CHAPTER I

INTRODUCTION

1.1 OPTIMIZATION AND MATHEMATICAL PROGRAMMING:

Any problem that requires a positive decision to be made can be classified as an operations research (OR) type problem. According to the Journal of Operations Research Society, U.K. - Operations Research is the application of the modern methods of mathematical sciences to complex problems arising from the direction and management of large systems of man, machines materials and money in industry, business, government and defence. The distinctive approach is to develop a scientific model of system, incorporating measurements of factors such as chance and risk with which to predict and compare the outcomes of alternative decisions, strategies or controls. The purpose is to help management to determine its policy and direction scientifically.

Optimization and uncertainty are dominant themes in operations research. It is the application of mathematical
analysis to managerial problems. The operations researcher makes a contribution to this problem solving effort by mathematical techniques to obtain a solution. These optimization methods are collectively known as mathematical programming.

Mathematical programming is a term coined by Robert Dorfman around 1950, and now is a generic term encompassing linear programming (LP), integer programing (IP), convex programming (CP), nonlinear programming (NLP), dynamic programming (DP), programming under uncertainty, etc.

Programming problems in general may either belong to the deterministic class or probabilistic class. By deterministic class it is meant that if certain actions are taken then it can be predicted with certainty that what will be (a) the requirements to carry out the actions and (b) the outcome of any actions. Programs involving uncertainty of a given action may depend on some chance event such as the weather, traffic delays, government policy, employment levels, or the rise and fall of customer demand. Sometimes the distribution of the chance event is known, sometimes it is unknown or partially known. Throughout this thesis by mathematical programming the deterministic type of mathematical programming is meant.
1.2 THE GENERAL MATHEMATICAL PROGRAMMING PROBLEM:

The standard form of the general mathematical programming problem may be taken as,

\[
\begin{align*}
\text{Minimize } f(x) \\
\text{Subject to,}
\end{align*}
\]

\[
\begin{align*}
g_j(x) & \geq 0 ; j=1,2,\ldots,m \\
X_i & \geq 0 ; i=1,2,\ldots,n
\end{align*}
\]

where \( X'=(x_1,x_2,\ldots,x_n) \) is the vector of unknown decision variables and \( f(X), g_j(X) \) are real valued functions of the \( n \) real variables \( x_1, x_2, \ldots, x_n \).

The function \( f(x) \) is called the objective function, the inequalities \( g_j(x) \geq 0 \) are referred to as the constraints, and the restrictions \( X_i \geq 0 \) are called non-negativity restrictions.

The mathematical programming problem stated in (1.2.1) is also known as a constrained optimization problem. An optimization problem without any constraint is called an unconstrained optimization problem.
1.3 ADVANCEMENTS IN MATHEMATICAL PROGRAMMING

TECHNIQUES:

Since the end of World War II, mathematical programming has developed rapidly as a new field of study dealing with applications of the scientific method to business operations and management decision making. But we can trace the existence of optimization methods to the days of Newton, Lagrange and Cauchy. The differential calculus methods of optimization was introduced by Newton and Leibnitz. The foundation of calculus of variation was laid by Bernoulli, Euler, Lagrange and Weirstress. Lagrange introduced his famous Lagrange Multiplier Technique to solve the constrained optimization problems. Cauchy made the first application of the Steepest Descent method to solve unconstrained minimization problems. In spite of these early contributions, very little progress was made until the middle of the twentieth century.

Linear programming was developed in 1947 by George B. Dantzig, Marshall Wood and their associates, as a tool for finding optimal solutions to military planning problems for the United States Air Force. The early applications were primarily limited to problems involving military operations, such as military logistics problems, military transportation problems, procurement problems, and other
related fields. The numerical procedure for solving a linear programming problem introduced by Dantzig is known as Simplex Method. But the method was not available until it was published in the Cowles Commission Monograph No.13 in 1951.

Kuhn H. W. and Tucker A. W. (1952) published their important paper dealing with necessary conditions, popularly known as K-T conditions, to be satisfied by an optimal solution to a mathematical programming problem, which laid the foundations for a great deal of later work in nonlinear programming.

Charnes and Lemke (1954) published an approximation method of treating problems with separable objective function subject to linear constraints. Later the technique was generalized by Miller (1963) to include separable constraints.

A number of papers by various authors dealing with the quadratic programming began to appear after 1955. Beale (1959) gave a method for solving a quadratic programming problem. Wolfe (1959) transformed the quadratic programming problem into an equivalent linear programming problem using K-T conditions, which could be solved by Simplex method. Other authors who gave techniques for solving quadratic

Bellman (1957) made the major original contribution to
the development of the dynamic programming technique. Dynamic programming problems paved the way for development of the methods of constrained optimization. The contributions of Rosen (1960, 1961), Zoutendijk (1966), to nonlinear programming have been very significant. Although till date no single technique has been found to be universally applicable to all nonlinear programming problems. The work of Carrol (1961), Fiacco and McCormick (1968) made many a difficult problem to be solved by using the well-known techniques of unconstrained optimization. Powell (1964) gave an efficient method for finding the minimum of a function of a several variables without calculating derivatives. Other authors who made contribution for unconstrained optimization are Fletcher and Reeves (1964), Davidon (1968), Fletcher (1970) etc. Grandinetti (1982) gave an updating formula for quasi-newton minimization algorithm.

Geometric programming was developed by Duffin, Peterson and Zener (1967). Geometric programming provides a systematic method for formulating and solving the class of optimization problems that tend to appear mainly in engineering designs. Ermer (1971) used geometric programming for optimization the constrained machinery economics problem. Dembo (1982) applied sensitivity analysis in geometric programming.
Dantzig (1955), Charnes and Cooper (1959,1960) developed stochastic programming techniques and solved problems by assuming design parameters to be independent and normally distributed. Some other authors who contributed in stochastic programming are Evers (1967), Greenberg (1968), etc.


Developments of new techniques for solving mathematical programming are still going on. Kachian (1979) gave a polynomial algorithm for linear programming. Karmarkar (1984) gave the polynomial time algorithm which is an excellent method for solving linear programming problem. A number of other authors including Anstreicher (1986), Gay (1987), Tomlin (1987), Shanno and Marsten (1985) etc. have worked on Karmarkar's algorithm.

1.4 APPLICATIONS OF MATHEMATICAL PROGRAMMING:

Mathematical programming models are widely used to solve a variety of military, economic, industrial and social problems. The early applications were primarily limited to problems involving military operations, such as
military logistics problems, military transportation problems, procurement problems, and other related fields. Some of the works are due to, Brackmen and Burnham (1968), Wollmer (1970), Dellinger (1971), Miercort and Soland (1971), Eckler and Burr (1972), Furman and Greenberg (1973), Burr, Folk and Karr (1985) etc.

In addition, mathematical programming was applied to interindustry economic problems, based on Wassily Leontiefs input-output analysis - by Koopman (1951), Leontief (1951) and Morgenstern (1954) with successful military applications, it was carried over to business uses. A 1976 survey in America to determined the use of mathematical programming by American companies shows that 74% of them use mathematical programming techniques to solve their various problems.

The need for optimal decision making arises from the relative scarcity of productive resources. The allocation of limited resources among competitive uses is of major interest to business decision makers. By an efficient allocation of resources, an attempt is made to achieve a specific business objective. In recent years, mathematical programming has received wide acclaim among business decision makers and industry management as a tool helpful in achieving their business objectives. Few contributions in this area are due to Dantzig, Johnson and White (1958), Smith (1961), Catchpole (1962), Arabeyre, Fearnley and et al (1969), Lee (1972), Rubin (1973), Belton (1985), Vijaylakshimi (1987), Silverman, Steuer and Wishman (1988) etc., etc.

Mathematical programming are widely used in field of production-scheduling, planning and inventory control. Many authors applied mathematical programming technique to the above areas such as, DeBoer and Vandersloot (1962), Efroymson and Ray (1966), Silver (1967), Von Lantenauer (1970), etc.

Service companies such as restaurants, hotels, gas-station and convenience stores are often composed of many distributed facilities. Such companies constantly make decisions about expanding their operations by establishing
new facilities and/or expanding existing ones. The use of optimization models for capacity expansion, both under deterministic demand and stochastic demand, has been an area of active investigation. Luss (1983) provide comprehensive literature surveys in this area. Recent articles include Ganz and Berman (1992), Bean, Higel and Smith (1992) etc.

Problems dealing with personnel assignment and training are another area where mathematical programming techniques are successfully applied. Such as physician manpower requirements of most developing countries can often be met by developing local training facilities, sending indigenes to train abroad, and by recruiting expatriate physicians. Ikem and Reisman (1990) developed a manpower planning model which coordinates physician manpower requirements of a developing country with its capacity to train such physicians along with national objectives to contain costs. Some others who contributed in this area are Rothstein (1972), Byrne and Potts (1973), Martel (1973), Segal (1974) etc.

In design, construction and maintenance of any engineering system, engineers have to take many technological and managerial decisions at several stages. The ultimate goal of all such decisions is either to
minimize the effort required or to maximize the desired benefit. Mathematical programming can be applied to solve many engineering problems also. The application of optimization methods in the design of thermal systems was presented by Stoker (1971). The works of Haug and Arora (1979), Johnson (1980), Krisch (1981) deal with the optimum design of machine and structural systems. Hussain and Gangiah (1976), Ray and Szekely (1973) discuss the applications of optimization techniques to chemical and metallurgical engineering problems. Rao (1973) gave the minimum cost design of concrete beams.

Apart from the applications of mathematical programming just listed above, it can be used as an adjunct to or in place of methods that are now associated with the fields of artificial intelligence (AI) and information technology. Ten Dyke (1990) identifies these problems. The analyst seeks to take raw data and transform them into useful information. In medical diagnosis also mathematical programming techniques are used where in we seek to associate a set of input attributes to a specific outcome or response. Few works are due to, Klepper (1966), Henschke and Flehinger (1967), Bahr et al (1968), Redpath and et al (1975,1976,), McDonald and Rubin (1977), Emerson (1975), Sonderman and Abrahamson (1985) etc., etc..

Other application of mathematical programming are in
Statistical technology plays an indispensable role in almost every possible sphere of human activity in the modern world. In fact all statistical procedures are, solutions to suitably formulated optimization problems. Whether it is designing a scientific experiment or planning a large-scale survey for collection of data, or choosing a stochastic model to characterize observed data, or drawing inference from available data, such as estimation, testing of hypothesis, and decision making. One has to choose an objective function and minimize or maximize it subject to given constraints on unknown parameters and inputs such as the costs involved. The classical optimization methods based on differential calculus are too restrictive and are either inapplicable or difficult to apply in many situations that arise in statistical work. The lack of suitable numerical algorithms for solving optimizing equations, has placed severe limitations on the choice of
objective functions and constraints and led to the development and use of some inefficient statistical procedures.

Attempts have therefore been made during the last three decades to find other optimization techniques that have wider applicability and can easily be implemented with the available computing power. One such technique that has potential for increasing the scope for application of efficient statistical methodology is mathematical programming, [C.R. Rao in Arthanari and Dodge (1981)]. The fundamental paper by Charnes, Cooper and Ferguson (1955) introduced the application of mathematical programming to statistics. As an alternative to the least-square approach to linear regression, they choose to minimize the sum of the absolute deviations (MINMAD), and showed the equivalence between the MINMAD problem and a linear programming problem. Wagner (1959) suggested solving the problem through the dual approach. An efficient modification of the simplex method by Barrodale and Roberts (1973) increased the possibility of using MINMAD regression as an alternative to classical regression.

Other areas of applications of mathematical programming in statistics developed simultaneously. Lee, Judge and Zellner (1968) applied it in maximum likelihood

Mathematical programming techniques are applicable to so wide variety of problems emerging from almost every branch of science, engineering, industry, management planning, medical science, military, etc. that it is not possible to even describe all the applications briefly in a single thesis.

1.5 MATHEMATICAL PROGRAMMING IN SAMPLING:

Sampling theory deals with problems associated with the selection of samples from a population according to certain probability mechanism. The problem of deriving
statistical information on population characteristics, based on sample data, can be formulated as an optimization problem in which we wish to minimize the cost of the survey, which is a function of the sample size, size of the sampling unit, the sampling scheme, and the scope of the survey, subject to the restriction that the loss in precision arising out of making decisions on the basis of the survey results is within a certain prescribed limit. Or alternatively, we may minimize the loss in precision, subject to the restriction that the cost of the survey is within the given budget. Thus we are interested in finding the optimal sample size and the optimal sampling scheme which will enable us to obtain estimates of the population characteristics with prescribed properties. Mathematical programming techniques are also applied to certain estimation problems related to sampling.


The subsequent chapters of this thesis are devoted to
the use of mathematical programming to some problems arising in univariate and multivariate stratified sampling.