CHAPTER 2

POROUS MEDIA FLOWS
Before discussing the work involved in the present investigation, it was decided to present work tries to study some of the models for percolation through porous media. The present chapter therefore, gives discription to some of the important impirical studies along with their results and, if possible, to establish a range of validity of the linear seepage law for various models investigated. This chapter therefore limits itself only to study of the linear and nonlinear regimes.

2.1. LINEAR SEEPAGE FLOW

2.1.1. DARCY'S LAW

The discussion regarding Darcy's well known formula for flow through porous materials published in 1856 (26) is found in literature from many branches of science and engineering such as the text books dealing with the flow through porous media (21, 47, 78, 83, 90), soil mechanics (85, 100, 107) and ground water hydrology (111, 112). The results which Darcy deduced from his classical experiments made on sand-bed filters form the basis of the theory of laminar flow through porous beds. A modern discussion of Darcy's experiments has been given in an article by Hubbert (50).

Darcy's law stated that for laminar flow conditions in a saturated sand bed the rate of flow or discharge per unit time is directly proportional to the hydraulic gradient, the area
of cross-section of bed normal to the flow and inversely proportional to the length of the path of flow or thickness of the bed. The Darcy’s law can be expressed mathematically as follows:

\[ Q = K_i A \]  
\[ \text{or} \quad V = \frac{U}{A} = K_i \]

where, \( Q \) = Rate of flow or total volume of fluid percolating in unit time.

\( K \) = Darcy’s coefficient of permeability.

\( i \) = Hydraulic gradient.

\( A \) = Total cross-sectional area of soil mass perpendicular to the direction of flow.

\( V \) = Velocity of flow or average velocity.

After replacing \( \frac{h_1 - h_2}{L} = i \) = hydraulic gradient, Equation (2.1) can be written as:

\[ Q = K_i A \frac{(h_1 - h_2)}{L} \]  

where, \( h_1 \) = The head of fluid on upstream side of the bed.

\( h_2 \) = The head of fluid on downstream side of the bed.

\( L \) = The length of the sand bed through which head lost is \( (h_1 - h_2) \).

The relationship (2.3) is known as Darcy’s law.

The coefficient of permeability has the units of velocity and depends upon the properties of percolating fluid and of the porous medium. If \( K \) is to be replaced by the 'specific
The specific or intrinsic permeability of porous medium.

But this relationship was not generally accepted until it was popularised by Wyckoff et al. (125). The verification of it consists in the innumerable successful determinations of permeability that have been performed upon its basis. The specific permeability can be determined on the basis of structure of porous medium, and has the dimension of a length squared.

Though limited in its application, Darcy’s law is of great significance and applicable to various problems from different branches of science and engineering. Since its publication a great deal of research work has been postulated in different fields where percolation phenomenon is important. A brief discussion of relevant studies of some of the important attempts is given below.

2.1.2. DERIVATION OF DARCY’S LAW FROM NAVIER-STOKES EQUATIONS

The Darcy’s law (Eqns. 2.1, 2.2) in no way describes the state of flow within individual pores. It represents the statistical macroscopic equivalent of the Navier-Stokes
equations of motion for the viscous flow of ground water (85). Hall (43) has given a derivation for the Darcy’s law in general form for non-isotropic porous media from Newton’s law of motion and viscosity. His analysis is based on the following assumptions.

1. That the inertial forces of the system are negligible.
2. That the liquid films are continuous, and
3. That a volume element of fluid porous medium system can be selected which is small compared to the gross dimensions of the system, yet large enough that the surface area of the matrix can be uniformly distributed throughout the volume element. The form of the equation developed is same as Darcy’s equation. The value of the coefficient of permeability introduced can be determined experimentally.

Hubbert (51) observed that each particle of fluid moving through a porous medium follows a continuously curvilinear path at a continuously varying speed and therefore with a continuously varying acceleration. He derived directly Darcy’s equation from the Navier-Stokes equations for steady motion taking the flow fields to be kinematically similar at different rates of flow. He showed this condition to be true as long as the effects of inertial forces are negligible in comparison to the viscous forces. The Darcy’s law follows in terms of dimensionless coefficient of proportionality, which depends upon the grain size, shape of the pores, density and velocity of the fluids flowing through porous materials.

Since it is very difficult to solve the Navier-Stokes
equations for an actual porous medium, Philip (81) replaced it by an idealized two dimensional array of squares, and solved the equations for this. Adopting the suggestion of Muskat (78) and Lamb (68), he assumed that the inertial terms could be reasonably neglected below a Reynolds number of unity. This assumption simplifies the solution of Navier-Stokes equations, but it means that the validity of Darcy's law cannot be investigated.

In applying Darcy's law, it is important to know the range of validity within which it is applicable. The Darcy's law is valid for a wide range of materials and flow conditions both for coarse and fine grained sands (100). In short the Darcy law is valid only for laminar flow. For the ground water flow occurring in nature and normally encountered in soil, the law is generally within its validity limit.

Watson (121) followed on the same lines as Philip, but he did not avoid the inertial terms. He introduced a computer solution of the problem for the values of Reynolds number ranging from zero to five, thus making it possible to check the validity of Darcy's law. The solution is of no practical utility, as the model considered consists of a two dimensional array of squares.

Scheidegger (90) collected data to show that the critical Reynolds number may vary from 0.1 to 75 for the Darcy law to be valid. Such a wide variation may be partly due to the different interpretations for the characteristic diameter used in the equations for Reynolds number.
2.1.3. **SEMI-EMPIRICAL CORRELATIONS FOR DARCY'S LAW**

The problem of fluid flow through porous medium is difficult to be treated by theoretical consideration due to the complexity of its nature. Therefore, the problem must be studied either by empirical or semi-empirical methods. For many years the empirical and semi-empirical attempts have been made to establish correlations for the permeability to solve the problem of porous media flow. An attempt in this direction will be significant only if parameters such as porosity, tortuosity, specific surface, grain size distribution, angularity, packing and orientation of the constituent grains (96) are taken into account.

Theoretical considerations create complications for its coincidence with empirical results. Yet these attempts attach physical significance when they are analysed microscopically. It will subsequently finalize the empirical tests of properties of flow. These involve the derivation of experimental laws of percolation from basic hydrodynamic or hydraulic considerations. The various works based on these considerations have been attempted in a number of ways each being compared to a different flow system. These approaches have been named the models of the main flow system. The various models to study the correlation between K and other parameters are discussed in the following paragraphs. The various models proposed by various research workers are:

5. Viscous Drag Models.
7. Unit Cell Models.

A detailed study of different types of models has been given in references (58, 90, 96) and a brief description is found in reference (7).

1. STRAIGHT CAPILLARIC MODEL

This simplest straight capillaric model of the linear case is one representing a porous medium by a bundle of straight parallel capillary tubes of uniform diameter. An idea was adopted from these models to correlate the permeability with either an average pore size or with the pore size distribution curve. Further modifications of the simplest straight capillary model give rise to the parallel, series and branching types of model discussed by Scheidegger (90, 96). A detailed study of these models will be given in chapter third.

2. HYDRAULIC RADIUS MODEL

As an actual porous media can not be truly represented by straight capillary tubes, more elaborate models must be used in order to obtain a satisfactory understanding of hydro-mechanics of fluid in porous media. Hydraulic radius model is based upon the assumption that a porous medium is equivalent to a series of channels, and hydraulic radius of these channels is used as a characteristic length parameter in place of the
The basic concept of this model is a result of dimensional consideration. On the basis of hydraulic radius model Scheidegger (90) proposed the following basic mathematical expression for the permeability:

\[ K = \frac{C \bar{R}^2}{F(P)} \]  

(2.5)

where, \( \bar{R} \) = The hydraulic radius of pore channels.

\( F(P) \) = The porosity factor.

and \( C \) = Some dimensionless constant which could be incorporated into the porosity factor.

If expressions for \( \bar{R} \) and \( F(P) \) are obtained in terms of known physical quantities, Eq. 2.5 can be used for predicting the permeability of the medium. Discussions regarding this are given in the next chapter.

3. STATISTICAL OR RANDOM WALK MODELS

All the capillaric models are based on such simplifying assumptions that they do not represent a true porous media. So the statistical mechanics is used to obtain more accurate results. Random walk model being one of them.

There are two fundamental random walk models possible:

(a) In the first one the motion of the fluid particle is taken to be random.

(b) In the second model the porous medium is assumed to be random in nature.

Many attempts have been made to solve the problem statistically (16, 93, 94). In the first model the attempt
to use random walk principle was made by Scheidegger (94). It is based on the assumption that a fundamental distribution function \( \gamma(r, x, y, z) \) exists which describes the probable position of fluid particle at certain time steps. The probability distribution is assumed to be Gaussian so that a 'dispersion' is introduced.

In the second approach when we put randomness on to the porous medium we arrive at a graph theoretical model. In the graph theoretical model, as a unit mass of fluid is injected into a porous medium, it will spread out in a geometrical form, is represented by a bifurcating graph.

We then consider not only one graph but the ensemble of all possible graphs with a definite number of free vertices. The total number \( \bar{N} \) of possible graphs with a definite number \( \bar{n} \) of free vertices can be calculated by the formula given in Ref.(71)

\[
\bar{N} = \frac{1}{2^{\bar{n}-1}} \left( \frac{2^{\bar{n}} - 1}{\bar{n}} \right) \quad (2.6)
\]

It shows that, as \( \bar{n} \) gets large, the total number of possible graphs becomes phenomenal. Therefore the best technique has to be used to randomly generate a certain number of graphs on a computer through which the required expectation values can be calculated is of Monte Carlo. It gives the result that a dispersion effect again occurs when a fluid is introduced into another in a porous medium. This dispersion has been verified experimentally.

A brief discussion is also found in the references (17, 84, 91, 92, 113) to solve the problem statistically. The
method of statistical mechanics has been applied by Scheidegger (91) on the following assumptions.

1. The porous medium is isotropic and averaged over the ensemble homogeneous.

2. The external forces on the fluid are homogeneous and time independent. This may be realized by a constant pressure drop along a given direction.

3. Different parts of one sample are 'macroscopically' identical.

Scheidegger then reached at an equation which, after setting certain factors equal to zero, reduces to a Darcy type equation. These factors describe a new macroscopic effect, i.e. the effect of 'dispersion' as discussed above. The individual particles of fluid do not only move along the stream-lines resulting from Darcy’s equation, but they also get dispersed sideways. The fluid flow in porous media is, therefore, may be determined by three macroscopic constants of the latter i.e. porosity, permeability and dispersivity and not by only two as assumed heretofore.

4. **Fingers Type Models**

By drawing an analogy with an immiscible displacement process, a useful model for treating the overall dynamics of (finger) has been suggested by Scheidegger and Jhonson (95). They considered a linear displacement experiment parallel to the $x$ direction with time ($t$). For certain value of $x$ and $t$ there will be some fingers present.

While drawing this analogy with an immiscible displacement
process Scheidegger and Johnson (95) made a rather specific assumption regarding the 'Fictitious' relative permeability for fluid, and reached at a Buckley-Leverett type equation given as:

\[ r(m_\perp) = \frac{\mu_2/\mu_1}{(\mu_2/\mu_1-1)+1/m_\perp} \quad (2.7) \]

where \( \mu_2/\mu_1 \) is the viscosity of the two fluids involved. According to thermodynamic analogy, they considered the evolution of \( m_i(x,t) \), for a solitary dispersion. Therefore, a complete relation between fingering and solitary dispersion theory was derived as below:

\[ J = \frac{\mu_2/\mu_1}{(\mu_2/\mu_1-1)+1/m_\perp} - m_\perp \quad (2.8) \]

where, \( m_\perp \) represents the average relative area (mass).

\( J \) is the function of \( m \) or a function of gradient of \( m \) (a constant i.e. saturation).

This equation refer to the Buckley-Leverett limit of solitary dispersion theory as applied to fingering. The actual behaviour of the finger may be expected to lie between the two limits i.e. solitary dispersion and diffusivity limits.

Thus, we have the result that the phenomenological microscopic theory of fingering in a unidirectional displacement process does not permit a steady state to occur.

5. **VISCOUS DRAG MODELS**

A different approach was introduced by Emerseleben (30) to Kozeny's physical explanation of permeability which may also be termed as viscous drag theory of permeability. In this model, the walls of the pores are taken to be obstacles to an otherwise
straight flow of the viscous fluid. The drag of the fluid on each portion of the walls is calculated from the Navier-Stokes equations and the sum of all the drag is thought to be equal to the resistance of the porous medium to the flow (90).

Iberall (55) assumed a model consisting of randomly distributed circular cylindrical fibres of the same diameter and accounts for the permeability on the basis of the drag on individual elements. Neglecting the inertial forces and using Emersleben's (30) derivation for the drag forces per unit length of a single fibre, surrounded by similar fibres all oriented along the direction of flow i.e.

\[ F_p = 4 \pi \mu U \]  

(2.9)

where, \( U \) = The pore velocity = \( V/P \) ; and \( F_p \) = The force on a single fibre per unit of its length. Assuming a constant velocity between different fibres, Iberall reached at the Darcy type equation.

\[ i = \frac{16\mu}{3\gamma \rho d_p^2} \cdot \frac{1-P}{2 \log \left( \frac{\rho v d_p}{\mu p} \right)} \cdot \frac{4-\log \left( \frac{\rho v d_p}{\mu p} \right)}{2 \log \left( \frac{\rho v d_p}{\mu p} \right)} \]  

(2.10)

where, \( i \) = Hydraulic gradient,

\( V \) = The specific velocity of the fluid i.e. the discharge divided by the total cross-sectional area.

\( \gamma \) = Specific weight of the fluid.

\( P \) = Porosity of the porous medium.

\( \rho \) = The mass density of the fluid ; and \( d_p \) = the diameter of the fibre.

Other symbols have their usual significance.
In the above equation, it can be seen that the permeability of a porous medium changes with the flow velocity even if the fluid viscosity is the same. It is associated with the inertial effects as the fluid velocity increases.

A different thought based on this model has been followed by Brinkman in a series of papers (9,10,11). The model underlying his treatment is that of a spherical particle embedded in a porous medium. The flow through the medium is described by a modification in the light of Darcy's law. On account of being empirical in nature the result of the latter can not be taken as rigorous solution of the problem. The equation for intrinsic permeability is:

\[ \bar{K} = \frac{R_1^2}{18} \left[ 3 + \frac{4}{1-P} - \frac{3}{2(1-P)} - \frac{3}{3} \right]^{1/2} \]  \hspace{1cm} (2.11)

where, \[ \frac{4}{3} \pi N R_1^3 = (1-P) \]

\( N \) = The number of particles per unit volume of the bed.

\( R_1 \) = The radius of the particle.

From the equation (2.11), it is clear that when \( P = \frac{1}{2} \), \( \bar{K} = 0 \), which represents an impossible physical situation.

Other important works based on drag theory are found in the articles by Happel (45), Mott (76) and DeGroot (28).

6. FISSURED ROCK MODELS

A model was prepared by Irmay (58) on the basis of similarity between a porous media and a rock fissured along three mutually inclined or perpendicular planes, known as fissured rock model. He applied the Poiseuille formula for each fissure and obtained Darcy's equation for an anisotropic
media. Irmay claims that the result obtained are found to be within 9% of the result obtained by Kozency-Carman equation for a range of porosity from 25% to 65%.

A detailed description of this model is given also by Bear et al (6) and Verma (117). Special attention was given by Bear (6) to determine the parameters affecting the flow i.e. the piezometric head gradients, the capillary pressure in the blocks (imbibition) and fissures and gravity.

7. UNIT CELL MODELS

To solve the problem of seepage through porous media Uchida (115) and Happel (46) have employed unit cell models. According to this technique that an assemblege can be divided into a number of identical cells, one particle occupying each cell. The boundary value problem thus reduces to the consideration of a single particle and its bounding envelop Gupta (42). The technique was particularly developed for regular arrays of particles, but it can be extended, in some stochastic sense, to random particle arrays as well.

8. AVERAGED NAVIER-STOKES EQUATIONS

This model was proposed by Irmay (57). He used the Navier-Stokes equations for viscous flow through a porous media. Using the space averaged values of velocities and their derivatives in different directions he reached at Darcy's equation for low values of Reynolds number. For high values of Reynolds number he suggested.

\[ i = aV + bV^2 + c. \frac{\delta V}{\delta t} \]  

(2.12)

This relation is similar to the one proposed by Polubarinova-
Kochina (83).

Here, \( V \) = Scepage velocity and \( a, b, c \) are coefficients given as

\[
a = \beta_0 \frac{(1-P)^2}{P^3} \frac{\nu}{g d_p^2},
\]

\[
b = \alpha_0 \frac{1-P}{P^3} \frac{1}{g d_p},
\]

and \( c = \frac{1}{P g} \)

where \( \alpha_0, \beta_0 \) are unknown shape factors, \( \nu \) is the kinematic viscosity of the fluid and \( g \) is the acceleration due to gravity.

Suggesting that the effects of unsteadyness in the flow are felt for a fraction of a second, Irmay modifies Eq. 2.12 to

\[
i = aV + bV^2
\]

(2.13)

which is the same as Forchheimer's empirical equation.

2.2. RELATIONSHIPS BETWEEN HYDRAULIC GRADIENT AND VELOCITY OF FLOW

In the previous part of this review the Darcy's law has been accepted as a fundamental law governing seepage through porous media. Since the Darcy's law is valid only in a certain 'seepage velocity' domain, outside which more general flow equations must be used to describe the flow correctly.

Some of the important works in this field are described below under the following headings.

2.2.1. Relationships of the type \( i = aV + bV^2 \)

2.2.2. Exponential Relationships i.e. \( i = aV^n \)

2.2.1. RELATIONSHIPS OF THE TYPE \( i = aV + bV^2 \)

By analogy with the flow through pipes, Forchheimer (38), modified Darcy's law for higher velocities by including second
order term in the velocity, i.e.

\[ i = aV = bV^2 \]  \hspace{1cm} (2.13)

and later by adding a third order term to

\[ i = aV + bV^2 + cV^3 \]  \hspace{1cm} (2.14)

Here, 'a', 'b' and 'c' are experimentally determined coefficients.

Polubarinova-Kochina (83) modified equation (2.13) to contain a time dependent term as

\[ i = aV + bV^2 + c \frac{\delta V}{\delta t} \]  \hspace{1cm} (2.12)

Similar expression was obtained by Irmay (57) theoretically, but according to him the third term is negligible for a steady flow through porous media, thus making equations (2.12) and (2.13) alike.

An important expression and more applicable both to linear and non-linear regimes of flow was introduced by Marcom (74) given below:

\[ i = \frac{\alpha \mu V}{\rho g d_p^2} + \frac{\beta V^2}{g d_p} \]  \hspace{1cm} (2.15)

where '\alpha' and '\beta' are coefficients dependent upon the characteristics of the packing.

By using the specific surface of the particles as the characteristic length Ergun and Oring (32) deduced a similar expression including porosity terms as well.

\[ i = 2\alpha \left( \frac{1-P}{p} \right)^2 \cdot \left[ \frac{\mu S_o V}{p \rho g} \right] + \frac{\beta}{8} \cdot \frac{1-P}{p^3} \cdot \left[ \frac{V^2 S_o}{g} \right] \]  \hspace{1cm} (2.16)

Here, \( S_o \) = specific surface of the solids.

Other symbols have their usual significance.
A similar relationship has been originated by Ungelund (31). But he did not include a porosity term clearly in his expression, and ‘a’ and ‘β’ depend upon grain shape and porosity. Ahmad and Sunada (3) and Ward (120) used the square root of intrinsic permeability as characteristic length parameter. Ward modified the equation to:

\[ i = \left( \frac{\gamma V}{\rho g K} \right) + \left( \frac{C_1 \rho V^2}{K^{1/2}} \right) \quad (2.17) \]

where, ‘C_1’ is dimensionless constant and same for all porous media. Ahmad and Sunada also reported the same equation. A brief mention regarding the above discussions is found in reference (126).

2.2.2. EXPO\textit{N}\textit{E}N\textit{T}IAL \textit{T}YPE \textit{R} ELATIONSHIPS

An exponential type of relationship between the hydraulic gradient and velocity of flow was derived by Spaugh (105) and White (122). The general relationship is given here:

\[ i = aV^n \quad (2.18) \]

where, ‘a’ and ‘n’ are constants which depend on the characteristics of the medium and on the regime of flow. White obtained \( n = 1.30 \), whereas Spaugh obtained values between 1.60 and 1.32 for \( n \) for the various materials investigated by them.

Experiments were performed by Escande (34) on crushed rocks of approximately 2" mean size. He introduced \( n = 2 \) in the above equation on the assumption that fully turbulent flow must occur at large values of Reynolds number, though there is no theoretical justification for adopting this value of ‘n’.

A value of \( n = 2 \) was used by Cohen de Lara (20) in his
equation for flow through porous media:

\[ i = \frac{f}{\eta d_p} \cdot \frac{y^2}{2g} \left( \frac{1+e}{e} \right)^5 \]  

(2.19)

Here, \( f \) = Friction factor.
\( e \) = The void ratio of the bed material;
\( \eta \) = The angularity coefficient, which according to him equals one for rounded and 0.65 for roughest crushed material.

Therefore, in Equation (2.19) 'a' is a function of shape and size of the material and porosity of the bed, beside the Reynolds number.

In the same way Vander Tuin (116) conducted experiments in an open channel on basalt boulders upto 200 kgs in weight and about 38.5 cms mean size. He reported his results in various ways, equation (2.18) is one of them. The value of 'n' was reported between 1.92 to 2.23 for very large materials.

Both types of relationships between the hydraulic gradient and velocity of flow are supported by Muskat (79). With the present knowledge it should be emphasized that both the Forchheimer type as well as exponential type relationships are based on a very limited amount of data and are good approximation over certain limited flow ranges. The constants 'a' and 'n' are actually functions of other variables including the rate of flow. Until the experiments covering a very wide range of flow as well as other parameters are conducted, 'a' and 'n' as well as 'a' and 'b' should be determined from the tests on the material under consideration for conditions which simulate the field situations.
2.3. LIMITS OF VALIDITY OF LINEAR SEEPAGE LAW

In applying Darcy’s law it is important to know the limits of validity within which it is applicable. The limits of linear law have been variously reported by various investigators. The linear seepage law is not applicable beyond certain lower as well as the upper limits of the Reynolds number. Since the author’s concern is to study some of the models responsible for non-linearity in porous media flow beyond Darcy regime a brief discussion about the upper limit of the applicability of Darcy’s law as well as the causes of non-linearity is essential. The discussion regarding the upper limit is given in the following paragraphs and the causes of non-linearity are taken up in the next chapter.

2.3.1. UPPER LIMIT OF THE VALIDITY OF DARCY’S LAW

There is no general agreement about the limit of the validity of linear law. Experiments conducted by Fancher, Lewis and Barnes (37) arrived at an upper limit of Reynolds number of one. Plain and Morrison (82) who conducted experiments on beds of spherical glass beads found that Darcy’s law holds upto Reynolds number of 75. Muskat (78) reports a value of one. Nielson (80) showed the deviation from the linear law at \( Re = 0.1 \). This large range shows the deficiency of the usual form of parameter in accounting for the variables involved in porous media flow, such as shape, size, grading, roughness, packing and porosity. Lindquist (72) found a value of four for critical Reynolds number on the basis of his experiment on bed composed of lead shots of uniform size and packed at 38% porosity.

Bakhmeteff and Feodoroff (5) suggested a value of five for
the upper limit. Engelund (31) reports values between 3.55 to 5.40. Similarly Schnecbeli (102) reported a value of 2 for angular grains and 5 for spherical particles.

Tek (109) reports values between 0.5 and 8.0 for different types of sands, whereas Hubbert (51) suggests a value of one for critical Re.

Dudgeon (29) conducted experiments on different materials and showed the critical Reynolds number to vary between 2 and 60. Bringing in porosity and shape factor in the expressions for the Reynolds number and friction factor, Brownell and Katz (12) tried to give a definite value to the critical Reynolds number, but without any success.

For a two dimensional array of squares, Watson (121) reported a theoretical value of critical Reynolds number as 5 but his result is unimportant from practical considerations as it is based on idealized model with simplified boundary conditions. A brief discussion is found in the references (1, 2, 119) to solve the problem satisfactory, but without any success.

A more systematic work was carried out by Gupta (42) to check the upper limit of linear law for the limestone and crushed quartzite. In both the cases, it was found that the friction factor-Reynolds number curve starts deviating from straight line at Re = 10, which shows the upper limit of the validity of linear law is at Reynolds number approximately 10.