
<th>9(^{th}) %</th>
<th>11(^{th}) %</th>
<th>THD %</th>
</tr>
</thead>
<tbody>
<tr>
<td>180°</td>
<td>0.45016</td>
<td>33.33</td>
<td>14.2857</td>
<td>11.1111</td>
<td>9.0939</td>
<td>7.68</td>
</tr>
<tr>
<td>144°</td>
<td>0.428131</td>
<td>20.604</td>
<td>8.8257</td>
<td>11.1097</td>
<td>9.092</td>
<td>4.75</td>
</tr>
<tr>
<td>108°</td>
<td>0.335535</td>
<td>33.329</td>
<td>14.288</td>
<td>11.1088</td>
<td>9.0928</td>
<td>7.69</td>
</tr>
</tbody>
</table>

Table 3.12 Comparison of fundamental adjacent line voltages and its harmonic content

<table>
<thead>
<tr>
<th>Conduction Mode</th>
<th>Fundamental rms (p.u.)</th>
<th>3(^{rd}) %</th>
<th>5(^{th}) %</th>
<th>7(^{th}) %</th>
<th>9(^{th}) %</th>
<th>11(^{th}) %</th>
<th>THD %</th>
</tr>
</thead>
<tbody>
<tr>
<td>180°</td>
<td>0.529221</td>
<td>53.929</td>
<td>23.11</td>
<td>11.105</td>
<td>9.096</td>
<td>12.44</td>
<td>63.85</td>
</tr>
<tr>
<td>144°</td>
<td>0.503309</td>
<td>33.33</td>
<td>14.28</td>
<td>11.107</td>
<td>9.098</td>
<td>7.69</td>
<td>41.59</td>
</tr>
<tr>
<td>108°</td>
<td>0.394437</td>
<td>53.93</td>
<td>23.11</td>
<td>11.111</td>
<td>9.091</td>
<td>12.44</td>
<td>63.85</td>
</tr>
</tbody>
</table>
Table 3.13. Comparison of fundamental Non-adjacent line voltages and its harmonic content.

<table>
<thead>
<tr>
<th>Conduction Mode</th>
<th>Fundamental rms (p.u.)</th>
<th>3rd %</th>
<th>5th %</th>
<th>7th %</th>
<th>9th %</th>
<th>11th %</th>
<th>THD %</th>
</tr>
</thead>
<tbody>
<tr>
<td>180°</td>
<td>0.856263</td>
<td>20.604</td>
<td>8.82</td>
<td>11.109</td>
<td>9.092</td>
<td>4.75</td>
<td>28.86</td>
</tr>
<tr>
<td>150°</td>
<td>0.814354</td>
<td>12.73</td>
<td>5.45</td>
<td>11.109</td>
<td>9.092</td>
<td>2.94</td>
<td>22.15</td>
</tr>
<tr>
<td>120°</td>
<td>0.638231</td>
<td>20.599</td>
<td>8.82</td>
<td>11.108</td>
<td>9.093</td>
<td>4.75</td>
<td>28.86</td>
</tr>
</tbody>
</table>

It is observed from the Tables 3.11-3.13 that the harmonic content in output for 180° and 108° conduction modes are identical and a significant reduction in lower order harmonics are achieved in 144° conduction mode. There is a marginal loss of fundamental in case of 144° compared to 180° conduction mode (almost 5% drop). The loss in fundamental in case of 108° conduction mode is significant (25% drop). Thus it may be concluded that during stepped operation of inverter 144° conduction mode may be used instead of conventional 180° or ten-step mode.

3.2.2 PWM MODE OF OPERATION

If a five-phase VSI is operated in PWM mode, apart from the already described ten states there will be additional 22 switching states. These remaining twenty two switching states encompass three possible situations: all the states when four switches from upper (or lower) half and one from the lower (or upper) half of the inverter are on (states 11-20); two states when either all the five upper (or lower) switches are ‘on’ (states 31 and 32); and the remaining states with three switches from the upper (lower) half and two switches from the lower (upper) half in conduction mode (states 21-30). The corresponding space vectors for 11-30 are obtained using equation (3.1) and it is seen that the total of 32 space vectors, available in the PWM operation, fall into four distinct categories regarding the magnitude of the available output phase voltages. The phase voltage space vectors are summarised in Table 3.14 for all 32 switching states and are shown in Fig. 3.18.

Table 3.14. Phase-to-neutral voltage space vectors for states 1-32.

<table>
<thead>
<tr>
<th>Space vectors</th>
<th>Value of the space vectors</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{1\text{phase to } V_{1\text{phase}} } )</td>
<td>( \frac{2}{5} V_{DC} 2 \cos(\pi/5) \exp(jk \pi/5) ) for ( k = 0, 1, 2, \ldots, 9 )</td>
</tr>
<tr>
<td>( V_{1\text{phase to } V_{2\text{phase}} } )</td>
<td>( \frac{2}{5} V_{DC} \exp(jk \pi/5) ) for ( k = 0, 1, 2, \ldots, 9 )</td>
</tr>
<tr>
<td>( V_{2\text{phase to } V_{1\text{phase}} } )</td>
<td>( \frac{2}{5} V_{DC} 2 \cos(2 \pi/5) \exp(jk \pi/5) ) for ( k = 0, 1, 2, \ldots, 9 )</td>
</tr>
<tr>
<td>( V_{3\text{phase to } V_{1\text{phase}} } )</td>
<td>0</td>
</tr>
</tbody>
</table>
Since Adjacent line voltage yield lower output value, only non-adjacent line voltages are elaborated in this section as well. The same procedure as that of ten-step mode is adopted to determine the non-adjacent line voltage space vectors for states 11-30 and are given by the expressions (3.35) and (3.36).
Fig. 3.18. Phase-to-neutral voltage space vectors for states 1-32 (states 31-32 are at origin) in d-q plane.

Since Adjacent line voltage yield lower output value, only non-adjacent line voltages are elaborated in this section as well. The same procedure as that of ten-step mode is adopted to determine the non-adjacent line voltage space vectors for states 11-30 and are given by the expressions (3.35) and (3.36).
\[ \begin{bmatrix} v_{1\text{IL}} \\ v_{1\text{2L}} \\ v_{1\text{3L}} \\ v_{1\text{4L}} \\ v_{1\text{5L}} \\ v_{1\text{6L}} \end{bmatrix} = \frac{2}{5} V_{DC} 2 \cos \left( \frac{\pi}{10} \right) \begin{bmatrix} e^{j\pi/10} \\ e^{j3\pi/10} \\ e^{j5\pi/10} \\ e^{j7\pi/10} \\ e^{j9\pi/10} \end{bmatrix} \]

\[ \begin{bmatrix} v_{2\text{IL}} \\ v_{2\text{2L}} \\ v_{2\text{3L}} \\ v_{2\text{4L}} \\ v_{2\text{5L}} \\ v_{2\text{6L}} \end{bmatrix} = \frac{2}{5} V_{DC} 2 \cos \left( \frac{3\pi}{10} \right) \begin{bmatrix} e^{j\pi/10} \\ e^{j3\pi/10} \\ e^{j5\pi/10} \\ e^{j7\pi/10} \\ e^{j9\pi/10} \end{bmatrix} \]

3.3 MODEL TRANSFORMATION USING DECOUPLING MATRIX

Since the system under discussion is a five-phase one, the complete model can be only be elaborated in five-dimensional space. The first two-dimensional spaces are \( d-q \), the second one is \( x-y \) and the last is zero sequence component which is absent due to assumption of isolated neutral. On the basis of the general decoupling transformation matrix for an \( n \)-phase system inverter voltage space vectors in the second two-dimensional sub-space (\( x-y \)) are determined with equation (3.52),

\[ v_{xy}^{\text{INV}} = \frac{2}{5} \left( v_a + a^2 v_b + a^4 v_c + a v_d + a^3 v_e \right) \] (3.52)

Thus 32 space vectors of phase-to-neutral voltage in the \( x-y \) plane are obtained using equation (3.52) and are illustrated in Fig. 3.19.
It can be seen from Figs. 3.18 and 3.19 that the outer decagon space vectors of the $d-q$ plane map into the inner-most decagon of the $x-y$ plane, the inner-most decagon of $d-q$ plane forms the outer decagon of $x-y$ plane while the middle decagon space vectors map into the same region. Further, it is observed from the above mapping that the phase sequence $a,b,c,d,e$ of $d-q$ plane corresponds to $a,c,e,b,d$, in $x-y$ which are basically the third harmonic voltages.

### 3.4 EXPERIMENTAL ANALYSIS OF A FIVE-PHASE VSI IN 180°, 144° AND 108° CONDUCTION MODES

The concept of stepped operation of inverter developed in the previous sections is validated using experimental results reported in this section. A prototype IGBT based multi-phase inverter is developed in the laboratory. The inverter is modular in nature and can be operated as single to nine-
Fig. 3.19. Phase-to-neutral voltage space vectors for states 1-32 (states 31-32 are at origin) in $x-y$ plane.

It can be seen from Figs. 3.18 and 3.19 that the outer decagon space vectors of the $d-q$ plane map into the inner-most decagon of the $x-y$ plane, the inner-most decagon of $d-q$ plane forms the outer decagon of $x-y$ plane while the middle decagon space vectors map into the same region. Further, it is observed from the above mapping that the phase sequence $a,b,c,d,e$ of $d-q$ plane corresponds to $a,c,e,b,d$, in $x-y$ which are basically the third harmonic voltages.

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phases. At first it is operated as a Five-phase voltage source inverter. The inverter is operated in 180°, 144° and 108° conduction modes and the performance is evaluated in terms of their harmonic content.

3.4.1 EXPERIMENTAL SET-UP

The inverter's control logic is implemented using Analog circuit and its complete block diagram is shown in Fig. 3.20 and its pictorial view is presented in Fig. 3.21.

![Fig. 3.20. Block diagram of the complete Inverter.](image-url)
Supply is taken from a single-phase supply and is converted to 9-0-9 V using a transformer, which is fed to the phase shifting circuit shown in Fig. 3.22, to provide appropriate phase shift for operation at various conduction angle. The phase shifted signal is then fed to the inverting/non-inverting Schmitt trigger circuit and wave shaping circuit (Fig. 3.23, 3.24). The processed signal is then fed to the isolation and driver circuit shown in Fig. 3.25, which is then finally given to the gate of IGBTs. There are two separate circuits for upper and lower legs of the inverter.

The power circuit is made up of IGBT SGW20N60 having a rating of 20 A and 600 V dc, with snubber circuit consisting of series combination of a resistance and a capacitor with a diode in parallel with the resistance.

![Diagram of the power circuit](image_url)

**Fig. 3.22. Phase shifting network.**
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Fig. 3.23. Non-inverting shmitt trigger and wave shaping circuit.

Fig. 3.24. Inverting shmitt trigger and wave shaping circuit.

Fig. 3.25. Gate driver circuit.
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Non-inverting Schmitt Trigger & Wave Shaping Circuit

Fig. 3.23. Non-inverting schmitt trigger and wave shaping circuit.

Inverting Schmitt Trigger & Wave Shaping Circuit

Fig. 3.24. Inverting schmitt trigger and wave shaping circuit.

Fig. 3.25. Gate driver circuit.
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3.5 EXPERIMENTAL RESULTS

Experiment is conducted for stepped operation of inverter with 180°, 144° and 108° conduction modes for star connected Five-phase resistive load. A single-phase supply is given to the control circuit through the phase shifting network. The output of the phase shifting circuit provides the required Five-phase output voltage by appropriately tuning it as shown in Fig. 3.26. This Five-phase signals are then further processed to generate the gate drive circuit.

![Fig. 3.26. Five-phase output obtained from phase shifting network.]

3.5.1 RESULTS OF 180° CONDUCTION MODE

The output from the Schmitt trigger circuit is presented in Fig. 3.27. The driving control gate/base signals for the ten-step mode for legs A -B of the inverter are illustrated in Fig. 3.28. The corresponding phase voltage thus obtained are shown in Fig. 3.29, keeping the dc link voltage at 60
Fig. 3.27. Output of wave shaping circuit for 180 degree conduction mode for leg A-B.

Fig. 3.28. Gate Drive signals for legs A-B for 180° conduction mode.

Fig. 3.29. Output phase 'a-d' voltages for 180° conduction mode.
Fig. 3.27. Output of wave shaping circuit for 180 degree conduction mode for leg A-B

Fig. 3.28. Gate Drive signals for legs A-B for 180° conduction mode

Fig. 3.29. Output phase ‘a-d’ voltages for 180° conduction mode.
Fig. 3.39. Non-adjacent line voltage for 108° conduction mode with dc link voltage equal to 180 V.

Fig. 3.40. AC side current for 108° conduction mode.

3.5.4 RESULT ANALYSIS

This section presents the comprehensive analysis of experimental results. The performance of three different conduction modes are elaborated in terms of the harmonic content in the phase voltages, line voltages and the distortion in the ac side line current.

The harmonic analysis of phase voltage, line voltage and input ac side line current is carried out for different conduction modes and the resulting waveforms are shown in Fig. 3.41-3.49.
3.5.4 RESULT ANALYSIS

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The harmonic analysis of phase voltage, line voltage and input ac side line current is carried out for different conduction modes and the resulting waveforms are shown in Fig. 3.41-3.49.
Fig. 3.41. Spectrum of phase 'a' voltage for 180° conduction mode for dc link voltage of 220 V.

Fig. 3.42. Spectrum of non-adjacent line 'a-c' voltage for 180° conduction mode for dc link voltage of 180 V.

Fig. 3.43. Spectrum of ac side input current for 180° conduction mode.
Fig. 3.41. Spectrum of phase ‘a’ voltage for 180° conduction mode for dc link voltage of 220 V.

Fig. 3.42. Spectrum of non-adjacent line ‘a-c’ voltage for 180° conduction mode for dc link voltage of 180 V.

Fig. 3.43. Spectrum of ac side input current for 180° conduction mode.
Fig. 3.44. Spectrum of phase ‘a’ voltage for 144° conduction mode for dc link voltage of 220 V.

Fig. 3.45. Spectrum of non-adjacent line ‘a-c’ voltage for 144° conduction mode for dc link voltage of 180 V.

Fig. 3.46. Spectrum of input ac side current for 144° conduction mode.
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Fig 3.44  Spectrum of phase 'a' voltage for 144° conduction mode for dc link voltage of 220 V

Fig 3.45  Spectrum of non-adjacent line 'a-c' voltage for 144° conduction mode for dc link voltage of 180 V

Fig 3.46  Spectrum of input ac side current for 144° conduction mode
Fig. 3.47. Spectrum of phase ‘a’ voltage for 108° conduction mode for dc link voltage of 220 V.

Fig. 3.48. Spectrum of non-adjacent line ‘a-c’ voltage for 108° conduction mode for dc link voltage of 60 V.

Fig. 3.49. Spectrum of ac side input line current for 108° conduction mode.
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Fig. 3.47 Spectrum of phase ‘a’ voltage for 108° conduction mode for dc link voltage of 220 V

Fig. 3.48 Spectrum of non-adjacent line ‘a-c’ voltage for 108° conduction mode for dc link voltage of 60 V

Fig. 3.49 Spectrum of ac side input line current for 108° conduction mode
Performance comparison in terms of harmonic content in output phase voltage, output non-adjacent line voltage and input ac side current for different conduction modes are presented in Figs. 3.50-3.52. It is clearly seen that the harmonic content reduces drastically with reduction in conduction angle. The harmonic content is largest in 180 degree conduction mode and it is least in 144 degree conduction mode. However, the best utilisation of available dc link voltage is possible with conventional ten step mode (180 degree conduction mode). This finding is in agreement with the analytical and simulation approach used in preceding sections. It can thus be concluded that a compromise between the loss in fundamental and corresponding gain in terms of lower harmonic content in output waveform is obtained by using 144 degree conduction mode. Hence it is recommended to operate inverter feeding a motor drive in 144° conduction mode to obtain optimum performance of the drive system.

![Phase Voltage Harmonic Comparison](image)

Fig. 3.50. Harmonic content in output phase voltage for different conduction mode.
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Fig. 3.51. Harmonic content in output phase voltage for different conduction mode.

Fig. 3.52. Harmonic content in input side ac current for different conduction mode.
3.6 DSP IMPLEMENTATION OF STEP MODE OF OPERATION

The results obtained in the previous section 3.5 are verified using implementation through TMS320F2812 DSP under same operating conditions. Control code is written in C++ and run in PC. It is transferred to the DSP using serial communication cable RS232. The DSP generate 10 gating signals which is fed to the power module of the inverter. The detail experimental set up is provided in the next chapter. All the three conduction angles are implemented. The developed algorithm is verified using a star-connected resistive load and a five-phase induction machine, except for 108° degree mode where the motor load is not used as the response is poor due to insufficient dc voltage reserve.

3.6.1 180° CONDUCTION MODE

The inverter is operated in 180° conduction mode and a five-phase star-connected resistive load is connected across the output terminal. The resulting phase voltage, non-adjacent line voltages are illustrated in Fig. 3.53 and Fig. 3.54, respectively.

Fig. 3.53. Output Phase voltage for 180° conduction mode.
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Fig. 3.53. Output Phase voltage for 180° conduction mode.
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It is observed that the phase voltage generated using cheap analog circuit based inverter shown in Fig. 3.29 is identical to the one obtained using DSP as shown in Fig. 3.53. Similarly, the non-adjacent line voltage of Fig. 3.30 is identical to the one shown in Fig. 3.54. This verifies the correct design of the analog based inverter and also verifies the DSP code. The same study is carried out using a five-phase induction motor as a load. The resulting voltage and stator current waveforms are presented in Fig. 3.55. The waveform is typical for such load.

Fig. 3.54. Non-adjacent line voltage for 180° conduction mode.

Fig. 3.55. Non-adjacent line voltage and stator current.
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3.6.2 144° CONDUCTION MODE

The operation of inverter for 144° conduction mode is investigated as well. The resulting waveforms for resistive load and five-phase motor load are presented in Fig. 3.56 and Fig. 3.57, respectively.

![Waveform Diagram](image)

Fig. 3.56. 144° conduction mode, a. Phase voltages, b. non-adjacent line voltage.
3.6.2 **144° CONDUCTION MODE**

The operation of inverter for 144° conduction mode is investigated as well. The resulting waveforms for resistive load and five-phase motor load are presented in Fig. 3.56 and Fig. 3.57, respectively.

![Waveform Diagram for 144° Conduction Mode](image)

**Fig 3.56. 144° conduction mode, a. Phase voltages, b. non-adjacent line voltage**
The waveforms obtained using DSP based inverter and analog circuit based inverter provide almost similar results.

### 3.6.3 108° CONDUCTION MODE

The operation of inverter for 108° conduction mode is investigated as well. The resulting waveforms for star-connected resistive load are presented in Fig. 3.58. The waveforms are better in case of DSP based system since the setting of 108 degree in analog circuit based inverter is not very accurate.

### 3.7 SUMMARY

This chapter develops a space vector model of a Five-phase voltage source inverter and it is shown that two sets of space vector exist namely $d$-$q$ and $x$-$y$. The second set of space vector termed here as $x$-$y$ space is essentially third harmonic of the space vector in $d$-$q$ plane. Altogether there exists thirty two space vectors out of which 30 are active and two are zero vectors. Complete mapping of the space vectors are provided in the chapter. Stepped operation of the inverter is further elaborated in terms of different conduction modes. 180, 144 and 108 degrees are taken up as
Fig. 3.57. Non-adjacent line voltage and stator current for motor load for 144° conduction mode.

The waveforms obtained using DSP based inverter and analog circuit based inverter provide almost similar results.

3.6.3 108° CONDUCTION MODE

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conduction angles. Fourier analysis is presented in terms of analytical expressions and simulation results are provided to support the analytical expressions. Experimental set-up is elaborated and the results for stepped operation are illustrated. The theoretical and experimental results matches to good extent.

Fig. 3.58. 108° conduction mode, a. Phase voltages, b. non-adjacent line voltage.
conduction angles Fourier analysis is presented in terms of analytical expressions and simulation results are provided to support the analytical expressions. Experimental set-up is elaborated and the results for stepped operation are illustrated. The theoretical and experimental results matches to good extent.

![Diagram of phase voltages and non-adjacent line voltage](image)

**Fig 3.58** 108° conduction mode, a) Phase voltages, b) non-adjacent line voltage