CHAPTER 3
OA-C FILTERS: DIRECT FORM SYNTHESIS

3.0 Introduction

This chapter is concerned with the realization of second- and higher-order OA-C filters, using the direct form synthesis approach. This synthesis technique should be such that it realizes the final network in the form suitable for the microminiaturization in the MOS technology. This implies that the circuits should contain only the active device, viz., operational amplifiers, and capacitors in their realizations. Moreover, all the important parameters of the network should be a function of ratioed-Cs.

Three techniques are proposed here for such a purpose and are critically studied. These are:

i) L-Based Approach to OA-C synthesis;

ii) FDNR-Based Approach to OA-C synthesis; and

iii) FDNCAP-Based Approach to OA-C synthesis.

In any direct form synthesis technique, the starting point is the passive RLC-prototype network, whose elements are to be realized in a manner so that the final realization contains only those elements, which are suitable for convenient micro-miniaturization. In the present case, only the OAs and Cs

† The material presented in this chapter has led to the publications of the author’s paper no. 3, and 4 listed on page vii.
are permitted in the final realizations. The restriction on the use of only OAs and capacitors requires the simulation of two different types of circuit elements in each of the three proposed techniques. The application of a particular technique will largely depend upon the configuration of the prototype. Each technique will be discussed in details in the subsequent section. Here we only present their outlines.

O In the L-based Approach to OA-C Synthesis, the R and L elements are simulated and directly substituted in the prototype to give an equivalent realization in the OA-C form.

O In the FDNR-based Approach to OA-C Synthesis, the admittance of the prototype network \(N\) is first scaled by the complex-frequency variable(s) to give the transformed network \(N'\). The transformation results in converting an RLC-network into a corresponding CRD-network\(^{16}\). The final realization is then obtained by simulating the R and D elements by the OA-C simulators. It may be noted that the transformation does not affect the transfer function of the network.

O In the FDNCAP-based Approach to OA-C Synthesis, the admittance of the prototype RLC-network \(N\) are scaled by the function, \(s^2\), to give a corresponding CDF-network \(N''\), without affecting the transfer function. The final realization boils down to the simulation of D and F elements in the OA-C form.

For all the three techniques, the OA-C simulators,
already studied in Chapter 2, are used. As the simulation of two different type of elements generally requires a large count of active and passive components, methods are also suggested for the optimization of component count.

The three approaches are critically studied for the realization of second-order and higher-order filters, respectively in Sections 3.1, 3.2 and 3.3. Suggestions for the optimization of component count and suitability of structures for a particular technique are clearly examined. In Section 3.4, experimental results on some circuits are included. The concluding remarks are given in Section 3.5.

3.1 FDNR-Based Approach to OA-C Filter Realizations

In the Frequency Dependent Negative Resistance (FDNR)-based approach to OA-C synthesis, the passive RLC prototype circuit describing the transfer function is first transformed into an equivalent network by scaling each admittance of the original network (N) by the complex-variable(s), as shown in Fig. 3.1. The new network (N') is obtained by converting a resistor to a capacitor, an inductor to a resistor and a capacitor to a frequency dependent negative resistance element. The FDNR has an impedance of the form, \(1/s^2D\), where D is parameter of the element with the units of square farad-ohm \((\text{f}^2\Omega)\). Thus, the RLC-network (N) is converted to a CRD-network (N') without affecting its transfer function and the
Fig. 3.1 C, R, D equivalents of R-L-C elements.
synthesis problem basically becomes that of the realization of resistors and FDNRs in the OA-C form.

It may be noted that in this technique, the final network realization requires the simulation of two different types of circuit element, viz., FDNRs and Rs. This can be achieved by either replacing each component by its ideal OA-C immittance simulator or by replacing a set of components by the equivalent non-ideal OA-C component simulators. Generally the clever use of the second approach proves out to be more attractive from the considerations of minimizing active and passive components and in the realizations having attractive sensitivity properties.

In this section, the FDNR-based technique is used for the realization of (i) standard second-order OA-C filters and (ii) higher-order OA-C filters. In Section 3.1.1, a standard second-order OA-C band-pass filter is realized by employing ideal FDNR simulator. This is followed by the realizations of standard second-order OA-C band-elimination and all-pass filters using the non-ideal FDNR-based component simulators. The advantages associated with the use of non-ideal simulators are clearly demonstrated. The technique is also employed in Section 3.1.3 in the realization of higher-order filter.
3.1.1 Realization of a second-order sections with ideal FDNR-simulator

The importance of standard second-order filter sections in active synthesis hardly needs any elaboration. In Sections 3.1.1 and 3.1.2 the realization of second-order filters, using the FDNR-based approach, is demonstrated. These employ the FDNR-based simulator of Table 2.2.

The realization of a second-order band-pass filter is first demonstrated by employing an ideal FDNR simulator. Consider the passive prototype second-order band-pass (BP) filter, shown in Fig. 3.2(a). Its scaling by the complex frequency 's' leads to the FDNR-based version, shown in Fig. 3.2(b). For the final realization of the BP filter in the OA-C form, the elements in the shunt arm are to be replaced by the appropriate component simulators. In this case, the ideal FDNR is replaced by the ideal OA-C FDNR simulator of Fig. 2.7 and R:C shunt-branch is replaced by the immittance simulator of Fig. 2.18. The complete realization is shown in Fig. 3.2(c). Direct analysis of the circuit, yields the transfer function as:

\[
T_{bp}(s) = \frac{V_{bp}}{V_i} = \frac{H_{bp} \left( \frac{w_0}{Q} \right) s}{s^2 + \left( \frac{w_0}{Q} \right) s + \frac{w_0^2}{Q}} = \frac{H_{bp} \left( \frac{w_0}{Q} \right) s}{D(s)} \tag{3.1}
\]

† As an alternative, the resistance of the shunt branch may be replaced by ideal R-simulator of Fig. 2.16. This however requires matching constraint and large component count.
Fig. 3.2 (a) Passive RLC prototype band-pass filter (BPF).  
(b) FDNR-version of prototype BPF.  
(c) OA–C version of Fig. 3.2(b).  
(d) C-MOS inverter.
where, $H_{bp}$, $w_o$ and $Q$ are, respectively, the mid-band gain, the pole-frequency and the pole-$Q$ of the BP filter, having the following relationships:

$$H_{bp} = \frac{C_o}{C_{ol}}, \quad \omega_o = \left[ B_0 B_1 \frac{C_1 C_2}{C_2 C_{34}} \right]^{1/2}$$

and

$$Q = \frac{C_2}{C_{ol}} \left[ \frac{B_0}{B_1} \cdot \frac{C_1 C_{34}}{C_2} \right]^{1/2} \quad (3.2)$$

where, $C_{jk} = C_j + C_k$. As the output has not been taken at the output terminal of an OA, the response will be load sensitive. In order to obviate this problem, it is recommended to use a buffer stage constituted by C-MOS inverter shown in Fig. 3.2(d). An important point to be noted is that the derived circuit may provide, in addition, other standard responses at the outputs of the OAs employed in the realization. For example, the BPF of Fig. 3.2(c) realizes standard second-order low pass response at the output of $OA_0$, with the transfer function given by

$$T_{lp}(s) = \frac{V_{lp}}{V_i} = \frac{H_{lp} w_o^2}{D(s)} \quad (3.3)$$

where, the low pass gain

$$H_{lp} = -\frac{C_o}{C_1} \quad (3.4)$$

From (3.2) to (3.4) it is seen that all the filter parameters are in terms of capacitors ratios, which is a highly
desirable feature for the implementation of the circuit in MOS-technology. The circuit possesses independent tuning of the pole-frequency \( w_o \) by bias-voltage control. The pole-Q is independently tunable by capacitor \( C_o \), without disturbing the pole-frequency.

The sensitivity study of the filter was also carried out. It yielded the sensitivity figures of filter parameters of interest, as given in Table 3.1. The expressions clearly demonstrate the sensitivity figures to be attractive, being no more than unity in magnitude. However, it may be recalled that the realization of ideal FDNR simulator itself requires critical component matching in \( B_s \), i.e., \( B_2 = B_3 \). This in-turn makes the overall realization fairly sensitive to \( B_2 \) and \( B_3 \), which is not evident from Table 3.1.

The study of the section demonstrates how a second-order OA-C filter may be realized using ideal FDNR simulator. The problem associated with the use of ideal FDNR are: (i) the circuits normally tend to require a higher component count. The present circuit uses four OAs, five capacitors and a buffer and (ii) the ideal simulator requires matching of components for its realization, making the overall circuit sensitive to such parameters.
Table 3.1: Sensitivity figures for BPF of Fig. 3.2(c)

<table>
<thead>
<tr>
<th>Sensitivity</th>
<th>$H_{bp}$</th>
<th>$H_{ip}$</th>
<th>$w_0$</th>
<th>$Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_0$</td>
<td>0</td>
<td>0</td>
<td>1/2</td>
<td>1/2</td>
</tr>
<tr>
<td>$B_1$</td>
<td>0</td>
<td>0</td>
<td>1/2</td>
<td>-1/2</td>
</tr>
<tr>
<td>$C_0$</td>
<td>$\frac{C_1}{C_{01}},(\langle \mid 1 \mid \rangle)$</td>
<td>1</td>
<td>0</td>
<td>$-\frac{C_0}{C_{01}},(\langle \mid 1 \mid \rangle)$</td>
</tr>
<tr>
<td>$C_1$</td>
<td>$-\frac{C_1}{C_{01}},(\langle \mid 1 \mid \rangle)$</td>
<td>-1</td>
<td>1/2</td>
<td>$-\frac{1}{2} \cdot \frac{C_1-C_0}{C_{01}},(\langle \mid 1/2 \mid \rangle)$</td>
</tr>
<tr>
<td>$C_2$</td>
<td>0</td>
<td>0</td>
<td>-1/2</td>
<td>1/2</td>
</tr>
<tr>
<td>$C_3$</td>
<td>0</td>
<td>0</td>
<td>$\frac{1}{2} \cdot \frac{C_4}{C_{34}},(\langle \mid 1/2 \mid \rangle)$</td>
<td>$-\frac{1}{2} \cdot \frac{C_4}{C_{34}},(\langle \mid 1/2 \mid \rangle)$</td>
</tr>
<tr>
<td>$C_4$</td>
<td>0</td>
<td>0</td>
<td>$-\frac{1}{2} \cdot \frac{C_4}{C_{34}},(\langle \mid 1/2 \mid \rangle)$</td>
<td>$\frac{1}{2} \cdot \frac{C_4}{C_{34}},(\langle \mid 1/2 \mid \rangle)$</td>
</tr>
</tbody>
</table>
3.1.2 Realization of second-order sections with non-ideal FDNR simulators

The section demonstrates the attractive features of filter realization using the non-ideal FDNR simulators. The realization of two standard second-order filters are considered as examples, viz., band-elimination (BE), and all-pass (AP) filters. The details of the realization and study of these filters are discussed as follows:

0 Band elimination filter - The prototype second-order band-elimination filter is shown in Fig. 3.3(a) and its transformed FDNR version is given in Fig. 3.3(b). To realize it in the OA-C form, the shunt branch is to be simulated. The non-ideal FDNR simulator of Fig. 2.10 has been used in the direct simulation of the entire shunt arm (shown within the dotted lines) of the transformed network. The final realization of the BE filter in the OA-C form is shown in Fig. 3.3(c). The analysis of the circuit yields the transfer function as:

\[
T_{be}(s) = \frac{V_{be}}{V_i} = H_{be} \cdot \frac{s^2 + \omega_n^2}{s^2 + \left(\frac{\omega_n}{Q}\right)s + \omega_n^2} = \frac{N(s)}{D(s)} \quad (3.5)
\]

where

\[H_{be} = \frac{C_0}{C_{ol}} \quad \omega_n = \omega_n = (B_1B_2 \cdot \frac{C_2}{C_3})^{1/2}\]

and

\[Q = (1 + \frac{C_0}{C_1}) \cdot \left(\frac{B_2}{B_1} \cdot \frac{C_2}{C_3}\right)^{1/2} \quad (3.6)\]
Fig. 3.3 (a) Prototype-RLC band-elimination filter (BEF)
(b) Scaled version of BEF
(c) OA-C version of BEF of Fig 3.3 (b)
Note, once again the filter parameters of interest are having the attractive feature of being in the form of capacitor-ratios. The circuit enjoys the multifunctional capability and provides two additional standard responses of band-pass (BP) and low-pass (LP) at the outputs of OA_1 and OA_2, respectively. The transfer function of the BP and LP filters are:

\[ T_{bp}(s) = \frac{V_{bp}}{V_i} = \frac{H_{bp} \frac{w_o}{Q} s}{D(s)} \]  
\[ T_{lp}(s) = \frac{V_{lp}}{V_i} = \frac{H_{lp} w_o^2}{D(s)} \]  

where, the respective filters gain are given by

\[ H_{bp} = -\frac{C_o}{C_1} \]  
\[ H_{lp} = \frac{C_o}{C_{ol}} \]  

These are also in the form of C-ratios. Obviously, the expressions for the \( w_o \) and Q remain unaltered for the additional responses.

The circuit possesses independent bias-voltage tuning of pole-frequency \( w_o \) (= \( w_n \)). Also, independent passive tuning of pole-Q is possible with \( C_o \) and/or \( C_1 \). The sensitivity figures of the circuit were derived out and the final results are included in Table 3.2. It is seen that active and passive sensitivities are, once again, within unity in magnitude. Note,
<table>
<thead>
<tr>
<th>Sensitivity</th>
<th>$H_{bp}$</th>
<th>$H_{lp}$</th>
<th>$Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_1$</td>
<td>0</td>
<td>0</td>
<td>1/2</td>
</tr>
<tr>
<td>$B_2$</td>
<td>0</td>
<td>1/2</td>
<td>0</td>
</tr>
<tr>
<td>$C_0$</td>
<td>1</td>
<td>0</td>
<td>1/2</td>
</tr>
<tr>
<td>$C_1$</td>
<td>$-{C_0, (</td>
<td>11</td>
<td>)}$</td>
</tr>
<tr>
<td>$C_2$</td>
<td>$-{C_0, (</td>
<td>11</td>
<td>)}$</td>
</tr>
<tr>
<td>$C_3$</td>
<td>$-{C_0, (</td>
<td>11</td>
<td>)}$</td>
</tr>
</tbody>
</table>

Table 3.2: Sensitivity figures for OA-C filter of Fig. 3.2(c)
high-valued resistors are required across capacitors, $C_1$ and $C_2$, to provide dc biasing to the inverting-terminals of the OAs.

**All-pass filter** - The second-order prototype of an all-pass (AP) filter is shown in Fig. 3.4(a) and its transformed version is given in Fig. 3.4(b). For the OA-C realization, the complete shunt arm, shown within the dotted lines, may be simulated by directly using the non-ideal FDNR-simulator of Fig. 2.23. The final OA-C version of the filter is given in Fig. 3.4(c). The use of non-ideal (FDNR) results in an appreciable optimization of active and passive components. Analysis of the filter yields the transfer function:

$$T_{ap}(s) = H_{ap} \frac{s^2 - a_1s + a_0}{s^2 + b_1s + b_0} \quad (3.10)$$

where,

$$H_{ap} = \frac{C_o}{C_{o1}}, \quad a_0 = b_0 = B_1B_2, \quad \frac{C_3}{C_{23}}, \quad \frac{C_4}{C_{45}}$$

and the AP-response is realized by satisfying the component condition:

$$a_1 = b_1 = \frac{C_2}{C_{23}}, \quad \text{when} \quad \frac{C_o}{C_1} = \frac{1}{2} \left( \frac{C_3}{C_2} - 1 \right) \quad (3.11)$$

This condition can conveniently be satisfied, particularly if the circuit is implemented in monolithic form in the MOS technology. As in earlier case, the circuit has multifunctional
Fig. 3.4 (a) Prototype all-pass filter (APF)  
(b) Transformed FDNR-version of APF  
(c) OA-C form of APF
capability and provides additional standard second-order BP and LP responses simultaneously at the outputs of OA1 and OA2, respectively. The various pertinent parameters of LP and BP responses are given by:

\[ H_{lp} = \frac{C_0}{C_{ol}}, \quad H_{bp} = -\frac{2C_0}{C_1}, \]

\[ \omega_0 = (B_1B_2 \cdot \frac{C_3}{C_{23}} \cdot \frac{C_4}{C_{45}})^{1/2}. \quad (3.12) \]

and

\[ Q = 2(1+\frac{C_2}{C_1})(\frac{B_2}{B_1} \cdot \frac{C_3}{C_{23}} \cdot \frac{C_4}{C_{45}})^{1/2}. \quad (3.12) \]

Once again, the filter parameters have the attractive feature of being in terms of C-ratios. The sensitivity figures for various filter parameters with respect to the active and passive elements are analysed and the final results included in Table 3.3. The table shows that the circuit enjoys attractive sensitivity performance, being less than or equal to unity in magnitude. The circuit also enjoys independent electronic tuning of pole-frequency. This filter can realize high-Q values due to the additional multiplication factor, \((1+C_0/C_1)\), involved in the expression of \(Q\). To provide the dc bias at the inverting-inputs of both the OAs, high-valued resistors are required across the capacitors \(C_1\) and \(C_4\):

It is important to note that both the circuits, realized with the non-ideal FDNR based simulators, have very attractive
Table 3.3: Sensitivity figures for OA-C filter of Fig. 3.4(c)

<table>
<thead>
<tr>
<th>Sensitivity</th>
<th>$w_0$</th>
<th>$H_{ap} = H_{lp}$</th>
<th>$H_{bp}$</th>
<th>$Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_1$</td>
<td>$1/2$</td>
<td>0</td>
<td>0</td>
<td>-1/2</td>
</tr>
<tr>
<td>$B_2$</td>
<td>$1/2$</td>
<td>0</td>
<td>0</td>
<td>1/2</td>
</tr>
<tr>
<td>$C_0$</td>
<td>0</td>
<td>$\frac{C_1}{C_{o1}},(&lt;1)$</td>
<td>1</td>
<td>$\frac{C_0}{C_{o1}},(&lt;1)$</td>
</tr>
<tr>
<td>$C_1$</td>
<td>0</td>
<td>$\frac{C_1}{C_{o1}},(&lt;1)$</td>
<td>-1</td>
<td>$\frac{C_0}{C_{o1}},(&lt;1)$</td>
</tr>
<tr>
<td>$C_2$</td>
<td>$\frac{1}{2},\frac{C_2}{C_{23}},(&lt;1/2)$</td>
<td>0</td>
<td>0</td>
<td>$\frac{C_2}{C_{23}},(&lt;1)$</td>
</tr>
<tr>
<td>$C_3$</td>
<td>$\frac{1}{2},\frac{C_2}{C_{23}},(&lt;1/2)$</td>
<td>0</td>
<td>0</td>
<td>$\frac{C_2}{C_{23}},(&lt;1/2)$</td>
</tr>
<tr>
<td>$C_4$</td>
<td>$\frac{1}{2},\frac{C_5}{C_{45}},(&lt;1/2)$</td>
<td>0</td>
<td>0</td>
<td>$\frac{C_5}{C_{45}},(&lt;1)$</td>
</tr>
<tr>
<td>$C_5$</td>
<td>$\frac{1}{2},\frac{C_5}{C_{45}},(&lt;1/2)$</td>
<td>0</td>
<td>0</td>
<td>$\frac{C_5}{C_{45}},(&lt;1)$</td>
</tr>
</tbody>
</table>
sensitivity properties. In addition, they normally do not require component matching in their realizations, and their total active and passive components are considerably lower. This illustrates that, whenever possible, clever use of non-ideal simulators should be made in the realization of networks based on the FDNR approach.

3.1.3 Realization of higher-order ladders

Higher-order filters are frequently realized in the form of ladder structures. In this section, the FDNR based approach is applied to the active-C realization of an n-th order all-pole ladder. The voltage transfer function of the LC ladder, shown in Fig. 3.5 (a), is given by

$$T(s) = \frac{K bo}{s^n + b_{n-1} s^{n-1} + \ldots + b_1 s + b_0}$$  \hspace{1cm} (3.13)

where, $b_0$, $b_1$, ..., $b_{n-1}$ and $K$ are the appropriately chosen constants, depending upon the filter characteristics. Low-pass filters, like, Butterworth, Chebyshev, etc., have transfer functions of this type, differing only in the choice of the coefficients. The LC-ladder may be realized in the OA-C form by using ideal floating inductance simulators, along with grounded passive $C$s. This is, however, not attractive from

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† The C-matching in the AP-case has been imposed by the basic structure from which the AP-filter is realized.
Fig. 3.5 (a) nth-order prototype LC low-pass ladder. (b) FDNR-version of the prototype low-pass ladder.
the consideration of practical implementation of the circuit. The ideal floating $L_s$ require a large count of active and passive components, besides the requirement of critical matching conditions. Therefore, the FDNR-based realization of the ladder is considered. The ladder network is scaled by the complex frequency 's', which transforms it into the CRD-version, shown in Fig. 3.5(b), without affecting the transfer function. The problem then reduces to the realization of ideal grounded FDNRs and floating $R_s$ in the OA-C form. However, as it has been pointed out in Chapter 2, the ideal simulators require large component count. Therefore, the active-C version of the ladder is realized in this section by employing non-ideal immittance simulators, which within certain constraints, realize their ideal versions. It may be noted that these circuits were not included in Chapter 2.

Consider the non-ideal grounded FDNR and the floating $R$ simulators, respectively shown in Fig. 3.6(a) and (b). The non-ideal grounded FDNR basically realizes the driving-point impedance function

$$Z(s) = \frac{s + B}{s^2C} = \frac{1}{sC} + \frac{B}{s^2C} \quad (3.14)$$

This circuit realizes a series combination of an FDNR and a capacitor. However, in the frequency range, $\omega \ll B$, the circuit realizes an effective ideal-FDNR with
Fig. 3.6  Tm OA–C circuits realizing (a) ideal grounded FDNR (b) ideal floating $R$; in the frequency range $\omega \ll B$. 

\[ D = \frac{C}{B} \]

\[ R = \frac{1}{B C} \]
Similarly, analysis of the floating-R simulator of Fig. 3.6(b) realizes the admittance-parameter matrix:

\[
\begin{bmatrix}
(s+B) & -B \\
-B & (s+B)
\end{bmatrix}
\]

In the frequency range, \(w \ll B\), the matrix reduces to

\[
\begin{bmatrix}
1 & -1 \\
-1 & 1
\end{bmatrix}
\]

and realizes an effective ideal-floating resistor with \(R = 1/CB\).

These low-component simulators are subsequently used in the active-C realization of the ladder, employing the transformed circuit of Fig. 3.5(b). The final OA-C version of the ladder is given in Fig. 3.7. It may be noted that the circuit uses a low component count of only \(\frac{3}{2}n\) OAs and \(\left(\frac{3}{2}n+2\right)\) capacitors for even \(n\), and of \(\frac{1}{2}(3n-1)\) OAs and \(\frac{3}{2}(n+1)\) capacitors for odd \(n\) ladders. The only restriction with the circuit is to use it in the limited frequency range, \(w \ll B\), which restricts its use to the AF-range. This section demonstrates how component saving may be achieved at the cost of some constraints imposed on the working frequency-range of filters.
Fig. 3.7 OA-C version of low-pass ladder of Fig. 3.5(b)
3.2 \textit{L}-Based Approach to OA-C Synthesis

In the inductance (L)-based approach to OA-C synthesis, the R and L elements of the prototype filter describing the transfer function are required to be simulated in the OA-C form and then substituted for the corresponding passive components. This results in the OA-C version of the filter. As has been seen in the preceding section, the use of ideal component simulators normally requires a large count of active and passive components. In addition, the use of such component simulators, particularly the floating ones, may result in undesirable sensitivity properties and the circuit having unaccountable parasites, when the matching conditions are not strictly satisfied. Alternatively, the use of non-ideal component simulators for replacing a set of components is more attractive from the considerations mentioned above.

In this section, the second approach is therefore used in the OA-C realizations of (i) standard second-order filters and (ii) higher-order ladders. In Section 3.2.1, a standard second-order OA-C high-pass (HP) filter is realized by employing non-ideal L-simulator. The technique is then applied in Section 3.2.2 in the realization of higher-order OA-C ladders.
3.2.1 Realization of second-order filter with non-ideal L-simulator

This section demonstrates the realization of a second-order high-pass filter from its prototype, by using a non-ideal OA-C grounded L-simulator. The passive HP-prototype is shown in Fig. 3.8(a). For obtaining its OA-C version, the RLC-tuned circuit is directly simulated and replaced by the OA-C simulator of Fig. 2.3. The resulting circuit is shown in Fig. 3.8(b). Its analysis gives the transfer function:

\[ T_{hp}(s) = \frac{V_{hp}}{V_i} = \frac{H_{hp}s^2}{s^2 + \left(\frac{w_o}{Q}\right)s + w_o^2} = \frac{N(s)}{D(s)} \]  

(3.18)

where

\[ H_{hp} = \frac{C^o}{C_{ol}}, \quad w_o = \left[ B_1B_2, \frac{C_2}{C_{ol2}}, \frac{C_3}{C_{34}} \right]^{1/2} \]

and

\[ Q = \frac{C_2}{C_1} \left[ \frac{B_2}{B_1}, \frac{C_{ol2}}{C_2}, \frac{C_3}{C_{34}} \right]^{1/2} \]  

(3.19)

This filter enjoys the multifunctional capability and provides additional standard second-order BP and LP responses at the outputs of OA1 and OA2, respectively. The BP and LP gains are given by:

\[ H_{bp} = -\frac{C^o}{C_1}, \quad H_{lp} = -\frac{C^o}{C_2} \]  

(3.20)

and the other parameters of interest, such as, pole-\(w_o\) and pole-\(Q\) remain unaltered. The filter parameters have the
Fig. 3.8 (a) Prototype high-pass filter.
(b) OA–C version of the prototype HPF.
attractive feature of being in terms of C-ratios. The sensitivity figures, for various filter parameters with respect to the active and passive elements, are analysed and the final results are included in Table 3.4. Expressions in the table show that the filter enjoys attractive sensitivity properties; again all the sensitivities being less than or equal to unity in magnitude. The circuit also enjoys independent electronic tuning of pole-frequency. However, independent passive control of filter parameters is not present in this case. In order to provide dc biasing and stability, high-valued resistors are required across the capacitors $C_2$ and $C_3$.

3.2.2 Realization of higher-order ladders

Inductance-based simulators of Table 2.1 may also be employed in the realization of various ladder structures in the OA-C form, this technique will particularly be suitable for topologies, which have inductance in the shunt-arms. Two examples are first considered for the realization of OA-C ladders having the Ls in the shunt-arms.

Consider the $n$-pole band-pass filter of Fig. 3.9(a). Its realization in the OA-C form requires the replacement of the grounded LC-arms by their simulated versions. The grounded LC-simulator of Fig. 2.2 is used in the realization of active-C version of the $n$-pole BP filter, as shown in Fig. 3.9(b). The realized filter only requires $2n$ OAs and $(4n-1)$ capacitors.
Table 3.4: Sensitivity figures for OA-C filter of Fig. 3.B(b)

<table>
<thead>
<tr>
<th>Sensitivity</th>
<th>$H_{hp}$</th>
<th>$H_{bp}$</th>
<th>$H_{lp}$</th>
<th>$w_0$</th>
<th>$Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B(_1)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1/2</td>
<td>-1/2</td>
</tr>
<tr>
<td>B(_2)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1/2</td>
<td>1/2</td>
</tr>
<tr>
<td>C(_2)</td>
<td>$\frac{C_2}{C_{o12}}, (&lt;1)$</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>C(_3)</td>
<td>$\frac{C_3}{C_{o12}}, (&lt;1)$</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>C(_4)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1/2</td>
<td>-1/2</td>
</tr>
<tr>
<td>C(_5)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
</tr>
</tbody>
</table>
Fig. 3.9 (a) Prototype n-pole BP-filter  
(b) OA-C version of n-pole BP filter.
As a second example, consider the n-th order linear phase BP filter, shown in Fig. 3.10(a). Its active-C realization requires the simulation and corresponding replacement of the passive RLC shunt-arms. The grounded tuned circuit simulator of Fig. 2.3 is employed for the purpose. The final realization is shown in Fig. 3.10(b), where an n-th order filter requires 2n OAs and (5n-1) capacitors in its realization.

The inductance-based simulation technique may also be employed attractively, with certain constraints, in realizing structures, which have Ls in the floating arms, provided multiple component replacement is possible. To demonstrate this, an n-th order Cauer-Chebyshev (CC) low-pass ladder of Fig. 3.11(a) is considered. Its realization in the OA-C form requires the simulation and subsequent replacement of the floating LC-tuned circuit. The floating LC-simulator of Fig. 2.25(b) already studied in Section 2.5 is used for the job. The resulting OA-C version of the CC low-pass ladder is shown in Fig. 3.11(b). This circuit uses a low count of only 2(n-1) OAs and $\frac{1}{2}(3n-1)$ capacitors for an n-th order ladder. The basic limitation of the circuit is in its use over a restricted frequency range of operation, $\omega \ll B$, as discussed in Section 3.1.3.

† Note: In the OA-C ladder realizations, high-valued resistors may appropriately be used for providing dc bias to the OAs.
Fig. 3.10 (a) Prototype $n^{th}$ order linear phase BP filter. (b) OA-C version of linear-phase BP filter.
Fig. 3.11 (a) $n^{th}$ order Cauer Chebyshev (CC) type low-pass filter.
(b) OA–C version of CC-type LP filter.
3.3 **FDNCAP-Based Approach to OA-C synthesis**

In this section, a new approach to direct form synthesis, called the FDNCAP-Based Approach, is considered for the realization of second-order and higher-order filters in the OA-C form. In this technique, the passive RLC-prototype network describing the transfer function is first transformed into an equivalent network by scaling each admittance of the original network \(N\) by the square of the complex-variable \((s^2)\), as shown in Fig. 3.12. The resulting network \(N^*\) is thus obtained by converting a resistor to an FDNR, an inductor to a capacitor and a capacitor to a frequency dependent negative capacitance (FDNCAP) element\(^{53}\). The FDNCAP has impedance of the form, \(1/s^3F\), where \(F\) is parameter of the element having the units of \((\text{farad})^3(\text{ohm})^2(f^3.\Omega^2)\). Thus, the RLC-network \(N\) is converted to a DCF-network \(N'\) and synthesis problem becomes that of the realization of FDNRs and FDNCAPs in the OA-C form.

As will be seen in the subsequent section, the technique will prove out to be very attractive in the realization of OA-C versions of those structures which have excessive number of inductances in the series, as well as, the shunt-arms. In this method also, either ideal or non-ideal FDNCAP circuits may be used in the realization of OA-C circuits. However, as has been pointed out earlier, the use of non-ideal FDNCAP-simulators for multiple-component replacement will prove out
Fig. 3.12 RLC elements transformation to DCF-version by $s^2$-scaling on each admittances.
to be much more attractive. In Section 3.3.1, the technique is used in the realization of second-order low pass-filter and in Section 3.3.2 for the realization of higher-order ladders.

3.3.1 Realization of second-order section with non-ideal FDNCAP-simulators

In this section the realization of standard second-order low-pass filter is described using an FDNCAP-based simulator. The prototype second-order LP filter is shown in Fig. 3.13(a). Its scaling by \( s^2 \) leads to the FDNCAP-based version of the circuit, shown in Fig. 3.13(b). It may be noticed that the transformed network may completely be simulated by the FDNCAP-based simulator of Fig. 2.15 by taking the responses at the output of OA\(_2\). In the resulting circuit, capacitor \( C \) is split as a parallel-combination of \( C_1 \) and \( C_2 \), i.e., \( C = C_1 C_2 / C_12 \). The final OA-C realization is given in Fig. 3.13(c). Analysis of the circuit yields the transfer function:

\[
T_{lp}(s) = \frac{V_{ip}}{V_i} = \frac{H_{lp} w_o^2}{s^2 + \left(\frac{w_o}{Q}\right)s + w_o^2} = \frac{N(s)}{D(s)} \tag{3.21}
\]

where

\[
H_{lp} = 1, \quad w_o = \left[B_1 B_2 \cdot \frac{C_2}{C_{12}} \cdot \frac{C_4}{C_{34}} \right]^{1/2}
\]

and
Fig. 3.13 (a) Prototype RLC low-pass filter (LPF).
(b) FDNCAP—version of LPF.
(c) OA–C—version of LPF.
Once again, the filter parameters have the attractive feature of being in terms of C-ratios and the pole-frequency being independently tunable with the bias-control. However, independent passive control of filter parameters is not present. As the filter's output is available at the output of OA, additional buffer is not needed, which is generally required in the circuits obtained by the direct form approach.

The sensitivity figures for various filter parameters, with respect to active and passive elements, are analysed and the final results are included in Table 3.5. The table shows that filter enjoys attractive sensitivity properties, being less than or equal to unity in magnitude. In order to provide dc biasing to the OAs, high-valued resistors are required across $C_2$ and $C_3$.

3.3.2 Realization of higher-order ladders

Sometimes in the ladder realizations, the L-based and the FDNR-based techniques do not prove out to be attractive, particularly when the structures contain inductances in both the series and the shunt-arms. In such cases, the FDNCAP-based approach generally leads to attractive realization of the circuit in the OA-C form. In this section, we shall consider two examples for the realization of the n-th order
Table 3.5: Sensitivity figures for OA-C filter of Fig. 3.13(c)

<table>
<thead>
<tr>
<th>Sensitivity TO DF</th>
<th>$H_{1p}$</th>
<th>$w_0$</th>
<th>$Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_1$</td>
<td>0</td>
<td>1/2</td>
<td>1/2</td>
</tr>
<tr>
<td>$B_2$</td>
<td>0</td>
<td>1/2</td>
<td>-1/2</td>
</tr>
<tr>
<td>$C_1$</td>
<td>0</td>
<td>- $\frac{1}{2} \cdot \frac{C_1}{C_{12}}, (&lt;</td>
<td>1/2</td>
</tr>
<tr>
<td>$C_2$</td>
<td>0</td>
<td>$\frac{1}{2} \cdot \frac{C_1}{C_{12}}, (&lt;</td>
<td>1/2</td>
</tr>
<tr>
<td>$C_3$</td>
<td>0</td>
<td>- $\frac{1}{2} \cdot \frac{C_3}{C_{34}}, (&lt;</td>
<td>1/2</td>
</tr>
<tr>
<td>$C_4$</td>
<td>0</td>
<td>$\frac{1}{2} \cdot \frac{C_3}{C_{34}}, (&lt;</td>
<td>1/2</td>
</tr>
</tbody>
</table>
ladders, based on the FDNCAP approach. The first example is based on the use of non-ideal FDNCAP for the simulation of a set of components. Therefore, it results in a low component realization with no matching and frequency restrictions. In the second realization, an ideal grounded FDNCAP is required. However, rather than using an ideal grounded FDNCAP circuit with a large component count and matching conditions, the use of non-ideal FDNCAP as an ideal one within certain frequency constraints, is suggested. This once again results in the optimization of components and elimination of critical matching conditions.

In the first case, consider the elliptic ladder filter of Fig. 3.14(a). Its transformed FDNCAP-version is shown in Fig. 3.14(b). For the realization of the ladder in the OA-C form, two types of non-ideal grounded simulators; I and II, are required. The first type of simulators (I) are used for the replacement of the series C:F shunt-arms, except for last arm. The FDNCAP-based simulator realizing grounded C:F-function is given in Fig. 2.13. This circuit is used for the replacement of the shunt-arms in the transformed circuit. Rather than simulating an ideal FDNR for the last shunt-arm, the complete L-section consisting of C and D is simulated by the non-ideal FDNR-simulator (II) of Fig. 2.9. The final OA-C version of the ladder is shown in Fig. 2.14(c). It is seen that the realization does not require any matching or frequency constraints, and moreover, it uses a low count of
Fig. 3.14 (a) Prototype elliptic ladder filter. 
(b) FDNCAP-version of elliptic ladder filter. 
(c) OA–C version of elliptic ladder filter.
only \(n\) OAs and \(2n\) Cs.

In the second example, the low-pass LC ladder of Fig. 3.15(a) is considered. Its transformed FDNCAP-version is shown in Fig. 3.15(b). Now to realize the circuit in the OA-C form, the grounded ideal FDNCAPs are required to be replaced by the corresponding OA-C simulators. The non-ideal FDNCAP of Fig. 2.13 in the frequency range, \(w \ll B\), acts as an ideal grounded FDNCAP-simulator, with the impedance function, \(Z(s) = 1/s^3 F\) and \(F = C_1/B_1 B_2\). Therefore, for the realization of the ladder in the OA-C form, this FDNCAP-simulator is used for the replacement of all the shunt-arms, except for the last arm, which is replaced by the FDNR-simulator of Fig. 3.6(a). The OA-C version of the Ladder is shown in Fig. 3.15(c). It is seen that the OA-C ladder uses only \((n+1)\) OAs and \((2n+1)\) Cs for even-\(n\), while \(n\) OAs and \(2n\) Cs for odd-\(n\) filters. Matching constraints are not present in this realization, but frequency restriction, \(w \ll B\), is present in the circuit.

3.4 Experimental Results

In this section experimental verifications of the three OA-C direct form synthesis techniques, studied in the preceding sections, have been included. This is done by experimentally verifying, one circuit each, realized through the different direct form synthesis approaches. Since the filters realized in this chapter are based on the component simulators studied
Fig. 3.15 (a) Prototype singly terminated low-pass ladder.
(b) FDNCAP-version of the LP-ladder.
(c) OA-C version of the LP-ladder.
in Chapter 2, the experimental results also provide the verifications on some of the component simulators of the previous chapter. Lacking IC fabrication facilities in the Department, the experimental verifications were done on the discrete versions of the circuits. The reported results have been obtained without resorting to tuning.

3.4.1 Design and testing of BE-filter (FDNR-Based Approach)

The discrete version of the second-order band-elimination filter (BEF) of Fig. 3.3(c), realized through the FDNR-based technique, was experimentally verified in the laboratory. Initially, the BE filter was designed for a centre frequency ($f_0$) of 32.9 KHz and quality factor of 10.5 with OAs of 741-type and discrete Cs of 1 percent tolerance. The use of the design equation (3.6), along with the measured value of $B$ of each OA as 0.9 MHz at $V_{cc} = \pm 9$ volts, yielded the following component values: $C_0 = 65$ nF, $C_1 = 226$ pF, $C_2 = 99.5$ pF, and $C_3 = 74$ nF. The designed circuit of Fig. 3.16(a) was tested with frequency response shown in Fig. 3.16(b). The experimental results show good conformity with design.

The multifunctional capability of the filter was also verified experimentally. The previous filter, with the given specifications and design, additionally provides the band-pass characteristics at the output of $OA_1$. The frequency response of this circuit, along with the designed and observed values of $f_0$ and $Q$, are included in Fig. 3.17. The low-pass
Fig. 3.16 (a) Designed Band-Elimination Filter of Fig. 3.3 (c).
(b) Frequency response of BEF of Fig. 3.16 (a).
Fig. 3.17 (a) Designed BPF of Fig. 3.3 (c)
(b) Frequency response of Fig. 3.17 (a)
response is also available at the output terminal of OA₂.

The filter was then slightly redesigned to yield the Butterworth response at the output of OA₂ for previous pole frequency, \( f_0 = 32.9 \text{ KHz} \). The modified values of capacitors become: \( C_0 = 5.3 \text{ nF} \) and \( C_1 = 290 \text{ pF} \); the other C-values remaining unaltered. The low-pass filter is shown in Fig. 3.18(a). Its experimental frequency response, along with the theoretical and observed values of parameters, are included in Fig. 3.18(b). The agreement between theory and experiment is obvious.

The variation of pole - \( f_0 \) and pole-\( Q \) with bias voltage (\( V_{cc} \)) for the BEF of Fig. 3.16(a) was also experimentally verified. The results are shown in Fig. 3.19. It demonstrates the independent electronic tuning of pole-frequency with bias-voltage over a considerable range of about 81 per cent, with \( Q \) practically remaining unaltered.

In all the cases, the experimental values were found to be in close conformity with the theory. Note, high-valued resistors, each of 2.2 Mohm, were employed across the capacitors \( C_1 \) and \( C_2 \) to provide the dc bias and stability to the OAs.

3.4.2 Design and testing of HP-filter (L-Based Approach)

The second-order high-pass filter of Fig. 3.8(b), realized through the L-based technique, was designed and
Fig. 3.18 (a) Designed low-pass filter circuit of Fig. 3.3(c).
(b) Frequency response of Fig. 3.18(a).
Fig. 3.19 Variation of pole-frequency ($f_0$) and pole-Q with bias-voltage ($V_{cc}$) for BEF of Fig.3.16(a).
experimentally verified by employing Bi-MOSFET input OAs, CA 3140. The measured value of $B$ at a bias of $\pm 9\text{V}$ was found to be $4.2 \text{ MHz}$. The HP-filter was designed for Butterworth response at a cut-off frequency, $f_o = 420 \text{ kHz}$. Since the operational frequency was in hundred of kilohertz range, it was essential to use the predistortion technique, discussed in Chapter 4. The circuit design incorporating predistortion, along with (3.9), yielded the following capacitors values: $C_0 = 1.02 \text{nF}$, $C_1 = 0.97 \text{nF}$, $C_2 = 4.6 \text{nF}$, $C_3 = 0.99 \text{nF}$, and $C_4 = 71.2 \text{nF}$. This circuit is shown in Fig. 3.20(a). It was experimentally verified and the frequency response is given in Fig. 3.20(b). The results once again exhibit good conformity with the theory.

3.4.3 Design and testing of LP-filter (FDNCAP-Based Approach)

The discrete version of the second-order low-pass filter of Fig. 3.13(c), realized through the FDNCAP-based technique, was designed and experimentally verified employing Bi-MOSFET input OAs, CA 3140. The measured value of $B$ at a bias of $\pm 15\text{V}$ was $4.5 \text{ MHz}$. The low-pass filter was designed for Butterworth response at cut-off frequency, $f_o = 100 \text{ kHz}$. Using (3.22), the design yields the following capacitors values: $C_1 = 220 \text{nF}$, $C_2 = 112 \text{ pF}$, $C_3 = 3.2 \text{nF}$, and $C_4 = 100.8 \text{nF}$. The designed circuit is given in Fig. 3.21(a). The LPF was then experimentally verified and the frequency response is shown in Fig. 3.21(b). Once again, the results
Fig. 3.20 (a) Designed high-pass filter circuit of Fig. 3.8 (b). (b) Frequency response of Fig. 3.20 (a).
Fig. 3.21 (a) Designed LPF of Fig. 3.13 (c).
(b) Frequency response curve of LPF of Fig. 3.21 (a).
were found in close conformity with the theory.

3.5 Concluding Remarks

In this chapter, the direct form OA-C synthesis has been considered in details. Three synthesis techniques, viz., (i) L-Based Approach, (ii) FDNR-Based Approach, and (iii) FDNCAP-Based Approach, have been described in detail and critically studied for the realization of second-order and higher-order networks in the OA-C form, starting from the passive RLC-prototype. For the simulation of components, ideal and non-ideal immittance simulators, already studied in Chapter 2, were employed. In all the three approaches, two different types of component simulators were required. It has been shown that use of ideal simulators, particularly in the floating mode, leads to circuit realizations with a number of undesirable features. These are: (i) use of large count of active and passive components, (ii) requirement of critical component matching, and (iii) in some cases, a restricted working range of frequency.

Therefore, in cases where ideal component simulators are essential, it has been suggested to use the corresponding non-ideal simulators within a frequency range, \( w \ll B \), where the non-ideal circuits can be assumed to provide the behaviour of ideal simulators. The use of such non-ideal simulators, within the restricted frequency range, provides circuit realizations with low component count and without requiring
critical matching constraints.

An alternative technique of using non-ideal component simulators, without frequency restriction, proves out to be still more attractive. In this method, the non-ideal component simulators used for the replacement of multiple-elements, either of the prototype or its transformed version. Such realizations use a low count of active and passive components, and do not have matching and frequency restrictions.

The suitability of a particular OA-C direct form synthesis approach, as is the case with conventional active-RC synthesis, depends on the structure of the basic prototype network. For example, the L-based technique proves out to be attractive when structures have $C(s)$ in the floating-arm(s) and $L$ or $LC$ or $RLC$-network(s) in the shunt arm(s). The FDNR based approach, generally leads to the realizations with a larger component count. However, the use of non-ideal component simulators, within the restricted frequency range, sometimes realizes filters with low component count. Structures which require, both floating and grounded inductances, are generally suitable to realize with the direct form FDNCAP approach. This is particularly so if multiple-component simulator is employed. The resulting OA-C networks have low component count and also do not suffer from matching constraints and frequency restrictions.

The discussed techniques provided a useful method for
the realization of biquadratic, as well as, higher-order filters, suitable for monolithic implementation in the MOS technology. The parameters of all the realized filter were found to be in terms of ratioed-C. This is very attractive feature for MOS implementation. The realized second order filters were generally found to provide additional standard responses, besides the one for which it was realized. The pole-frequencies of all second-order filters were found to be tunable electronically with the bias-voltage control without affecting the $Q$ of the filters. Also in many cases, the circuit incorporates independent passive control of pole-$Q$. The sensitivity studies of the second-order filters showed them to have attractive sensitivity performance. Finally, the experimental results on the filters, realized by the three synthesis techniques, gave experimental results which were in good conformity with the theory, without resorting to tuning.