CHAPTER 5

MODELLING AND CONTROL OF A SIX-PHASE VSI IN QUASI SIX-PHASE CONFIGURATION

5.1 INTRODUCTION

After the description of a five-phase voltage source inverter the next odd number is seven. But the most commonly used motor phase number in multi-phase group is six. There are two system of six-phases; quasi six-phase and symmetrical six-phase. In quasi six-phase two set of three-phase windings are displaced by 30° while in symmetrical six-phase this phase shift angle is 60°. Thus this chapter describes in detail a six-phase VSI whose output is quasi six-phase. This chapter discusses The modelling and control of a quasi six-phase voltage source inverter for square wave operation. Since a lot has been taken up on PWM of a six-phase VSI in the literature, it is skipped here. The inverter operation is elaborated for various conduction angles. At first the conventional 180° conduction mode is elaborated followed by two other conduction angles; 150° and 120°. Simulation results are provided to validate the theoretical findings. Experimental setup and results are provided to support the simulation results.

5.2 MODELLING OF A SIX-PHASE VSI

Power circuit topology of a six-phase voltage source inverter is shown in Fig. 5.1. Each switch in the circuit consists of two power semiconductor devices, connected in anti-parallel. One of these is a fully controllable semiconductor, such as a bipolar transistor, MOSFET or IGBT, while the second one is a diode. The input of the inverter is a dc voltage, which is regarded further on as being constant. The inverter outputs are denoted in Fig. 1 with lower case symbols \((a_1, b_1, c_1, a_2, b_2, c_2)\), while the points of connection of the outputs to inverter legs have symbols in capital letters \((A_1, B_1, C_1, A_2, B_2, C_2)\). The basic operating principles of the six-phase VSI are developed in what follows assuming the ideal commutation and zero forward voltage drop. The same inverter can supply symmetrical six phase and quasi six-phase output with little modification in the switching signal provided to the gate drive.
Further, if the block shown with dotted line box, if kept inoperative, will provide three-phase supply.

\[ V_{dc} \]

\[ A_1 \quad A_2 \quad B_1 \quad B_2 \quad C_1 \quad C_2 \]

\[ a_1 \quad b_1 \quad b_2 \quad c_1 \quad c_2 \]

\[ n \]

Fig. 5.1. A general power circuit topology for six-phase VSI.

Thus Fig. 5.1 is a general topology for all the three configurations (symmetrical six-phase, quasi six-phase and three-phase). Operation of quasi six-phase configuration is discussed in the following section assuming simple square wave control technique for 180°, 150° and 120° conduction modes. The symmetrical configuration is described in the next chapter.

### 5.3 180° CONDUCTION MODE

This is the most common type of power switch firing where each switch conducts for half of the fundamental cycle i.e. 180°. The same mode is used for three-phase VSI and is extended here for a six-phase VSI. Since the operation of each leg is complementary, a small dead band is provided to avoid short circuit of the leg, although it is not shown in the switching waveform. The switch signals for the upper legs are shown in Fig. 5.2 and the corresponding switches being on are listed in Table 5.1. Leg-voltages waveform for quasi-six phase VSI is shown in Fig. 5.3.

It is seen that three switches being on from the upper leg and three from lower leg. One leg changes state after an interval of 30°.
Table 5.1. Switches position in each mode for 180° conduction mode.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Duration</th>
<th>Switches On</th>
<th>Polarity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0-30°</td>
<td>1,5,6,8,9,10</td>
<td>$A_1^r A_2^r B_2^2 C_1^r C_2^r$</td>
</tr>
<tr>
<td>2</td>
<td>30°-60°</td>
<td>1,2,5,6,9,10</td>
<td>$A_1^r A_2^r B_2^2 C_1^r C_2^r$</td>
</tr>
<tr>
<td>3</td>
<td>60°-90°</td>
<td>1,2,6,9,10,11</td>
<td>$A_1^r A_2^r B_2^2 C_1^r C_2^r$</td>
</tr>
<tr>
<td>4</td>
<td>90°-120°</td>
<td>1,2,9,10,11,12</td>
<td>$A_1^r A_2^r B_1^2 C_1^r C_2^r$</td>
</tr>
<tr>
<td>5</td>
<td>120°-150°</td>
<td>1,2,3,10,11,12</td>
<td>$A_1^r A_2^r B_1^2 C_1^r C_2^r$</td>
</tr>
<tr>
<td>6</td>
<td>150°-180°</td>
<td>1,2,3,4,11,12</td>
<td>$A_1^r A_2^r B_1^2 C_1^r C_2^r$</td>
</tr>
<tr>
<td>7</td>
<td>180°-210°</td>
<td>2,3,4,7,11,12</td>
<td>$A_1^r A_2^r B_1^2 C_1^r C_2^r$</td>
</tr>
<tr>
<td>8</td>
<td>210°-240°</td>
<td>3,4,7,8,11,12</td>
<td>$A_1^r A_2^r B_1^2 C_1^r C_2^r$</td>
</tr>
<tr>
<td>9</td>
<td>240°-270°</td>
<td>3,4,5,7,8,12</td>
<td>$A_1^r A_2^r B_1^2 C_1^r C_2^r$</td>
</tr>
<tr>
<td>10</td>
<td>270°-300°</td>
<td>3,4,5,6,7,8</td>
<td>$A_1^r A_2^r B_1^2 C_1^r C_2^r$</td>
</tr>
<tr>
<td>11</td>
<td>300°-330°</td>
<td>4,5,6,7,8,9</td>
<td>$A_1^r A_2^r B_1^2 C_1^r C_2^r$</td>
</tr>
<tr>
<td>12</td>
<td>330°-360°</td>
<td>5,6,7,8,9,10</td>
<td>$A_1^r A_2^r B_1^2 C_1^r C_2^r$</td>
</tr>
</tbody>
</table>
Phase-to-neutral voltages of the star connected load are most easily found by defining a voltage difference between the star point $n$ of the load and the negative rail of the dc bus $N$. The following correlation then holds true

$$\begin{align*}
V_{A1} &= v_{a1} + v_{nN} \\
V_{B1} &= v_{b1} + v_{nN} \\
V_{C1} &= v_{c1} + v_{nN} \\
V_{A2} &= v_{a2} + v_{nN} \\
V_{B2} &= v_{b2} + v_{nN} \\
V_{C2} &= v_{c2} + v_{nN}
\end{align*}$$

(5.1)

Since the phase voltages in a star connected load sum to zero, summation of the equations (5.1) yields

$$v_{nN} = \left(\frac{1}{6}\right)(v_{A1} + v_{B1} + v_{C1} + v_{A2} + v_{B2} + v_{C2})$$

(5.2)

Substitution of (5.2) into (5.1) yields phase-to-neutral voltages of the load in the following form:

$$v_{a1} = \left(\frac{5}{6}\right)v_{a1} - \left(\frac{1}{6}\right)(v_{B1} + v_{C1} + v_{A2} + v_{B2} + v_{C2})$$
\[ v_{b1} = \left( \frac{5}{6} \right) v_{B1} - \left( \frac{1}{6} \right) \left( v_{A1} + v_{C1} + v_{A2} + v_{B2} + v_{C2} \right) \]
\[ v_{c1} = \left( \frac{5}{6} \right) v_{C1} - \left( \frac{1}{6} \right) \left( v_{A1} + v_{B1} + v_{A2} + v_{B2} + v_{C2} \right) \]
\[ v_{a2} = \left( \frac{5}{6} \right) v_{A2} - \left( \frac{1}{6} \right) \left( v_{A1} + v_{B1} + v_{C1} + v_{B2} + v_{C2} \right) \]
\[ v_{b2} = \left( \frac{5}{6} \right) v_{B2} - \left( \frac{1}{6} \right) \left( v_{A1} + v_{B1} + v_{C1} + v_{A2} + v_{C2} \right) \]
\[ v_{c2} = \left( \frac{5}{6} \right) v_{C2} - \left( \frac{1}{6} \right) \left( v_{A1} + v_{B1} + v_{C1} + v_{A2} + v_{B2} \right) \]

(5.3)

Fig. 5.4. Phasors of phase-to-neutral voltages for quasi six-phase system.

The values of leg voltages obtained from Fig. 5.2 are substituted in equation (5.3) to determine the phase-to-neutral voltages, shown in Fig. 5.5. It is seen that the phase-to-neutral voltages takes on six different values as \( \pm \frac{1}{2}V_\phi, \pm \frac{1}{3}V_\phi \) and \( \pm \frac{2}{3}V_\phi \), resulting in twelve stepped voltage waveform. It is tabulated in Table 5.3.

It is seen from the Fig. 5.4, depicting the positions of six-phase voltages, that there exist three different set of line voltages, namely, adjacent \( (v_{a1a2} = v_{a1n} - v_{a2n}) \), 1\(^{st}\) non-adjacent \( (v_{a1b1} = v_{a1n} - v_{b1n}) \) and 2\(^{nd}\) non-adjacent \( (v_{a1b2} = v_{a1n} - v_{b2n}) \).
\[ v_{b1} = \left( \frac{5}{6} \right) v_{B1} - \left( \frac{1}{6} \right) \left( v_{A1} + v_{C1} + v_{A2} + v_{B2} + v_{C2} \right) \]
\[ v_{c1} = \left( \frac{5}{6} \right) v_{C1} - \left( \frac{1}{6} \right) \left( v_{A1} + v_{B1} + v_{A2} + v_{B2} + v_{C2} \right) \]
\[ v_{a2} = \left( \frac{5}{6} \right) v_{A2} - \left( \frac{1}{6} \right) \left( v_{A1} + v_{B1} + v_{C1} + v_{A2} + v_{C2} \right) \]
\[ v_{b2} = \left( \frac{5}{6} \right) v_{B2} - \left( \frac{1}{6} \right) \left( v_{A1} + v_{B1} + v_{C1} + v_{A2} + v_{B2} \right) \]
\[ v_{c2} = \left( \frac{5}{6} \right) v_{C2} - \left( \frac{1}{6} \right) \left( v_{A1} + v_{B1} + v_{C1} + v_{A2} + v_{B2} \right) \]  \hfill (5.3)

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In order to relate the input dc link voltage of the inverter with the output phase-to-neutral, Fourier analysis of the voltage waveforms is undertaken. Observing that the waveforms of the phase to neutral voltage possess half-wave symmetry. In case of phase-to-neutral voltage $V_{a1}$, shown in Fig. 5.5, the Fourier series is:

$$V(t) = \sum_{n=1}^{\infty} \frac{V_{dc}}{3(2n-1)} \sqrt{1^2 + m^2} \sin[(2n-1)\omega t + \phi]$$

Where $l = 2\sin(2n-1)\frac{\pi}{6} - \sin(2n-1)\frac{\pi}{2}$ and $m = 5 + 2\cos(2n-1)\frac{\pi}{3}$; and $\phi = \tan^{-1}\left(\frac{l}{m}\right)$

The expression in equation (5.4) equals to zero for all the harmonics whose order is divisible by 6. Hence one can write the phase-to-neutral voltages as:

$$V(t) = \frac{V_{dc}}{\pi} \left[ 2\sin\omega t + \sqrt{\frac{2}{3}} \sin(3\omega t + \frac{\pi}{4}) + \frac{2}{5} \sin(5\omega t) + \frac{2}{7} \sin(7\omega t) + \frac{\sqrt{2}}{9} \sin(9\omega t - \frac{\pi}{4}) + \frac{2}{11} \sin(11\omega t) + \frac{2}{13} \sin(13\omega t) + \frac{\sqrt{2}}{15} \sin(15\omega t + \frac{\pi}{4}) + \frac{2}{17} \sin(17\omega t) + \frac{2}{19} \sin(19\omega t) + \frac{\sqrt{2}}{21} \sin(21\omega t - \frac{\pi}{4}) + \ldots \right]$$

<table>
<thead>
<tr>
<th>Mode</th>
<th>Duration</th>
<th>$V_{a1}$</th>
<th>$V_{a2}$</th>
<th>$V_{b1}$</th>
<th>$V_{b2}$</th>
<th>$V_{c1}$</th>
<th>$V_{c2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0°-30°</td>
<td>$\frac{1}{2}V_{dc}$</td>
<td>$-\frac{1}{2}V_{dc}$</td>
<td>$-\frac{1}{2}V_{dc}$</td>
<td>$\frac{1}{2}V_{dc}$</td>
<td>$\frac{1}{2}V_{dc}$</td>
<td>$\frac{1}{2}V_{dc}$</td>
</tr>
<tr>
<td>2</td>
<td>30°-60°</td>
<td>$\frac{1}{3}V_{dc}$</td>
<td>$\frac{1}{3}V_{dc}$</td>
<td>$\frac{2}{3}V_{dc}$</td>
<td>$-\frac{2}{3}V_{dc}$</td>
<td>$\frac{1}{3}V_{dc}$</td>
<td>$\frac{1}{3}V_{dc}$</td>
</tr>
<tr>
<td>3</td>
<td>60°-90°</td>
<td>$\frac{1}{2}V_{dc}$</td>
<td>$\frac{1}{2}V_{dc}$</td>
<td>$-\frac{1}{2}V_{dc}$</td>
<td>$-\frac{1}{2}V_{dc}$</td>
<td>$\frac{1}{2}V_{dc}$</td>
<td>$\frac{1}{2}V_{dc}$</td>
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<tr>
<td>4</td>
<td>90°-120°</td>
<td>$\frac{2}{3}V_{dc}$</td>
<td>$\frac{2}{3}V_{dc}$</td>
<td>$-\frac{1}{2}V_{dc}$</td>
<td>$-\frac{1}{2}V_{dc}$</td>
<td>$\frac{2}{3}V_{dc}$</td>
<td>$\frac{2}{3}V_{dc}$</td>
</tr>
<tr>
<td>5</td>
<td>120°-150°</td>
<td>$\frac{1}{2}V_{dc}$</td>
<td>$\frac{1}{2}V_{dc}$</td>
<td>$\frac{2}{3}V_{dc}$</td>
<td>$\frac{2}{3}V_{dc}$</td>
<td>$-\frac{1}{2}V_{dc}$</td>
<td>$-\frac{1}{2}V_{dc}$</td>
</tr>
<tr>
<td>6</td>
<td>150°-180°</td>
<td>$\frac{1}{3}V_{dc}$</td>
<td>$\frac{1}{3}V_{dc}$</td>
<td>$\frac{1}{3}V_{dc}$</td>
<td>$\frac{1}{3}V_{dc}$</td>
<td>$\frac{2}{3}V_{dc}$</td>
<td>$\frac{2}{3}V_{dc}$</td>
</tr>
<tr>
<td>7</td>
<td>180°-210°</td>
<td>$-\frac{1}{2}V_{dc}$</td>
<td>$-\frac{1}{2}V_{dc}$</td>
<td>$\frac{2}{3}V_{dc}$</td>
<td>$\frac{2}{3}V_{dc}$</td>
<td>$-\frac{1}{2}V_{dc}$</td>
<td>$-\frac{1}{2}V_{dc}$</td>
</tr>
<tr>
<td>8</td>
<td>210°-240°</td>
<td>$-\frac{1}{3}V_{dc}$</td>
<td>$-\frac{1}{3}V_{dc}$</td>
<td>$\frac{2}{3}V_{dc}$</td>
<td>$\frac{2}{3}V_{dc}$</td>
<td>$-\frac{1}{3}V_{dc}$</td>
<td>$-\frac{1}{3}V_{dc}$</td>
</tr>
<tr>
<td>9</td>
<td>240°-270°</td>
<td>$-\frac{1}{2}V_{dc}$</td>
<td>$-\frac{1}{2}V_{dc}$</td>
<td>$\frac{2}{3}V_{dc}$</td>
<td>$\frac{2}{3}V_{dc}$</td>
<td>$-\frac{1}{2}V_{dc}$</td>
<td>$-\frac{1}{2}V_{dc}$</td>
</tr>
<tr>
<td>10</td>
<td>270°-300°</td>
<td>$-\frac{2}{3}V_{dc}$</td>
<td>$-\frac{2}{3}V_{dc}$</td>
<td>$\frac{1}{3}V_{dc}$</td>
<td>$\frac{1}{3}V_{dc}$</td>
<td>$\frac{2}{3}V_{dc}$</td>
<td>$\frac{2}{3}V_{dc}$</td>
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<tr>
<td>11</td>
<td>300°-330°</td>
<td>$-\frac{1}{2}V_{dc}$</td>
<td>$-\frac{1}{2}V_{dc}$</td>
<td>$-\frac{1}{2}V_{dc}$</td>
<td>$-\frac{1}{2}V_{dc}$</td>
<td>$\frac{1}{2}V_{dc}$</td>
<td>$\frac{1}{2}V_{dc}$</td>
</tr>
<tr>
<td>12</td>
<td>330°-360°</td>
<td>$-\frac{1}{3}V_{dc}$</td>
<td>$-\frac{1}{3}V_{dc}$</td>
<td>$-\frac{1}{3}V_{dc}$</td>
<td>$-\frac{1}{3}V_{dc}$</td>
<td>$\frac{2}{3}V_{dc}$</td>
<td>$\frac{2}{3}V_{dc}$</td>
</tr>
</tbody>
</table>
From (5.5) it follows that the fundamental sinusoidal component of the output phase-to-neutral voltages has an RMS value equal to

$$V_{a1} = \frac{\sqrt{2}}{\pi} V_{DC} = 0.45V_{DC}$$  \hspace{1cm} (5.6)

From Fig. 5.5, mean square value is determined as:

$$\text{Mean Square Value} = \frac{1}{\pi} V_{DC} \left[ \left( \frac{1}{2} \right)^2 + \left( \frac{1}{3} \right)^2 + \left( \frac{2}{3} \right)^2 + \left( \frac{1}{2} \right)^2 + \left( \frac{1}{3} \right)^2 \right] \times \frac{\pi}{6} = \frac{17}{72} V_{DC}^2$$  \hspace{1cm} (5.7)

$$V_{rms} = \sqrt{\text{Mean Square Value}} = \frac{\sqrt{34}}{12} V_{DC}$$  \hspace{1cm} (5.8)

Total harmonic r.m.s. voltage (p.u.) is given by

$$V_{Hrms} = \sqrt{(V_{rms})^2 - (V_1)^2} = \sqrt{\left( \frac{\sqrt{34}}{12} \right)^2 - \left( \frac{\sqrt{2}}{\pi} \right)^2} = 0.1829446469$$  \hspace{1cm} (5.9)
Hence total harmonic distortion is

$$THD = \frac{r.m.s. \text{ total harmonic voltage}}{r.m.s. \text{ total voltage}}$$

$$= \frac{0.1829446469}{\sqrt{34/12}} = 0.3764969772 \text{ or } 37.65\%$$

(5.10)

FFT analysis of inverter phase ‘Vα1’ voltage is performed using Matlab/Simulink code and is shown in Fig. 5.6 and this validates equation (5.5).

Fig. 5.6. Inverter phase ‘Vα1’ voltage time domain waveform and its harmonic spectrum in frequency domain (Fundamental frequency is 50 Hz, Fundamental : 0.45196 ; Highest harmonics : Orders=[3 5] Values=[23.0731% 19.461%]

Line-to-line voltages are taken up next. There are four systems of line voltages; adjacent, next adjacent, first non-adjacent and the second non-adjacent. The magnitude of second non-adjacent line voltages are maximum and the magnitude of adjacent line voltages are minimum. Adjacent line voltages are not taken up for discussion. Only two systems of non-adjacent voltages are elaborated in the subsequent sections.

First non-adjacent line-to-line voltages are determined next. Table 5.3 summarizes the values of the first non-adjacent line-to-line voltages in the twelve 30° of equal intervals. Wave shapes are shown in Fig. 5.7. The conduction angle of the switches are 120° resulting in increased fundamental voltage compared to adjacent line voltage.
Hence total harmonic distortion is

\[
THD = \frac{r ms\ total\ harmonic\ voltage}{r ms\ total\ voltage}
\]

\[
= \frac{0.1829446469}{\sqrt{34/12}} = 0.3764969772 \text{ or } 37.65\%
\]  

(5.10)

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![Inverter phase 'V_a' voltage time domain waveform and its harmonic spectrum in frequency domain](image)

- Fig 5.6 Inverter phase ‘V_a’ voltage time domain waveform and its harmonic spectrum in frequency domain (Fundamental frequency is 50 Hz, Fundamental 0.45196, Highest harmonics Orders=[3 5] Values=[23.0731% 19.461%])

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Table-5.3 1st Non-Adjacent Line-Line Voltages (180°).

<table>
<thead>
<tr>
<th>Mode</th>
<th>Duration</th>
<th>$V_{a1b1}$</th>
<th>$V_{a2b2}$</th>
<th>$V_{b1c1}$</th>
<th>$V_{b2c2}$</th>
<th>$V_{c1a1}$</th>
<th>$V_{c2a2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0-30°</td>
<td>$V_{DC}$</td>
<td>0</td>
<td>-$V_{DC}$</td>
<td>-$V_{DC}$</td>
<td>0</td>
<td>$V_{DC}$</td>
</tr>
<tr>
<td>2</td>
<td>30°-60°</td>
<td>$V_{DC}$</td>
<td>$V_{DC}$</td>
<td>-$V_{DC}$</td>
<td>-$V_{DC}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>60°-90°</td>
<td>$V_{DC}$</td>
<td>$V_{DC}$</td>
<td>0</td>
<td>-$V_{DC}$</td>
<td>-$V_{DC}$</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>90°-120°</td>
<td>$V_{DC}$</td>
<td>$V_{DC}$</td>
<td>0</td>
<td>0</td>
<td>-$V_{DC}$</td>
<td>-$V_{DC}$</td>
</tr>
<tr>
<td>5</td>
<td>120°-150°</td>
<td>0</td>
<td>$V_{DC}$</td>
<td>$V_{DC}$</td>
<td>0</td>
<td>-$V_{DC}$</td>
<td>-$V_{DC}$</td>
</tr>
<tr>
<td>6</td>
<td>150°-180°</td>
<td>0</td>
<td>$V_{DC}$</td>
<td>$V_{DC}$</td>
<td>0</td>
<td>-$V_{DC}$</td>
<td>-$V_{DC}$</td>
</tr>
<tr>
<td>7</td>
<td>180°-210°</td>
<td>-$V_{DC}$</td>
<td>0</td>
<td>$V_{DC}$</td>
<td>$V_{DC}$</td>
<td>0</td>
<td>$V_{DC}$</td>
</tr>
<tr>
<td>8</td>
<td>210°-240°</td>
<td>-$V_{DC}$</td>
<td>-$V_{DC}$</td>
<td>$V_{DC}$</td>
<td>$V_{DC}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>240°-270°</td>
<td>-$V_{DC}$</td>
<td>-$V_{DC}$</td>
<td>0</td>
<td>$V_{DC}$</td>
<td>$V_{DC}$</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>270°-300°</td>
<td>-$V_{DC}$</td>
<td>-$V_{DC}$</td>
<td>0</td>
<td>0</td>
<td>$V_{DC}$</td>
<td>$V_{DC}$</td>
</tr>
<tr>
<td>11</td>
<td>300°-330°</td>
<td>-$V_{DC}$</td>
<td>-$V_{DC}$</td>
<td>0</td>
<td>-$V_{DC}$</td>
<td>0</td>
<td>$V_{DC}$</td>
</tr>
<tr>
<td>12</td>
<td>330°-360°</td>
<td>0</td>
<td>0</td>
<td>-$V_{DC}$</td>
<td>-$V_{DC}$</td>
<td>0</td>
<td>$V_{DC}$</td>
</tr>
</tbody>
</table>

Fourier analysis is done for the waveform of the first non-adjacent voltage and the coefficients of the Fourier series of $V_{a1b1}$ as they possess odd quarter-wave symmetry, are:

$$B_{2n-1} = \frac{4V_{DC}}{\pi(2n-1)} \left[ \cos\left(\frac{2n-1}{2}\right) \right] \quad \text{where } n = 1, 2, 3, \ldots \tag{5.11}$$

And hence, the series is:

$$V(t) = \frac{4V_{DC}}{(2n-1)\pi} \cos\left(\frac{2n-1}{6}\right) \sin(2n-1)\alpha t \quad \text{where } n = 1, 2, 3, \ldots \tag{5.12}$$

From (5.12) it follows that the fundamental sinusoidal component of the output first non-adjacent line-to-line voltages has an RMS value equal to

$$V_{a1b1} = \sqrt{\frac{6}{\pi}} V_{DC} = 0.779697 V_{DC} \tag{5.13}$$

From Fig. 5.7, mean square value is determined as:

$$\text{Mean Square Value} = \frac{1}{\pi} \left( V_{DC}^2 + \frac{2\pi}{3} \right) = \frac{2}{3} V_{DC}^2 \tag{5.14}$$

$$V_{rms} = \sqrt{\text{Mean Square Value}} = \sqrt{\frac{6}{3}} V_{DC} \tag{5.15}$$

Total harmonic r.m.s. voltage (p.u.) is given by

$$V_{Hrms} = \sqrt{\left( V_{rms} \right)^2 - \left( V_1 \right)^2} = \sqrt{\left( \frac{\sqrt{6}}{3} \right)^2 - \left( \frac{\sqrt{6}}{\pi} \right)^2} = 0.2423624658 \tag{5.16}$$

Hence total harmonic distortion is
\[ THD = \frac{r.m.s. \, total \, harmonic \, voltage}{r.m.s. \, total \, voltage} \]
\[ = \frac{0.2423624658}{\sqrt{6/3}} = 0.296832187 \text{ or } 29.68\% \] (5.17)

FFT analysis of inverter first non-adjacent line-to-line ‘\(V_{a1b1}\)’ voltage is performed using simulation (Matlab/Simulink code) to verify the analytical expressions developed above, and the resulting waveform is shown in Fig. 5.8. The results obtained are in full compliance with equations (5.12) to (5.13), validating the analytical approach.
Chapter 5: Modelling and Control of a Six-Phase VSI in Quasi Six-Phase Configuration

\[
THD = \frac{r.m.s. \text{ total harmonic voltage}}{r.m.s. \text{ total voltage}}
\]

\[
= \frac{0.2423624658}{\sqrt{6/3}} = 0.296832187 \text{ or } 29.68\%
\]  

(5.17)

FFT analysis of inverter first non-adjacent line-to-line ‘\(V_{a1b1}\)’ voltage is performed using simulation (Matlab/Simulink code) to verify the analytical expressions developed above, and the resulting waveform is shown in Fig. 5.8. The results obtained are in full compliance with equations (5.12) to (5.13), validating the analytical approach.

Fig. 5.7. First non-adjacent line voltages for 180° conduction mode.
Fig 5.8. Inverter first non-adjacent line-to-line \(^{\text{Va1b1}}\) voltage time domain waveform and its harmonic spectrum in frequency domain (Fundamental frequency is 50 Hz, Fundamental order 7 78432, Highest harmonics orders=[5 7] Values=[20 1552% 14 1073%].

Second set of line voltages are determined next. Table 5.4 summarizes the values of the second non-adjacent line-to-line voltages in the twelve 30° of equal intervals. Waveshapes are shown in Fig 5.9, resulting in further higher fundamental voltages due to increased conduction period (150°) compared to the first non-adjacent line voltages.

Table 5.4 2\(^{\text{nd}}\) Non-Adjacent Line-Line Voltages (180°)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Duration</th>
<th>(V_{a1b2})</th>
<th>(V_{a2c1})</th>
<th>(V_{b1c2})</th>
<th>(V_{b2a1})</th>
<th>(V_{c1a2})</th>
<th>(V_{c2b1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0-30°</td>
<td>(V_{DC})</td>
<td>(-V_{DC})</td>
<td>(-V_{DC})</td>
<td>(-V_{DC})</td>
<td>(V_{DC})</td>
<td>(V_{DC})</td>
</tr>
<tr>
<td>2</td>
<td>30°-60°</td>
<td>(V_{DC})</td>
<td>0</td>
<td>(-V_{DC})</td>
<td>(-V_{DC})</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>60°-90°</td>
<td>(V_{DC})</td>
<td>(V_{DC})</td>
<td>(-V_{DC})</td>
<td>(-V_{DC})</td>
<td>(-V_{DC})</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>90°-120°</td>
<td>(V_{DC})</td>
<td>(V_{DC})</td>
<td>0</td>
<td>(-V_{DC})</td>
<td>(-V_{DC})</td>
<td>(-V_{DC})</td>
</tr>
<tr>
<td>5</td>
<td>120°-150°</td>
<td>(V_{DC})</td>
<td>(V_{DC})</td>
<td>(V_{DC})</td>
<td>(-V_{DC})</td>
<td>(-V_{DC})</td>
<td>(-V_{DC})</td>
</tr>
<tr>
<td>6</td>
<td>150°-180°</td>
<td>0</td>
<td>(V_{DC})</td>
<td>(V_{DC})</td>
<td>0</td>
<td>(-V_{DC})</td>
<td>(-V_{DC})</td>
</tr>
<tr>
<td>7</td>
<td>180°-210°</td>
<td>(-V_{DC})</td>
<td>(V_{DC})</td>
<td>(V_{DC})</td>
<td>(V_{DC})</td>
<td>(-V_{DC})</td>
<td>(-V_{DC})</td>
</tr>
<tr>
<td>8</td>
<td>210°-240°</td>
<td>(-V_{DC})</td>
<td>0</td>
<td>(V_{DC})</td>
<td>(V_{DC})</td>
<td>0</td>
<td>(-V_{DC})</td>
</tr>
<tr>
<td>9</td>
<td>240°-270°</td>
<td>(-V_{DC})</td>
<td>(-V_{DC})</td>
<td>(V_{DC})</td>
<td>(V_{DC})</td>
<td>(V_{DC})</td>
<td>(-V_{DC})</td>
</tr>
<tr>
<td>10</td>
<td>270°-300°</td>
<td>(-V_{DC})</td>
<td>(-V_{DC})</td>
<td>0</td>
<td>(V_{DC})</td>
<td>(V_{DC})</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>300°-330°</td>
<td>(-V_{DC})</td>
<td>(-V_{DC})</td>
<td>(V_{DC})</td>
<td>(V_{DC})</td>
<td>(V_{DC})</td>
<td>(V_{DC})</td>
</tr>
<tr>
<td>12</td>
<td>330°-360°</td>
<td>0</td>
<td>(-V_{DC})</td>
<td>(-V_{DC})</td>
<td>0</td>
<td>(V_{DC})</td>
<td>(V_{DC})</td>
</tr>
</tbody>
</table>
The coefficients of the Fourier series for second Non-adjacent line-to-line voltages \((V_{ab2})\) as they possess odd quarter-wave symmetry, are:

\[
B_{2n-1} = \frac{4V_{DC}}{\pi (2n-1)} \left[ \cos(2n-1) \frac{\pi}{12} \right] \quad \text{where } n = 1, 2, 3, \ldots \tag{5.18}
\]

And hence, the series is:

\[
V(t) = \frac{4V_{DC}}{(2n-1)\pi} \cos(2n-1) \frac{\pi}{12} \sin(2n-1)\omega t \quad \text{where } n = 1, 2, 3, \ldots \tag{5.19}
\]

From (5.33) it follows that the fundamental sinusoidal component of the output second non-adjacent line-to-line voltages has an RMS value equal to

\[
V_{ab2} = \frac{\sqrt{3} + 1}{\pi} V_{DC} = 0.86964 V_{DC} \tag{5.20}
\]

From Fig. 5.12, mean square value is determined as:

\[
\text{MeanSquareValue} = \frac{1}{\pi} (V_{DC}^2 \times \frac{5\pi}{6}) = \frac{5}{6} V_{DC}^2 \tag{5.21}
\]
The coefficients of the Fourier series for second Non-adjacent line-to-line voltages $(V_{a1b2})$ as they possess odd quarter-wave symmetry, are:

$$B_{2n-1} = \frac{4V_{DC}}{\pi(2n-1)} \left[ \cos(2n-1) \frac{\pi}{12} \right] \quad \text{where } n = 1, 2, 3, \ldots$$

And hence, the series is:

$$V(t) = \frac{4V_{DC}}{(2n-1)\pi} \cos(2n-1) \frac{\pi}{12} \sin(2n-1)\omega t \quad \text{where } n = 1, 2, 3, \ldots$$

From (5.33) it follows that the fundamental sinusoidal component of the output second non-adjacent line-to-line voltages has an RMS value equal to

$$V_{a1b2} = \frac{\sqrt{3} + 1}{\pi} V_{DC} = 0.86964V_{DC}$$

From Fig. 5.12, mean square value is determined as:

$$\text{MeanSquareValue} = \frac{1}{\pi} (V_{DC}^2 \times \frac{5\pi}{6}) = \frac{5}{6} V_{DC}^2$$
\[ V_{\text{rms}} = \sqrt{\text{Mean Square Value}} = \sqrt{\frac{5}{6}}V_{\text{DC}} \] (5.22)

Total harmonic r.m.s. voltage (p.u.) is given by

\[ V_{H\text{rms}} = \sqrt{(V_{\text{rms}})^2 - (V_1)^2} = \sqrt{\left(\frac{5}{6}\right)^2 - \left(\frac{\sqrt{3} + 1}{\pi}\right)^2} = 0.2775999331 \] (5.23)

Hence total harmonic distortion is

\[ \text{THD} = \frac{\text{r.m.s. total harmonic voltage}}{\text{r.m.s. total voltage}} \]

\[ = \frac{0.2775999331}{\sqrt{\frac{5}{6}}} = 0.3040954906 \text{ or } 30.41\% \] (5.24)

FFT analysis of inverter second Non-adjacent line-to-line 'V_{a1b2}' voltage is performed using simulation study (Matlab/Simulink code) and is shown in Fig. 5.10 and this validates equation (5.33).

**Fig. 5.10. Inverter second Non-adjacent line-to-line 'V_{a1b2}' voltage time domain waveform and its harmonic spectrum in frequency domain (Fundamental frequency is 50 Hz, Fundamental : 0.869608 ; Highest harmonics : Orders=[3 11] Values=[24.393% 9.09472%].**

### 5.4 150° CONDUCTION MODE

A conduction mode is proposed called 150° conduction mode, in which each switch remains on for 150° or 41.67% of the fundamental cycle. One of the important advantages of
\[ V_{rms} = \sqrt{MeanSquareValue} = \sqrt{\frac{5}{6}} V_{DC} \]  

(5.22)

Total harmonic r m s voltage (p u) is given by

\[ \sqrt{V_{rms}} \equiv \sqrt{(\frac{5}{6})} \left( \frac{\sqrt{3} + 1}{\pi} \right)^2 = 0.2775999331 \]  

(5.23)

Hence total harmonic distortion is

\[ THD = \frac{rms \ total \ harmonic \ voltage}{rms \ total \ voltage} \]

\[ = \frac{0.2775999331}{\sqrt{\frac{5}{6}}} = 0.3040954906 \text{ or } 30.41\% \]  

(5.24)

FFT analysis of inverter second Non-adjacent line-to-line ‘V_{a1b2}’ voltage is performed using simulation study (Matlab/Simulink code) and is shown in Fig. 5.10 and this validates equation (5.33).

**Fig 5.10 Inverter second Non-adjacent line-to-line ‘V_{a1b2}’ voltage time domain waveform and its harmonic spectrum in frequency domain (Fundamental frequency is 50 Hz, Fundamental 0.869608, Highest harmonics Orders=[3 11] Values=[24 393% 9 09472%])**

**5.4 150° CONDUCTION MODE**

A conduction mode is proposed called 150° conduction mode, in which each switch remains on for 150° or 41.67% of the fundamental cycle. One of the important advantages of
this conduction mode is that the dead band of 30° is provided inherently providing a safe mode of operation. The gating signal for the twelve switches are shown in Fig. 5.11 and corresponding switches being on are listed in Table 5.5.

![Gate Drive signal for 150° conduction mode.](image)

**Table 5.5. Switches position in each mode for 150° conduction mode.**

<table>
<thead>
<tr>
<th>Mode</th>
<th>Duration</th>
<th>Switches On</th>
<th>Polarity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0-30°</td>
<td>1, 5, 6, 9, 10</td>
<td>(A_1^p [A_1^p] B_1^p B_2^p C_1^p C_2^p)</td>
</tr>
<tr>
<td>2</td>
<td>30°-60°</td>
<td>1, 2, 6, 9, 10</td>
<td>(A_1^p A_2^p B_1^p B_2^p [C_1^p] C_2^p)</td>
</tr>
<tr>
<td>3</td>
<td>60°-90°</td>
<td>1, 2, 9, 10, 11</td>
<td>(A_1^p A_2^p B_1^p B_2^p C_1^p [C_2^p])</td>
</tr>
<tr>
<td>4</td>
<td>90°-120°</td>
<td>1, 2, 10, 11, 12</td>
<td>(A_1^p A_2^p [B_1^p] B_2^p C_1^p C_2^p)</td>
</tr>
<tr>
<td>5</td>
<td>120°-150°</td>
<td>1, 2, 3, 11, 12</td>
<td>(A_1^p A_2^p B_1^p [B_2^p] C_1^p C_2^p)</td>
</tr>
<tr>
<td>6</td>
<td>150°-180°</td>
<td>2, 3, 4, 11, 12</td>
<td>([A_3^p] A_2^p B_1^p B_2^p C_1^p C_2^p)</td>
</tr>
<tr>
<td>7</td>
<td>180°-210°</td>
<td>3, 4, 7, 11, 12</td>
<td>(A_1^p [A_3^p] B_1^p B_2^p C_1^p C_2^p)</td>
</tr>
<tr>
<td>8</td>
<td>210°-240°</td>
<td>3, 4, 7, 8, 12</td>
<td>(A_1^p A_2^p B_1^p B_2^p [C_1^p] C_2^p)</td>
</tr>
<tr>
<td>9</td>
<td>240°-270°</td>
<td>3, 4, 5, 7, 8</td>
<td>(A_1^p A_2^p B_1^p B_2^p C_1^p [C_2^p])</td>
</tr>
<tr>
<td>10</td>
<td>270°-300°</td>
<td>4, 5, 6, 7, 8</td>
<td>(A_1^p A_2^p [B_1^p] B_2^p C_1^p C_2^p)</td>
</tr>
<tr>
<td>11</td>
<td>300°-330°</td>
<td>5, 6, 7, 8, 9</td>
<td>(A_1^p A_2^p B_1^p [B_2^p] C_1^p C_2^p)</td>
</tr>
<tr>
<td>12</td>
<td>330°-360°</td>
<td>5, 6, 8, 9, 10</td>
<td>([A_3^p] A_2^p B_1^p B_2^p C_1^p C_2^p)</td>
</tr>
</tbody>
</table>
The symbol \([x]\) in Table 5.6 represent the floating mode, similar to one of chapter 3. Thus it is seen that in every interval of 30° one of the legs is idle. It is seen from Fig. 5.11 and Table 5.6 that five switches (2 from upper and 3 from lower leg or vice versa) are operative in this mode of operation contrary to 180° conduction mode, where 6 switches are in operation in every 30° interval. Phase-to-neutral voltages are determined using equation (5.3) and Fig. 5.11 and are tabulated in Table 5.6, the resulting waveform is shown in Fig. 5.12.

Table 5.6 Phase Voltages (150°)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Duration</th>
<th>(V_{a1})</th>
<th>(V_{a2})</th>
<th>(V_{b1})</th>
<th>(V_{b2})</th>
<th>(V_{c1})</th>
<th>(V_{c2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0-30°</td>
<td>(\frac{2}{5}V_{DC})</td>
<td>0</td>
<td>(-\frac{3}{5}V_{DC})</td>
<td>(-\frac{2}{5}V_{DC})</td>
<td>(\frac{2}{5}V_{DC})</td>
<td>(\frac{2}{5}V_{DC})</td>
</tr>
<tr>
<td>2</td>
<td>30°-60°</td>
<td>(\frac{2}{5}V_{DC})</td>
<td>(\frac{2}{5}V_{DC})</td>
<td>(-\frac{3}{5}V_{DC})</td>
<td>(-\frac{3}{5}V_{DC})</td>
<td>0</td>
<td>(\frac{2}{5}V_{DC})</td>
</tr>
<tr>
<td>3</td>
<td>60°-90°</td>
<td>(\frac{3}{5}V_{DC})</td>
<td>(\frac{3}{5}V_{DC})</td>
<td>(-\frac{2}{5}V_{DC})</td>
<td>(-\frac{3}{5}V_{DC})</td>
<td>(-\frac{2}{5}V_{DC})</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>90°-120°</td>
<td>(\frac{3}{5}V_{DC})</td>
<td>(\frac{3}{5}V_{DC})</td>
<td>0</td>
<td>(-\frac{2}{5}V_{DC})</td>
<td>(-\frac{2}{5}V_{DC})</td>
<td>(-\frac{2}{5}V_{DC})</td>
</tr>
<tr>
<td>5</td>
<td>120°-150°</td>
<td>(\frac{2}{5}V_{DC})</td>
<td>(\frac{2}{5}V_{DC})</td>
<td>(\frac{2}{5}V_{DC})</td>
<td>0</td>
<td>(-\frac{3}{5}V_{DC})</td>
<td>(-\frac{2}{5}V_{DC})</td>
</tr>
<tr>
<td>6</td>
<td>150°-180°</td>
<td>0</td>
<td>(\frac{2}{5}V_{DC})</td>
<td>(\frac{2}{5}V_{DC})</td>
<td>(\frac{2}{5}V_{DC})</td>
<td>(-\frac{3}{5}V_{DC})</td>
<td>(-\frac{3}{5}V_{DC})</td>
</tr>
<tr>
<td>7</td>
<td>180°-210°</td>
<td>(-\frac{2}{5}V_{DC})</td>
<td>0</td>
<td>(\frac{3}{5}V_{DC})</td>
<td>(\frac{3}{5}V_{DC})</td>
<td>(-\frac{2}{5}V_{DC})</td>
<td>(-\frac{3}{5}V_{DC})</td>
</tr>
<tr>
<td>8</td>
<td>210°-240°</td>
<td>(-\frac{2}{5}V_{DC})</td>
<td>(-\frac{2}{5}V_{DC})</td>
<td>(\frac{3}{5}V_{DC})</td>
<td>(\frac{3}{5}V_{DC})</td>
<td>0</td>
<td>(-\frac{2}{5}V_{DC})</td>
</tr>
<tr>
<td>9</td>
<td>240°-270°</td>
<td>(-\frac{3}{5}V_{DC})</td>
<td>(-\frac{2}{5}V_{DC})</td>
<td>(\frac{2}{5}V_{DC})</td>
<td>(\frac{2}{5}V_{DC})</td>
<td>(\frac{2}{5}V_{DC})</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>270°-300°</td>
<td>(-\frac{3}{5}V_{DC})</td>
<td>(-\frac{3}{5}V_{DC})</td>
<td>0</td>
<td>(\frac{2}{5}V_{DC})</td>
<td>(\frac{2}{5}V_{DC})</td>
<td>(\frac{2}{5}V_{DC})</td>
</tr>
<tr>
<td>11</td>
<td>300°-330°</td>
<td>(-\frac{2}{5}V_{DC})</td>
<td>(\frac{3}{5}V_{DC})</td>
<td>(-\frac{3}{5}V_{DC})</td>
<td>0</td>
<td>(\frac{3}{5}V_{DC})</td>
<td>(\frac{3}{5}V_{DC})</td>
</tr>
<tr>
<td>12</td>
<td>330°-360°</td>
<td>0</td>
<td>(-\frac{2}{5}V_{DC})</td>
<td>(-\frac{2}{5}V_{DC})</td>
<td>(-\frac{2}{5}V_{DC})</td>
<td>(\frac{3}{5}V_{DC})</td>
<td>(\frac{3}{5}V_{DC})</td>
</tr>
</tbody>
</table>
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Fig. 5.12. Phase-to-neutral voltages for 150° conduction mode.

It is seen from the Fig. 5.12 that the phase-to-neutral voltage is an eight stepped voltage waveform taking on five different voltage levels $\pm \frac{2}{5} V_{dc}$, $\pm \frac{3}{5} V_{dc}$ and 0.

In order to relate the input dc link voltage of the inverter with the output phase-to-neutral, Fourier analysis of the voltage waveforms is undertaken. Observing that the waveforms of the phase to neutral voltage possess half-wave symmetry. In case of phase-to-neutral voltage $V_{a1}$, shown in Fig. 5.16, the Fourier series is:

$$V(t) = \frac{2V_{dc}}{5(2n-1)\pi} \left( \frac{\sqrt{I^2 + m^2}}{2} \right) \sin((2n-1)\omega t + \phi)$$  \hspace{1cm} (5.25)

Where $I = [1 + 2\cos(2n-1)\frac{\pi}{6}] \cos(2n-1)\frac{\pi}{6}$,

$m = \sin(2n-1)\frac{\pi}{6}$; And $\phi = \tan^{-1}\left(\frac{m}{I}\right)$

Hence one can write the phase-to-neutral voltages as:
It is seen from the Fig. 5.12 that the phase-to-neutral voltage is an eight stepped voltage waveform taking on five different voltage levels $\pm \frac{2}{5}V_{dc}$, $\pm \frac{3}{5}V_{dc}$ and 0.

In order to relate the input dc link voltage of the inverter with the output phase-to-neutral, Fourier analysis of the voltage waveforms is undertaken. Observing that the waveforms of the phase to neutral voltage possess half-wave symmetry. In case of phase-to-neutral voltage $V_{a1}$, shown in Fig. 5.16, the Fourier series is:

$$V(t) = \frac{2V_{DC}}{5(2n-1)\pi} \left( \frac{1}{5} + \frac{m^2}{5} \right) \sin((2n-1)\omega t + \phi)$$  \hspace{1cm} (5.25)

Where $l = [1 + 2\cos(2n-1)\pi 6] \cos(2n-1)\pi 6$;

$$m = \sin(2n-1)\frac{\pi}{6}; \text{ And } \phi = \tan^{-1}\left( \frac{m}{l} \right)$$

Hence one can write the phase-to-neutral voltages as:
\[ V(t) = \left( \frac{2V_{DC}}{5\pi} \right) \left[ \sqrt{13 + 6\sqrt{3}} \sin(\omega t + \tan^{-1}((3 + \sqrt{3})^{-1})) + \right. \]
\[ \frac{2}{3} \sin(3\omega t + \frac{\pi}{2}) + \frac{\sqrt{13 - 6\sqrt{3}}}{5} \sin(5\omega t + \tan^{-1}((3 - \sqrt{3})^{-1})) + \]
\[ \frac{\sqrt{13 + 6\sqrt{3}}}{7} \sin(7\omega t - \tan^{-1}((3 - \sqrt{3})^{-1})) + \frac{2}{9} \sin(9\omega t - \frac{\pi}{2}) + \]
\[ \frac{\sqrt{13 + 6\sqrt{3}}}{11} \sin(11\omega t - \tan^{-1}((3 + \sqrt{3})^{-1})) + \]
\[ \frac{\sqrt{13 + 6\sqrt{3}}}{13} \sin(13\omega t + \tan^{-1}((3 + \sqrt{3})^{-1})) + \frac{2}{15} \sin(15\omega t + \frac{\pi}{2}) + \]
\[ \frac{\sqrt{13 - 6\sqrt{3}}}{17} \sin(17\omega t + \tan^{-1}((3 - \sqrt{3})^{-1})) + \]
\[ \frac{\sqrt{13 - 6\sqrt{3}}}{19} \sin(19\omega t - \tan^{-1}((3 - \sqrt{3})^{-1})) + \frac{2}{19} \sin(19\omega t - \frac{\pi}{2}) + \ldots \] \tag{5.26}\

From (5.26) it follows that the fundamental sinusoidal component of the output phase-to-neutral voltages has an RMS value equal to

\[ V_{al} = \frac{2\sqrt{13 + 6\sqrt{3}}}{5\pi \sqrt{2}} V_{DC} = 0.435443 V_{DC} \tag{5.27} \]

It is interesting to note that the loss in fundamental in 150° conduction mode is almost 5% compared to 180° conduction mode. The same amount of reduction is observed in 144° conduction mode compared to 180° conduction mode in a Five-phase VSI.

From Fig. 5.12, mean square value is determined as:

\[ Mean\ Square\ Value = \frac{1}{\pi} \left[ \left( \frac{2}{5} V_{DC} \right)^2 \times \frac{\pi}{3} + \left( \frac{3}{5} V_{DC} \right)^2 \times \frac{\pi}{3} + \left( \frac{2}{5} V_{DC} \right)^2 \times \frac{\pi}{6} \right] = \frac{1}{5} V_{DC}^2 \tag{5.28} \]

\[ V_{rms} = \frac{\sqrt{5}}{5} V_{DC} \tag{5.29} \]

Total harmonic r.m.s. voltage (p.u.) is given by

\[ V_{hrms} = \sqrt{(V_{rms})^2 - (V_1)^2} = \sqrt{\left( \frac{\sqrt{5}}{5} \right)^2 - \left( \frac{2\sqrt{13 + 6\sqrt{3}}}{5\pi \sqrt{2}} \right)^2} = 0.1019270268 \tag{5.30} \]

Hence total harmonic distortion is

\[ THD = \frac{r.m.s.\ total\ harmonic\ voltage}{r.m.s.\ total\ voltage} \]
\[ = \frac{0.1019270268}{\sqrt{5/5}} = 0.2279157607 \text{ or } 22.79\% \tag{5.31} \]
FFT analysis of inverter phase ‘V₁₁’ voltage is performed using Matlab/Simulink code and is shown in Fig. 5.13 and this validates equation (5.26).

![Fig 5.13 Inverter phase ‘V₁₁’ voltage time domain waveform and its harmonic spectrum in frequency domain (Fundamental frequency is 50 Hz, Fundamental: 0.435549 ; Highest harmonics: Orders=[3 11] Values=[13.7733% 9.06421%].)

First Non-Adjacent Line-to-line voltages are elaborated next. First Non-Adjacent Line-to-Line voltages are determined using Leg voltage of Fig. 5.11. The result, thus obtained is used to draw their waveform as shown in Fig. 5.14.

The coefficients of the Fourier series for first Non-adjacent line-to-line voltages \((V_{a₁b₁})\) as they possess half-wave symmetry, are:

\[
A_{2n-1} = \sin(2n-1)\frac{2\pi}{3}; \quad B_{2n-1} = \left[2\cos^2(2n-1)\frac{5\pi}{6} - 3\cos(2n-1)\frac{5\pi}{6}\right] \quad \text{where } n = 1, 2, \ldots \quad (5.32)
\]

And hence, the series is:

\[
V(t) = \frac{4V_Dc}{\pi(2n-1)} \sqrt{l^2 + m^2} \sin((2n-1)\omega t + \phi) \quad \text{where } n = 1, 2, 3, \ldots \text{ and} \quad (5.33)
\]

\[
l = \sin(2n-1)\frac{2\pi}{3}; \quad m = \left[2\cos^2(2n-1)\frac{5\pi}{6} - 3\cos(2n-1)\frac{5\pi}{6}\right] \text{and } \phi = \tan^{-1}\left(\frac{l}{m}\right)
\]

From (5.33) it follows that the fundamental sinusoidal component of the output first non-adjacent line-to-line voltages \((V_{a₁b₁})\) has an RMS value equal to
FFT analysis of inverter phase ‘Vai’ voltage is performed using Matlab/Simulink code and is shown in Fig 5.13 and this validates equation (5.26).

Fig 5.13 Inverter phase ‘Vai’ voltage time domain waveform and its harmonic spectrum in frequency domain (Fundamental frequency is 50 Hz, Fundamental = 0.435549, Highest harmonics Orders = [3 11], Values = [13.7733% 9.06421%]).

First Non-Adjacent Line-to-line voltages are elaborated next. First Non-Adjacent Line-to-Line voltages are determined using Leg voltage of Fig 5.11. The result, thus obtained is used to draw their waveform as shown in Fig 5.14.

The coefficients of the Fourier series for first Non-adjacent line-to-line voltages (Vab1) as they possess half-wave symmetry, are

\[A_{2n-1} = \sin(2n-1) \frac{2\pi}{3}, \quad B_{2n-1} = \left[ 2 \cos^2(2n-1) \frac{5\pi}{6} - 3 \cos(2n-1) \frac{5\pi}{6} \right]\]

where \(n = 1, 2, \ldots, \) (5.32)

And hence, the series is

\[V(t) = \frac{417}{5\pi(2n-1)} \sqrt{l^2 + m^2} \sin((2n-1)\omega t + \phi) \quad \text{where} \quad n = 1, 2, 3, \quad \text{and} \quad (5.33)\]

\[l = \sin(2n-1) \frac{2\pi}{3}, \quad m = \left[ 2 \cos^2(2n-1) \frac{5\pi}{6} - 3 \cos(2n-1) \frac{5\pi}{6} \right] \quad \text{and} \quad \phi = \tan^{-1}\left( \frac{l}{m} \right)\]

From (5.33) it follows that the fundamental sinusoidal component of the output first non-adjacent line-to-line voltages (Vab1) has an RMS value equal to
Fig. 5.14. First non-adjacent line-to-line voltages for 150° conduction mode.

\[
V_{abl} = \frac{2\sqrt{39} + 18\sqrt{3}}{5\pi \sqrt{2}} V_{DC} = 0.75421 V_{DC} \tag{5.34}
\]

From Fig. 5.14, mean square value is determined as:

\[
\text{Mean Square Value} = \frac{1}{\pi} \left[ \left( \frac{2}{5} V_{DC} \right)^2 \times \frac{\pi}{6} + \left( V_{DC} \right)^2 \times \frac{\pi}{2} + \left( \frac{3}{5} V_{DC} \right)^2 \times \frac{\pi}{6} \right] = \frac{44}{75} V_{DC}^2 \tag{5.35}
\]

\[
V_{rms} = \frac{2\sqrt{33}}{15} V_{DC} \tag{5.36}
\]

Total harmonic r.m.s. voltage (p.u.) is given by

\[
V_{Hrms} = \sqrt{\left( V_{rms} \right)^2 - \left( V_1 \right)^2} = \sqrt{\left( \frac{2\sqrt{33}}{15} \right)^2 - \left( \frac{2\sqrt{39} + 18\sqrt{3}}{5\pi \sqrt{2}} \right)^2} = 0.1335440865 \tag{5.37}
\]
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From Fig. 5.14, mean square value is determined as:

\[
\text{Mean Square Value} = \frac{1}{\pi} \left[ \left( \frac{2\sqrt{33}}{5} \right)^2 \times \frac{\pi}{6} + \left( \sqrt{\frac{3}{5}} V_{DC} \right)^2 \times \frac{\pi}{2} + \left( \frac{2}{5} \right)^2 \times \frac{\pi}{6} \right] = \frac{44}{75} V_{DC}^2
\]  

(5.35)

\[
V_{\text{rms}} = \frac{2\sqrt{33}}{15} V_{DC}
\]  

(5.36)

Total harmonic r.m.s. voltage (p.u.) is given by

\[
V_{\text{Hrms}} = \sqrt{(V'_{\text{rms}})^2 - (V'_1)^2} = \sqrt{\left( \frac{2\sqrt{33}}{15} \right)^2 - \left( \frac{2\sqrt{39} + 18\sqrt{3}}{5\pi\sqrt{2}} \right)^2} = 0.1335440865
\]  

(5.37)
Hence total harmonic distortion is

\[
THD = \frac{r.m.s. \text{ total harmonic voltage}}{r.m.s. \text{ total voltage}}
\]

\[
= \frac{0.1335440865}{2\sqrt{33/15}} = 0.1743528116 \text{ or } 17.44\%
\]

It is observed that the THD reduces significantly in first non-adjacent line voltages compared to 180° conduction mode. FFT analysis of inverter First Non-adjacent line-to-line ‘Va1b1’ voltage is performed using Matlab/Simulink code and is shown in Fig. 5.15 and this validates equation (5.33).

![Graph showing First non-adjacent Line-Line Voltage 'Va1b1' and its harmonic spectrum](image)

Fig. 5.15. Inverter First Non-adjacent line-to-line ‘Va1b1’ voltage time domain waveform and its harmonic spectrum in frequency domain (Fundamental frequency is 50 Hz, Fundamental : 0.754335 ; Highest harmonics : Orders=[11 13] Values=[9.07259% 7.70726%]).

Second set of line voltages are determined next. The same approach as that of 180° conduction mode is utilized to determine the wave shapes and are shown in Fig. 5.16.
Hence total harmonic distortion is

$$\text{THD} = \frac{\text{rms total harmonic voltage}}{\text{rms total voltage}}$$

$$= \frac{0.1335440865}{2\sqrt{33}/15} = 0.1743528116 \text{ or } 17.44\%$$ (5.38)

It is observed that the THD reduces significantly in first non-adjacent line voltages compared to 180° conduction mode FFT analysis of inverter First Non-adjacent line-to-line ‘Vaib1’ voltage is performed using Matlab/Simulink code and is shown in Fig 5.15 and this validates equation (5.33).

![Image of waveform and spectrum](image)

**Fig 5.15** Inverter First Non-adjacent line-to-line ‘Vaib1’ voltage time domain waveform and its harmonic spectrum in frequency domain (Fundamental frequency is 50 Hz, Fundamental 0.754335, Highest harmonics Orders=[1 1 13] Values=[9.07259% 7.70726%])

Second set of line voltages are determined next. The same approach as that of 180° conduction mode is utilized to determine the wave shapes and are shown in Fig 5.16.
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Fig. 5.16. Second non-adjacent line-to-line voltages for 150° conduction mode.

The coefficients of the Fourier series for second Non-adjacent line-to-line voltages ($V_{aib2}$) as they possess odd quarter-wave symmetry, are:

$$B_{2n-1} = \frac{4I_{dc}}{5\pi(2n-1)} \left[ 2 + 3 \cos(2n-1) \frac{\pi}{6} \right] \quad \text{where } n = 1, 2, 3, \ldots$$

(5.39)

And hence, the series is:

$$V(t) = \frac{4I_{dc}}{5\pi(2n-1)} \left[ 2 + 3 \cos(2n-1) \frac{\pi}{6} \right] \sin(2n-1)\omega t \quad \text{where } n = 1, 2, 3, \ldots$$

(5.40)

From (5.40) it follows that the fundamental sinusoidal component of the output second non-adjacent line-to-line voltages ($V_{aib2}$) has an RMS value equal to

$$V_{aib2} = \frac{3\sqrt{6} + 4\sqrt{2}}{5\pi} V_{DC} = 0.8279446V_{DC}$$

(5.41)
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The coefficients of the Fourier series for second Non-adjacent line-to-line voltages \((V_{a1b2})\) as they possess odd quarter-wave symmetry, are:

\[
B_{2n-1} = \frac{4V_{in}}{5\pi(2n-1)} \left[ 2 + 3 \cos(2n-1) \frac{\pi}{6} \right] \quad \text{where } n = 1, 2, 3, \ldots 
\]  
(5.39)

And hence, the series is:

\[
V(t) = \frac{4V_{in}}{5\pi(2n-1)} \left[ 2 + 3 \cos(2n-1) \frac{\pi}{6} \right] \sin(2n-1)\omega t \quad \text{where } n = 1, 2, 3, \ldots 
\]  
(5.40)

From (5.40) it follows that the fundamental sinusoidal component of the output second non-adjacent line-to-line voltages \((V_{a1b2})\) has an RMS value equal to

\[
V_{a1b2} = \frac{3\sqrt{3} + 4\sqrt{2}}{5\pi} V_{DC} = 0.8279446V_{DC} 
\]  
(5.41)
From Fig. 5.23, mean square value is determined as:

\[
\text{Mean Square Value} = \frac{1}{\pi} \left[ \left( \frac{2}{5} V_{DC} \right)^2 \times \frac{\pi}{6} + \left( 2 \pi \times \frac{2}{3} \right) \times \frac{\pi}{6} + \left( \frac{2}{5} V_{DC} \right) \times \frac{\pi}{6} \right] = \frac{18}{25} V_{DC}^2 \tag{5.70}
\]

\[
V_{rms} = \frac{3\sqrt{2}}{5} V_{DC} \tag{5.71}
\]

Total harmonic r.m.s. voltage (p.u.) is given by

\[
V_{Hrms} = \sqrt{\left( V_{rms} \right)^2 - \left( V_{1} \right)^2} = \sqrt{\left( \frac{3\sqrt{2}}{5} \right)^2 - \left( \frac{3\sqrt{6} + 4\sqrt{2}}{5\pi} \right)^2} = 0.1857625565 \tag{5.42}
\]

Hence total harmonic distortion is

\[
THD = \frac{r.m.s. \text{ total harmonic voltage}}{r.m.s. \text{ total voltage}} = \frac{0.1857625565}{3\sqrt{2}/5} = 0.2189232723 \text{ or } 21.89\% \tag{5.43}
\]

It is observed that there is a loss of almost 5% in fundamental compared to 180° conduction mode, however, there is a significant improvement in the quality of the output which is evident from the reduced value of THD. FFT analysis of inverter second Non-adjacent line-to-line ‘Vaib2’ voltage is performed using Matlab/Simulink code and is shown in Fig. 5.17 and this validates equation (5.68).
From Fig. 5.23, mean square value is determined as

\[
\text{Mean Square Value} = \frac{1}{\pi} \left[ \left( \frac{2}{5} V_{\text{DC}} \right)^2 + \frac{2\pi}{3} \left( \frac{2}{5} V_{\text{DC}} \right)^2 \right] = \frac{18}{25} V_{\text{DC}}^2
\] (5.70)

\[
V_{\text{rms}} = \frac{3\sqrt{2}}{5} V_{\text{DC}}
\] (5.71)

Total harmonic r m s voltage (p u) is given by

\[
V_{\text{rms}} = \sqrt{\left( V_{\text{rms}} \right)^2 - \left( V_1 \right)^2} = \sqrt{\left( \frac{3\sqrt{2}}{5} \right)^2 - \left( \frac{3\sqrt{6} + 4\sqrt{2}}{5\pi} \right)^2} = 0.1857625565
\] (5.42)

Hence total harmonic distortion is

\[
\text{THD} = \frac{r m s \text{ total harmonic voltage}}{r m s \text{ total voltage}}
\] (5.43)

\[
\text{THD} = \frac{0.1857625565}{3\sqrt{2}/5} = 0.2189232723 \text{ or } 21.89\%
\]

It is observed that there is a loss of almost 5% in fundamental compared to 180° conduction mode, however, there is a significant improvement in the quality of the output which is evident from the reduced value of THD. FFT analysis of inverter second Non-adjacent line-to-line ‘V_{a1b2}’ voltage is performed using Matlab/Simulink code and is shown in Fig. 5.17 and this validates equation (5.68).

![Fig 5.17 Inverter second Non-adjacent line-to-line ‘V_{a1b2}’ voltage time domain waveform and its harmonic spectrum in frequency domain(Fundamental frequency is 50 Hz, Fundamental 0.828075 , Highest harmonics Orders=[3 11] Values=[14 5281% 907335%)]
5.5. **120° CONDUCTION MODE**

The approach of a three-phase VSI is adopted here to investigate the performance of a six-phase VSI in 120° conduction mode, where each power switch is assumed to conduct for 120°. The gate drive signal for this conduction mode is depicted in Fig. 5.18 and corresponding switches being on are listed in Table 5.7.

![Gate Drive signals for 120° conduction mode.](image)

Fig. 5.18. Gate Drive signals for 120° conduction mode.

It is seen from Fig. 5.18 that each leg conducts for 240° and remain idle for 120° leaving ample time to provide dead band for safe operation of each leg. Moreover it is seen that two legs remain idle in every 30° interval. Further it is observed that only two upper and two lower switches remains on. Phase-to-neutral voltages are determined for this operating mode using equation (5.3) and Fig. 5.18 and the resulting waveform is shown in Fig. 5.19. and the values are tabulated in Table 5.8.
Table 5.7. Switches position in each mode for 120° conduction mode

<table>
<thead>
<tr>
<th>Mode</th>
<th>Duration</th>
<th>Switches On</th>
<th>Polarity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0-30°</td>
<td>1,6,9,10</td>
<td>( A_1^* [A_2^* B_1^* B_1^* C_1^0 C_2^1] )</td>
</tr>
<tr>
<td>2</td>
<td>30°-60°</td>
<td>1,2,9,10</td>
<td>( A_1^* A_2^* B_1^* C_1^0 C_2^1 )</td>
</tr>
<tr>
<td>3</td>
<td>60°-90°</td>
<td>1,2,10,11</td>
<td>( A_1^* A_2^* [B_1^0 B_1^0 C_1^1 C_2^2] )</td>
</tr>
<tr>
<td>4</td>
<td>90°-120°</td>
<td>1,2,11,12</td>
<td>( A_1^* A_2^* [B_1^0 B_1^0 C_1^1 C_2^2] )</td>
</tr>
<tr>
<td>5</td>
<td>120°-150°</td>
<td>2,3,11,12</td>
<td>( A_1^* A_2^* [B_1^0 B_1^0 C_1^1 C_2^2] )</td>
</tr>
<tr>
<td>6</td>
<td>150°-180°</td>
<td>3,4,11,12</td>
<td>( A_1^* A_2^* [B_1^0 B_1^0 C_1^1 C_2^2] )</td>
</tr>
<tr>
<td>7</td>
<td>180°-210°</td>
<td>3,4,7,12</td>
<td>( A_1^* A_2^* [B_1^0 B_1^0 C_1^1 C_2^2] )</td>
</tr>
<tr>
<td>8</td>
<td>210°-240°</td>
<td>3,4,7,8</td>
<td>( A_1^* A_2^* [B_1^0 B_1^0 C_1^1 C_2^2] )</td>
</tr>
<tr>
<td>9</td>
<td>240°-270°</td>
<td>4,5,7,8</td>
<td>( A_1^* A_2^* [B_1^0 B_1^0 C_1^1 C_2^2] )</td>
</tr>
<tr>
<td>10</td>
<td>270°-300°</td>
<td>5,6,7,8</td>
<td>( A_1^* A_2^* [B_1^0 B_1^0 C_1^1 C_2^2] )</td>
</tr>
<tr>
<td>11</td>
<td>300°-330°</td>
<td>5,6,8,9</td>
<td>( A_1^* A_2^* [B_1^0 B_1^0 C_1^1 C_2^2] )</td>
</tr>
<tr>
<td>12</td>
<td>330°-360°</td>
<td>5,6,9,10</td>
<td>( A_1^* A_2^* [B_1^0 B_1^0 C_1^1 C_2^2] )</td>
</tr>
</tbody>
</table>

Table 5.8. Phase Voltage for 120° conduction mode.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Duration</th>
<th>( V_{a1} )</th>
<th>( V_{a2} )</th>
<th>( V_{b1} )</th>
<th>( V_{b2} )</th>
<th>( V_{c1} )</th>
<th>( V_{c2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0-30°</td>
<td>( \frac{1}{2} V_{dc} )</td>
<td>0</td>
<td>-( \frac{1}{2} V_{dc} )</td>
<td>-( \frac{1}{2} V_{dc} )</td>
<td>0</td>
<td>( \frac{1}{2} V_{dc} )</td>
</tr>
<tr>
<td>2</td>
<td>30°-60°</td>
<td>( \frac{1}{2} V_{dc} )</td>
<td>( \frac{1}{2} V_{dc} )</td>
<td>-( \frac{1}{2} V_{dc} )</td>
<td>-( \frac{1}{2} V_{dc} )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>60°-90°</td>
<td>( \frac{1}{2} V_{dc} )</td>
<td>( \frac{1}{2} V_{dc} )</td>
<td>0</td>
<td>-( \frac{1}{2} V_{dc} )</td>
<td>-( \frac{1}{2} V_{dc} )</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>90°-120°</td>
<td>( \frac{1}{2} V_{dc} )</td>
<td>( \frac{1}{2} V_{dc} )</td>
<td>0</td>
<td>0</td>
<td>-( \frac{1}{2} V_{dc} )</td>
<td>-( \frac{1}{2} V_{dc} )</td>
</tr>
<tr>
<td>5</td>
<td>120°-150°</td>
<td>0</td>
<td>( \frac{1}{2} V_{dc} )</td>
<td>( \frac{1}{2} V_{dc} )</td>
<td>0</td>
<td>-( \frac{1}{2} V_{dc} )</td>
<td>-( \frac{1}{2} V_{dc} )</td>
</tr>
<tr>
<td>6</td>
<td>150°-180°</td>
<td>0</td>
<td>0</td>
<td>( \frac{1}{2} V_{dc} )</td>
<td>( \frac{1}{2} V_{dc} )</td>
<td>-( \frac{1}{2} V_{dc} )</td>
<td>-( \frac{1}{2} V_{dc} )</td>
</tr>
<tr>
<td>7</td>
<td>180°-210°</td>
<td>( -\frac{1}{2} V_{dc} )</td>
<td>0</td>
<td>( \frac{1}{2} V_{dc} )</td>
<td>( \frac{1}{2} V_{dc} )</td>
<td>0</td>
<td>-( \frac{1}{2} V_{dc} )</td>
</tr>
<tr>
<td>8</td>
<td>210°-240°</td>
<td>( -\frac{1}{2} V_{dc} )</td>
<td>( -\frac{1}{2} V_{dc} )</td>
<td>( \frac{1}{2} V_{dc} )</td>
<td>( \frac{1}{2} V_{dc} )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>240°-270°</td>
<td>( -\frac{1}{2} V_{dc} )</td>
<td>( -\frac{1}{2} V_{dc} )</td>
<td>0</td>
<td>( \frac{1}{2} V_{dc} )</td>
<td>( \frac{1}{2} V_{dc} )</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>270°-300°</td>
<td>( -\frac{1}{2} V_{dc} )</td>
<td>( -\frac{1}{2} V_{dc} )</td>
<td>0</td>
<td>0</td>
<td>( \frac{1}{2} V_{dc} )</td>
<td>( \frac{1}{2} V_{dc} )</td>
</tr>
<tr>
<td>11</td>
<td>300°-330°</td>
<td>0</td>
<td>( -\frac{1}{2} V_{dc} )</td>
<td>( -\frac{1}{2} V_{dc} )</td>
<td>0</td>
<td>( \frac{1}{2} V_{dc} )</td>
<td>( \frac{1}{2} V_{dc} )</td>
</tr>
<tr>
<td>12</td>
<td>330°-360°</td>
<td>0</td>
<td>0</td>
<td>( -\frac{1}{2} V_{dc} )</td>
<td>( -\frac{1}{2} V_{dc} )</td>
<td>( \frac{1}{2} V_{dc} )</td>
<td>( \frac{1}{2} V_{dc} )</td>
</tr>
</tbody>
</table>
The phase-to-neutral voltage takes on the voltage value $\pm 0.5V_a$ and conduction period is $120^\circ$.

In order to relate the input dc link voltage of the inverter with the output phase-to-neutral, Fourier analysis of the voltage waveforms is undertaken. Observing that the waveforms of the phase to neutral voltage possess odd quarter-wave symmetry. In case of phase-to-neutral voltage $V_{a1}$, shown in Fig. 5.19, the coefficients of the Fourier series are:

$$B_{2n-1} = \frac{2V_{in}}{(2n-1)\pi} \cos\left(\frac{\pi}{6}(2n-1)\right)$$

where $n = 1, 2, 3, \ldots$. (5.44)

The Fourier series is given by:

$$V(t) = \frac{2V_{DC}}{(2n-1)\pi} \cos\left(\frac{\pi}{6}(2n-1)\right) \sin(2n-1)\omega t$$

where $n = 1, 2, 3, \ldots$. (5.45)

From (5.45) it follows that the fundamental sinusoidal component of the output phase-to-neutral voltages ($V_{a1}$) has an RMS value equal to

$$V_{a1} = \frac{\sqrt{6}}{2\pi} V_{DC} = 0.3898484V_{DC}$$

From Fig. 5.19, mean square value is determined as:
The phase-to-neutral voltage takes on the voltage value \( \pm 0.5V_{dc} \) and conduction period is 120°.

In order to relate the input dc link voltage of the inverter with the output phase-to-neutral, Fourier analysis of the voltage waveforms is undertaken. Observing that the waveforms of the phase to neutral voltage possess odd quarter-wave symmetry. In case of phase-to-neutral voltage \( V_{a1} \), shown in Fig. 5.19, the coefficients of the Fourier series are:

\[
B_{2n-1} = \frac{2V_{dc}}{(2n-1)\pi} \cos(2n-1)\pi \frac{\pi}{6} \quad \text{where } n = 1, 2, 3, \ldots.
\]  

The Fourier series is given by:

\[
V(t) = \frac{2V_{dc}}{(2n-1)\pi} \cos(2n-1)p_i \sin(2n-1)\omega t \quad \text{where } n = 1, 2, 3, \ldots.
\]  

From (5.45) it follows that the fundamental sinusoidal component of the output phase-to-neutral voltages \( V_{a1} \) has an RMS value equal to

\[
V_{a1} = \frac{\sqrt{6}}{2\pi} V_{DC} = 0.389484V_{DC}
\]  

From Fig. 5.19, mean square value is determined as:
Mean Square Value

\[ \text{Mean Square Value} = \frac{1}{\pi} \left[ \left( \frac{1}{2} V_{DC} \right)^2 \times \frac{2\pi}{3} \right] = \frac{1}{6} V_{DC}^2 \quad (5.47) \]

\[ V_{rms} = \frac{1}{\sqrt{6}} V_{DC} \quad (5.48) \]

Total harmonic r m s voltage (p u ) is given by

\[ V_{thrms} = \sqrt{\left( t_{rms} \right)^2 - \left( t_1 \right)^2} = \sqrt{\left( \frac{1}{\sqrt{6}} \right)^2 - \left( \frac{\sqrt{6}}{2\pi} \right)^2} = 0.1211812329 \quad (5.49) \]

Hence total harmonic distortion is

\[ THD = \frac{\text{r m s total harmonic voltage}}{\text{r m s total voltage}} \]

\[ \frac{0.1211812329}{1/\sqrt{6}} = 0.296832187 \text{ or } 29.68\% \quad (5.50) \]

FFT analysis of inverter phase ‘V_a1’ voltage is performed using Matlab/Simulink code and is shown in Fig. 5.20 and this validates equation (5.45).

![Inverter phase 'V_a1' voltage time domain waveform and its harmonic spectrum](image)

**Fig 5.20**: Inverter phase ‘V_a1’ voltage time domain waveform and its harmonic spectrum in frequency domain (Fundamental frequency is 50 Hz, Fundamental 0.390644, Highest harmonics Orders=[5 7] Values=[19.7269% 14.4324%]
First Non-Adjacent Line-to-line voltages are elaborated next. First Non-Adjacent Line-to-Line voltages are determined using Leg voltage of Fig. 5.18. The result, thus obtained is used to obtain the waveform which is shown in Fig. 5.21.

![Fig. 5.21. First non-adjacent line-to-line voltages for 120° conduction mode.](image)

The coefficients of the Fourier series for first Non-adjacent line-to-line voltages \( (V_{a_1b_1}) \) as they possess odd quarter-wave symmetry, are:

\[
B_{2n-1} = \frac{4V_{DC}}{\pi(2n-1)} \left[ \cos^2 \left( \frac{2n-1}{6} \pi \right) \right]; \text{ where } n = 1, 2, 3, \ldots
\]  

(5.51)

And hence, the series is:

\[
V(t) = \frac{4V_{DC}}{\pi(2n-1)} \left[ \cos^2 \left( \frac{2n-1}{6} \pi \right) \sin(2n-1) \omega t \right]; \text{ where } n = 1, 2, 3, \ldots
\]  

(5.52)

From (5.52) it follows that the fundamental sinusoidal component of the output first non-adjacent line-to-line voltages \( (V_{a_1b_1}) \) has an RMS value equal to

\[
V_{a_1b_1} = \frac{3\sqrt{2}}{2\pi} V_{DC} = 0.6752372V_{DC}
\]  

(5.53)
First Non-Adjacent Line-to-line voltages are elaborated next. First Non-Adjacent Line-to-Line voltages are determined using Leg voltage of Fig. 5.18. The result, thus obtained is used to obtain the waveform which is shown in Fig. 5.21.

![Waveform diagram showing first non-adjacent line-to-line voltages for 120° conduction mode.](image)

Fig. 5.21. First non-adjacent line-to-line voltages for 120° conduction mode.

The coefficients of the Fourier series for first Non-adjacent line-to-line voltages \( (V_{a1b1}) \) as they possess odd quarter-wave symmetry, are:

\[
B_{2n-1} = \frac{4V_{DC}}{\pi(2n-1)} \left[ \cos^2\left(\frac{2\pi}{3}\right) \frac{\pi}{6} \right] \quad \text{where } n = 1, 2, 3, \ldots
\]  

(5.51)

And hence, the series is:

\[
V(t) = \frac{4V_{DC}}{\pi(2n-1)} \left[ \cos^2\left(\frac{2\pi}{3}\right) \frac{\pi}{6} \right] \sin(2n-1)\omega t \quad \text{where } n = 1, 2, 3, \ldots
\]  

(5.52)

From (5.52) it follows that the fundamental sinusoidal component of the output first non-adjacent line-to-line voltages \( (V_{a1b1}) \) has an RMS value equal to

\[
V_{a1b1} = \frac{3\sqrt{2}}{2\pi} V_{DC} = 0.6752372V_{DC}
\]  

(5.53)
From Fig. 5.21, mean square value is determined as:

\[
\text{Mean Square Value} = \frac{1}{\pi}\left[\left(\frac{1}{2}V_{\text{DC}}\right)^2 \times \frac{\pi}{3} + V_{\text{DC}}^2 \times \frac{\pi}{3} + \left(\frac{1}{2}V_{\text{DC}}\right)^2 \times \frac{\pi}{3}\right] = \frac{1}{2}V_{\text{DC}}^2
\]  
(5.54)

\[
V_{\text{rms}} = \frac{\sqrt{2}}{2}V_{\text{DC}}
\]  
(5.55)

Total harmonic r.m.s. voltage (p.u.) is given by

\[
V_{\text{Hrms}} = \sqrt{(V'_{\text{rms}})^2 - (V'_{\text{DC}})^2} = \sqrt{\left(\frac{\sqrt{2}}{2}\right)^2 - \left(\frac{3\sqrt{2}}{2\pi}\right)^2} = 0.2098920523
\]  
(5.56)

Hence total harmonic distortion is

\[
\text{THD} = \frac{\text{r.m.s. total harmonic voltage}}{\text{r.m.s. total voltage}} = \frac{0.2098920523}{\sqrt{2}/2} = 0.296832187 \text{ or } 29.68\%
\]  
(5.57)

FFT analysis of inverter First Non-adjacent line-to-line ‘va1b1’ voltage is performed using Matlab/Simulink code and is shown in Fig. 5.22 and this validates equation (5.22).

Fig. 5.22. Inverter First Non-adjacent line-to-line ‘va1b1’ voltage time domain waveform and its harmonic spectrum in frequency domain(Fundamental frequency is 50 Hz, Fundamental \(0.676504\); Highest harmonics. Orders=[5 7] Values=[19.764% 14.4326%].

Second set of line voltages are determined next. The same approach as that of 150° and 120° conduction modes are used here and the Wave shapes are shown in Fig. 5.23.
The coefficients of the Fourier series for second Non-adjacent line-to-line voltages $(V_{a1b2})$ as they possess odd quarter-wave symmetry, are:

$$B_{2n-1} = \frac{4V_{DC}}{\pi(2n-1)} \left[ \frac{\cos(2n-1) \pi}{6} \cdot \frac{\cos(2n-1) \pi}{12} \right]; \text{ where } n = 1, 2, 3, \ldots$$  \hspace{1cm} (5.58)

And hence, the series is:

$$V(t) = \frac{4V_{DC}}{\pi(2n-1)} \left[ \frac{\cos(2n-1) \pi}{6} \cdot \frac{\cos(2n-1) \pi}{12} \right] \sin(2n-1) \omega t; \text{ where } n = 1, 2, 3, \hspace{1cm} (5.59)$$

From (5.59) it follows that the fundamental sinusoidal component of the output second non-adjacent line-to-line voltages $(V_{a1b2})$ has an RMS value equal to

$$V_{a1b2} = \frac{3 + \sqrt{3}}{2\pi} V_{DC} = 0.75313V_{DC}$$  \hspace{1cm} (5.60)

From Fig. 5.23, mean square value is determined as:
The coefficients of the Fourier series for second non-adjacent line-to-line voltages \((V_{a1b2})\) as they possess odd quarter-wave symmetry, are:

\[
B_{2n-1} = \frac{4V_{DC}}{\pi(2n-1)} \left[ \cos(2n-1) \frac{\pi}{6} \cos(2n-1) \frac{\pi}{12} \right]; \text{ where } n = 1, 2, 3, \ldots \tag{5.58}
\]

And hence, the series is:

\[
V(t) = \frac{4V_{DC}}{\pi(2n-1)} \left[ \cos(2n-1) \frac{\pi}{6} \cos(2n-1) \frac{\pi}{12} \right] \sin(2n-1)\omega t; \text{ where } n = 1, 2, 3, \tag{5.59}
\]

From (5.59) it follows that the fundamental sinusoidal component of the output second non-adjacent line-to-line voltages \((V_{a1b2})\) has an RMS value equal to

\[
V_{a1b2} = \frac{3+\sqrt{3}}{2\pi} V_{DC} = 0.75313 V_{DC} \tag{5.60}
\]

From Fig. 5.23, mean square value is determined as:
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\[ \text{Mean Square Value} = \frac{1}{\pi} \left[ \left( \frac{1}{2} V_{DC} \right)^2 \times \frac{\pi}{6} + \left( V_{DC} \right)^2 \times \frac{\pi}{2} \right] = \frac{7}{12} V_{DC}^2 \]  

\[ V_{rms} = \frac{\sqrt{21}}{6} V_{DC} \]  

Total harmonic r.m.s. voltage (p.u.) is given by

\[ V_{Hrms} = \sqrt{(V_{rms})^2 - (V_1)^2} = \sqrt{\left( \frac{\sqrt{21}}{6} \right)^2 - \left( 3 + \sqrt{3} \right)^2} = 0.1270024625 \]  

Hence total harmonic distortion is

\[ \text{THD} = \frac{\text{r.m.s. total harmonic voltage}}{\text{r.m.s. total voltage}} \]  

\[ \frac{0.1270024625}{\sqrt{21}/6} = 0.1662852565 \text{ or } 16.29\% \]  

It can be seen that the quality of waveform has increased significantly. FFT analysis of inverter second Non-adjacent line-to-line ‘Va1b2’ voltage is performed using Matlab/Simulink code and is shown in Fig. 5.24 and this validates equation (5.59).

Fig. 5.24. Inverter second Non-adjacent line-to-line ‘Va1b2’ voltage time domain waveform and its harmonic spectrum in frequency domain (Fundamental frequency is 50 Hz, Fundamental : 0.754447; Highest harmonics: Orders=[11 13] Values=[8.87387% 7.82075%].
\[ Mean\ Square\ Value = \frac{1}{\pi} \left[ \left( \frac{1}{2} V_{DC} \right)^2 + \left( V_{DC} \right)^2 + \frac{\pi}{6} V_{DC} \right] = \frac{7}{12} V_{DC}^2 \] (5.61)

\[ V_{rms} = \frac{\sqrt{21}}{6} V_{DC} \] (5.62)

Total harmonic rms voltage (p.u.) is given by

\[ V_{Hrms} = \sqrt{\left( V_{rms} \right)^2 - \left( V_{r} \right)^2} = \sqrt{\left( \frac{\sqrt{21}}{6} \right)^2 - \left( \frac{3+\sqrt{3}}{2\pi} \right)^2} = 0.1270024625 \] (5.63)

Hence total harmonic distortion is

\[ THD = \frac{r.m.s\ total\ harmonic\ voltage}{r.m.s\ total\ voltage} \]

\[ = \frac{0.1270024625}{\sqrt{21}/6} = 0.1662852565 \text{ or } 16.29\% \] (5.64)

It can be seen that the quality of waveform has increased significantly. FFT analysis of inverter second non-adjacent line-to-line ‘V_{a1b2}’ voltage is performed using Matlab/Simulink code and is shown in Fig. 5.24 and this validates equation (5.59).

Fig 5.24 Inverter second non-adjacent line-to-line ‘V_{a1b2}’ voltage time domain waveform and its harmonic spectrum in frequency domain (Fundamental frequency is 50 Hz, Fundamental 0.754447, Highest harmonics Orders=[11 13] Values=[8 87387% 7.82075%]
5.6 COMPARISON OF SIMULATION RESULTS

A comparison of simulation results are carried out and presented in the tabular form in Table 5.9-5.10. Comparison in terms of harmonic content is phase voltages and second non-adjacent line-to-line voltage are done. It is observed that the lowest order harmonic in phase voltages in case of 180° and 150° conduction modes are 3rd while it is 5th in case of 120° conduction mode. The other lower order harmonics are almost same in 180° and 120° conduction modes. The harmonic contents in 150° conduction mode is drastically reduced compared to the other two modes. This is an interesting finding as the loss in fundamental is nearly less than 5% and the gain in THD is quite high. The 3rd harmonic in the two phases cancel each other and thus no 3rd harmonic is seen in the line-to-line voltages. The better harmonic performance of 150° is once again evident.

<table>
<thead>
<tr>
<th>Conduction Mode</th>
<th>Fundamental rms (p.u.)</th>
<th>3rd %</th>
<th>5th %</th>
<th>7th %</th>
<th>9th %</th>
<th>THD %</th>
</tr>
</thead>
<tbody>
<tr>
<td>180°</td>
<td>0.4506</td>
<td>23.07</td>
<td>19.46</td>
<td>14.53</td>
<td>8.41</td>
<td>8.59</td>
</tr>
<tr>
<td>150°</td>
<td>0.435</td>
<td>13.77</td>
<td>6.61</td>
<td>4.83</td>
<td>4.59</td>
<td>9.06</td>
</tr>
<tr>
<td>120°</td>
<td>0.39</td>
<td>19.72</td>
<td>14.437</td>
<td>0</td>
<td>8.81</td>
<td>29.21</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Conduction In deg.</th>
<th>Fundamental rms (p.u.)</th>
<th>3rd %</th>
<th>5th %</th>
<th>7th %</th>
<th>9th %</th>
<th>THD %</th>
</tr>
</thead>
<tbody>
<tr>
<td>180</td>
<td>0.778</td>
<td>0</td>
<td>20.155</td>
<td>14.1</td>
<td>0</td>
<td>9.19</td>
</tr>
<tr>
<td>150</td>
<td>0.753</td>
<td>0</td>
<td>6.63</td>
<td>4.81</td>
<td>0</td>
<td>9.07</td>
</tr>
<tr>
<td>120</td>
<td>0.675</td>
<td>0</td>
<td>19.764</td>
<td>14.43</td>
<td>0</td>
<td>8.86</td>
</tr>
</tbody>
</table>

5.7 EXPERIMENTAL RESULTS

A modular multi-phase inverter is developed in the laboratory which can operate from single to nine phases. It is operated as a Five-phase VSI and the results are presented in Chapter 3. Here it is operated as a Six-phase in quasi six-phase configuration. Experiment is conducted for stepped operation of inverter with 180°, 150° and 120° conduction modes for star connected six-phase resistive load. A single-phase supply is given to the control circuit through the phase shifting network. The output of the phase shifting circuit provides the required quasi six-phase output voltage by appropriately tuning it as shown in Fig. 5.25. These quasi six-phase signals are then further processed to generate the gate drive circuit. The complete block diagram of the control circuit is already shown in chapter 3 in Fig. 3.20.
The distorted waveform is due to the distortion in the mains. The same is reflected here as well. Only four currents are shown due to limitation of the oscilloscope which has only four channels.

5.7.1 180° CONDUCTION MODE

Each switch is assumed to conduct for 180°, leading to the operation in the ten-step mode. Phase delay between firing of two switches in any subsequent two phases is equal to 30° and 90°. The output from the Schmitt trigger circuit is presented in Fig. 5.26. The driving control gate/base signals for legs A1 – A2 of the inverter are illustrated in Fig. 5.27. The corresponding phase voltage thus obtained are shown in Fig. 5.28, keeping the dc link voltage at 60 V.
The distorted waveform is due to the distortion in the mains. The same is reflected here as well. Only four currents are shown due to limitation of the oscilloscope which has only four channels.

### 5.7.1 180° Conduction Mode

Each switch is assumed to conduct for 180°, leading to the operation in the ten-step mode. Phase delay between firing of two switches in any subsequent two phases is equal to 30° and 90°. The output from the Schmitt trigger circuit is presented in Fig. 5.26. The driving control gate/base signals for legs A₁ – A₂ of the inverter are illustrated in Fig. 5.27. The corresponding phase voltage thus obtained are shown in Fig. 5.28, keeping the dc link voltage at 60 V.
Fig. 5.26. Output of wave shaping circuit for 180° conduction mode for leg A1-A2.

Fig. 5.27. Gate Drive signals for legs A1-A2 for 180° conduction mode.

Fig. 5.28. Output phase 'a1-b2' voltages for 180° conduction mode.
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Fig. 5.26. Output of wave shaping circuit for 180° conduction mode for leg A1-A2

Fig. 5.27. Gate Drive signals for legs A1-A2 for 180° conduction mode

Fig. 5.28. Output phase 'a1-b2' voltages for 180° conduction mode
There are two systems of line voltages in a quasi six-phase system namely adjacent (a1a2 of 30°, a2b1 of 90° phase shifts) and non-adjacent (a1b1 of 120°, a1b2 of 150° phase shifts). First Non-adjacent line voltage thus obtained is shown in Fig. 5.29. All currents are measured using ac/dc current probe giving output of 100 mV/A.

Fig. 5.29. First Non-adjacent line voltage (V_{aibi}) for 180° conduction mode.

The ac side input current is also measured and is depicted in Fig. 5.30. The analysis is presented in section (5.8).

Fig. 5.30. AC side input current for 180° conduction mode
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There are two systems of line voltages in a quasi six-phase system namely adjacent (a1a2 of 30°, a2b1 of 90° phase shifts) and non-adjacent (a1b1 of 120°, a1b2 of 150° phase shifts). First Non-adjacent line voltage thus obtained is shown in Fig. 5.29. All currents are measured using ac/dc current probe giving output of 100 mV/A.

![Fig. 5.29. First Non-adjacent line voltage (Vaibi) for 180° conduction mode.](image)

The ac side input current is also measured and is depicted in Fig. 5.30. The analysis is presented in section (5.8).

![Fig. 5.30. AC side input current for 180° conduction mode.](image)
5.7.2 150° CONDUCTION MODE

The gate drive signal is such that each power switch remains on for 150° (or 41.67% duty cycle) and remains floating for 30° (or 8.33% duty cycle). This mode thus provides an inherent dead band in the switching of two power switches of the same leg. The output from the wave shaping circuit and the gate drive for two legs are shown in Fig. 5.31 and Fig. 5.32, respectively. The corresponding phase-to-neutral output voltage for phase ‘a1’ is shown in Fig. 5.33. First Non-adjacent line voltage is presented in Fig. 5.34 and the input ac side current are shown in Fig. 5.35.

Fig. 5.31. Output of wave shaping circuit for 150° conduction mode for leg A1-A2.

Fig. 5.32. Gate Drive signals for leg A1-A2 for 150° conduction mode.
5.7.2 150° CONDUCTION MODE

The gate drive signal is such that each power switch remains on for 150° (or 41.67% duty cycle) and remains floating for 30° (or 8.33% duty cycle). This mode thus provides an inherent dead band in the switching of two power switches of the same leg. The output from the wave shaping circuit and the gate drive for two legs are shown in Fig. 5.31 and Fig. 5.32, respectively. The corresponding phase-to-neutral output voltage for phase ‘a1’ is shown in Fig. 5.33. First Non-adjacent line voltage is presented in Fig. 5.34 and the input ac side current are shown in Fig. 5.35.

![Fig. 5.31 Output of wave shaping circuit for 150° conduction mode for leg A1-A2.](image)

![Fig. 5.32 Gate Drive signals for leg A1-A2 for 150° conduction mode.](image)
Fig. 5.33. Output phase ‘a1-b2’ voltage for 150° conduction mode.

Fig. 5.34. Non-adjacent line voltage (V_{a1b1}) for 150° conduction mode.

Fig. 5.35. AC side input current for 150° conduction mode.
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Fig. 5.33 Output phase ‘a1-b2’ voltage for 150° conduction mode

Fig. 5.34 Non-adjacent line voltage (V_{a1b1}) for 150° conduction mode

Fig. 5.35 AC side input current for 150° conduction mode.
5.7.3 108° CONDUCTION MODE

The gate drive signal is such that each power switch remains on for 120° (or 33.33% duty cycle) and remains floating for 60° (or 16.67% duty cycle). This mode thus also provide an inherent dead band in the switching of two power switch of the same leg. The gate drive signals for two legs are shown in Fig. 5.36. The corresponding phase-to-neutral voltage is presented in Fig. 5.37. First Non-adjacent line voltage and the input ac side current are depicted in Fig. 5.38 and 5.39, respectively.

![Fig. 5.36. Gate Drive signals for legs A1-A2 for 120° conduction mode.](image1)

![Fig. 5.37. Output phase 'a1-b2' voltages for 120° conduction mode.](image2)
5.7.3 108° CONDUCTION MODE

The gate drive signal is such that each power switch remains on for 120° (or 33.33% duty cycle) and remains floating for 60° (or 16.67% duty cycle). This mode thus also provide an inherent dead band in the switching of two power switch of the same leg. The gate drive signals for two legs are shown in Fig 5.36. The corresponding phase-to-neutral voltage is presented in Fig 5.37. First Non-adjacent line voltage and the input ac side current are depicted in Fig. 5.38 and 5.39, respectively.

---

**Fig 5.36** Gate Drive signals for legs A1-A2 for 120° conduction mode

---

**Fig 5.37** Output phase ‘a1-b2’ voltages for 120° conduction mode
5.8 RESULT ANALYSIS

This section presents the comprehensive analysis of experimental results. The performance of three different conduction modes are elaborated in terms of the harmonic content in the phase voltages, line voltages and the distortion in the ac side line current. The harmonic waveforms are illustrated in Figs. 5.40-5.48.
5.8 RESULT ANALYSIS

This section presents the comprehensive analysis of experimental results. The performance of three different conduction modes are elaborated in terms of the harmonic content in the phase voltages, line voltages and the distortion in the ac side line current. The harmonic waveforms are illustrated in Figs. 5.40-5.48.
Fig. 5.40. Spectrum of phase ‘a1’ voltage for 180° cond. mode for dc link voltage of 220 V.

Fig. 5.41. Spectrum of non-adjacent line ‘a1-b1’ voltage for 180° conduction mode.

Fig. 5.42. Spectrum of ac side input current for 180° conduction mode.
Chapter 5: Modelling and Control of a Six-Phase VSI in Quasi-Six-Phase Configuration

Fig. 5.40. Spectrum of phase ‘a1’ voltage for 180° cond. mode for dc link voltage of 220 V.

Fig. 5.41. Spectrum of non-adjacent line ‘a1-b1’ voltage for 180° conduction mode.

Fig. 5.42. Spectrum of ac side input current for 180° conduction mode.
Fig. 5.43. Spectrum of phase ‘$a_1$’ voltage for $150^\circ$ cond. mode.

Fig. 5.44. Spectrum of non-adjacent line ‘$a_1$-$b_1$’ voltage for $150^\circ$ conduction mode.

Fig. 5.45. Spectrum of input ac side current for $150^\circ$ conduction mode.
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Fig. 5.43. Spectrum of phase 'a1' voltage for 150° cond. mode.

Fig. 5.44 Spectrum of non-adjacent line 'a1-b1' voltage for 150° conduction mode.

Fig. 5.45 Spectrum of input ac side current for 150° conduction mode.
Fig. 5.46. Spectrum of phase ‘a1’ voltage for 120° conduction mode.

Fig. 5.47. Spectrum of non-adjacent line ‘a1-bi’ voltage for 120° conduction mode.

Fig. 5.48. Spectrum of ac side input line current for 120° conduction mode.
Chapter 5 Modelling and Control of a Six-Phase VSI in Quasi Six-Phase Configuration

Fig 5.46 Spectrum of phase ‘a1’ voltage for 120° conduction mode

Fig 5.47 Spectrum of non-adjacent line ‘a1-b1’ voltage for 120° conduction mode

Fig 5.48 Spectrum of ac side input line current for 120° conduction mode
Performance comparison in terms of harmonic content in output phase voltage, output non-adjacent line voltage and input ac side current for different conduction modes are presented in Figs. 5.49-5.51. It is clearly seen that the harmonic content reduces drastically with reduction in conduction angle. The harmonic content is largest in 180 degree conduction mode and it is least in 150 degree conduction mode except 11th harmonic which is smallest for 120°. However, the best utilisation of available dc link voltage is possible with conventional 180 degree conduction mode. It can thus be concluded that a compromise between the loss in fundamental and corresponding gain in terms of lower harmonic content in output waveform is obtained by using 150 degree conduction mode.

Fig. 5.49. Harmonic content in output phase voltage for different conduction mode.

Fig. 5.50. Harmonic content in output phase voltage for different conduction mode.
Chapter 5: Modelling and Control of a Six-Phase VSI in Quasi Six-Phase Configuration

5.9 SUMMARY

This chapter elaborates the operation of a six-phase voltage source providing quasi six-phase output. Brief description of modeling is given and the operation of inverter in square wave mode is elaborated. Three different conduction modes are taken up; 180°, 150° and 120°. Analytical and simulation approach is at first used to explain the operation and performance of the inverter. This approach is reinforced with the experimental results. Complete experimental results are provided and a comparison if performance for three different modes are presented.

Fig. 5.51. Harmonic content in input side ac current for different conduction mode.