CHAPTER 6

WIDE RANGE ELECTRONICALLY TUNABLE OTA-C FILTERS†

6.0 Introduction

The previous chapters were, basically, concerned with the realization and study of filters and oscillators, which were suitable for high frequency applications and were also attractive to implement in the monolithic MOS-technology. The use of dynamic response of operational amplifiers in the design resulted in the reliable high frequency performance, and the use of only operational amplifiers and ratioed-capacitors made them attractive for microminiaturization in the MOS technology. A feature which is very important to an analog integrated circuit is its wide-range electronic tunability, since variation of passive parameters is not possible in ICs. Most of the OA-C circuits exhibit limited tunability with bias-voltage control. The tunable range being generally limited to less than a decade. Moreover, parameter control by varying the bias-voltage is not a very desirable method.

In this and the subsequent chapter, we study another class of filters and oscillators, henceforth, called as the OTA-C circuits. These circuits employ Operational Transcon-

† The material presented in this chapter has led to the publication of author’s paper Nos.10, 11 and 12, listed on page vii.
ductance Amplifier (OTA) as the active device, along with capacitors/capacitor-ratios. This makes the OTA-C networks suitable for implementation in the monolithic MOS technology. Also, the large bandwidth of a practical OTA (BW ~ 2 to 10 MHz makes these circuits suitable for reliable high frequency performance. Two additional attractive features of the device are:

(i) Wide-range linear tunability (~ 3 to 6 decades) of its transconductance gain ($g_m$) with control-current or voltage;

(ii) Convenient conversion from differential voltage controlled current source (DVCCS) to differential voltage controlled voltage source (DVCVS) by the use of a buffer-stage. The buffer may be available on IC-chip itself, e.g., LM 13600/LM 13700.

The first feature is responsible for the attractive wide-range electronic tunability of the OTA-based circuits. This makes them highly suitable for applications in automatic control, music synthesis, speech synthesis, instrumentation systems, communication systems, etc. The second feature of using OTAs as either a controlled current source or as a controlled voltage source generally makes the OTA-based realizations economical in component count. Probably no existing approach to the design of monolithic analog circuits simultaneously enjoys the desirable features discussed above.
The present and the next chapter, deal with OTA-based filters and oscillators, respectively. The organisation of this chapter is as follows. The basic building blocks for the realization of OTA-C circuits are considered in Section 6.1. These consist of programmable integrator (PI), first-order filter sections and C-multiplier. The circuit realizations, as-well-as, detailed studies are included. In Section 6.2, second-order OTA-C filters are realized and their critical performance studies are made. Three categories of circuits, based on the number of standard responses available from them, have been considered. The realization of higher-order filters is included in Section 6.3. Finally, the concluding remarks are given in Section 6.4.

6.1 Basic Building Blocks for OTA-C Circuits

In this section, those basic building blocks (BBBs), which are extensively used in the realization of OTA-C filters and oscillators, are considered. These are:

i) OTA-C programmable integrator (PI);

ii) OTA-C first-order filter sections, realizing low-pass, high-pass and all-pass characteristics; and

iii) OTA-based C-multiplier.

These circuits are now examined in details.
6.1.1 OTA-C programmable integrator

The circuit of OTA-based programmable integrator (PI) is shown in Fig. 6.1. In the realization, the OTA is loaded by a capacitor to create the desired integrating effect. In order to make the output load insensitive, a buffer stage may be employed in the realization. An OTA is a differential-pair with variable bias. The relationship between its transconductance \( g_m \) and bias-control current \( I_B \) is given by

\[
g_m = \frac{I_o}{V_i} = \frac{I_B}{2V_T}
\]

where, \( V_T \) is the equivalent thermal-voltage. The transfer function \( \frac{V_o}{V_i} \) of the PI may easily be derived as

\[
T_{PI}(s) = \frac{V_o}{V_i} = \frac{g_m}{sC} = \left[ \frac{I_B}{2C V_T} \right] \frac{s}{s} = k
\]

where, \( k = \frac{I_B}{2C V_T} \) is the integration constant. The expression shows the circuit to realize an ideal integrator with a programmable-gain \( k \), directly proportional to the bias-control current \( I_B \). This makes the circuit electronically tunable over several decades. This PI will be employed extensively as a basic building block in the realization of filters and oscillators.
Fig. 6.1: (a) Programmable Integrator and its (b) symbolic representation.
5.1.2 OTA-C first-order filter sections

As has already been pointed out in Section 4.1, the first-order low-pass, high-pass and all-pass sections play an important role in the realization of odd-order and higher-order filters. In this chapter, some of them also have been utilized in the realization of standard second-order filters. In addition, these sections are useful in instrumentation and communication applications. The first-order OTA-C sections have the added important feature of wide-range electronic tunability with $I_B/V_B$-control.

In Figs. 6.2 to 6.4, low-pass (LP), high-pass (HP) and all-pass (AP) first-order OTA-C circuits are shown. Routine analysis has been used in deriving their transfer functions, which are included in Table 6.1.

**Low-pass section**

Two first-order OTA-C sections, realizing LP-responses, are shown in Fig. 6.2. The circuit of Fig. 6.2 (a) realizes, LP-characteristics, with electronically tunable gain ($H_I$) and pole-frequency ($w_o$). However, independent tunability of $H_I$ and $w_o$ is not present in this section. The circuit of Fig. 6.2(b) employs an additional OTA. However, this increase in the active component is justified by the circuit not only having tunable-parameters, but also having independent electronic tunability of pole-frequency with $I_{B_2}$ (as, $g_{m2}$ $I_{B_2}$), and of the gain with $I_{B_1}$ (as, $g_{m1}$ $I_{B_1}$).
FIG. 6.2: First order OTA-C low-pass section with (a) inter-dependent gain and (b) independent adjustable gain.

FIG. 6.3: First-order OTA-C high-pass section.

FIG. 6.4: Two First-order OTA-C all-pass sections.
Table 6.1: Characteristics of first order OTA-C sections

<table>
<thead>
<tr>
<th>Fig. No.</th>
<th>Type</th>
<th>Transfer function</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.2(a)</td>
<td>Low-pass</td>
<td>[ T_{lp}(s) = \frac{g_m/C_1}{s+g_m/C_1} = \frac{H_1}{s+w_o} ] where, ( H_1 = w_o = g_m/C_1 ).</td>
</tr>
<tr>
<td>6.2(b)</td>
<td>Low-pass</td>
<td>[ T_{lp}(s) = \frac{g_m/C_1}{s+g_m/C_1} = \frac{H_1}{s+w_o} ] where, ( H_1 = g_{m1}/C_1, w_o = g_{m2}/C_1 ).</td>
</tr>
<tr>
<td>6.3</td>
<td>High-pass</td>
<td>[ T_{hp}(s) = \frac{s}{s+g_m/C_1} = \frac{H_h s}{s+w_o} ] where, ( H_h = 1, w_o = g_m/C_1 ).</td>
</tr>
<tr>
<td>6.4(a)</td>
<td>All-pass</td>
<td>[ T_{ap}(s) = \frac{s-g_{m2}/C_2}{s+g_m/C_1} = \frac{H_a s-w'_o}{s+w_o} ] where, ( H_a = 1, w'_o = g_m/C_2, w_o = g_m/C_1 ). With ( C_1 = C_2 = C, w_o = w'_o = g_m/C ).</td>
</tr>
<tr>
<td>6.4(b)</td>
<td>All-pass</td>
<td>[ T_{ap}(s) = \frac{s-g_{m1}/C_1}{s+g_m/C_2} = \frac{H_a s-w'<em>o}{s+w_o} ] where, ( H_a = 1, w'<em>o = g</em>{m1}/C_1, w_o = g</em>{m2}/C_1 ). With ( g_{m1} = g_m = g_m, and C_1 = C ). ( w_o = w'_o = g_m/C ).</td>
</tr>
</tbody>
</table>

† In all the cases, \( g_{m1} \propto I_{p1} \).
**High-pass section**

The circuit of Fig. 6.3 realizes high-pass characteristics with a constant gain \(H_h = 1\) and electronically tunable cut-off frequency \(w_o\). The circuit uses a low component count of only one OTA and one capacitor in its realization.

**All-pass section**

Two OTA-C circuits for the realization of AP-response are included in Fig. 6.4. The first circuit employs only one OTA and two equal-valued capacitor, whereas, the second circuit uses two OTAs of matched gains and one capacitor. These circuits provide unity-gain and the phase relationship is given by:

\[
\phi = \pi - 2 \tan^{-1}\left(\frac{wC}{g_m}\right) = \pi - 2 \tan^{-1}\left[\frac{2wCV_T}{I_B}\right].
\]

The circuits will find wide applications as electronically tunable two-quadrants phase-shifters. It may be noted that on interchanging the input-terminals of OTA\(_1\) in Fig. 6.4(b), the bass equalizer of Reference 79 is obtained, with the transfer function given by

\[
T(s) = \frac{sC + g_{m1}}{sC + g_{m2}}.
\]

**Performance study**

The important performance aspects of the first-order
sections are now considered. First, the incremental sensitivity stability and tunability aspects are examined. The circuits then designed and verified experimentally.

**Sensitivity study** - Incremental sensitivities of important circuit parameters have been evaluated with respect to active and passive components. The results of such an analysis have been included in Table 6.2.

It is evident from the table that the nominal values of the gain-sensitivity \( S_H \), the \( w_o \)-sensitivity \( S_{x}^{w_o} \) and the \( w_0 \)-sensitivity \( S_{x}^{w_0} \), with respect to the active parameters, as-well-as, to the capacitors are either equal to zero or unity in magnitude. Also, for each parameter, the sum of sensitivities with respect to active and passive components is found to be zero. The results may be summarized as:

\[
|S(H, w_o, w_o' ; x_1)| \leq 1
\]

and

\[
\sum_F S_{x_1}^F \neq 0, \quad (6.5)
\]

where, \( F = H, w_o, \) and \( w_o' \). Thus, the circuits exhibit attractive sensitivity properties.

**Stability** - The transfer functions, included in Table 6.1, clearly demonstrates the circuits to be unconditionally stable, where the poles are always confined to the left-half of the s-plane.
Table 6.2: Sensitivity figures of first-order sections of Figs. 6.4 to 6.6

<table>
<thead>
<tr>
<th>Sensitivity TO</th>
<th>Fig. 6.4(a)</th>
<th>Fig. 6.4(b)</th>
<th>Fig. 6.5</th>
<th>Fig. 6.6(a)</th>
<th>Fig. 6.6(b)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$H_1$</td>
<td>$w_0$</td>
<td>$H_1$</td>
<td>$w_0$</td>
<td>$H_a$</td>
</tr>
<tr>
<td>$C_1$</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>$C_2$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>$g_{m_1}$</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$g_{m_2}$</td>
<td>-</td>
<td>-</td>
<td>0</td>
<td>1</td>
<td>-</td>
</tr>
</tbody>
</table>


- **Tunability** - The OTA-C first-order filters have an important advantages over the corresponding OA-C sections in having wide-range electronic tunability of important circuit parameters with current or voltage control. This aspect has already been described earlier.

**Experimental results**

The low-pass, high-pass and all-pass sections were designed and experimentally verified on the discrete version of the circuits. In all the cases, CA3030E, OTAs and discrete capacitors of measured values within 1 percent tolerance were employed.

- **LP and HP sections** - The LP and HP filter sections of Fig. 6.2(a) and 6.3, respectively, were initially designed, each for a cut-off frequency, \( f_o = 3.27 \text{ KHz} \), at \( I_B = 10 \mu A \). The capacitor values were obtained from the appropriate design equations given in Table 6.1 for \( I_B = 10 \mu A \) and \( V_T = 26 \text{ mV} \) (at 28°C). In both the cases, \( C_1 \) came out to be of 9.32 nF. The circuit was then assembled and tested. The cut-off frequencies were tuned by varying the bias-current, \( I_B \). For \( I_B = 10 \mu A, 100 \mu A \) and \( 1000 \mu A \), the cut-off frequencies were respectively found to be 3.278 KHz, 32.78 KHz and 327.8 KHz, for both the LP- and HP-sections. The results of both cases, for the three sets of control current, are respectively shown in Figs. 6.5 and 6.6. It is obvious that the circuits present
FIG. 6.5: Frequency response of first-order Low-pass filter section of Fig. 6.2(a)
Fig. 6.6: Frequency response curves of first-order high-pass filter section of fig. 6.3
good agreement between theory and the experiment.

• **AP-section** - Initially, the AP-circuit of Fig. 6.4(a), was designed for a phase shift of $85^\circ$ at a frequency of $f = 1$ KHz and $I_B = 30 \mu A$. At room temperature of $28^\circ C$, $V_T = 26$ mV. Use of eqn. (6.3) with the given value of $V_T$ yielded the capacitor value, $C = 100$ nF. The control current was then varied from $1 \mu A$ to $1000 \mu A$ to cover a wide-range of phase angles in the first two quadrants, viz., from $35^\circ$ to $176^\circ$. The variations of phase-angle and with bias-current, and magnitude with frequency are shown in Fig. 6.7. Once again, a close conformity is seen between the design and the experimental results for the AP-case. Note, a high-valued resistor ($\sim 2.2$ Mohm) was connected across the capacitor ($C_2$) to provide dc-path and stability to the circuits, as in the case of some OA-C circuits.

6.1.3 **OTA-based C-multiplier**

Subsequently, it will be shown that the OTA-based filters and oscillators have the integrating capacitors in the denominator of expressions for pole-frequency and frequency of oscillations, i.e.,

$$w_p \propto \left( \frac{1}{C_{i1} \cdot C_{i2}} \right)^{1/2}$$

where $C_{i1}$s are the loading capacitors of the PIs employed in
FIG. 6.7: Curves of A P-section of fig. 6.4 (a) giving:

(i) Phase-variation with bias-current and
(ii) Frequency response.
the circuit. In MOS implementation, very low-valued capacitors are desirable in the circuit fabrication. This makes the realizations convenient and attractive for higher frequency applications. However, for low frequency applications, the requirement of high-valued capacitors will make the circuit difficult to implement in the IC form. This problem may be circumvented by using OTA-based C-multiplier, discussed in this section. The C-multiplier may also be employed in instrumentation applications, where wide-range, linear, electronic tunability or large-valued C's are required.

0 The circuit realization

The proposed circuit of C-multiplier is shown in Fig. 6.8. Its routine analysis yields the driving-point admittance as:

\[ Y(s) = s \cdot \left[ 1 + \frac{g_{m1}}{g_{m2}} \right] C \]  

(6.6a)

or

\[ Y(s) = s \cdot \left[ l + \frac{I_{B1}}{I_{B2}} \right] C = s \cdot C_e . \]  

(6.6b)

This shows that the circuit realizes an effective capacitance

\[ C_e = \left[ 1 + \frac{I_{B1}}{I_{B2}} \right] C. \]  

(6.7)

The following attractive features are evident from the above equations:

(i) the effective capacitance (\(C_e\)) is temperature-invariant,
FIG. 6.8 The OTA-based C-multiplier

FIG. 6.9 Variation of effective capacitance ($C_e$) with control-current ($I_{B_1}$)
(ii) $C_e$ can be tuned over several decades with the bias-current, $I_{B_1}$, keeping $I_{B_2}$ fixed at a small value,

(iii) the characteristics of both the OTAs will track, if dual-OTAs on a chip are used. This will provide highly stable multiplying factors, even under varying environmental conditions, and

(iv) very large $C$-values may conveniently be realized over a limited chip-area.

The electronically tunable wide-range $C$-multipliers will provide a convenient method for range extension of OTA-based filters and oscillators. In addition, it will also find applications in many instrumentation systems for extending the range of measurements at low cost.

**Experimental results**

Experimental verification of the circuit was obtained on a discrete version of the $C$-multiplier, using dual and buffered OTA, LM 13600N. Equation (b.7) gives the multiplying factor, $(1 + I_{B_1}/I_{B_2})$, which was controlled by varying the bias-current ($I_{B_1}$) of OTA$_1$ from 1 $\mu$A to 1000 $\mu$A, after keeping bias-current ($I_{B_2}$) of OTA$_2$ fixed at 1 $\mu$A. The results with a capacitor, $C = 10$ pF, at a frequency, $f = 1$ KHz, are shown in Fig.6.9. It is observed that the designed and experimental values are in close conformity over a wide range of 3-decades. Non-linearity is also exhibited at low values of multiplying factors, as dictated by the governing equation.
6.2 **Realization of Second-Order OTA-C Filters**

In the analog system design, the importance of second-order filters has already been emphasized in Chapter 3. In this section, three basic schemes, based on the analog computer method, are discussed for the realization of second-order OTA-C filters. These circuits, besides incorporating all the desirable features of OA-C second-order filters, have the added advantage of independent electronic tunability over several decades. The availability of multiple responses, along with wide-range electronic tunability, makes them suitable for the realization of complex analog systems using the modular-approach to system design. In such systems the parameters may also be programmed digitally. The circuit realizations have been categorized under three sections, based on their number of standard responses.

The first scheme of Section 6.2.1 provides two standard responses, viz., band-pass (BP) and low-pass (LP). Two circuits, Filter-A and Filter-B, have been obtained from the scheme. Non-interacting electronic control of pole-frequency and pole-Q is feasible in both the cases.

The second scheme, given in Section 6.2.2, provides three standard responses simultaneously at different nodes of the circuit. The three responses are AP, BP and LP. The same scheme with slight modification realizes band-elimination (BE) characteristic in place of the AP-response, with the
other two responses remaining unaltered. Independent control of pole-Q and pole-frequency \( (w_0) \) is possible in the later case, while in the former one, only \( w_0 \) can independently be tuned electronically.

The third scheme considered in Section 6.2.3 yields four standard second-order responses, viz., HP, LP, BP and BE. In this scheme, pole-Q and pole-frequency can independently be tuned electronically for all the four responses.

In the realization of the schemes in the OTA-C form, the basic building blocks already discussed in Section 6.1 have been used. The circuits are realized with the help of PI and first-order sections. In the case of low-frequency applications, convenient tunability can be provided with the help of C-multiplier.

The circuits are also critically studied in the respective sections for their performance based on tunability, sensitivity and stability. Experimental verifications on the discrete-version of some of the circuits are given in Section 6.2.4 with convincing results.

6.2.1 OTA-C filters with two standard responses

The scheme realizing two standard second-order responses, viz., BP and LP, is shown in Fig. 6.10. It consists of a first-order LP-section and an inverting integrator, along with,
Fig. 6.10 - The basic scheme realizing two standard second-order responses.

Fig. 6.11: OTA-C Version of Fig. 6.10: Filter-A
scaling factors, \( \alpha \) and \( \beta \). Routine analysis yields the two transfer functions:

\[
T_{BP}(s) = \frac{v_1}{v_i} = \frac{s \cdot H_{BP} \cdot \frac{w_o}{Q}}{s^2 + s \cdot \left(\frac{w_o}{Q}\right) + w_o^2}
\]  

(6.8)

of a band-pass filter at node 1, and

\[
T_{LP}(s) = \frac{v_2}{v_i} = \frac{H_{LP} \cdot w_o^2}{s^2 + s \cdot \left(\frac{w_o}{Q}\right) + w_o^2}
\]

(6.9)

of a low-pass filter at node 2, with the circuit parameter relationships given by:

\[
w_o = (a_0 \beta k)^{1/2}, \quad Q = \frac{(a_0 \beta k)^{1/2}}{b_o}
\]

\[
H_{BP} = a \frac{a_0}{b_o}, \quad H_{LP} = -\frac{a}{b_o}.
\]

(6.10)

For the implementation of the scheme in the OTA-C form, the programmable integrator of Fig. 6.1 and the first-order LP-sections of Fig. 6.2 have been employed, which yield Filter-A and Filter-B, discussed as follows.

O Filter-A

The circuit uses the PI of Fig. 6.1 in inverting-mode and first-order LP-section of Fig. 6.2(b). The \( \alpha \) and \( \beta \)-factors are implemented with C-ratios. The complete circuit is shown in Fig. 6.11. It realizes second-order BP and LP
characteristics at nodes 1 and 2, respectively, as given by (6.8) and (6.9). The final parameter values are:

\[
\begin{align*}
    w_0 &= \left( \frac{g_{m1} \cdot g_{m2}}{C_0 C_1} \cdot \frac{C_2}{C_1 + C_2} \right)^{1/2}, \quad BW = \frac{g_{m0}}{C_0}, \\
    Q &= \frac{C_0}{g_{m0}} \left( \frac{g_{m1} \cdot g_{m2}}{C_0 C_1} \cdot \frac{C_2}{C_1 + C_2} \right)^{1/2} \\
    H_{BP} &= \frac{g_{m1}}{g_{m0}} \cdot \frac{C_1}{C_1 + C_2} \quad \text{and} \quad H_{LP} = -\frac{C_1}{C_2} \quad (6.11)
\end{align*}
\]

where, \( g_{m_j} = \frac{I_{B_j}}{2V_T} \) is the transconductance gain, \( I_{B_j} \) is bias-current of the jth OTA and \( V_T \) is the thermal-equivalent voltage. If \( g_{m1} = g_{m2} = g_m \) or, \( I_{B1} = I_{B2} = I_B \), and \( C_0 = C_1 \) and \( C_1 = C_2 = C \), then (6.11) reduces to:

\[
\begin{align*}
    w_0 &= \frac{I_B}{2V_T C_1^{1/2}}, \quad BW = \frac{I_B}{2V_T C_1}, \quad Q = \frac{1}{\sqrt{2}} \cdot \frac{I_B}{I_{B0}} \\
    H_{BP} &= \frac{1}{2} \cdot \frac{I_B}{I_{B0}} \quad \text{and} \quad H_{LP} = -1. \quad (6.12)
\end{align*}
\]

A critical study of Filter-A for its performance based on tunability, stability and sensitivity aspects is given as under.

- **Tunability aspect**: It is evident from (6.12) that all the filter parameters, except for \( H_{LP} \), related either directly to the bias-current or to the ratio of bias-current, can conveniently be tuned electronically over a wide-range. The following cases are of significance:
(i) Independent electronic tunability of pole-frequency ($w_0$) and bandwidth (BW) may respectively be exercised through the control of $I_B$ and $I_{B_0}$. Thus, Filter-A realizes a widely tunable filter with variable bandwidth.

(ii) Simultaneous independent electronic control of $Q$ and $w_0$ is not possible in Filter-A. However, for a fixed-setting of $w_0$, $Q$ may be varied electronically over a wide range with $I_{B_0}$. It may also be noted that in this case, independent passive control of $w_0$ is possible with $C_1$.

(iii) It is interesting that the circuit directly realizes LP-Butterworth characteristics, if $I_B$ and $I_{B_0}$ are supplied from the same source, i.e., $I_B = I_{B_0}$. Thus, it is very convenient to realize a widely tunable Butterworth characteristics from Filter-A.

(iv) In case the integrating capacitors ($C_i$'s) are realized through C-multiplier of Section 6.1.3, the following additional advantages are obtained:

(a) Independent electronic tunability of $w_0$ and $Q$ now becomes possible.

(b) Using low and MOS-compatible values of capacitors, convenient low-frequency operation becomes possible with Filter-A.

- **Stability and sensitivity aspects** - Equations (6.8), (6.9) and (6.11) clearly exhibit Filter-A to be unconditionally stable.
The sensitivity relationships of all the important circuit parameters, viz., $w_o$, $Q$, $BW$, $H_{BP}$ and $H_{LP}$, with respect to the active and passive components have been derived and their nominal values included in Table 6.3. It is evident from the table that the $w_o$-sensitivities are less than or equal to 1/2 in magnitude. The $Q$, $BW$ and gain sensitivities are also fairly low, being lesser than unity in magnitude. Moreover, the sum of sensitivities for each parameter, with respect to active and passive components, is also found to be zero for the various parameters considered in the table. The overall results may be summarized as:

$$|S(Q, BW, H_{LP}, H_{BP}; x_i)| \leq 1$$

$$|S(w_o; x_i)| \leq 1/2$$

and

$$\sum F S_{x_i} \triangleq \sum F = 0 \quad (6.13)$$

where, $F = w_o$, $BW$, $Q$, $H_{BP}$, and $H_{LP}$.

It is concluded that the sensitivities of the circuits are, in general, low and attractive. However, it may be noted that the active parameter ($g_m$) is inversely proportional to temperature, which makes the corresponding sensitivity figures of 1/2 and unity appreciable, particularly, under varying environmental conditions. This requires stabilization of $g_m$, which is discussed in Section 7.5.
Table 6.3: Sensitivity figures for Filter-A

<table>
<thead>
<tr>
<th>Sensitivity TO OF</th>
<th>( w_0 )</th>
<th>( Q )</th>
<th>( \text{BW} )</th>
<th>( H_{BP} )</th>
<th>( H_{LP} )</th>
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<tr>
<td>( g_{m_0} )</td>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>( g_{m_1} )</td>
<td>1/2</td>
<td>1/2</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>( g_{m_2} )</td>
<td>1/2</td>
<td>1/2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( c_{0} )</td>
<td>-1/2</td>
<td>1/2</td>
<td>-1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( c_{1} )</td>
<td>-1/2</td>
<td>-1/2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( c_{2} )</td>
<td>(-\frac{1}{2}\cdot\frac{c_{1}}{c_{1}+c_{2}}\langle</td>
<td>\frac{1}{2}</td>
<td>\rangle)</td>
<td>(-\frac{1}{2}\cdot\frac{c_{1}}{c_{1}+c_{2}}\langle</td>
<td>\frac{1}{2}</td>
</tr>
<tr>
<td></td>
<td>(\frac{1}{2}\cdot\frac{c_{1}}{c_{1}+c_{2}}\langle</td>
<td>\frac{1}{2}</td>
<td>\rangle)</td>
<td>(\frac{1}{2}\cdot\frac{c_{1}}{c_{1}+c_{2}}\langle</td>
<td>\frac{1}{2}</td>
</tr>
</tbody>
</table>
0 Modified circuit: Filter-B

A slight modification of the basic scheme of Fig. 6.10 results in Filter-B, shown in Fig. 6.12. In this circuit realization, PI of Fig. 6.1 and LP-section of Fig. 6.2(a) have been used. The filter realizes the BP and LP characteristics given by (6.8) and (6.9) with the following parameters relationships:

\[ w_0 = \left( \frac{g_{m_1} \cdot g_{m_2}}{C_0 C_1} \right)^{1/2}, \quad BW = \frac{g_{m_0}}{C_0} \]

\[ Q = \frac{g_{m_1} \cdot g_{m_2}}{g_{m_0} C_0 C_1}^{1/2} \]

\[ H_{BP} = 1 \quad \text{and} \quad H_{LP} = -1. \]

(6.14)

It may be noted that in Filter-B, two capacitors have been minimized over Filter-A. In case, \( g_{m_1} = g_{m_2} = g_m \), i.e., the OTA's are biased from the same source with \( I_{B_1} = I_{B_2} = I_B \), and equi-valued integrating capacitors are used, \( C_0 = C_i \), (6.14) reduces to:

\[ w_0 = \frac{I_B}{2V T C_i}, \quad BW = \frac{I_{B_0}}{2V T C_i}, \quad Q = \frac{I_B}{I_{B_0}} \]

\[ H_{BP} = 1, \quad \text{and} \quad H_{LP} = -1. \]

(6.15)

The above equations show that though both the filters realize similar responses, convenient realization of Butterworth
Fig. 6.12: The Filter-B
response, tunable over a wide range is not present for Filter-B. Also, the gain $H_{BP}$ of the filter is fixed and equal to unity. In some applications, this feature is attractive where fixed BP-gain, irrespective of other parameter values, is required. The other tunability and stability aspects of Filter-B are similar to those of Filter-A and shall not be repeated.

- Sensitivity aspects - The sensitivity study, similar to that for Filter-A, was performed and the results are included in Table 6.4. The sensitivity performance of the circuit, as well as, the corresponding comments are once again similar to those of Filter-A. It may be noted that all the gain-sensitivities are zero in this case.

Table 6.4: Sensitivity figures for Filter-B

<table>
<thead>
<tr>
<th>Sensitivity TO</th>
<th>$g_m_0$</th>
<th>$g_m_1$</th>
<th>$g_m_2$</th>
<th>$C_0$</th>
<th>$C_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_0$</td>
<td>0</td>
<td>$1/2$</td>
<td>$1/2$</td>
<td>-$1/2$</td>
<td>-$1/2$</td>
</tr>
<tr>
<td>$Q$</td>
<td>-1</td>
<td>$1/2$</td>
<td>$1/2$</td>
<td>1/2</td>
<td>1/2</td>
</tr>
<tr>
<td>$BW$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>$H_{BP}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$H_{LP}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
6.2.2 OTA-C filter with three standard responses

The basic scheme for realizing three standard second-order responses is shown in Fig. 6.13. Its analysis yields the following standard second-order voltage transfer functions:

\[
T_{LP}(s) = \frac{v_1}{v_i} = \frac{\alpha k_1 k_2}{D(s)} \quad (6.16)
\]

where

\[
D(s) = s^2 + s(1-\alpha-\beta) k_1 + k_1 k_2(1-\beta), \quad (6.17)
\]

realizing LP-response at node 1;

\[
T_{BP}(s) = \frac{v_2}{v_i} = \frac{-\alpha k_1 s}{D(s)} \quad (6.18)
\]

realizing BP-response at node 2; and

\[
T(s) = \frac{v_3}{v_i} = \alpha \cdot \frac{s^2 - s\beta k_1 + k_1 k_2 (1-\beta)}{D(s)} \quad (6.19)
\]

at node 3. In case the factors \( \alpha \) and \( \beta \) satisfy the condition

\[
\alpha + 2\beta = 1 \quad (6.20)
\]

the circuit realizes AP-characteristic at node 3, given by

\[
T_{AP}(s) = \frac{v_3}{v_i} = \alpha \cdot \frac{s^2 - s\beta k_1 + k_1 k_2 (1-\beta)}{s^2 + s\beta k_1 + k_1 k_2 (1-\beta)} \quad (6.21)
\]

Thus, standard LP, BP and AP responses are simultaneously realizable from the circuit, having the characteristic polynomial...
**Fig. 6.13:** The scheme realizing three standard second-order responses.

**Fig. 6.14:** The Filter-C
\[ D(s) = s^2 + s\beta k_1 + k_1k_2(1-\beta). \] (6.22)

O Filter C

Direct implementation of the scheme is possible by using two PIs and passive summers, realized through C-ratios. The implementation of the scheme in the OTA-C form is shown in Fig. 6.14. Henceforth, this circuit will be referred to as Filter-C. The corresponding LP, BP, and AP functions realized at nodes 1, 2, and 3, respectively, are

\[ T_{LB}(s) = \frac{v_1}{v_1} = \frac{C_1}{C_1+C_2} \cdot \frac{k_1k_2}{s^2 + s\frac{C_3}{C_3+C_4} k_1 + k_1k_2 \frac{C_4}{C_3+C_4}}, \] (6.23)

\[ T_{BP}(s) = \frac{v_2}{v_1} = -\frac{C_1}{C_1+C_2} \cdot \frac{k_1s}{s^2 + s\frac{C_3}{C_3+C_4} k_1 + k_1k_2 \frac{C_4}{C_3+C_4}}, \] (6.24)

and

\[ T_{AP}(s) = \frac{v_3}{v_1} = \frac{C_1}{C_1+C_2} \cdot \frac{s^2+s\frac{C_3}{C_3+C_4} k_1 + k_1k_2 \frac{C_4}{C_3+C_4}}{s^2+s\frac{C_3}{C_3+C_4} k_1 + k_1k_2 \frac{C_4}{C_3+C_4}}, \] (6.25)

with the conditions:

\[ \frac{C_4}{C_3} = 2 \frac{C_1}{C_2} + 1, \text{ and } C_4 > C_3. \] (6.26)

In the circuit, only the LP and the BP responses are available at buffered outputs. In case load invariant AP-response is desired, an additional buffer may be employed at node 3. The various filter parameters, viz., pole-\( \omega_0 \), the BW, pole-Q and
gains of the filter may be expressed as:

\[ w_o = \left[ k_1 k_2 \cdot \frac{C_4}{C_3 + C_4} \right]^{1/2}, \quad BW = k_1 \cdot \frac{C_3}{C_3 + C_4} \]

\[ Q = \frac{C_4}{C_3} \cdot \left[ \frac{k_2}{k_1} \left( 1 + \frac{C_3}{C_4} \right) \right]^{1/2} \]

\[ H_{LP} = \frac{C_1}{C_1 + C_2} \cdot \left[ 1 + \frac{C_2}{C_4} \right] \]

\[ H_{BP} = - \frac{C_1}{C_1 + C_2} \cdot \left[ 1 + \frac{C_4}{C_3} \right] \quad \text{and} \quad H_{AP} = \frac{C_1}{C_1 + C_2} \quad (6.27) \]

where, \( k_j = \frac{g_m j}{C_{ij}^2 V_T} \), \( j = 1, 2 \).

Without losing generality, the circuit may be biased from the same source, i.e., \( I_{B1} = I_{B2} = I_B \), and the integrating capacitors may also be made equal, i.e., \( C_{i1} = C_{i2} = C_i \). This simplifies the parameters as:

\[ w_o = \frac{I_B}{2C_i V_T} \cdot \left( \frac{C_4}{C_3 + C_4} \right)^{1/2}, \quad BW = \frac{I_B}{2C_i V_T} \cdot \frac{C_3}{C_3 + C_4} \]

\[ Q = \frac{C_4}{C_3} \cdot \left[ 1 + \frac{C_2}{C_4} \right]^{1/2}, \quad H_{LP} = \frac{C_1}{C_{12}} \cdot \frac{C_3}{C_4} \]

\[ H_{BP} = - \frac{C_1}{C_{12}} \cdot \frac{C_3}{C_3}, \quad \text{and} \quad H_{AP} = \frac{C_1}{C_{12}} \quad (6.28) \]

where, \( C_{jk} = C_j + C_k \).
Equations (6.27) and (6.28) clearly exhibit the following attractive features of the circuit, for its convenient MOS-implementation:

(i) The circuit only employs OTAs and ratioed-capacitors/capacitors in its realization.

(ii) It has multifunctional capability and provides non-inverting LP- and AP-responses and inverting BP-response.

(iii) Most of the circuit parameters, viz., $Q$, $H_{AP}'$, and $H_{LP}'$, are only in terms of C-ratios, which can be realized in high precision and stability in MOS technology.

(iv) The circuit is electronically tunable over a wide frequency range, without affecting its $Q$ and gains.

A critical performance study of Filter-C, based on tunability, stability and sensitivity is given as follows.

- **Tunability aspect**: As has been pointed out, the circuit has independent electronic tunability of $\omega_0$ with respect to $Q$ and the gains of the filter. However, electronic tunability in the other circuit parameters is missing. This makes Filters- A and B more attractive from the consideration of multiparameter tunability. This draw-back will however be eliminated in the modified circuit (Filter-D), obtained from the same basic scheme.

- **Stability and sensitivity**: By ensuring the conditions given
in (6.26) for the realization of AP-response, the poles of the TFs are confined to the left-half of s-plane and stable operation of the circuit is ensured. The nominal values of the incremental sensitivity, after the substitution of conditions for AP-realization, are given in Table 6.5. For ensuring reasonably low sensitivities, particularly for the parameter, $H_{BP}$, with respect to $C_1$ and $C_2$, an additional design constraint is

$$\frac{C_1}{C_2} > 1.$$  \hspace{1cm} (6.29)

Use of eqns. (6.26) and (6.29), along with the sensitivity expressions of Table 6.5, give the following sensitivity results:

$$|S(Q, BW, H_{BP} ; x_1)| < 2,$$

$$|S(w_o ; x_1)| \leq 1/2$$

$$|S(H_{AP}, H_{LP} ; x_1)| \leq 1$$

and

$$\Sigma = 0.$$  \hspace{1cm} (6.30)

(Any Parameter)

It may be noted that the above results clearly demonstrate the maximum sensitivity values for Filter-C to be slightly greater than those of Filters-A and B. However, these figures are fairly reasonable and may still be insured to be much lower than the indicated maximum values through a careful design. The constraints imposed in (6.26) and (6.28), however, make the filter design only possible for $Q > 1$. 
### Table 6.5: Sensitivity figures of Filter-C

<table>
<thead>
<tr>
<th>Sensitivity TO OF</th>
<th>( w_0 )</th>
<th>( Q )</th>
<th>( BW )</th>
<th>( H_{AP} )</th>
<th>( H_{BP} )</th>
<th>( H_{LP} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g_{m1} )</td>
<td>( \frac{1}{2} )</td>
<td>( -\frac{1}{2} )</td>
<td>( 1 )</td>
<td>( 0 )</td>
<td>( 0 )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>( g_{m2} )</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{2} )</td>
<td>( 0 )</td>
<td>( 0 )</td>
<td>( 0 )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>( C_{11} )</td>
<td>( -\frac{1}{2} )</td>
<td>( \frac{1}{2} )</td>
<td>( -1 )</td>
<td>( 0 )</td>
<td>( 0 )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>( C_{12} )</td>
<td>( -\frac{1}{2} )</td>
<td>( -\frac{1}{2} )</td>
<td>( 0 )</td>
<td>( 0 )</td>
<td>( 0 )</td>
<td>( 0 )</td>
</tr>
<tr>
<td>( C_1 )</td>
<td>( 0 )</td>
<td>( \frac{2C_1}{C_{12}}, (\langle 2 \rangle) )</td>
<td>( \frac{-2C_1}{C_{12}}, (\langle 1 \rangle) )</td>
<td>( \frac{C_2}{C_{12}}, (\langle 1 \rangle) )</td>
<td>( \frac{-C_2^2}{C_{12}(2C_1-C_2)}, (\langle 1 \rangle) )</td>
<td>( \frac{C_2}{C_{12}}, (\langle 1 \rangle) )</td>
</tr>
<tr>
<td>( C_2 )</td>
<td>( 0 )</td>
<td>( \frac{-2C_1}{C_{12}}, (\langle 1 \rangle) )</td>
<td>( \frac{2C_1}{C_{12}}, (\langle 2 \rangle) )</td>
<td>( \frac{C_2}{C_{12}}, (\langle 1 \rangle) )</td>
<td>( \frac{C_2^2}{C_{12}(2C_1-C_2)}, (\langle 1 \rangle) )</td>
<td>( \frac{-C_2}{C_{12}}, (\langle 1 \rangle) )</td>
</tr>
<tr>
<td>( C_3 )</td>
<td>( \frac{-1}{2} \cdot \frac{C_3}{C_{34}}, (\langle \frac{1}{2} \rangle) )</td>
<td>( \frac{2C_3}{C_{34}}, (\langle 1 \rangle) )</td>
<td>( \frac{-C_4}{C_{34}}, (\langle 1 \rangle) )</td>
<td>( 0 )</td>
<td>( \frac{2C_3C_4}{C_{34}(C_3^2+C_4^2)}, (\langle 2 \rangle) )</td>
<td>( \frac{C_3}{C_{34}}, (\langle 1 \rangle) )</td>
</tr>
<tr>
<td>( C_4 )</td>
<td>( \frac{1}{2} \cdot \frac{C_3}{C_{34}}, (\langle \frac{1}{2} \rangle) )</td>
<td>( \frac{-2C_3}{C_{34}}, (\langle 1 \rangle) )</td>
<td>( \frac{C_4}{C_{34}}, (\langle 1 \rangle) )</td>
<td>( 0 )</td>
<td>( \frac{-2C_3C_4}{C_{34}(C_3^2+C_4^2)}, (\langle 2 \rangle) )</td>
<td>( \frac{C_3}{C_{34}}, (\langle 1 \rangle) )</td>
</tr>
</tbody>
</table>
Modified circuit: Filter-D

The basic scheme of Fig. 6.13 yields a different set of three standard responses on making the feed-back factor, $\beta = 0$. This eliminates a feedback path from node 2, and also ensures a unity feedback from node 1. The resulting system gives the following TFs:

$$T_{LP}(s) = \frac{v_1}{v_i} = \frac{\alpha k_1 k_2}{D(s)} \quad (6.31)$$

at node 1,

$$T_{BP}(s) = \frac{v_2}{v_i} = -\frac{\alpha k_1 s}{D(s)} \quad (6.32)$$

at node 2, and

$$T_{BE}(s) = \frac{v_3}{v_i} = \frac{s^2 + k_1 k_2}{D(s)} \quad (6.33)$$

at node 3, where

$$D(s) = s^2 + s(1-\alpha) k_1 + k_1 k_2. \quad (6.34)$$

It is evident from the equations that with $\beta = 0$, the scheme realizes LP, BP and BE-responses, respectively, at nodes 1, 2, and 3. The resulting OTA-C version, obtained from the modified scheme, is shown in Fig. 6.15. The corresponding second-order filter responses may be expressed as:

$$T_{LP}(s) = \frac{v_1}{v_i} = \frac{C_1}{C_1 + C_2} \cdot \frac{k_1 k_2}{D(s)}, \quad (6.35)$$

$$T_{BP}(s) = \frac{v_2}{v_i} = -\frac{C_1}{C_1 + C_2} \cdot \frac{sk_1}{D(s)}, \quad (6.36)$$
Fig. 6.15: The Filter-D
and
$$T_{BE}(s) = \frac{C_1}{C_1 + C_2}, \quad \frac{s^2 + k_1 k_2}{D(s)} \quad (6.37)$$

where
$$D(s) = s^2 + s\left(\frac{C_2}{C_1 + C_2}k_1\right) + k_1 k_2. \quad (6.38)$$

It may be noted that in the process, two capacitors, $C_3$ and $C_4$, have been reduced. The following parameter relationships are obtained for the filter:

$$w_0 = w_n = \left[k_1 k_2\right]^{\frac{1}{2}}, \quad Q = (1 + \frac{C_1}{C_2})\left(\frac{k_2}{k_1}\right)^{\frac{1}{2}}$$

$$BW = \frac{C_2}{C_1 + C_2} \cdot k_1, \quad H_{BE} = H_{LP} = \frac{C_1}{C_1 + C_2}$$

and
$$H_{BP} = -\frac{C_1}{C_1 \cdot C_2}. \quad (6.39)$$

On biasing the two OTAs from the same supply, i.e.,
$$I_{B_1} = I_{B_2} = I_B$$
and employing equi-valued integrating capacitors,
$$C_{i_1} = C_{i_2} = C_i$$
the parameter relationships reduce to:

$$w_0 = w_n = \frac{I_B}{2C_1 V_T}, \quad Q = 1 + \frac{C_1}{C_2}$$

$$BW = \frac{C_2}{C_1 + C_2} \cdot \frac{I_B}{2C_1 V_T}, \quad H_{LP} = H_{BE} = \frac{C_1}{C_1 + C_2}$$

and
$$H_{BP} = -\frac{C_1}{C_2}. \quad (6.40)$$
This circuit also enjoys the same attractive feature as those mentioned for Filter-C, leading to convenient micro-miniaturization in MOS technology. The performance features of the circuit are next considered.

- **Tunability aspects**: The tunability aspects of Filter-D are as follows,

  (i) It has independent wide-range electronic tunability of pole-$w_0$ with bias-current/voltage.

  (ii) Independent passive tunability of pole-$Q$ without disturbing $w_0 (= w_n)$ is present with $C_1$ and/or $C_2$. This however affects the gains of the filter.

- **Stability and sensitivity**: The filter realizes stable transfer functions with poles confined to the left-half of s-plane. The incremental sensitivities have also been evaluated and the results are given in Table 6.6. It may be noted that by making $\beta = 0$, the sensitivity performance of the circuit has improved considerably, which may be summarized as:

\[
\left| S(w_0, Q, H_{BE}, H_{LP}; x_j) \right| \leq \frac{1}{2},
\]

\[
\left| S(BW, H_{BP}; x_j) \right| \leq 1
\]

and

\[
\Sigma_F = 0 \quad (6.41)
\]

where, $F$ represents the parameters $w_0$, $Q$, $H_{BE}$, $H_{LP}$, $H_{BP}$, and $BW$. In fact, the circuit has $w_0$-sensitivities similar to those
Table 6.6: Sensitivity figures for Filter-D

<table>
<thead>
<tr>
<th>Sensitivity</th>
<th>( w_o )</th>
<th>Q</th>
<th>BW</th>
<th>( H_{BE} )</th>
<th>( H_{BP} )</th>
<th>( H_{LP} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g_{m1} )</td>
<td>1/2</td>
<td>-1/2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( g_{m2} )</td>
<td>1/2</td>
<td>1/2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( C_{i1} )</td>
<td>-1/2</td>
<td>1/2</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( C_{i2} )</td>
<td>-1/2</td>
<td>-1/2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( C_1 )</td>
<td>0</td>
<td>(-\frac{c_2}{c_1+c_2}),&lt;(1)</td>
<td>(-\frac{c_1}{c_1+c_2}),&lt;(1)</td>
<td>\frac{c_2}{c_1+c_2},&lt;(1)</td>
<td>1</td>
<td>\frac{c_2}{c_1+c_2},&lt;(1)</td>
</tr>
<tr>
<td>( C_2 )</td>
<td>0</td>
<td>\frac{c_2}{c_1+c_2},&lt;(1)</td>
<td>\frac{c_1}{c_1+c_2},&lt;(1)</td>
<td>\frac{c_2}{c_1+c_2},&lt;(1)</td>
<td>-1</td>
<td>\frac{c_2}{c_1+c_2},&lt;(1)</td>
</tr>
</tbody>
</table>
of Filter-C, but has improved Q and gain sensitivities.

6.2.3 OTA-C Filters with Four Standard Responses

Multifunctional capability and wide range electronic tunability constitute one of the most important requirements for the microminiaturization of filters. In the circuits considered so far, not more than three standard second-order responses were simultaneously available. In order to further increase the multifunctional capability, two methods are suggested:

(i) Realization of practically all the standard second-order responses by summing any two standard responses, obtained from a second-order filter block, along with the input signal \(^{32,36,52,71}\).

(ii) Realization of additional responses from a basic second-order filter block by the adjustment of circuit parameters through switching.

The use of summer (a CMOS-inverter in the case of MOS technology) in generating practically all the standard responses by summing-up the outputs of any two standard responses, generated from a basic filtering block, and the input has already been considered in Chapter 4 for the OA-C filters\(^{71}\). This technique is equally applicable in the case of OTA-C filters, through the use of compatible CMOS inverter. Therefore this method will not be discussed any further in this chapter.
The second technique for the realization of additional responses is based on the adjustment of suitable parameter(s) or coefficient(s), through electronic switching, of an appropriate second-order filtering block. As an example, Filters-C and D only differed in the adjustment of their coefficients in realizing AP, BP and LP responses, in the first case, and BE, BP and LP responses in the second case. This could have been achieved through MOS switches. In this section, the realization of four standard responses through switching is considered in detail by employing the scheme given in Fig. 6.16. Here the adjustment of coefficient is obtained through a single SPDT-MOS switch. Consider the voltage TFs at nodes 1, and 2, respectively, given by:

\[ T_1(s) = \frac{v_1}{i_1} = \frac{s^2 + \beta \gamma k}{s^2 + s\alpha + \beta k} \quad (6.42) \]

and

\[ T_2(s) = \frac{v_2}{i_1} = \frac{s(\gamma-1)k + \alpha \gamma k}{s^2 + \alpha s + \beta k} \quad (6.43) \]

where, \( k \) is the integrator's constant.

**Case 1** - In case \( \gamma = 0 \), the eqns. (6.42) and (6.43) reduce to

\[ T_{HP}(s) = \frac{v_1}{i_1} = \frac{s^2}{s^2 + \alpha s + \beta k} \quad (6.44) \]

and

\[ T_{BP}(s) = \frac{v_2}{i_1} = \frac{-ks}{s^2 + s\alpha + \beta k}, \quad (6.45) \]
Fig. 6.16: Basic scheme realizing four standard second-order responses.

Fig. 6.17: OTA-C Filter realized from the scheme of Fig. 6.16 (The Filter-E)
respectively, Thus, the scheme realizes standard second-order non-inverting HP-response and inverting BP-response at nodes 1, and 2 of the circuit.

Case 2 - If $\gamma = 1$, the voltage TFs at nodes 1 and 2, respectively, become

$$T_{BE}(s) = \frac{v_1}{v_i} = \frac{s^2 + \beta k}{s^2 + s\alpha + \beta k} \quad (6.46)$$

and

$$T_{LP}(s) = \frac{v_2}{v_i} = \frac{\alpha k}{s^2 + s\alpha + \beta k} \quad (6.47)$$

Now the same scheme realizes standard second-order BE and LP responses.

0 Filter-E

The practical implementation of the scheme is shown in Fig. 6.17. This simple circuit requires only three OTAs, two capacitors and single MOS switch for its implementation. It may be noted that all the circuit components are MOS integrable$^{63,82}$. The TFs realized by the filter, under the two conditions, are given as follows:

I. With switch (S) connected to A (ground), $\gamma = 0$, and the respective HP and BP responses realized at nodes 1 and 2, are:

$$T_{HP}(s) = \frac{v_1}{v_i} = \frac{s^2}{D(s)} \quad (6.48)$$

and
\[ T_{BP}(s) = \frac{v_2}{v_1} = \frac{-(g_m g_2/C_2)}{D(s)} \]  

where

\[ D(s) = s^2 + s \cdot \frac{g_m g_0}{C_1} + \frac{g_m g_2}{C_1 C_2}. \]

II. In the second case, when \( S \) is connected to \( B \) (input), \( \gamma = 1 \), and the BE and LP-responses, respectively, obtained at nodes 1 and 2, are given by:

\[ T_{BE}(s) = \frac{v_1}{v_1} = \frac{s^2 + (g_m g_1 g_m 2/C_1 C_2)}{D(s)} \]  

and

\[ T_{LP}(s) = \frac{v_2}{v_1} = \frac{g_m g_2/C_1 C_2}{D(s)}. \]

It is evident that standard buffered second-order LP, HP, BP, and BE responses are conveniently realized by the circuit through a single MOS-switch.

The important filter parameters are derived as follows:

\[ w_0 = \left[ \frac{g_m g_1 g_2}{C_1 C_2} \right]^{1/2}, \quad BW = \frac{g_m 0}{C_1}, \]

\[ Q = \frac{1}{g_m 0} \cdot \left[ \frac{C_1}{C_2} g_m 1 g_m 2 \right]^{1/2}, \quad H_{BP} = 1, \]

\[ H_{BP} = -\frac{g_m 2}{g_m 0}, \quad H_{BE} = 1, \quad \text{and} \quad H_{LP} = \frac{g_m 0}{g_m 1}. \]
If equi-valued capacitors, $C_1 = C_2 = C$, and OTAs on the same semiconductor-chip, biased from the same source, are used, eqn. (6.53) simplifies to:

$$w_o = \frac{I_B}{2VTC}, \quad BW = \frac{I_{BO}}{2VT'C}, \quad Q = \frac{I_B}{I_{BO}},$$

$$H_{HP} = 1, \quad H_{BP} = -\frac{I_B}{I_{BO}}, \quad H_{BE} = 1, \quad H_{LP} = -\frac{I_{BO}}{I_B}, \quad (6.54)$$

for $g_{m_1} = g_{m_2} = g_m$ when $I_{B1} = I_{B2} = I_B$.

**Tunability** - The tunability aspects of the filter are as follows:

(i) Independent wide-range electronic tunability of $w_o$ (or $w_n$) and BW are possible, respectively, with $I_B$ and $I_{BO}$ controls.

(ii) Electronic tunability of pole-Q at a fixed pole-$w_o$ (or $w_n$) is possible with $I_{BO}$, by keeping $I_B$ fixed at a suitable value.

(iii) Similarly, pole-$w_o$ (or $w_n$) may be tuned at a constant Q by varying $I_B$ and $I_{BO}$ simultaneously in constant proportion. This is conveniently possible by using a common, variable dc supply for all the three OTAs.

(iv) In case the biasing and switching are digitally controlled, may-be through a microprocessor-based control, multi-functional programmable filters over wide tuning ranges may be realized in the monolithic MOS form.
• **Stability and sensitivity** - The TFs of the filter inherently have poles confined to the left-half of s-plane, which makes the realization stable. The incremental sensitivity figures have also been evaluated and are available in Table 6.7. These figures indicate the circuit to have attractive sensitivity properties, similar to those of filters A, B and D. The summerized results given in eqn. (6.41) are also equally applicable here.

6.2.4 **Experimental results**

The theory developed for the second order OTA-C filters is verified for Filters-A and E. In both the cases, discrete versions of the circuits were designed and fabricated by using the general purpose dual, buffered-OTA, LM 13600 N, along with discrete capacitors of values measured within 1 percent tolerance.

0 **Results on Filter-A**

Filter-A of Fig. 6.11 realizes standard second-order BP and LP responses with the parameters relationships given in eqn. (6.12). Initially, the circuit was designed for a band-pass response with \( Q = 10 \) at a centre frequency, \( f_o = 65.5 \text{ KHz} \). At room temperature of 28°C, \( V_T = 26 \text{ mV} \). The use of expressions of \( w_o \) and \( Q \) in eqn. (6.12) with \( I_B = 100 \text{ \mu A} \) gave, \( C = 3.3 \text{ nF} \) and \( I_B = 7.07 \text{ \mu A} \). The observed frequency response of the BP filter is shown in Fig. 6.18. The experimental results are found to be in close proximity of the design,
Table 6.7: Sensitivity figures of Filter-E

<table>
<thead>
<tr>
<th>Sensitivity TO OF</th>
<th>$w_o$</th>
<th>$Q$</th>
<th>BW</th>
<th>$H_{HP}$</th>
<th>$H_{BP}$</th>
<th>$H_{BE}$</th>
<th>$H_{LP}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_{m_0}$</td>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$g_{m_1}$</td>
<td>$1/2$</td>
<td>$1/2$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>$g_{m_2}$</td>
<td>$1/2$</td>
<td>$1/2$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$C_1$</td>
<td>$-1/2$</td>
<td>$1/2$</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$C_2$</td>
<td>$-1/2$</td>
<td>$-1/2$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Fig. 6.18: The BP-frequency response of Filter—A

<table>
<thead>
<tr>
<th></th>
<th>Designed</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_0$</td>
<td>65.5 KHz</td>
<td>65.4 KHz</td>
</tr>
<tr>
<td>$Q$</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>
without even resorting to tuning.

Next, the Butterworth LP-characteristics (Q = 0.707) at a pole-frequency, \( f_0 = 3.275 \text{ KHz} \), was realized. Proceeding as earlier, eqn. (6.12) gives \( I_B = I_{B_0} = 5 \mu A \), with the other designed values remaining unaltered. The circuit was also found to provide the desired designed response. The cut-off frequency was then tuned by varying the bias-current (\( I_B \)). For \( I_B = 5 \mu A \), 50 \( \mu A \), and 500 \( \mu A \), the cut-off frequencies were respectively found to be 3.275 KHz, 32.75 KHz and 327.5 KHz. The experimental results, along with the designed values for the three sets of \( I_B \) are shown in Fig. 6.19. These present good agreement between theory and the experiment. Note, a high valued resistor (\( \sim 2.2 \text{ Mohm} \)) was required across the capacitor \( C_2 (= C) \), to provide the dc path and stability to the circuit.

0 Results on Filter-E

Filter-E was initially designed to provide Butterworth LP-and HP-characteristics (Q = 0.707) at a pole-frequency, \( f_0 = 46.37 \text{ KHz} \). At room temperature of 28°C, \( V_T = 26 \text{ mV} \). With \( I_B = 50 \mu A \), the expressions of \( w_0 \) and Q in eqn. (6.54) gave the following designed values: \( C_1 = C_2 = C = 3.3 \text{ nF} \) and \( I_{B_0} = 70.7 \mu A \). The switch S was first connected to A (ground) to obtain the HP-response at node 1. On simply switching S to B (input), the same filter provided corresponding LP-response at node 2. The observed LP and HP-responses are
Fig. 6.19: Butterworth LP—frequency response of Filter—A
shown in Fig. 6.20. These are found to give the designed and experimental values of $f_o$ in close agreement, once again without resorting to tuning. Next, the BP and BE-characteristics at, $f_o = 46.37$ KHz with $Q = 10$, were realized. Proceeding as earlier, (6.54) gave the value, $I_{B_0} = 5 \, \mu A$, with the other designed values remaining unaltered. This circuit was found to provide the desired BP-response at node 2, when $S$ was connected to $A$ (ground). By connecting $S$ to $B$ (input), the corresponding BE-response was observed at node 1. The observed frequency responses are shown in Fig. 6.21. The experimental results once again exhibit a good agreement between the theory and the experiment.

To demonstrate the tuning aspects of the circuit, the centre frequency of the BP-filter, which is directly dependent on bias-current ($I_B$), was practically tuned over a wide-range by varying $I_B$ at a constant bandwidth ($I_{B_0} = 10 \, \mu A$). In this case, the other designed values remained unchanged. The frequency response curves for different values of $f_o$ are shown in Fig. 6.22. The theoretical and practical values of $f_o$ and $Q$ at four different spot-values of $I_B$ are included in Table 6.8. These present a close conformity between the theory and the experiment.

The pole-$Q$ for the BP-response of Filter-E was also independently tuned by varying the bias-current ($I_{B_0}$) at a fixed frequency, $f_o = 185.4$ KHz ($I_B = 200 \, \mu A$). The theoretical
Fig. 6.20: The LP- and HP- responses of Filter-E
FIG. 6.21: The BP- and BE-responses of Filter-E.
Fig. 6.22 - Performance curves for BP-respose of Filter-E for variation of:
(i) Centre-frequency ($f_0$) with bias-current ($I_B$) at a constant band-width ($I_{B_0} = 10 \mu A$), and
(ii) $Q$ with centre-frequency ($f_0$).
Fig. 6.23- Variation of Q for BP-response of Filter-E with bias-current (I_{B_0}) at a constant mid-frequency (f_o).
and practical curves are shown in Fig. 6.23, which once again exhibits close agreement with the design. The obtained results adequately justify the claim of multifunctional versatility and non-interactive tuning of Filter-E over wide ranges.

Table 6.8: The results of variation of $f_o$ and $Q$ with $I_B$ for BP-response of Filter-E.

<table>
<thead>
<tr>
<th>$I_B$ (μA)</th>
<th>Theoretical</th>
<th></th>
<th>Experimental</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_o$ (KHz)</td>
<td>$Q$</td>
<td>$f_o$ (KHz)</td>
<td>$Q$</td>
</tr>
<tr>
<td>10</td>
<td>9.27</td>
<td>1.0</td>
<td>9.2</td>
<td>1.1</td>
</tr>
<tr>
<td>40</td>
<td>37.08</td>
<td>4.0</td>
<td>37.2</td>
<td>4.2</td>
</tr>
<tr>
<td>160</td>
<td>148.32</td>
<td>16.0</td>
<td>148.8</td>
<td>15.6</td>
</tr>
<tr>
<td>640</td>
<td>593.28</td>
<td>64.0</td>
<td>594.3</td>
<td>62.4</td>
</tr>
</tbody>
</table>

6.3 Realization of Analog Systems and High-order Filters

The extensive use of cascade approach in the OA-based realization of higher-order (say, nth-order) filters has already been considered in Section 4.3. It has been shown that for even $n^{\dagger}$, the overall transfer function $T(s)$ is first suitably decomposed into $n/2$ second-order functions, $T_1(s)$. From $T_1(s)$, corresponding second-order building blocks are then designed, which are suitably connected in a non-

$^{\dagger}$ for odd $n$, one first or third-order section will also be required.
interactive cascade to realize the overall nth-order system\textsuperscript{B7}. This technique is equally suitable for OTA-based realization of higher-order systems. The requirement for non-interactive cascade may be ensured by:

(i) taking the response from the output of a buffered-OTA, and
(ii) designing $T_i(s)$ for high input-impedance level.

The study of the second-order OTA-C filters has clearly demonstrated the following attractive features:

(a) The circuits have high versatility in realizing multifunctional responses, particularly, by the adjustment of suitable coefficient through a MOS-switch. Even four standard responses may be obtained from simple filtering block, as in the case of Filter-E.

(b) The circuits have convenient and wide-range electronic tunability of important circuit parameters.

(c) The circuits are ideally suited for IC implementation in the MOS technology.

These features suggest the attractive use of multifunctional OTA-C filter, as the basic building block, in the Modular Approach to analog system and higher-order filter design. A suitable OTA-C filter, such as Filter-E, may be used as the basic module, whose response, as-well-as, the parameters may be adjusted through digital/microprocessor-based control.
By incorporating an array of such building blocks, along with the switching arrangement on the same semiconductor chip, complex systems may be designed through the cascadec form approach. Also, the entire hybrid system, comprising of analog and digital circuits, may be fabricated on a single MOS-chip.

The starting point in the design of a higher-order system is the decomposition of the transfer function $T(s)$. This decomposition or pole-zero pairing of $T(s)$ into second-order terms, is not unique and its choice has significant effect on a filter's performance; the most important being its dynamic-range. For example, consider the realization of a six-pole band-pass filter, whose function $T_{BP}(s)$ is of the form:

$$T_{BP}(s) = \frac{Hs^3}{(s^2 + \frac{w_{p1}}{Q_{p1}} s + \frac{w_1}{p_{p1}})(s^2 + \frac{w_{p2}}{Q_{p2}} s + \frac{w_2}{p_{p2}})(s^2 + \frac{w_{p3}}{Q_{p3}} s + \frac{w_3}{p_{p3}})}$$

(6.55)

This function can be decomposed in three second-order sub-functions as

$$T_{BP}(s) = T_1(s).T_2(s).T_3(s).$$

(6.56)

The desired BP-response can be obtained through a cascade of second-order sections (of the type of Filter-E), either by designing all the sections as BP-sections only or by designing,
one section each, as LP, BP and HP. With Filter-E as the basic block, a careful consideration of eqn. (6.54) provides the following gain-relationships:

\[ |H_{BP}| = Q_{BP}, \quad H_{LP} = Q_{LP}^{-1}, \quad \text{and} \quad H_{HP} = 1. \]  

(6.57)

Now, in case \( Q_1, Q_2 \) and \( Q_3 \) requirements are large [eqn. (6.55)], use of only BP-sections in the cascade will give a very high overall filter gain and may result in serious distortion due to the dynamic-range limitations. Therefore, the second-form of decomposition using LP, BP and HP sections seems to be more attractive to use.

Next we consider the influence of the active device, viz., OTA, on the dynamic range of the realized filters. Basically, dynamic range depends on the ratio of maximum distortionless output voltage to the total rms noise at the output of a filter. For a good dynamic range, the active device used in the realization should have low noise voltage \( (e_N) \), and a good differential input voltage range (without producing non-linearity at the output). In general, the noise voltages of OTAs are found to be low, e.g., the CA3280 has an \( e_N \) of 8 nV/√Hz at 1 KHz. However, the differential input voltage range of traditional OTAs of 3080-type is low, being of the order of a few millivolts. Use of such OTAs will therefore restrict the dynamic range of the realized filters. This problem can now be obviated easily by using
the readily available improved OTAs, like, LM 13600, LM13700, CA 3280, with on-chip diode linearization. With such OTAs, the linear input signal level range is significantly improved to about 8 volts\(^89,90\). Thus, filters designed with such OTAs can easily achieve much improved dynamic ranges.

6.4 Concluding Remarks

The filters realized and studied in this chapter employ operational transconductance amplifier as the active device, along with, capacitors/capacitor-ratios. These circuits have been referred to as the 'OTA-C Circuits'. Three basic building blocks, viz., the programmable integrator, the first-order sections and the C-multiplier, have been realized and studied. The programmable integrator and the first-order sections were subsequently used in the actual implementation of schemes related to second-order responses. The C-multiplier has been found to be useful in conveniently extending the frequency range of operation to low frequency values. Three basic schemes have been studied for the realization of standard second-order responses. From the first scheme, Filter-A and Filter-B have been derived. Both the circuits realize two standard responses, viz., LP and BP. Filter-A has the attractive feature of convenient realization of Butterworth LP-characteristic, independently tunable over a wide frequency range. Filter-B uses only grounded capacitors and also employs two less capacitors in its realization over Filter-A. Its
general performance features are similar to those of Filter-A.

Filters C and D have been derived from the second scheme, which simultaneously provides three standard responses. With Filter-C, AP, BP and LP-responses are obtained. For the realization of AP-characteristics, matching-constraints in terms of ratioed-C are to be satisfied. This makes the sensitivity of the circuit slightly higher than those of the other filters discussed in this chapter. Filter-D yields BE, BP and LP-characteristics. Further, it does not require any matching-constraint and also minimizes the capacitor-count by two over Filter-C.

Filter-E is realized from the third scheme and provides four standard responses—HP, BP, BE and LP. The circuit uses a simple SPDT switch and at each of its setting provides two standard responses, viz., HP and BP at switch-position A (ground) and, BE and LP at switch-position B (input).

The details of the performance study illustrate the following general attractive features of OTA-C filters:

(i) All the circuits are characterized by wide-range electronic tunability.

(ii) The circuits possess the versatility of providing multifunctional responses.

(iii) The circuits have reliable high frequency performance.
Low frequency operations, within the constraint of MOS technology, are also possible through the use of C-multpliers. This makes the realization suitable for operation from low frequencies up to about 1 MHz range with the general purpose OTAs.

(iv) The use of OTAs, both as DVCCS and DVCVS, generally makes the circuit realizations simpler in terms of component-count, as-well-as, design.

(v) In general, the circuits are characterized by low sensitivity properties.

(vi) Only OTAs, capacitors/ratioed-capacitors and, in some cases, electronic switches are used in the filter implementation. All these components can conveniently be fabricated in the MOS technology\(^{58,82,91}\).

The first-order sections, C-multiplier, Filter-A, and Filter-E were also designed in the discrete form and tested in the laboratory. In each case, results were found to be in conformity with the theory.

The studies conducted in this chapter and the above mentioned advantages show the OTA-C filters, a potential class for the realization of monolithic filters. The multifunctional capabilities, along with wide-range electronic tunability also makes the circuits attractive, as a second-order basic building block, for the realization of higher-order analog filters and systems.