CHAPTER 5

ANALYSIS OF THE RESULTS
CHAPTER FIVE
ANALYSIS OF THE RESULTS

2.1. GENERAL

As mentioned in Chapter 3, the problem of percolation of fluids has been analysed based on pipe flow analogy and various models as discussed in Chapter 4 have been envisaged and studied experimentally to explain the causes of non-linearity in Darcy flow regime. The results of experimental investigations on flow through various types of models used in the present study are presented as plots between the dimensionless quantities i.e. friction factor and Reynolds number.

Figures 5.1 to 5.5 show the results of flow through smooth straight pipes of various diameters and lengths in terms of friction factor and Reynolds number plots. Figure (5.6) shows a graph between the diameter and the critical Reynolds number for straight pipes of different diameters.

Figures 5.7 to 5.16, 5.17 to 5.26, 5.27 to 5.36, 5.37 to 5.46 and 5.47 to 5.56 are the results obtained for curved pipes of different diameters used in model 1 (unidirectional flow) in the form of friction factor Reynolds number plots. Curves 5.77 to 5.81 show the results of experimental investigation for the same pipes of model 1 as R/D ratio vs critical Reynolds number curves. These curves are plotted together as a master curve in figure 5.62. The results are discussed later.

Plots 5.63 to 5.67, 5.68 to 5.72, 5.73 to 5.77, 5.78 to 5.82 and 5.83 to 5.87 show the results of the investigation of flow through curved pipes of model 2 (sinusoidal flow). Curves 5.88
to 5.92 represent the results to show the effect of curvatures on flow for the same pipes of model 2 in the terms of R/D ratio versus critical Reynolds number. The results of Figures 5.93 to 5.92 are plotted as master curve in Figure 5.93 and discussed later.

Figures 5.94 to 5.108 are friction factor-Reynolds number plots for fifteen different models of expansion and contraction type model i.e. model 3. A master curve 5.109 is obtained from these plots to show the effect of change in diameter on critical Reynolds number i.e. as \( \frac{d_1}{d_2} \) ratio versus critical Reynolds number plots.

A detailed discussion of the various results to investigate the various aspects of the problem is given below.

5.2. STRAIGHT PIPE MODELS

In order that the results of the experimental investigation can be expressed in the form of an equation or a well defined curve, inclusion of all the factors affecting the flow is essential. The analysis of flow through smooth straight pipes leads to parameters such as friction factor and Reynolds number which account for all the variables affecting the flow phenomenon through such pipes. While studying flow through straight pipes, these parameters have been used to predict whether the flow through the pipe is laminar or turbulent.

The friction factor and Reynolds number have already been defined by the equations as

\[ f = \frac{2 \rho d h f}{\nu^2} \quad (5.1) \]
and

\[ \text{Re} = \frac{\rho d V}{\mu} = \frac{V d}{\nu} \]  

(5.2)

All the symbols have their usual significance.

The problem of the determination of the law of resistance for flow through smooth straight pipes has been the subject of many exhaustive investigations. It is obvious that the law of resistance is clearly defined over an extremely wide range of those variables which influence flow through such smooth straight pipes. Expressing the coefficient of resistance in the form of friction factor and Reynolds number, it has been shown that the friction factor is not a constant but its value depends upon the relative roughness of the pipe surface and the Reynolds number of the flow. The general form of this function is given graphically by the experimental results.

In the beginning experiments were conducted with smooth straight pipes of different diameters and lengths. Since the pipes used were small in diameter and made of copper, therefore it was thought essential to check whether the existing laws for straight pipes are applicable to them or not. This was important as it will give an idea about the circularity and straightness of the tubes. The results of those experiments are plotted in the terms of friction factor and Reynolds number in Figures 5.1 to 5.5.

From the results of these plots, it can be seen that there is a definite discontinuity in the curves when the Reynolds number reaches the point \( \text{Re} \approx 2 \times 10^3 \), indicating the change from streamline to turbulent flow in all the pipe tests. Below these values
of critical Reynolds number, tests agree very closely with the law of $f = \frac{64}{\text{Re}}$. This equation represents the solution apart from its corresponding form for steady laminar flow in circular pipes. The straight lines in all the plots represent the laminar flow regime. In this regime the viscous forces predominate and the inertial forces and other effects are considered negligible.

From the plots it can be seen that above the laminar regime, plots start deviating from linear law due to the presence of inertial forces. These parts of the curves are called transition regime. This regime is generally considered between the Reynolds number 2000-4000. Fig.5.1 to 5.5 also support these observations.

The third regime for flows for which $\text{Re} > 4000$, indicates, of course, the existence of the second solution representing the turbulent flow regimes. In this regime the inertial forces are considered sufficiently large as compared to the viscous forces. The transition from laminar to turbulent flow is sudden in contrast to flow through porous materials.

The same results have been proposed by White (123), Crucitz (24), and also in references (75) and (39).

To study the effect of cross-sectional area (diameter) of pipe on flow through straight pipes, a comparative study of the results obtained for different pipes is made. The critical Reynolds numbers for different pipes are tabulated below.
### Table 5.1

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Pipe No.</th>
<th>Length of pipe in Meters</th>
<th>Radius of Curvature R is Infinity (Straight Pipe)</th>
<th>Diameter of pipe in cm</th>
<th>Critical Reynolds Number</th>
<th>Decr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>1</td>
<td>15.75</td>
<td>$R = \infty$</td>
<td>0.302</td>
<td>2260</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>2</td>
<td>14.46</td>
<td>,</td>
<td>0.45</td>
<td>2215</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>3</td>
<td>14.65</td>
<td>,</td>
<td>0.595</td>
<td>2195</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>4</td>
<td>12.00</td>
<td>,</td>
<td>0.817</td>
<td>2150</td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>5</td>
<td>12.50</td>
<td>,</td>
<td>1.033</td>
<td>2090</td>
<td></td>
</tr>
</tbody>
</table>

The results mentioned in the above table are in the form of critical Reynolds number against corresponding pipe diameters and have been obtained from the plots 5.1 to 5.5.

To see the effect of cross-sectional area on flow through straight pipes, a type curve (Fig. 5.6) is obtained from the results of Table 5.1 in the form of diameter versus critical Reynolds number plot. This plot shows a slight variation of critical Reynolds number with pipe diameter. This slight variation may be due to factors such as non-circularity of the pipe or the pipe not being perfectly straight. Besides this, the entry conditions in all the cases studied may not be the same. Therefore, no satisfactory conclusion can be drawn from these results regarding the effect of diameter on flow through straight pipes till more experiments under controlled conditions are performed. Since the variation in critical Reynolds number is not much, it can be said the existing laws of straight pipe are applicable to the tube used in this study.
5.3. MODEL I - CURVED PIPES (UNIDIRECTIONAL FLOW)

With regards to the influence of curvature, on flow through pipes comparatively not much work is available as already discussed in Chapter 3. Experimental tests were therefore conducted to study the effect of curvature upon the law of resistance up to a Reynolds number of about 10,000. The range of curvature is of particular interest from theoretical point of view, since it influences the change from streamline to turbulent flow.

The present experimental investigation has its origin from a re-examination, by the author, of some earlier experimental studies on curved pipes, which it was hoped might provide information concerning the circumstances determining the limits within which flow must be streamline in character. This investigation actually developed into a more general study to cover a wide range of flow through curved pipes of various diameters at different radii of curvatures.

The experimental part of the work consisted of tests on same five pipes used in straight pipe model, whose lengths, diameters, and curvatures are mentioned in Chapter 4. The results of these experiments obtained at various radii of curvatures for each pipe arc plotted in the form of friction factor and Reynolds number. Figures 5.7 to 5.16, 5.17 to 5.26, 5.27 to 5.36, 5.37 to 5.46 and 5.47 to 5.56 are shown for pipes numbered 1 to 5 in Table 4.1, respectively.

The general character of the results follow smooth curves. The individual test points lie with remarkable consistency on these curves. The results are also more interesting in respect
of change from linear to non-linear motion which appears to be dependent on curvature. The maximum difference between the values of critical Reynolds number is observed for individual pipes when results for maximum and minimum curvatures are compared.

For the purpose of comparison of the results for different pipes of Model 1, the following tables have been prepared with the help of experimental results of Figures 5.7 to 5.56. The Tables show the critical Reynolds numbers corresponding to their radii of curvature of each pipe used in this work. The standard figures for this model are given as Figures 4.2 to 4.11.

**TABLE 5.2**

**CURVATURE EFFECT STUDIES FOR MODEL 1 PIPE NO.1**

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Radius of Curvature of Pipe in Meters. R</th>
<th>Diameter of the Pipe in Meter. d</th>
<th>d/R</th>
<th>Critical Reynolds Numbers Recr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>25</td>
<td>0.302x10^{-2}</td>
<td>8.278x10^{3}</td>
<td>1995</td>
</tr>
<tr>
<td>2.</td>
<td>20</td>
<td>„</td>
<td>6.622x10^{3}</td>
<td>1790</td>
</tr>
<tr>
<td>3.</td>
<td>15</td>
<td>„</td>
<td>4.966x10^{3}</td>
<td>1565</td>
</tr>
<tr>
<td>4.</td>
<td>10</td>
<td>„</td>
<td>3.311x10^{3}</td>
<td>1335</td>
</tr>
<tr>
<td>5.</td>
<td>5</td>
<td>„</td>
<td>1.655x10^{3}</td>
<td>1000</td>
</tr>
<tr>
<td>6.</td>
<td>4</td>
<td>„</td>
<td>1.242x10^{3}</td>
<td>1005</td>
</tr>
<tr>
<td>7.</td>
<td>3</td>
<td>„</td>
<td>0.993x10^{3}</td>
<td>940</td>
</tr>
<tr>
<td>8.</td>
<td>2</td>
<td>„</td>
<td>0.662x10^{3}</td>
<td>870</td>
</tr>
<tr>
<td>9.</td>
<td>1</td>
<td>„</td>
<td>0.331x10^{3}</td>
<td>795</td>
</tr>
<tr>
<td>10.</td>
<td>0.5</td>
<td>„</td>
<td>0.165x10^{3}</td>
<td>720</td>
</tr>
</tbody>
</table>
### TABLE 5.3
**CURVATURE EFFECT STUDIES FOR MODEL 1 PIPE NO. 2**

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Radius of Curvature of Pipe in Meters, R</th>
<th>Diameter of Pipe in Meters, d</th>
<th>R/d</th>
<th>Critical Reynolds Number Recr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>25</td>
<td>0.45x10^-2</td>
<td>5.555x10^3</td>
<td>1950</td>
</tr>
<tr>
<td>2.</td>
<td>20</td>
<td>,</td>
<td>4.444x10^3</td>
<td>1750</td>
</tr>
<tr>
<td>3.</td>
<td>15</td>
<td>,</td>
<td>3.333x10^3</td>
<td>1522</td>
</tr>
<tr>
<td>4.</td>
<td>10</td>
<td>,</td>
<td>2.222x10^3</td>
<td>1280</td>
</tr>
<tr>
<td>5.</td>
<td>5</td>
<td>,</td>
<td>1.111x10^3</td>
<td>1025</td>
</tr>
<tr>
<td>6.</td>
<td>4</td>
<td>,</td>
<td>0.888x10^3</td>
<td>966</td>
</tr>
<tr>
<td>7.</td>
<td>3</td>
<td>,</td>
<td>0.666x10^3</td>
<td>905</td>
</tr>
<tr>
<td>8.</td>
<td>2</td>
<td>,</td>
<td>0.444x10^3</td>
<td>840</td>
</tr>
<tr>
<td>9.</td>
<td>1</td>
<td>,</td>
<td>0.222x10^3</td>
<td>770</td>
</tr>
<tr>
<td>10.</td>
<td>0.5</td>
<td>,</td>
<td>0.111x10^3</td>
<td>690</td>
</tr>
</tbody>
</table>

### TABLE 5.4
**CURVATURE EFFECT STUDIES FOR MODEL 1 PIPE NO. 3**

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Radius of Curvature of Pipe in Meters, R</th>
<th>Diameter of Pipe in Meters, d</th>
<th>R/d</th>
<th>Critical Reynolds Number Recr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>25</td>
<td>0.595x10^-2</td>
<td>4.201x10^3</td>
<td>1905</td>
</tr>
<tr>
<td>2.</td>
<td>20</td>
<td>,</td>
<td>3.561x10^3</td>
<td>1702</td>
</tr>
<tr>
<td>3.</td>
<td>15</td>
<td>,</td>
<td>2.521x10^3</td>
<td>1480</td>
</tr>
<tr>
<td>4.</td>
<td>10</td>
<td>,</td>
<td>1.630x10^3</td>
<td>1250</td>
</tr>
<tr>
<td>5.</td>
<td>5</td>
<td>,</td>
<td>0.840x10^3</td>
<td>1000</td>
</tr>
<tr>
<td>6.</td>
<td>4</td>
<td>,</td>
<td>0.672x10^3</td>
<td>945</td>
</tr>
<tr>
<td>7.</td>
<td>3</td>
<td>,</td>
<td>0.504x10^3</td>
<td>885</td>
</tr>
<tr>
<td>8.</td>
<td>2</td>
<td>,</td>
<td>0.356x10^3</td>
<td>820</td>
</tr>
<tr>
<td>9.</td>
<td>1</td>
<td>,</td>
<td>0.161x10^3</td>
<td>750</td>
</tr>
<tr>
<td>10.</td>
<td>0.5</td>
<td>,</td>
<td>0.034x10^3</td>
<td>670</td>
</tr>
</tbody>
</table>
### Table 5.5

**Curvature Effect Studies for Model 1 Pipe No. 4**

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Radius of Curvature of Pipe in Meters, R</th>
<th>Diameter of Pipe in Meter, d</th>
<th>R/d</th>
<th>Critical Reynolds Numbers Recr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>25</td>
<td>0.517x10^{-2}</td>
<td>3.059x10^3</td>
<td>1850</td>
</tr>
<tr>
<td>2.</td>
<td>20</td>
<td>,,</td>
<td>2.447x10^3</td>
<td>1658</td>
</tr>
<tr>
<td>3.</td>
<td>15</td>
<td>,,</td>
<td>1.839x10^3</td>
<td>1450</td>
</tr>
<tr>
<td>4.</td>
<td>10</td>
<td>,,</td>
<td>1.225x10^3</td>
<td>1220</td>
</tr>
<tr>
<td>5.</td>
<td>5</td>
<td>,,</td>
<td>0.611x10^3</td>
<td>932</td>
</tr>
<tr>
<td>6.</td>
<td>4</td>
<td>,,</td>
<td>0.489x10^3</td>
<td>930</td>
</tr>
<tr>
<td>7.</td>
<td>3</td>
<td>,,</td>
<td>0.367x10^3</td>
<td>970</td>
</tr>
<tr>
<td>8.</td>
<td>2</td>
<td>,,</td>
<td>0.244x10^3</td>
<td>905</td>
</tr>
<tr>
<td>9.</td>
<td>1</td>
<td>,,</td>
<td>0.122x10^3</td>
<td>735</td>
</tr>
<tr>
<td>10.</td>
<td>0.5</td>
<td>,,</td>
<td>0.061x10^3</td>
<td>640</td>
</tr>
</tbody>
</table>

### Table 5.6

**Curvature Effect Studies for Model 1 Pipe No. 5**

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Radius of Curvature of Pipe in Meters, R</th>
<th>Diameter of Pipe in Meter, d</th>
<th>R/d</th>
<th>Critical Reynolds Numbers Recr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>25</td>
<td>1.033x10^{-2}</td>
<td>2.303x10^3</td>
<td>1792</td>
</tr>
<tr>
<td>2.</td>
<td>20</td>
<td>,,</td>
<td>1.846x10^3</td>
<td>1606</td>
</tr>
<tr>
<td>3.</td>
<td>15</td>
<td>,,</td>
<td>1.535x10^3</td>
<td>1405</td>
</tr>
<tr>
<td>4.</td>
<td>10</td>
<td>,,</td>
<td>0.923x10^3</td>
<td>1190</td>
</tr>
<tr>
<td>5.</td>
<td>5</td>
<td>,,</td>
<td>0.461x10^3</td>
<td>943</td>
</tr>
<tr>
<td>6.</td>
<td>4</td>
<td>,,</td>
<td>0.369x10^3</td>
<td>398</td>
</tr>
<tr>
<td>7.</td>
<td>3</td>
<td>,,</td>
<td>0.277x10^3</td>
<td>345</td>
</tr>
<tr>
<td>8.</td>
<td>2</td>
<td>,,</td>
<td>0.134x10^3</td>
<td>780</td>
</tr>
<tr>
<td>9.</td>
<td>1</td>
<td>,,</td>
<td>0.092x10^3</td>
<td>705</td>
</tr>
<tr>
<td>10.</td>
<td>0.5</td>
<td>,,</td>
<td>0.046x10^3</td>
<td>603</td>
</tr>
</tbody>
</table>
Before the analysis of experimental results, the author was of the opinion that there will be a sudden change in the regime of flow in curved pipes as well, as happens in case of flow through straight pipes. But the experimental results when plotted in the form of friction factor-Reynolds number curves show a deviation from linear law before the actual turbulence starts. This may be due to the inertial effects in the flow due to the curvature of flow paths. This deviation from linear law starts earlier in pipes with greater curvature, i.e. the critical Reynolds number decreases with an increase in the radius of curvatures. This finding is true for all the pipes. A comparative study of the results for straight pipes as well as the results of Model 1, i.e. curved pipes is given below.

All the pipes of model 1 have already been investigated experimentally for flow under straight conditions and the results listed in Table (5.1) in the form of critical Reynolds number.

From the values of critical Reynolds numbers of Table (5.1), it can be seen that the plots 5.1 to 5.5 between ‘Re’ and ‘f’ start deviation from linear law at Re > 2 x 10^3, whereas Figures 5.7, 5.17, 5.27, 5.37 and 5.47 obtained for a radius of curvature of 25 m (i.e. minimum curvature) for each pipe of model 1 start deviating from linear law at Re < 2 x 10^3. It can be observed that when the critical Reynolds number of a straight pipe is compared with that obtained at a radius of curvature of 25 m for a pipe of same diameter, the difference between these values is not much. This difference becomes noticeable when the results of straight pipes are compared with the results for similar pipes of a radius of curvature of 0.5 m (maximum-curvature). These
values of critical Reynolds numbers for maximum curvature ranged from 600 to 720 for various pipes as it is obvious from tables (5.2) to (5.6).

From the above results it can be concluded that with an increase in curvature, the critical Reynolds number as defined in this work decreases. The results of White (123), Taylor (107), Cruetz (24), Shaukat and Seshadri (99) also lead to a similar conclusion for the effects of curvature.

Now if we compare the results of model 1 for pipe No.1 listed in Table (5.2) with the values obtained at the same curvatures for other pipes i.e. 2,3,4 and 5 shown in the Tables (5.3), (5.4), (5.5) and (5.6) respectively, it can be seen that the values of critical Reynolds number at particular curvature increase with a decrease in pipe diameter. This finding is exactly similar to that observed for straight pipes and may be due to the same reasons as discussed in case of straight pipe models discussed under (5.2) besides some experimental errors.

The critical Reynolds number values obtained for different pipes at various curvatures as listed in the Tables (5.2) to (5.6), have been plotted as curves between the dimensionless parameters R/d and critical Reynolds numbers i.e. Figures 5.57 to 5.61. They have been prepared for various pipes. These curves give a relationship between R/d and Re_{cr}. (critical Reynolds number) which obviously, is not a linear relationship. In an attempt to eliminate the effect of size of pipes, all the results of Figures 5.57 to 5.61 have been plotted as a master curve in Figure 5.62 but without any success. The R/d versus Re_{cr} curve plot shows a separate curve for each pipe, though the results
appear to converge into a single curve for smaller values of $A/d$.

The investigation was not extended to study the flow behaviour in the turbulent regime, as the main aim of the study was to investigate the various causes of non-linearity in the flow and the relative importance of each parameter on $Re_c$.

5.4. MODEL 2 - CURVED PIPES (SINUSOIDAL FLOW)

As already mentioned earlier that, the same five pipes used for straight type model and for Model 1, were also studied to investigate the effect of curvature with sinusoidal shape on the flow behaviour. Standard Figure (4.12) represents the shape of Model 2 for all the pipes. The models on this category were made with the radii of curvatures of 4 m, 3 m, 2 m, 1 m and 0.5 m for all the pipes studied. For a radius of curvature $> 4$ m, it was not possible to get an appreciable reversal in the direction of flow due to short lengths of the pipes.

The results of the experimental investigation of flow on these models for all the five pipes are reported in Figures 5.63 to 5.67, 5.68 to 5.72, 5.73 to 5.77, 5.78 to 5.82, 5.83 to 5.87 respectively for 0.302, 0.40, 0.59, 0.817 and 1.035 cm diameter pipes in the form of friction factor-Reynolds number plots.

The experimental results of these models cover a range of Reynolds numbers from 2 to 10,000. The author's main concern was to study the effect of curvature as well as change in the direction of flow on critical Reynolds number, therefore the critical Reynolds numbers for various pipes at different radii of curvature for Model 2 are given in the following table. These values of critical Reynolds numbers have been taken from
their corresponding 'I' versus 'Re' curves and represent the point of deviation from straight line law.

**TABLE 5.7**

**CURVATURE EFFECT STUDIES FOR MODEL 2-PIPE NO. 1**

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Radius of Curvature of Pipe in Meters, R</th>
<th>Diameter of Pipe in Meter, d</th>
<th>( \frac{d}{d} )</th>
<th>Critical Reynolds Numbers, ( \text{Re}_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>4</td>
<td>0.302x10^{-2}</td>
<td>1.324x10^5</td>
<td>962</td>
</tr>
<tr>
<td>2.</td>
<td>3</td>
<td>,</td>
<td>0.993x10^3</td>
<td>330</td>
</tr>
<tr>
<td>3.</td>
<td>2</td>
<td>,</td>
<td>0.662x10^3</td>
<td>791</td>
</tr>
<tr>
<td>4.</td>
<td>1</td>
<td>,</td>
<td>0.331x10^3</td>
<td>695</td>
</tr>
<tr>
<td>5.</td>
<td>0.5</td>
<td>,</td>
<td>0.163x10^3</td>
<td>590</td>
</tr>
</tbody>
</table>

**TABLE 5.8**

**CURVATURE EFFECT STUDIES FOR MODEL 2-PIPE NO. 2**

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Radius of Curvature of Pipe in Meters, R</th>
<th>Diameter of Pipe in Meter, d</th>
<th>( \frac{d}{d} )</th>
<th>Critical Reynolds Numbers, ( \text{Re}_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>4</td>
<td>0.45x10^{-2}</td>
<td>0.424x10^3</td>
<td>925</td>
</tr>
<tr>
<td>2.</td>
<td>3</td>
<td>,</td>
<td>0.665x10^3</td>
<td>550</td>
</tr>
<tr>
<td>3.</td>
<td>2</td>
<td>,</td>
<td>0.444x10^3</td>
<td>760</td>
</tr>
<tr>
<td>4.</td>
<td>1</td>
<td>,</td>
<td>0.222x10^3</td>
<td>660</td>
</tr>
<tr>
<td>5.</td>
<td>0.5</td>
<td>,</td>
<td>0.111x10^3</td>
<td>232</td>
</tr>
</tbody>
</table>

**TABLE 5.9**

**CURVATURE EFFECT STUDIES FOR MODEL 2-PIPE NO. 3**

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Radius of Curvature of Pipe in Meters, R</th>
<th>Diameter of Pipe in Meter, d</th>
<th>( \frac{d}{d} )</th>
<th>Critical Reynolds Numbers, ( \text{Re}_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>4</td>
<td>0.595x10^{-2}</td>
<td>0.672x10^3</td>
<td>910</td>
</tr>
<tr>
<td>2.</td>
<td>3</td>
<td>,</td>
<td>0.504x10^3</td>
<td>325</td>
</tr>
<tr>
<td>3.</td>
<td>2</td>
<td>,</td>
<td>0.353x10^3</td>
<td>730</td>
</tr>
<tr>
<td>4.</td>
<td>1</td>
<td>,</td>
<td>0.153x10^3</td>
<td>625</td>
</tr>
<tr>
<td>5.</td>
<td>0.5</td>
<td>,</td>
<td>0.084x10^3</td>
<td>505</td>
</tr>
</tbody>
</table>
The critical Reynolds numbers for various pipes at different radii of curvatures have been obtained from the plots 5.65 to 5.87 respectively. The highest value of critical Reynolds number in these results is 902 for \( d = 0.502 \times 10^{-2} \) m and \( R = 4 \) m and the lowest value is 419 for \( d = 1.035 \times 10^{-2} \) m and \( R = 0.5 \) m. In order to study the effect of curvature, we consider one set of results of pipe No.1 Table (5.7). It can be seen that the values of \( \text{Re}_\text{cr} \) are gradually decreasing with increase in the radii of curvature.
curvatures. In the Table (5.7), the maximum value of $\text{Re}_{cr}$ is obtained at a radius of curvature of 4 m (min curvature) i.e. $\text{Re}_{cr} = 962$ and the minimum $\text{Re}_{cr}$ is at $R = 0.5$ m (max. curvature) i.e. $\text{Re}_{cr} = 590$. Thus difference between these two values of $\text{Re}_{cr}$ obviously indicates the effect of curvature on critical Reynolds number since other factors are same in all the cases. The same nature of curvature effect is found in the experimental results for other four pipes listed in Table (5.8) to (5.11) respectively.

In order to study the effect of change in the direction of flow, the results of model 1 and 2 can be compared for a particular pipe at the same radius of curvature. For example if the smallest diameter pipe is chosen for a comparison of results then with a radius of curvature of 4 m, $\text{Re}_{cr}$ for model 1 is 1005 and the corresponding value for model 2 is 962, showing an early onset of inertial effects in sinusoidal flow as compared to unidirectional flow with same curvature. This is true for all the cases as can be seen from a comparison of Tables (5.2) to (5.6) with Tables (5.7) to (5.11). A comparison for all the pipes at a radius of curvature of 2 m is given below in the Table (5.12).
The above Table clearly indicates the effect of change in the direction of flow i.e. other factors remaining the same, a flow with changes in its direction experiences an early onset of inertial effects when compared to a unidirectional flow.

From a study of these tables, it is obvious that the values of critical Reynolds numbers are decreasing with an increase in the curvature of pipes which is in agreement with the results obtained in Model 1. In other words, we can say that the results on \( f \) versus \( Re \) plots for all the cases studied start deviating from straight lines earlier, indicating an early change from streamline to turbulent flow, when compared to results for straight pipes. Besides this, as can be seen from the results given in Tables (5.7) to (5.11), the critical Reynolds number is continuously dropping with an increase in radius of curvature.

Now on the basis of the values of critical Reynolds number listed in the above tables, the curves between dimensionless
parameter H/d versus Recr, i.e., Figures 5.83 to 5.92 are prepared for all the pipes. These plots do not follow a linear law, and appear to be similar in nature to those plotted for Model 1. From these figures it is obvious that Recr increases with an increase in H/d ratio. In an attempt to eliminate the effect of size of tubes, a master curve Figure (5.93) has been prepared. The results do not plot as a single curve and separate curves are obtained for all the pipes. The curves have a tendency to converge at lower values of H/d.

5.5. Model 3. Expansion and Contraction Type Model

A discussion regarding the geometrical parameters of various cases studied under this model has already been given in Chapter 4 and the models studied experimentally are shown diagramatically in Figures from (4.13) to (4.27). In order to study the effect of expansion and contraction on flow resistance experimentally, tests were conducted on various sizes of this model. The results of the experimental investigation are plotted as friction factor-Reynolds number graphs. The values of friction factor and Reynolds number are calculated by the equations as mentioned earlier. Since in the expansion-contraction type model, two pieces with diameters, i.e., d₁ and d₂ are involved, either d₁ or d₂ can be taken as characteristic length parameter for calculating the friction factor and Reynolds number values. The critical Reynolds numbers based on d₁ and d₂ are correlated with each other by the equation:

\[ \text{Re}_{cr1} = \left( \frac{d_1}{d_2} \right) \times \text{Re}_{cr2} \]  

(5.3)
OR

\[ \text{Re}_{c2} = \left( \frac{d_2}{d_1} \times \text{Re}_{c1} \right) \quad (5.4) \]

where, \( \text{Re}_{c1} \) = Critical Reynolds number based on larger diameter pipe \( (d_1) \).
\( \text{Re}_{c2} \) = Critical Reynolds number based on smaller diameter pipe \( (d_2) \).

\( d_1 \) and \( d_2 \) = Diameters of bigger and smaller pipes respectively.

In all, five sets of expansions and contractions as listed in Table (5.13) were tested. For all the models two sets of friction factor and Reynolds number values were determined, one with respect to diameter \( 'd_1' \) and the other with respect to diameter \( 'd_2' \) of the pipes involved. The results of the experiments on these models are shown in Figures 5.94 to 5.103 in terms of friction factors and Reynolds number plots.

These curves are similar in nature to those prepared for other cases investigated in this work, i.e. straight pipes, Model 1 and Model 2, except for the range of validity of linear law. From a study of Figures 5.94 to 5.103, it can be seen that for a given value of Reynolds number the frictional resistance increases with an increase in the diameter ratio \( d_1/d_2 \).

As the author's main interest is to investigate the effect of expansion-contraction as well as other parameters already covered by Models 1 and 2, on critical Reynolds number, the critical Reynolds numbers for various cases studied in this model along with the diameters of the pipes used in a particular case are given in the following Table. Critical values have been
obtained from their corresponding 'f' versus 'Re' curves, i.e. from Figures, 5.94 to 5.109.

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Set No.</th>
<th>Diameter of pipe in cm. (d_1)</th>
<th>Diameter of Pipe in cm. (d_2)</th>
<th>(\frac{d_1}{d_2})</th>
<th>Critical Reynolds Numbers (Re_{cr1})</th>
<th>Critical Reynolds Numbers (Re_{cr2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>1</td>
<td>1.021</td>
<td>0.317</td>
<td>1.750</td>
<td>1300.4</td>
<td>2250</td>
</tr>
<tr>
<td>2.</td>
<td>2</td>
<td>0.595</td>
<td>0.595</td>
<td>1.716</td>
<td>838.7</td>
<td>1525</td>
</tr>
<tr>
<td>3.</td>
<td>3</td>
<td>0.45</td>
<td>0.317</td>
<td>2.269</td>
<td>387.3</td>
<td>630</td>
</tr>
<tr>
<td>4.</td>
<td>4</td>
<td>0.302</td>
<td>0.302</td>
<td>3.380</td>
<td>133.4</td>
<td>620</td>
</tr>
<tr>
<td>5.</td>
<td>5</td>
<td>0.201</td>
<td>0.201</td>
<td>5.080</td>
<td>56.2</td>
<td>433</td>
</tr>
<tr>
<td>6.</td>
<td>6</td>
<td>0.817</td>
<td>0.595</td>
<td>1.373</td>
<td>1500.2</td>
<td>2060</td>
</tr>
<tr>
<td>7.</td>
<td>7</td>
<td>0.45</td>
<td>0.45</td>
<td>1.816</td>
<td>729.8</td>
<td>1325</td>
</tr>
<tr>
<td>8.</td>
<td>8</td>
<td>0.302</td>
<td>0.302</td>
<td>2.705</td>
<td>299.4</td>
<td>310</td>
</tr>
<tr>
<td>9.</td>
<td>9</td>
<td>0.201</td>
<td>0.201</td>
<td>4.065</td>
<td>130.4</td>
<td>530</td>
</tr>
<tr>
<td>10.</td>
<td>10</td>
<td>0.595</td>
<td>0.45</td>
<td>1.322</td>
<td>1474.8</td>
<td>1950</td>
</tr>
<tr>
<td>11.</td>
<td>11</td>
<td>0.302</td>
<td>0.302</td>
<td>1.970</td>
<td>571.0</td>
<td>1175</td>
</tr>
<tr>
<td>12.</td>
<td>12</td>
<td>0.201</td>
<td>0.201</td>
<td>2.960</td>
<td>231.7</td>
<td>74</td>
</tr>
<tr>
<td>13.</td>
<td>13</td>
<td>0.45</td>
<td>0.302</td>
<td>1.490</td>
<td>1248.3</td>
<td>1860</td>
</tr>
<tr>
<td>14.</td>
<td>14</td>
<td>0.201</td>
<td>0.201</td>
<td>2.759</td>
<td>244.4</td>
<td>963</td>
</tr>
<tr>
<td>15.</td>
<td>15</td>
<td>0.302</td>
<td>0.201</td>
<td>1.525</td>
<td>1199.3</td>
<td>1302</td>
</tr>
</tbody>
</table>

It is worth mentioning here that the various models in this case were prepared by keeping the diameter of one of the pipes
constant and varying the diameter of the other pipes. It can be seen clearly from Table (5.13) and also from the Figures 4.15 to 4.27.

From the results of the plots, it is obvious that the range of Reynolds number obtained in this study was from $Re < 20$ to $R > 3 \times 10^4$. The maximum critical Reynolds number value is at $Re_{cr} = 2250$ obtained from $d_1/d_2 = 1.250$ and the minimum $Re_{cr} = 453$ at $d_1/d_2 = 5.030$ as can be seen from the Table (5.13).

From a study of the above Table from the point of view of the effect of expansion ratio $d_1/d_2$ on critical Reynolds number, it appears that the critical Reynolds number values are decreasing gradually with an increase in the $d_1/d_2$ ratio e.g., taking the results of first set of models in the Table in which $d_1$ is constant and $d_2$ is gradually varied. The experimental results of the Table show that a difference exists between the values of $Re_{cr_1}$ and $Re_{cr_2}$ corresponding to diameters $d_1$ and $d_2$. For example, at the minimum $d_1/d_2$ ratio (1.25) $Re_{cr_1}$ has a value of 1300.4 and $Re_{cr_2} = 2250$, but at the maximum $d_1/d_2$ ratio (5.030) the values obtained are $Re_{cr_1} = 36.2$ and $Re_{cr_2} = 453.0$ listed in the Table.

This difference between the values of $Re_{cr_1}$ and $Re_{cr_2}$ for an individual model test is just due to the definition of $Re_{cr_1}$ and $Re_{cr_2}$ and, in the author's opinion, it is not a parameter which can be correlated with the nature of the flow in any way.

Further from the results of the Table, it is obvious that the maximum values of critical Reynolds numbers are obtained for Set 1 at minimum $d_1/d_2$ ratio and the minimum values of $Re_{cr}$ are for the same set at maximum $d_1/d_2$ ratio. Throughout the experimental results as shown in the Table, it can be seen that $Re_{cr}$
decreases with an increase in the expansion ratio \( d_1/d_2 \). A comparison of the results of this model with those of Models 1 and 2, indicate that expansions and contractions in the flow path are more effective in causing non-linearity in the flow than the curvature - unidirectional or sinusoidal. In this model the inertial effects become dominant much earlier than in the previous models.

In order to establish a relationship between the expansion ratio \( d_1/d_2 \) and the critical Reynolds number, the results listed in Table (5.13) are plotted in Figure 5.109. The two critical Reynolds numbers \( \text{Rec}_{r1} \) and \( \text{Rec}_{r2} \) are plotted against their corresponding \( d_1/d_2 \) ratio values. Two separate curves are obtained for \( \text{Rec}_{r1} \) and \( \text{Rec}_{r2} \). From these two curves, it is clear that the divergence between the points of these curves is increasing gradually as \( d_1/d_2 \) ratio increases and the maximum separation between these curves is obtained at maximum \( d_1/d_2 \) ratio (5.030). The two curves coincide with each other when \( d_1/d_2 \) approaches unity, at which the two critical Reynolds numbers become equal. From the above plot it can be seen that at \( d_1/d_2 = 1 \), the model represents a straight tube and the corresponding value of \( \text{Rec}_{r} \) obtained from Figure (5.109) is approximately 2500, which agrees very well with the already established results.

Therefore, on the basis of these experimental results we can draw the following conclusions, that is the expansions and contractions in the flow affect the critical Reynolds number more than does, the curvature of flow paths. Secondly the critical Reynolds numbers decrease with an increase in the expansion ratio \( d_1/d_2 \).
5.6. ACCURACY OF THE EXPERIMENTAL RESULTS

Though it was aimed to conduct all the experiments with maximum accuracy but the results of the experimental study may be in error due to the following sources:

1. Those involved in making a model, and
2. Those involved in recording the pressure, discharge etc.

5.6.1. ERROR INVOLVED IN MAKING A MODEL

Each pipe was tested at various curvatures especially in Model 1 and Model 2. It would appear that because of the change in the model curvature, a number of kinks might be produced due to the softness of the material. The cross-sectional area of the pipe might have differed slightly each time due to the bending of pipe, resulting in an error in the results. It is possible, that if the tests were conducted with proper care against these factors, results might improve.

5.6.2. OBSERVATIONAL ERRORS

The above mentioned error will, in fact, vary from observation to observation. At high rates of flow the error will be less compared to low rates of flow. This infact is due to the following reasons.

1. For high rates of flow, the mercury-water differential manometer was used. The accuracy of the manometer reading in the case was from \( \pm \frac{0.1}{100} \) to \( \pm \frac{0.1}{10} \), since the range of head of 100 cms to 10 cms was read up to an accuracy of \( \pm 0.1 \) cm.

2. For recording medium pressure differences, an air-water manometer was used. The minimum accuracy of the manometer reading in this case was \( \pm \frac{0.1}{10} \), since minimum head of 10 cms was taken to an accuracy of \( \pm 0.1 \) cm which is same as the minimum
for mercury-water manometer readings.

3. For a very low pressure differences, a paraffin-water manometer was used and readings upto 1 cm were taken. An error of 1 mm in observation (which was the least count of the scale) will result in a 10% error in the corresponding resistance coefficient. This is more than the error obtained from other manometers discussed above.

4. At low differential pressures, the surface tension forces between the oil and manometer tubes affected the movement of the meniscus.

5. The specific gravity of the oil changed with time as some impurities got dissolved in it. Though the oil was changed quite frequently, the results still appear to be affected due to the variations in the specific gravity.

6. No attempt was made to check the apparatus against temperature changes. It can change the viscosity of water for each observation and causes error in the results.

7. At low pressure differences, the air bubbles in the manometer if any can be more effective in causing the experimental error in the results.

Because of the above mentioned reasons the experimental results of friction factor-Reynolds number plots at low values of Reynolds number are not very accurate.

5.7. RANGE OF REYNOLDS NUMBERS

As already mentioned earlier that, the highest value of Reynolds number achieved in the present investigation is that for Model 3, which is about $3 \times 10^4$. For other models the highest
figure is slightly less than $10^4$. And the highest value of critical Reynolds number achieved in the present study is that for the straight type model and for model 3, at minimum $d_1/d_2$ ratio which are $2 \times 10^3$ and 2250 respectively. For Model 1, the highest value of critical Reynolds number is $< 2 \times 10^2$, i.e. $Re_c = 1995$, whereas the minimum $Re_c = 603$.

Similarly the maximum and minimum values of critical Reynolds number obtained for Model 2, were as $Re_c = 962$ and $Re_c = 419.0$ respectively.

The minimum value of critical Reynolds number is slightly above 66.0 obtained only for Model 3.

The experimental results of straight pipe, model 1 and model 2 cover a range of Reynolds number from 25-10000, but the points starting from Reynolds number approximately 100 have been included on the graphs corresponding to the above model. This was done simply to reduce the length of linear part of the $'f'$ versus $'Re'$ plot since linear relationship covered a very wide range in the experimental results. This in any case does not affect the critical point.
$f$ vs $Re$ curve for Model 1
$R = \infty$
$d = 0.302 \text{ Cm}$

$FIG. 5.1$

$f$ vs $Re$ curve for Model 1
$R = \infty$
$d = 0.45 \text{ cm}$
f vs. Re curve for Model 1

$R = \infty$

$d = 0.595 \text{ cm}$

$Re$

FIG 5.3

f vs. Re curve for Model 1

$R = \infty$

$d = 0.817 \text{ cm}$

$Re$

FIG 5.4
f vs. Re curve for Model 1

\[ R = \infty \]
\[ d = 1.083 \text{ cm} \]

FIG. 5.5

\[ 'd' \text{ vs. } Re_{Cr} \text{ curve for straight pipes (} R = \infty \) \]

FIG. 5.6
f vs. Re curve for Model 1
R = 2.5 m
d = 0.302 Cm

FIG. 5-7

f vs. Re curve for Model 1
R = 20 m
d = 0.302 Cm

FIG. 5-8
**f vs. Re curve for Model 1**

- **$R = 15 \text{ m}$**
- **$d = 0.302 \text{ cm}$**

![Graph showing the relationship between f and Re for Model 1 with R = 15 m and d = 0.302 cm.]

**f vs. Re curve for Model 1**

- **$R = 10 \text{ m}$**
- **$d = 0.302 \text{ cm}$**

![Graph showing the relationship between f and Re for Model 1 with R = 10 m and d = 0.302 cm.]

**FIG. 59**
f vs. Re curve for Model 1
R = 5 m
d = 0.302 cm

f vs. Re curve for Model 1
R = 4 m
d = 0.302 cm
f vs. Re curve for Model 1

R = 3 m

\( d = 0.302 \text{ cm} \)

FIG. 5.13

f vs. Re curve for Model 1

R = 2 m

\( d = 0.302 \text{ cm} \)

FIG. 5.14
FIG 5-15

f vs. Re curve for Model 1
R = 1.0 m
d = 0.302 cm

f vs. Re curve for Model 1
R = 0.5 m
d = 0.302 cm
f vs. Re curve for Model I

R = 25 m
d = 0.45 cm.

f vs. Re curve for Model I

R = 20 m
d = 0.45 cm.
f vs Re curve for Model 1

R = 15 m

d = 0.45 cm

FIG. 5.19

f vs Re curve for Model 1

R = 10 m

d = 0.45 cm

FIG. 5.20
$f$ vs $Re$ curve for Model 1

$R = 5 \text{m}$

$d = 0.45 \text{ cm}$

$10^2$ $10^3$ $10^4$

$10^{-1}$ $10^{-2}$ $10^{-3}$

$Re$

FIG. 5.21

$f$ vs. $Re$ curve for Model 2

$R = 4 \text{m}$

$d = 0.45 \text{ cm}$

$10^2$ $10^3$ $10^4$

$10^{-1}$ $10^{-2}$ $10^{-3}$

$Re$

FIG. 5.22
$f$ vs $Re$ curve for Model 1

$R = 3\text{ m}$
$\delta = 0.45\text{ cm}$

$f$ vs $Re$ curve for Model 1

$R = 2\text{ m}$
$\delta = 0.45\text{ cm}$
$f$ vs. $Re$ curve for Model 1

$R = 1 \text{ m}$
$d = 0.45 \text{ cm}$

$R = 0.5 \text{ m}$
$d = 0.45 \text{ cm}$
$f$ vs. $Re$ curve for Model 1

$R = 25 \, m$

$d = 0.595 \, cm.$

---

$Re$

$F IG. 5.27$

$F IG. 5.28$
$f$ vs $Re$ curve for Model 1

$R = 15 \text{ m}$
$d = 0.595 \text{ cm}$

$R = 10 \text{ m}$
$d = 0.595 \text{ cm}$
f vs Re curve for Model 1

$R = 5 \text{ m}$

$d = 0.595 \text{ cm}$

f vs Re curve for Model 1

$R = 4 \text{ m}$

$d = 0.595 \text{ cm}$
f vs Re curve for Model 1

R = 3 m
\( d = 0.595 \text{ cm} \)

f vs Re curve for Model 1

R = 2 m
\( d = 0.595 \text{ cm} \)
f vs Re curve for Model 1

$R = 1 \text{ m}$
$d = 0.595 \text{ cm}$
f vs. Re curve for Model 1

$R = 25 \text{ m}$
$d = 0.817 \text{ cm.}$

$F(8.5:37)$

f vs. Re curve for Model 1

$R = 20 \text{ m}$
$d = 0.817 \text{ cm.}$

$F(6.5:38)$
f vs. Re curve for Model 1

$R = 15\text{ m}$

$d = 0.817\text{ cm}$

FIG 5.39

f vs. Re curve for Model 1

$R = 10\text{ m}$

$d = 0.817\text{ cm}$

FIG 5.40
f vs. Re curve for Model 1

$R = 5 \text{ m}$
$d = 0.817 \text{ cm}$

FIG. 5.41

f vs. Re curve for Model 1

$R = 4 \text{ m}$
$d = 0.817 \text{ cm}$

FIG. 5.42
$f$ vs $Re$ curve for Model 1

$R = 3\, m$
$d = 0.817\, cm$

$Re$

$10^2$ $10^3$ $10^4$

FIG. 543

$f$ vs $Re$ curve for Model 1

$R = 2\, m$
$d = 0.817\, cm$

$Re$

$10^2$ $10^3$ $10^4$

FIG. 544
f vs. Re curve for Model 1

\( R = 1 \text{ m} \)
\( d = 0.817 \text{ cm} \)

FIG. 5.45

f vs. Re curve for Model 1

\( R = 0.5 \text{ m} \)
\( d = 0.817 \text{ cm} \)

FIG. 5.46
f vs. Re curve for Model 1
R = 2.5 m
d = 1.083 cm

f vs. Re Curve for Model 1
R = 20 m
d = 1.083 cm.
f vs. Re Curve for Model 1
R = 15 m
d = 1.083 Cm

\[ \text{FIG. 5-49} \]

f vs. Re Curve for Model 1
R = 10 m
d = 1.083 Cm

\[ \text{FIG. 5-50} \]
$f$ vs. $Re$ curve for Model 1

$R = 5 \text{ m}$

$d = 1.083 \text{ cm}$

$Re$

$10^2$ to $10^4$

$f$

$0.1$ to $10$

$0.01$ to $1.0$

Fig 5.51

$f$ vs. $Re$ Curve for Model 1

$R = 4 \text{ m}$

$d = 1.083 \text{ cm}$
f Vs. Re Curve for Model 1
R = 3 m
d = 1.083 cm

Re

f

0.1

0.01

10^2

10^3

10^4

FIG. 5-53

f Vs. Re Curve for Model 1
R = 2 m
d = 1.083 cm

f

0.1
f Vs. Re Curve for Model 1
R = 1 m
d = 1.083 cm

FIG. 5-55

f Vs. Re Curve for Model 1
R = 0.5 m
d = 1.083 cm

FIG. 5-56
\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig5_57.png}
\caption{Re \(_{Cr}\) vs. \(\frac{R}{d}\) curve for Model 1 \(d = 0.302\, \text{cm.}\)}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig5_58.png}
\caption{Re \(_{Cr}\) vs. \(\frac{R}{d}\) curve for Model 1 \(d = 0.45\, \text{cm.}\)}
\end{figure}
$\frac{R}{d}$ vs. $Re_{Cr}$ curve for Model 1

d = 0.595 cm.

FIG. 5.59

$\frac{R}{d}$ vs. $Re_{Cr}$ curve for Model 1

d = 0.817 cm.

FIG. 5.60
\[ \frac{R}{d} \text{ vs. } \text{Re}_{Cr} \text{ curve for Model 1} \]

\( d = 1.083 \text{ cm.} \)

---

\[ \frac{R}{d} \text{ vs. } \text{Re}_{Cr} \text{ curve for Model 1} \]

- (o) \( d = 0.302 \text{ cm.} \)
- (b) \( d = 0.450 \text{ cm.} \)
- (●) \( d = 0.595 \text{ cm.} \)
- (△) \( d = 0.817 \text{ cm.} \)
- (▲) \( d = 1.083 \text{ cm.} \)
$f$ vs. $Re$ curve for Model 2

$R = 3.0\,\text{m}$
$d = 0.302\,\text{cm}$

$R = 4\,\text{in}$
$d = 0.302\,\text{cm}$
$f$ vs. $Re$ curve for Model 2

$R = 2.0 \text{m}$

$d = 0.302 \text{Cm}$
$f$ vs $Re$ curve for Model 2
$R = 0.5m$
$d = 0.302cm$

$R = 4m$
$d = 0.45$ cm
f vs Re curve for Model 2

$R = 3 \, m$

$d = 0.45 \, cm$

$R = 2 \, m$

$d = 0.45 \, cm$
f vs Re curve for Model 2

$R = 1\ m$
$\ d = 0.45\ cm$

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$R = 0.5\ m$
$\ d = 0.45\ cm$
$f$ vs. $Re$ curve for Model 2

$R = 4\text{ m}$
$d = 0.595\text{ cm}$

$F I G. 5.73$

$f$ vs. $Re$ curve for Model 2

$R = 3\text{ m}$
$d = 0.595\text{ cm}$

$F I G. 5.74$
$f$ vs. $Re$ curve for Model 2

$R = 2 \text{ m}$
$d = 0.595 \text{ cm}$

$Re$

$10^{-1}$
$10^0$
$10^1$
$10^2$
$10^3$
$10^4$

$10^{-1}$
$10^0$
$10^1$
$10^2$
$10^3$
$10^4$

$Re$

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FIG. 5.77

$f$ vs. $Re$ curve for Model 2

$R = 0.5 \text{ m}$
$\text{d} = 0.595 \text{ cm}$

FIG. 5.78

$f$ vs. $Re$ curve for Model 2

$R = 4 \text{ m}$
$\text{d} = 0.817 \text{ cm}$
f vs. Re curve for Model 2

$R = 3 \text{ m}$
$d = 0.817 \text{ cm}$

FIG. 5.79

f vs. Re curve for Model 2

$R = 2 \text{ m}$
$d = 0.817 \text{ cm}$

FIG. 5.80
$f$ vs. $Re$ curve for Model 2

$R = 1\, \text{m}$

$d = 0.817\, \text{cm}$

$R = 0.5\, \text{m}$

$d = 0.817\, \text{cm}$
$f$ vs. $Re$ curve for Model 2

$R = 3$ m
$d = 1.083$ cm

${\text{FIG. 5.83}}$
f Vs. Re Curve for Model 2
R = 2 m
d = 1.083 Cm

f Vs. Re Curve for Model 2
R = 1 m
d = 1.083 Cm
$f$ vs. $Re$ curve for Model 2

$R = 0.5$ m

$d = 1.083$ cm

$\frac{R}{d}$ vs. $Re_{Cr}$ curve for Model 2

$d = 0.302$ cm
Figure 5.89:
\[ \frac{R}{d} \] vs. \( \text{Re}_{Cr} \) curve for Model 2
\( d = 0.45 \text{ cm.} \)

Figure 5.90:
\[ \frac{R}{d} \] vs. \( \text{Re}_{Cr} \) curve for Model 2
\( d = 0.595 \text{ cm.} \)
$\frac{R}{d}$ vs. $Re_{Cr}$ curve for Model 2

$d = 0.817 \text{ cm.}$

**FIG. 5.91**

$\frac{R}{d}$ vs. $Re_{Cr}$ curve for Model 2

$d = 1.083 \text{ cm.}$

**FIG. 5.92**
$R/d$ vs. $Re_{Cr}$ curve for Model 2

- $\varnothing$ $d = 0.302$ cm.
- $\triangle$ $d = 0.450$ cm.
- $\bullet$ $d = 0.595$ cm.
- $\Delta$ $d = 0.817$ cm.
- $\blacktriangle$ $d = 1.083$ cm.

FIG. 5.93
$f$ vs $Re$ curve for Model 3

$d_1 = 1.02$ cm

$d_2 = 0.817$ cm
f vs Re curve for Model 3

d_1 = 1.021 cm
d_2 = 0.595 cm

Based on d_2

Based on d_1

Scale for 2

Scale for 1

FIG 595
$f$ vs $Re$ curve for Model 3

$d_1 = 1.021$ cm
$d_2 = 0.45$ cm
f vs Re curve for Model 3

$\text{d}_1 = 1.021 \text{ cm}$
$\text{d}_2 = 0.302 \text{ cm}$

FIG 5.97
$f$ vs $Re$ curve for Model 3

d_1 = 0.817 cm
d_2 = 0.505 cm

FIG. 5-99
$f \text{ vs } Re \text{ curve for Model 3}$

$d_1 = 0.017 \text{ cm}$

$d_2 = 0.45 \text{ cm}$

FIG 5 100
$f$ vs $Re$ curve for Model 3

$d_1 = 0.017 \text{ cm}$

$d_2 = 0.302 \text{ cm}$

Scale for $f$

Scale for $Re$

Based on $d_1$

Based on $d_2$
f vs Re curve for Model 3

\[ d_1 = 0.817 \text{ cm} \]
\[ d_2 = 0.201 \text{ cm} \]
$f$ vs $Re$ curve for Model 3

$d_1 = 0.595$ cm
$d_2 = 0.45$ cm

FIG 5.103
$f$ vs Re curve for Model 3

d_1 = 0.595 \text{ cm}

d_2 = 0.302 \text{ cm.}

FIG. 5.10A
Fig. 5-105

f vs Re curve for Model 3

\( d_1 = 0.595 \text{ cm} \)

\( d_2 = 0.201 \text{ cm} \)
f vs Re curve for Model 3
\(d_1 = 0.45 \, \text{cm}\)
\(d_2 = 0.302 \, \text{cm}\)

FIG 5106
FIG 5107

\( f \) vs \( Re \) curve for Model 3

- \( d_1 = 0.45 \text{ cm} \)
- \( d_2 = 0.201 \text{ cm} \)

Scale for \( d_1 \)

Scale for \( d_2 \)

Markers indicate points based on \( d_1 \) and \( d_2 \).
f vs Re curve for Model 3

\[ d_1 = 0.302 \text{ cm} \]
\[ d_2 = 0.201 \text{ cm} \]
\[
\frac{d_1}{d_2} \text{ vs. } \text{Re}_\text{Cr} \text{ curves for Model 3}
\]

\[
\text{Re}_\text{Cr}_1 = \left( \frac{d_2}{d_1} \times \text{Re}_\text{Cr}_2 \right)
\]

\[
\text{Re}_\text{Cr}_2 = \left( \frac{d_1}{d_2} \times \text{Re}_\text{Cr}_1 \right)
\]

**FIG. 5.109**