Chapter VI

MATHEMATICAL GEOGRAPHY

Alberuni had an unabating and unflinching devotion to the branch of mathematical geography and he himself testified to it when he said, 'I am deeply interested in such observations, for they are more preferable to me than all other ambitions'.¹ Astronomical observations seem to be a part and parcel of his being and it appears that he was by some inner urge as also by his undiminishin© thirst for knowledge to devote himself to such studies. Unfortunately, sometimes he had no instrument of precision to accomplish the task, but even then we find him engrossed in his study and he manages to get on with the crudest forms of instrument which he could contrive. He writes: 'on the day of writing this chapter (IV), Thesday, the first of Jamada II, the year four hundred nine of the Hijra, I was in Jayfur, a village adjacent to Kabul. I had a keen interest in making observations for determining the latitudes of these places, though I was in strain and agony. I think that even Noah and Lot -- Peace be granted to them! -- did not suffer such agony, and I do hope, with God's munificence, to be their third in receiving His mercy for my salvation. I had no instrument then for measuring the altitude, and was not in possession of any of the material

from which one might be made. So I drew an arc of a circle on
the back of a computation board, and divided each degree into
six equal parts, so that each part represented an interval of
ten minutes, and suspended it vertically by plumb lines.²

In determining the latitudes and longitudes of places,
Alberuni followed the Ptolemic traditions, albeit with greater
skill, accuracy and proficiency. It may be noted that like
Marinus Ptolemy had computed the coordinates of latitudes and
longitudes of some 6,000 near and far places on the basis of
information gathered mainly from the accounts of travellers and
mariners. It was this not wholly inexact but largely doubtful
information that served as the foundation of Ptolemy's mathematical
geography. Alberuni, unlike Ptolemy, himself took the trouble
of visiting the places in person and then scientifically determined
their geometrical position on the terrestrial globe. He thus
revealed a greater amount of sincerity of purpose than did his
Greek master. It seems a routine scientific work with him that
no sooner did he set his foot on a new soil than he set before
himself the task of determining its position on the globe. This
was a practice which he followed from his early youth and instances
are numerous where we find him working on this tedious and
painstaking job with the seal and concentration of a mature mind.
In the year A.D. 997, when he was only about twentyfour, he is
found working on the determination of longitudinal difference
between Kath and Baghdad, in collaboration with the noted astronomer

² Ibid., p. 86.
Abul Wafa of Baghdad.

DETERMINATION OF LATITUDES

In the second chapter of Tahdid, Alberuni discusses at length the various methods which he considered suitable for the determination of latitude of cities. Likewise in his other great work Kitab al-Hind, which was written six years after Tahdid, he takes up the subject afresh. And all these with other observations have also been incorporated in his magnum opus, al-Qanun al-Mas'udi (Canon Masudicus), which is more in the nature of an astronomical treatise.

Alberuni, not only had access to the works of ancient Greek authors such as Aristotle, Eratosthenes, Posidonius, Hipparchus, Marinus and Ptolemy, whose writings on the subject were considered the intellectual recipes of his time, but he also had before him the Indian astronomical writings such as those of Aryabhata, Brahmagupta and Varahamihira. In his own times, Muslim astronomers like al-Mas'udi, al-Kindi, Abu Ma'ashar, al-Fazari, al-Battani, Sulaiman bin Ismat al-Samarqandi and Yahya bin Abu Mansur and a host of others, had already pushed the horizon of scientific observation. But it is interesting to surmise that the originality of his intellect never let him follow the results of his predecessors or his contemporaries blindly and we find him refuting and censuring them and charging them for their insincerity and lack of wisdom in a scientific work of such a high order. He did not spare even the Indian astronomers who seem to have the
greatest influence upon him. He does not discuss the Hindu methods for determining latitudes and shows ignorance of the methods employed by them. This is rather astonishing, for he had recourse to *Sindhind* or *Brahmasphutasidhanta*, the *al-Arkand* or the *Khandakhadyaka*, or other Sanskrit translations of al-Khwarizmi, al-Kindi and Abu Ma'ashar. He himself translated from the original Sanskrit texts *Paulisa Sidhanta* and the *Khandakhadyaka* and where the usual methods of determining the latitudes from equinoctial shadows or from sun's zenith distances and declinations have been discussed. Whatever be the reasons of this neglect they are at least unknown to us.

Alberuni conceived the idea of latitudes of places as 'the measures of their northern or southern displacements', and in order to assigned them the exact positions as they would be occupying geometrically on the globe, he constructed a hemisphere of ten cubits and marked on it the latitudes and longitudes of places from those distances which came to his knowledge either through the travellers who had actually done the traversing or through his own visits to such places. But on the whole he is always critical and never uses an information unless he himself has checked up its authenticity.

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6 ibid., p. 14.
ALBERUNI'S ACTUAL METHOD OF DETERMINING
THE LATITUDES OF CITIES

Amongst the Arabs of the ninth and tenth centuries the common practice of determining the latitude of any given town was by measuring the meridianal altitude of the sun at noon and its declination at that time. If the declination was towards the north, it was subtracted from the altitude; and added, if its position was towards the south. The results then gave the altitude of Aries or Libra and by further subtracting this from 90 degrees, the latitude of the said town was found. This method is similar to what modern geographers employ in the determination of latitudes.

Alberuni, however, discusses in his Tahdid the methods he considers most appropriate for such determination. He categorizes them into two broad heads: first, by observation of the fixed stars and, secondly, by observation of the sun. He classifies the former method into three types:

1) when their parallel circles are permanently above the local horizon, i.e., the permanent visible stars of the circumpolar stars;

2) when parallel circles touch the local horizon; and

3) when parallel circles intersect the local horizon.

7 Alvi, Z., Arab Geography in the Ninth and Tenth Centuries (Aligarh, 1965), pp. 48-49.
8 Ibid., p. 49.
9 The 'fixed stars' to the Arabs meant those shining spherical bodies which occupy the eighth sky and are other than the so-called 'planets' -- the moon, mercury, venus, the sun, mars, jupiter and saturn (in the order of their occupying the first seven skies).
He further divides each of the formentioned methods into three subheads according to the position of the zenith of the star. The zenith would be either (a) inside the parallel, or (b) on the parallel, or (c) outside the parallel. For the observations made from the position of the sun, he gives credence only to that premise when the sun's parallel intersects the local horizon, for, according to him, there is no civilisation where the sun's parallel circles would be above or touching the local horizon.

In the following pages, an attempt has been made to represent his different methods in algebraic form. He first takes up the case of circumpolar star, whose parallel always remains above the local horizon. Under this head he sets three different conditions:

i) when the maximum and minimum altitudes are in the same horizon (north);

ii) when their (north or south) horizons are different; and

iii) when one of the altitudes is exactly at 90 degrees.

The methods of finding latitude for the above three cases may be formulated respectively as below:

1) \[
\frac{L-B}{2} + B = \theta \quad \text{latitudes of the place}
\]

10 Alberuni, Tahdid, pp. 34-35.
11 ibid., p. 34.
12 The determination of latitude by the observation of the altitude of a circumpolar star and the declination of the zenith by altazimuth instrument is still the most convenient method. See Astronomy, Dugan, Ch.I, and also p. 72.
ii) $90 - L + 90 - B/2 + B = 0$ latitude of the place

iii) $90 - B/2 + B = 0$ latitude of the place

when $L$ is the maximum altitude and $B$ the minimum.

Furthermore, another simpler way of finding the latitude given by him, may be expressed thus:

iv) $L + B/2 = 0$ latitude of the place

As a concrete example for the fourth method, he takes the data from Mohammad and Ahmad, sons of Musa, who found the maximum and minimum altitudes of the nineteenth star in the Great Bear at Baghdad as 63 degrees 13 minutes and 3 degrees 45 minutes respectively. Inserting the data in the fourth formula given above:

$$63^d 13^m + 3^d 45^m / 2 = 33^d 29^m \text{ (which is the latitude of Baghdad)}$$

The following formulation would be applicable in the case when the star touches the local horizon as it crosses the meridian but the transit lies between the zenith and north point:

v) $L/2 = 0$ latitude of the place

But if the transit is between zenith and south point, the formula would be amended to:

vi) $180 - L/2 = 0$ latitude of the place

Alberuni goes on to discuss and enumerate the various situations of celestial stars which help in the determination of latitudes. At one place he refers to the procedure of finding the latitude by
observation of the sun but does not elaborate it. Instead, he takes up the problem of calculating the latitude of a place from the known latitude of another place. He puts forth four different situations and their respective solutions. In all the four cases the altitude of a fixed star is to be reckoned at both the places (of known and unknown latitudes), irrespective of time interval.

Case I: Star being on the meridian circle and on the same side of the zenith in each of them.
Solution: Difference of altitudes at both the places shall be equal to the unknown latitude.

Case II: Altitudes south of the zenith
Solution: If the altitude at the known place is bigger, add the difference of altitudes to the known latitude; if it is less, subtract the difference from the known latitude; and the resultant will be the latitude of unknown place.

Case III: Altitudes north of zenith, whether either in upper or in lower culmination.
Solution: Add the difference in altitudes to the known latitude, if its altitude is less; subtract, if the difference of the altitude is greater.

Case IV: Altitudes in reciprocal positions.
Solution: Calculate the sum of co-altitudes and add this to the known latitude, if its altitude is north of zenith; deduct the sum of co-altitudes from the latitude of known place if its altitude is in the south of zenith.

Thus he gives very convenient methods which could be employed for the determination of latitudes. These instances alone prove the sharpness of his mathematical brain and the originality of

13 Alberuni, Tahdid, p.34.
his intellect.

It is interesting to note that Alberuni himself determined the latitude of some of the Indian towns which he visited during his sojourn in the country. Some of the towns and their respective latitudes are given below:

- (1) Dunpur \(34^d20'm\)
- (2) Ghazna \(33^d35'm\)
- (3) Jailam \(33^d20'm\)
- (4) Kabul \(33^d47'm\)
- (5) Nandna \(32^d0'm\)
- (6) Purshawan \(34^d44'm\)
- (7) Kandi \(33^d55'm\)
- (8) Lamghan \(34^d43'm\)
- (9) Mandakakor \(31^d50'm\)
- (10) Multan \(29^d40'm\)
- (11) Salkot \(32^d56'm\)
- (12) Nainind \(34^d30'm\)

In addition to this list of 12 towns, he gives in his *al-Qanun al-Mas'udi* another list of about 600 towns with their latitudes. Most probably it is the same list that he refers in his *Kitab al-Hind*.  

It would appropriate at this place to refer to the astronomical observations which Alberuni carried out in India. In India Alberuni worked out the latitudes and longitudes of many towns such as Krukshetra, Ujjain, al-Hansa, Basanava and several others. He also mentions the findings of Hindu astronomers, and refers to the latitudes of Kanoj and Taneshar as given by Balabhadr. In connection with the latitude of Kashmir he makes a reference to the book *KARANA-SARA*.

17 ibid., p. 316.
He makes a rather surprising statement: 'In what way the Hindus determine the latitude of a place has not come to our knowledge.' It is surprising because Khandakhadyaka and Grahmasphutasidhanta as well as in Pancasidhantika, with each of which he was well acquainted, the methods of determining the latitude are explicitly given.  

**DETERMINATION OF LONGITUDES**

Alberuni devotes a considerable portion of his treatises on the determination of longitudes and their differences. At least one whole chapter in Tahdid and the second chapter of the sixth treatise of al-Qanun al-Mas‘udi deal with the methods which could be used for their determination. In Kitab al-Hind he again takes up the subject, though partially, in chapters XXIX, XXX and XXXI. Here he compares his own methods with the Indian methods and points out the shortcomings in the latter.

Alberuni starts his reckoning of longitudes from the eastern coast of the 'Circumambient Ocean' (Atlantic), and for this purpose selects Sus al-Aqsa on the western coast of al-Maghrib (Northern Africa) and points out the different choices of the Westerners and the Easterners writers in the selection of main

18 ibid., p.304.
19 Sen, op. cit.
20 Alberuni, Tahdid, Chapter V.
21 The second chapter of the fifth treatise of al-Qanun al-Mas‘udi is devoted to it. Though it is purely trigonometrical.
meridian. In the literature of the Alexandrian School of Greek geographers, the tradition had been to draw the prime meridian to pass through Alexandria, joining Syene and Meroe in the south and Troas and the mouth of Borysthenes in the north. Ptolemy, however, makes the prime meridian pass through the western island of Faroe in the Canary group islands -- which, according to him, lie two hundred farsaks west of the African coast. These islands are thus the 'initial inhabited locality'.

Some Arab astronomers followed Ptolemy in this respect while others took the western-most point of Africa as the starting point. The Indian astronomers, on the other hand, considered Lanks as the centre of the inhabited world, the extension of the inhabited world being ninety degrees each way. This prime meridian of the Hindus connected Ujjain, the fortress of Rohitaka, the Yamuna and the Cold Mountains (Meru).

As long as a prime meridian is only a line of reference for astronomical location of places eastward or westward, it is insignificant whether one starts from this or that end. However, since it is the basis of many astronomical calculations, its longitudinal accuracy is of greatest importance. A great discrepancy is observed in the determination of longitudes in the works of those authors who begin their reckoning with the

23 Alberuni, Tadbir, p. 120.
25 Ibid., p. 316. It appears that this description of the prime meridian was taken from Pulisa Siddhanta. Following the Hindus the Arabs, at least in the initial phase, adopted the meridian of Ujjain as reference line for longitude estimation. See Sen, S.N., op.cit.
western-most extremity of the inhabited world, so that the whole of the inhabited lies to the east of the prime meridian. In such instances the greater the eastward displacement the greater is the amount of error. Ptolemy's work is no exception to it, and Alberuni has committed gross mistakes in case of those places which lie in the eastern and the southeastern parts of the inhabited world.

RECTILINEAR CORRECTIONS

Alberuni express his doubt as to the reliability of Ptolemy, not on account of lack of accuracy of this author, but because Ptolemy had to rely for a good deal on certain reports from far-away countries. It may be noted that the spread of Islam had expanded the horizons of the known world and communications had much improved in comparison with those in the times of Ptolemy. Hence, Alberuni was on a better footing than his Greek predecessor. He himself rectifies his own results on the basis of corrected rectilinear distances. He allows a deduction of about one-tenth of the reported distance in order to get the straight distance between the places. But it seems that this deduction of one-tenth was not a constant quantity because in some places he makes an arbitrary deduction ranging from one-seventh to one-sixth of the traversed distance — as he has done.

27 Ibid., p. 189.
in the case of Baghdad and Ghazna. It seems that in this he was guided by the nature of the terrain of that particular tract and his personal observations.

RECTIFICATION OF ERROR

Besides the rectilinear corrections which he has applied to his calculations, another important step which he seems to have adopted in the calculation of longitudinal position is the selection of an intermediate station of reference. He takes Ghazna as the intermediary station — a place where he had carried out most of his astronomical observations. Ghazna was used for those places which were on the eastern extremity of the inhabited world. This was a remarkable step in as much as it proves his sincere and conscious effort to overcome the pitfalls of his predecessor, Ptolemy.

THE CONCEPT OF LONGITUDES

Alberuni considers that while latitudes have a determinate beginning and an end, longitudes are hypothetical lines and have no actual beginning or end. To elaborate this point he asserts that 'as the inhabited world does not spread over a complete circuit, there exist two extremities in longitude, an eastern extremity and a western one', covering from one extremity to the
other one hundred and eighty degrees. In keeping with the practice of the other Muslim astronomers of his times, Alberuni reckons the longitudes of places from the western end.

**DIVERGENT RESULTS**

There seems to have been a great deal of confusion in Alberuni's time regarding the initial and final meridians and he discusses the views of various Muslim astronomers on this issue. In practice, it matters little from which line of reference one begins the reckoning, but the situation was so confused that Alberuni was obliged to observe the longitude of a place as determined by the easterners was at a variance of about ten degrees from that determined by the westerners. He notes that al-Fazari has assumed the difference to be thirteen and a half degrees, and remarks that if al-Fazari's initial meridian were to pass through the Immortal Isles, then the final meridian should be $13^d30^m$ beyond the eastern extremity and if the initial meridian were to be reckoned from the east, then the final meridian in the west should fall beyond those isles by $13^d30^m$. Alberuni also furnishes the example of Baghdad, whose longitude is given by him as seventy degrees and by others as eighty degrees, and comments that this difference is due to the reasons given above.

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32 ibid., p. 121.
33 loc. cit.
34 loc. cit.
35 loc. cit.
CUPOLA OF THE EARTH

Muslim astronomers, following the tradition of their Hindu counterparts, used to assign the central position of the Oekumene to Lanka, and called it the Cupola of the Earth. To Alberuni this was just a metaphorical expression, for no place on the earth, in whatever form and shape the earth might be conjectured, could be denoted by the term 'Cupola'. This term could be an expression for denoting the equal extent of the inhabited world on either side. He remarks that what the Hindus meant by 'Cupola', which was wrongly translated in Arabic literature, was 'central position'. The line on the equator passing through Lanka was taken by the Muslim astronomers of Spain of a much later date as the prime meridian.

HINDU METHOD OF DETERMINING LONGITUDES

The Hindu method of reckoning longitudes involved the measurement of distances in yojanas, starting from the meridian of Ujain. 'And the more to the west the position of a place is, the greater is the number of yojanas; the more to the east, the smaller is this number.' Alberuni finds it immaterial whether one reckons the longitudinal difference in day-minutes, one-sixtieth of a circle, or in farsakhs or yojanas. But the Hindu methods of longitudinal determination, he finds 'totally different; and howsoever

37 Sen, op.cit.
39 Ibid., p. 311.
different they are, it is perfectly clear that none of them hits the right mark. 40

In his Kitab al-Hind, Alberuni elaborately refers to the different methods mentioned in the various ancient Sanskrit sources. One of the methods which he cites is the determination of 'deshantra', i.e., the 'difference between the places' by multiplying it by the mean daily motion of the sun and dividing the product by 4,800. The quotient then is to be added, in order to get the longitude of the place in question, to the 'mean place of the sun, as it has been found for noon or midnight of Ujain'. 41 The result obtained is in yojanas. This procedure is described by Brahmagupta in Khandakhadyaka. 42

Alberuni does not consider 4,800 yojanas as the correct length of the circumference of the earth. However, the different lengths assigned to the radius of the earth given by Pulisa and Brahmagupta must lead to different results in the computation of the circumference of the earth. 43 It is possible that this distance of 4,800 yojanas was considered as the circumference of the parallel of latitude passing through Ujain.

Of the Hindu methods, there is only one -- that given by Pulisa -- which he finds correct. This method involves the simultaneous observation of the lunar eclipse at two different places. The difference between the occurrence of the eclipse is

40 loc. cit.
41 ibid., p. 312.
42 See, op.cit.
multiplied by the circumference of the earth and the product is divided by sixty. The quotient is the number of yojanas between the two places. A method of finding the correct circumference is given in Karana-tileka, to which Alberuni refers. The method is based on the simple assertion that the radius of the parallel of circles decreases in the same ratio as the straight shadow of a gnomon increases poleward. There were two methods adopted by Hindu astronomers for the determination of the longitudinal differences:

i) from the time difference of an eclipse obtained from two places, and

ii) from the latitudinal differences of and the linear distance between two places.

Alberuni took note of both these methods given in Indian astronomical texts.

ALBERUNI'S METHODS OF DETERMINATION OF LONGITUDES

The various methods which Alberuni employed for the determination of longitudes seem to rest on the rationale of determining the longitudinal difference between different towns. He asserts that 'if we obtain that the difference, we do not need these final and initial meridians'. By the longitudinal difference he means 'the arc which is intercepted on the equator between their meridian circles, or that intercepted on any one of the parallel and similarly situated circles'.

44 Ibid., p. 314.
45 Sen, op.cit.
46 Alberuni, Tabijid, p.121
47 Ibid., p. 122.
quadrangle thus formed is then converted into a planimetrical
quadrangle 'by substituting for the spherical lines or arcs
corresponding chords, the length of which is expressed in radials
and their sexagesimal parts'.

The attempt to determine meridians of longitude involves
greater difficulties than the measurement of latitudes. Gnomon is
of great help in the absence of any precise instrument for
determining longitudes, mathematical devices have to be resorted
to. Alberuni did the same. His operations placed much reliance
on two factors: first, the latitudinal position of towns and,
secondly, the distance between them. He has no trust in the 'time
method' of Hipparchus which involved a comparative observation of
eclipses. He considers the 'distance method' to be superior to
the 'eclipse method', because 'the first appearance and the
end of the visibility of the eclipse, which are its most critical
moments (aghar awqatihi), can be observed only approximately'.

Alberuni realizes that the subject of longitude is intimately
connected with that of time, and consequently he directs his
investigation of longitude in that direction. He takes up three
different cases where the coordinate would differ and wherein
the moments of 'sun-rise', 'mid-day', 'sun-set' and 'mid-night'
would also differ. 'If we want to determine the longitudinal
difference between one city and another, we have to ascertain

48 Kramers, Al-Biruni Commemoration Volume, p.177.
49 ibid., p. 184.
50 loc. cit.
51 A usual method of time expression in Medieval times.
the same moment of time itself in both of them. Since the beginnings of daylights and nights as well as their middles and ends are different, it is impossible to determine the same moment of time relative to that part of daylight, or the night, that has passed, because it is simultaneously different in both of them unless the rising of the sun and its setting coincide with the points of intersection of their horizons'.

THE ECLIPSE OF THE MOON

The absence of chronometer and lack of facilities such as the modern wireless transmission to check up the time at two distant places as also the fact that an element of error may creep into observation with naked eye presented difficulties in the recording of the rising and the setting of the sun and led Alberuni to remark that it was necessary to 'rise from (the earth) into the air 'for the observation of celestial phenomena.'

The eclipses of the sun and the moon are two universally known phenomena. According to Alberuni, the eclipse of the moon has a far greater scope of observational accuracy than that of the sun. In his own words, 'It is truly more reliable, and the people of the profession have aimed at it for connecting longitudes'. He explains that from the study of the time of lunar eclipse different cases may arise. If we are given the time, recorded in each town, relative to midnight, we may have

52 Alberuni, Tahdīd, p. 128.
53 Ibid., p. 129.
54 loc. cit.
any one of the following six cases:

(i) If it is, at each of them, on the line of midheaven, the two towns are on the same meridian and there is no longitudinal difference between them.

(ii) If it is midnight at one of them but before midnight at the other, the first town is east of the second by an amount equal to the advance of the eclipse ahead of its midnight.

(iii) If it is after midnight at the other, then it is east of the first by the amount of lag of the eclipse behind its midnight.

(iv) If it is before midnight at both of them, the longitudinal difference between them is the difference between the hours remaining up to the respective midnight at each of them, and the one whose remaining time is bigger is west of the other.

(v) If it is after midnight at both of them, the longitudinal difference between them is the difference between the hours elapsed after the respective midnight at each of them, and the one whose elapsed time is bigger is east of the other. If there is no difference between the remaining times of the elapsed times, then there is no longitudinal difference between the two towns.

(vi) If it is after midnight at one of them and before midnight at the other, the longitudinal difference between them is the sum of the time elapsed after midnight at the one end and the time remaining until midnight at the other, and the town where the

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55 Ibid., pp. 152-53.
eclipse occurs after midnight is east of the other.

Alberuni employs his astronomical ingenuity in the determination of longitudinal differences between places by simple trigonometrical deliberations. And the results thus obtained are so near to modern values that they indicate a high degree of accuracy in his measurements. An example of this is the fact that he calculated the difference between the longitudes of Shiraz and Baghdad as amounting to $6^d33^m32^s$ which exceeds the modern difference by a very small fraction (the modern difference being $6^d16^m$).

Alboruni mentions in al-Qanun al-Mas'udi and Tahdid that in order to justify his claim of accuracy he had taken a large number of towns in pairs, e.g., Ghasna and Baghdad; Shiraz and Ghasna; Baghdad and al-Rayy; al-Jurjaniya and Ghasna; Ghasna and al-Iskandariya. In the case of the last pair, Alberuni was highly confident of being accurate, because it was at Ghasna that he had had his last observation of the sun and al-Iskandariya, was known to him as the place where Ptolemy himself had carried out his observations, and as such likely to be more reliable.

Of the towns whose longitudinal differences he was to ascertain, he first found out their road distances and after subjecting them to 'straightening corrections' applied them with the latitudinal differences in a simple trigonometrical operation. Thus, the formula to obtain the length of the arc between their

56 Sen, op.cit.
meridians at the equator was as follows:

\[
\text{longitudinal arc at the equator} = \frac{(\text{Distance})^2 - (\text{Latitudinal difference})^2}{\cos \text{of Lat. of one} \times \cos \text{of Lat. of the other town}}
\]

He arrived at this conclusion through fifteen arithmetical operations, mostly executed in the laborious sexagesimal system.

MEASUREMENT OF A TERRESTRIAL DEGREE

The measurement of the spherical surface of the earth and the related problems comes under the purview of the recent science of Geodesy, but the roots of this science go back quite deep in antiquity. The earliest recorded notions of the rotundity of the earth are those of ancient civilisations, which held the view that the earth was a circular plane. The early Greeks held a similar view and we find that in the Homeric poems the earth is conceived as a geometrical circle. With the passage of time, however, the earth came to be regarded as a sphere. Pythagoreans were probably the first to hold an empirical view that the earth was a sphere. Many natural philosophers and writers of a later date such as Hecateus and Herodotus did not stick to this view and it was left to the genius of Aristotle to expound and prove the sphericity of the earth. Most of the succeeding geographers and astronomers followed him and Strabo, being one such, very convincingly elaborated this point of view. Indian astronomers also conceived the earth to be a perfect sphere. Muslim astronomers, it may be noted, were influenced by both Greek and Indian sources and readily
accepted the Greek and Indian ideas in this respect and even went to adduce better and newer proofs.\textsuperscript{58}

ARAB'S INTERESTS

Due to the lack of standardization of the units of linear measurements such as stadia and yojanas, and on account of the inconsistencies in the results obtained by earlier astronomers, the Muslims took upon themselves the task of finding the accurate circumference of the earth. The attempts made by the Arabs in the measurement of the terrestrial degree were of great significance and came very close to modern computations.

The incentive which led the Muslims in this special field of geography largely came from the Caliph (A.H.198-218/ A.D. 813-833). He ordered the astronomers of his times to determine scientifically and precisely the measurement of an arc of the meridian, and there with the help of that measure ascertain the diameter and the circumference of the earth.\textsuperscript{59} Indeed, with al-Mamun begins the scientific period of Islamic culture,\textsuperscript{60} and the Translation Bureau which he established immensely helped the Muslim scholars in their acquaintance with the works of Greeks and Indians. al-Mamun was himself not satisfied with the translations and could see through discrepancies therein, so that he sought the help of his astronomers to attend to the practical side of the work and to remove inconsistencies and fill the gaps in

\textsuperscript{58} Burni, S.H., Muslim Researches in Geodesy, \textit{Al-Beruni Commemoration Volume}, p.3.

\textsuperscript{59} Burni, \textit{Al-Beruni Commemoration Volume}, p. 5.

\textsuperscript{60} Ahmad, M., \textit{Muslim Contribution to Geography} (Lahore, 1947), p.73.
knowledge. In this pursuit a number of Astronomical Tables (az-zij) were prepared by various astronomers. al-Mas'udi in his Murūj ud-Dhabab (Meadows of Gold) and Ibn Yunus in his Kitab al-Zij al-Kabir have given their assessment of this work. Sind bin 'Ali bin Abdul Malik al-Mawarudi, on the instance of al-Mamun, went to Raqqa and Tadmur (Palmyra) to calculate the length of one degree of the arc of meridian round the surface of the earth and this they found to be 57 miles.\textsuperscript{61} Habash reported the length as 56\(^{1}/4\) miles.\textsuperscript{62} It is evident from these instances that the results obtained by the Muslim astronomers were often divergent.

Driven by his keen sense of accuracy, al-Mamun sent to the plain of Sinjar another team of three, consisting of Khalid, al-Ustaqlabi and Ahmad bin al-Sukhtari ul-Zara. The members of the team observed the sun's altitude at the given meridian and then they divided themselves into two parties, Khalid in one and the latter two in the other. The parties then proceeded in opposite direction polewards keeping straight courses. They reached and spotted the place where the sun's declination was one degree and then they found out the linear distance between the two places. They found that the single arc of the earth measured as 56 miles.\textsuperscript{63}

Alberuni narrates this method of reckoning in his Tahdīd. He, however, observes that in Farghani's reports an arc is 56\(^{2}/3\) miles rather than 56 miles. He himself checked the accounts of Habash and found the former figure correct.

\textsuperscript{61} Burnil Al-Biruni Commemoration Volume, p.10.
\textsuperscript{62} I.e. cit.
\textsuperscript{63} Ibid., p.13.
In Kitab al-Tafhim li Sunna'at al-Tanjeem, written by
Alberuni in Arabic and Persian languages separately, he gives
another account of his observations. 64 He says:

If any one wants to know them (dimensions of the earth)
by such distances as are well known and in vogue amongst
the people, let him know that the Earth's diameter is
2,163 2/3 farsangs, and its circumference 6,800 farsangs,
and according to those calculations the total area of
the Earth's surface from outside in the manner in which
you multiply one cubit into one cubit, i.e., square
cubit, is 14,712,727 1/4 farsangs and the total area of
its whole body (as a globe) in the manner of multiplying
one cubit into one cubit square, i.e., cubic cubit, is
166,744,242 2/5. 65

ALBERUNI'S METHOD

Alberuni had the ambition of finding the length of an arc by
noting the dip of the horizon and was looking for a suitable
place which would fulfill the required conditions. He got this
longsought opportunity in India and in al-Qanun al-Mas'udi, he
states the method which he adopted in India for determining the
value of an arc of a degree. Resorting on the mountain of Nandana
in India, he determined its vertical altitude and then climbing
on the peak of the mountain, he observed the extent of the dip of
the horizon and found out the true value of the angle. He reckoned
one degree as of about 57 miles. 66 It is interesting to note that
his value of the length of an arc varies in different accounts.
Sometimes he takes the figure as 56 miles (as in Tahdid) and
sometimes as 56 2/3 miles (as in al-Qanun al-Mas'udi and al-Tafhim)

64 Ibid., p.14.
65 Ibid., p.15.
66 Ibid., p.18.
or 57 miles (as in some places in al-Qanun al-Mas'udi). The explanation may lie in the inconsistency in the standards of measuring units employed in such operations.

The method which Alberuni refers to in al-Qanun al-Mas'udi is a novel device of his own and he discusses it elaborately in his book on astrolabe - al-Kitab fi al-Astrolab. The method is:

You climb a mountain situated close to the sea or a level plain, and then observe the setting of the sun and find out the dip of the horizon we have already mentioned, and then find the value of the perpendicular of this mountain. You multiply this height into the sine of the complementary angle of the dip, and divide the total by the versed sine of this dip itself. Then multiply (the double of) the quotient into 22 and divide the result of this multiplication by 7. You will get the length of the Earth's circumference (in the same terms or proportion) in which the height of the mountain has been fixed.67

In Tabbid he reports his calculations, based on the above method. He made his observations on the mountain of Nandana and found the length of a single degree to be 55 miles 58' 55".68 The results thus obtained by Alberuni are, as he himself points out, very close to what were worked out by al-Mamun's astronomers.

 Conversion of Miles into Meridian Degrees and Vice Versa

In al-Qanun al-Mas'udi, Alberuni envisages a method of converting the miles of the distances into the degrees of the meridian circle, by multiplying the miles by 3 and dividing the multiplication by 170. For the conversion of degrees of a distance

67 Ibid., p. 33.
68 Ibid., p. 35.
into miles the reverse process may be applied, i.e., first multiplication by 170 and then division by 3. But since the one-third of a degree is 20', he suggests that instead of dividing by 170 may be multiplied by 20' so as to get the miles. 69

**OBLIQUITY OF THE ECLIPTIC**

The ecliptic and celestial equator are the traces upon the celestial sphere of the planes of the earth's orbit and equator, respectively, and the angle between these circles is called the obliquity of the ecliptic, and its value is determined to be 23.5 degrees. In other words, the earth's axis makes an angle of this magnitude with the vertical plane, that is, the axis is inclined to the plane of the orbit by 66.5 degrees. Since this inclination of the earth brings in its consequence many important phenomena such as the changes of season and the duration of day and night. Also, it is significant in the demarcation of the zones of Climates and the fixation of latitudes.

From the earliest times astronomers were concerned with the determination of the obliquity of the ecliptic. Ptolemy found that this inclination amounted to $23^d51^m20^s$. 70 and he held it as constant and invariable. The Muslim astronomers showed a similar concern and one of them, al-Battani, estimated the amount of declination to be $23^d35^m$. 71 On the authority of Alberuni, we also

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69 Ibid., pp. 30–32.
70 Alvi, *Arab Geography in the Ninth and Tenth Centuries*, p. 47. Alberuni reports Ptolemy's estimate as half of $47^d42^m30^s$ which comes to $23^d51^m15^s$. However, for constructing the table of declination, he took into account $23^d51^m20^s$. See *Tablid*, p. 59.
71 loc.cit.
know that Yahya bin Mansur calculated the maximum declination as $23^\circ 33^\prime$. This was done in the year A.H. 213, and al-Mamun, rejecting this observation, ordered Khalid bin 'Abd al-Malik al-Marwarudhi to work it out again at Damascus. In between 216 and A.H. 218, al-Marwarudhi carried out his observations and from these Alberuni derived two results about the declination of the earth: 23; 33; 57; 30 and 23; 34; 27; 30. He, however, disregards the latter result, because the period of observation was only approximately one year. Moreover, he further remarks that Samad bin 'Ali who supervised the work of Khalid has reported the figure of Khalid as 23; 33; 52, which is very close to Khalid's 23; 33; 57; 30. Mohammad and Ahmad, sons of Musa, in the year A.H. 243, observed the maximum declination at Surra-man-ra'a as 23; 34; 30 but again in 254 and 255 A.H. at Baghdad they found it to be $23^\circ 35^\prime$. This figure tallies with that which al-Battani obtained at al-Raqqa. Alberuni quotes a series of other observations carried out by astronomers before his time and he assesses them critically. He does not seem to be satisfied with the works of his predecessors and comments that 'an observer should keep alert, constantly scrutinizing his work, promoting his self-criticism, moderating his self-admiration, and pursuing his researches without impatience or boredom'.

ALBERUNI'S DEFINITION OF MAXIMUM DECLINATION

'The maximum declination is the magnitude of the angle formed by the intersection of the plane of the celestial equator and the

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72 Alberuni, Tahdid, p.60.
73 ibid., p.77.
plane of the ecliptic (sun's apparent path round the earth). It is also called the total declination, and is equal to the arc between the poles of those two circles. 74

METHODS OF DETERMINATION

The obliquity of the ecliptic, according to Alberuni, without the aid of the latitude of the place of observation is made by two methods:

1) by obtaining the maximum altitude of the sun in the celestial meridian at the place of observation and also its minimum altitude at that place. The half of the difference between them when they are on the same side of zenith or the half of the sum of their compliments when on opposite sides of zenith, gives the maximum declination.

2) by obtaining both the altitudes and also an altitude of the sun of known azimuth on that day. Alberuni considers the former as most reliable and it is interesting to note that almost all the astronomers, whose observations have been cited above, determined the declination by this method.

To cite an example from al-Marwarudhi:

Maximum altitude of the sun at Damascus (A.H.217) 60;3;55
Maximum altitude of the sun at Damascus (A.H.216) 32;56
Difference in altitude 47;7;55
Half of the difference 23;33;57;30 that is, the obliquity of the ecliptic.

74 ibid., p. 58
Observational Studies

Alberuni engaged himself for a number of years, specially since A.H. 384, in the pursuit of more accurate and satisfying results. The work fascinated him, and became one of his life missions. Many a times his work was interrupted by unfavourable circumstances. These interruptions (in between 384-65 A.H.) were of political nature and they not only hampered his work but also compelled him to forsake his place of work. However, as times change, we find him once again setting down to his work.

At Jayhun, he observes the declination to be 23; 35; 45 degrees and this finding rests on his own method. It is by this method that he later determined the maximum altitude at Jurjaniya as 71; 18 and the complement of the latitude as 47; 42; 10. The difference between them, which is 23; 35; 50 gave the maximum declination.

At Ghazna his researches gave the maximum declination as 23; 35. However, he concludes that 'all the testimonies that we have adduced point out collectively that the maximum declination is 23 degrees, plus one-third and one-quarter of a degree (i.e., 23; 35 degree). The slight excess or defect in some of the astronomers is due to the instrument (of observation)'

75 By this method Alberuni first determines the maximum altitude and the colatitude of the place of observation and then subtract one from the other. The difference gives the declination. See Tahdid, p. 77.
76 Alberuni, Tahdid, p. 79.
77 ibid., p. 83.
ALBERUNI'S CRITICISM OF INDIAN OBSERVATIONS

Alberuni is very critical of the result obtained by Indian astronomers. Their erroneous estimation of a maximum declination of 24 degrees excites him to comment that 'they (Indians) are far from being critical investigators, and that he does not have confidence in any claim to precision in their observations'.

He does not spare even those of his compatriots who side with Indians in their investigations. He has little trust in al-Makki's line of reasoning and he sees no validity in Makki's argument that the difference in Indian observations is due to the fact that the Indians referred their observations to the centre of the universe, while others refer to the surface of the earth. Alberuni in this connection remarks that if this were the case, the maximum declination, after making necessary adjustments for parallax, should have been less, and not more, than the declination measured by other observers.

DETERMINATION OF DISTANCES AND COORDINATES

As noted earlier, Alberuni did elaborate work on the determination of the longitudinal difference between places. It may be recalled that his methods for such determinations were purely astronomical or trigonometrical. We have already reviewed the astronomical methods and examined their utility and authenticity. Now we may take up a problem which was historically

78 ibid., p. 79.
79 loc. cit.
significant and its importance was realized by Alberuni himself. In fact, Alberuni devoted one of his works to this purpose and considered it to be a real contribution. This work was entitled Kitab Tahdid Nihayat al-Amakin li Tashih Masafat al-Masakin (The Determination of the Coordinates of Positions for the Correction of Distances between cities). It must be noted that Alberuni's researches in Kitab Tahdid were in pursuit of the determination of the longitude of Ghasnah, which he ultimately did very successfully. The significance of Alberuni's contributions lay in the fact that he was able to develop a sort of a relationship between four independent factors, namely, (i) the linear distance between the two towns; (ii) the longitudinal difference between them; (iii) and (iv) the respective latitudes of the two towns involved. Alberuni asserts that the interrelationship between these four factors is such that if three of these factors are known, the fourth can be deduced from them. Elaborating his contention he says that there are three combinations in this calculation: First, when the two latitudes and the longitudinal difference between them is known and the distance is to be ascertained. This has already been discussed. Secondly, when the two latitudes and the longitudinal difference are known and the distance is to be calculated from them. Thirdly, when one of the latitudes is to be determined by the known combination of distance, longitudinal difference and one of the latitudes. The last two combinations and

80 Alberuni, Tahdid, pp. 189-90 and 261.
81 Ibid., p. 360.
their problems were considered so significant by Alberuni that he was tempted to write *Tahdid*. It should be noted that Alberuni applied trigonometrical methods to find the solutions of the above mentioned problems. As the methods themselves are outside the scope of the present work, we would leave them out.

In his 'distance method', Alberuni takes the help of a few stations whose astronomical positions on the terrestrial globe had already been ascertained. These he called 'reference bases'. He considered Baghdad to be the most suitable reference base for the determination of longitudes as it was a renowned centre of learning where astronomical observations were recorded generally correctly. For other known latitudes, he took Shiraz, Sijistan, Rayy, Nishapur, Jurjaniya and Balkh. Besides these seven towns, he used a few others for corroborative evidences. In addition, Alberuni uses a few other towns merely for 'measuring one of them against another until (his) mind is fairly satisfied with the longitudes obtained' and thus he can gradually and successfully 'proceed to the intended base at Ghasna, because (his) observation and operations are based at it'.

The operation, which is actually completed in fifteen steps, is a tedious arithmetical contrivance, but each step has been very carefully laid down and verified trigonometrically. Initially, for the determination of longitudinal difference between the two towns, Alberuni lays down two requirements: first, the latitudes

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82 loc. cit.
83 loc. cit.
of the towns and, second, the straight terrestrial distance separating the towns. So far as the determination of the latitudinal positions of the towns is concerned it has either been determined by him personally or taken from earlier authentic works of renowned Arab astronomers. In this connection the authors whose findings are considered trustworthy by Alberuni, mention should be made of al-Rhwarismi, Habash al-Hasib, Abu 'Ali al-Sinawi (Avicenna), Sulaiman bin 'Ismat al-Samarqandi, Abu al-Hasan Ahmad bin Muhammad bin Sulaiman, Abu al-Fadl al-Hirawi and a few other authors of the gij. These reportings were, as usual, checked and re-checked by him and more than often were from indirect methods corroborating them from other sources.

The determination of linear distances between places was not always physically possible for him to ascertain personally and, therefore, in spite of his meticulous temperament, he had to depend on travellers' reports. But here again he moved with caution and never accepted a statement unless it had been verified. Another important step that Alberuni had taken in ascertaining straight horizontal distances was a certain amount of deduction made on measured distances. This deduction, which often ranged between one-tenth to one-sixth of the total distance, was not always wholly arbitrary but was made attuned with the curves, windings and diversions of the roads dependent upon the terrain of the land. For instance, while calculating the linear distance between Jurjaniya and Rayy the total deduction made was one-sixth on the measured distance of 185 farsakhs,84 while in the case of Darghan

84 ibid., p. 206.
and Amuya it amounted only to one-tenth of the distance. Alberuni claimed that as compared to the terrain between Baghdad and Rayy, the terrain of the route between Jurjaniya and Rayy is more rugged and difficult, and hence in this case one-sixth should be a reasonable deduction. On the contrary, the road between Darghan and Amuya is a straight stretch on level ground and therefore the deduction should be accordingly less, i.e., one-tenth. There are other instances also where the total deduction comes to even less than a tenth. In the case of Jurjaniya and Balkh, for example, Alberuni drops only one-fifteenth part of the distance. These examples will suffice to show that Alberuni kept a close watch on the terrain and the nature of the road while determining the straight distances which he efficiently used in his calculations. It may also be pointed out that after making necessary deductions from the total distance, Alberuni often rounded off the remainder, probably to facilitate calculations. An instance will make it clear. In the case of Jurjaniya and Balkh Alberuni suggests a deduction of one-third of one-fifth in the measured distance of 150 farsakh but instead of taking 141.25 farsakh he rounds it off to 140 farsakh.

Having determined the latitudes of the two towns and their distance in a straight stretch Alberuni sets out to determine the longitudinal difference between them in the following order.

85 Ibid., p. 222.
86 Ibid., p. 216.
87 loc. cit.
1. Calculates the latitudinal difference between the towns, say, A and B.
2. Determines its chord value, i.e., sin of arc in 1.
3. Calculates the square of the chord in 2.
4. Converts the linear distance between A and B into equatorial degrees.
5. Determines the chord value of 4.
6. Calculates the square of the value in 5.
7. Calculates the difference between the squares of the two chords, i.e., subtracts 3 from 6 or vice versa.
8. Multiplies the difference between the squares of the chords by cosine of one of the latitudes of the towns, say, of A.
9. Divides the product in 8 by the cosine of the latitude of the other town, i.e., B.
10. Extracts the square root of 9.
11. Multiplies the square root by total sine.
12. Divides the product in 11 by the cosine of latitude of town A.
13. Converts the chord value in 12 into an arc of the equator and this gives the longitudinal difference between the towns A and B.

Before taking up concrete examples from Alberuni's works it will be worthwhile to discuss in some length Alberuni's method for converting linear distances into equatorial degrees and also in the reciprocal order and for converting the arcs of the equator into chords. So far as the conversion of terrestrial miles into equatorial degrees is concerned, although Alberuni had his own values of 55; 53; 15 miles for a degree (which he found very

88 Ibid., p. 189.
close to Habash's estimates of 56 miles per degree), he somehow prefers to use the values computed by al-Farabi, i.e., \(56^{2/3}\) of a mile per degree. As has already been said elsewhere in this discussion, Alberuni determined his value of a degree while living in the fort of Mandana in India while other estimates were generally made in the plain of Sinjar near Mosul specially under the patronage of Caliph al-Ma'mun.

The method adopted by Alberuni for the conversion of an arc into a chord has been very elaborately explained by him in the seventh chapter of the third treatise of *al-Qanun al-Mas'udi*. Alberuni has condensed his results in the form of a table in chapter six (from which an extract has been given as an appendix (See Appendix I). As would be noted, the table of the sine has been worked out by him at a regular interval of quarter of a degree, beginning from 0 and ending at 90 degrees. The intermediary arcs and their sines have been determined by him by interpolation according to the method explained by him in chapter seven of the same treatise. For example, the value of arc 12 degree 36 minutes can be calculated by looking into the table between 12 degrees 30 minutes and 12 degrees 45 minutes, in between which it lies. The values inserted in the table for the arcs 12 degrees 30 minutes and 12 degrees 45 minutes are 12; 59; 30; 39 respectively, and from this the intermediary value of arc 12 degrees 36 minutes can be easily determined by the simple

89 Ibid., p. 179. See also Table, pp. 181-2.
90 Ibid., pp. 178-9.
method of interpolation. It would be observed that sometimes
Alberuni's inserted values do not tally which may be either
due to bad copying by the transcribers or due to his own slips
in the calculations. He had himself admitted at one place that
since he is mentally worried he is liable to commit errors in
his calculations. However, Kramers' note on the mistakes in
Alberuni's calculation for the chord value of an arc of 8 degrees
6 minutes seems preposterous as the sine values of high orders
cannot be obtained by doubling or trebling the lower orders
because the sine values, as is evident from Alberuni's sine table,
do not change uniformly and fixedly. Kramers should have searched
the value between 8 degrees (8 21 1 23) and 8 degrees 15 minutes
(8 36 34 24) instead of doubling the sine values of 4 degrees
(4 11 7 24) and 4 degrees 15 minutes (4 26 47 26) for obtaining
the sine of 8 degrees 6 minutes. Again, Kramers' quoted value
for 4 degrees 15 minutes does not tally with the correct value
given by Alberuni in his table (Kramers gives 4 26 27 26).92

In his eighteen odd examples cited in Tahdid Alberuni puts to
a good test his 'relationship method' for the determination of
any of the unknown factor amongst longitudinal difference, distance
and the latitudes of the two towns. Surprisingly his method is
generally successful, although here and there minor discrepancies
are met. The eighteen examples fall into three categories. The
first includes those cases where longitudinal difference between
the towns have been ascertained. The second consists of those

91 Ibid., p. 201.
cases where longitude and latitude of a town has been determined from the known longitude and latitude of another town. The last category includes those cases where the distance between the two towns has been calculated from other known data. We would give here at least one example from each category to show the worth of Alberuni's contribution.

DETERMINATION OF THE LATITUDINAL DIFFERENCE BETWEEN JURJANIYA AND RAYY

1. Alberuni's own determined latitude of Jurjaniya
2. Latitude of Rayy (as determined by Khujandi and confirmed by al-Hirawi)
3. Latitudinal difference
4. Its chord
5. Square of the chord
6. The linear distance 463 miles shown in degrees
7. Its chord
8. Its square
9. Difference between the squares
10. Multiplying 9 by cos lat. of Jurjaniya
11. Dividing the product by cos lat. of Rayy
12. Extracting square root of 11
13. Multiplying 12 by total sine
14. Dividing 13 by cos lat. of Jurjaniya
15. Converting the chord value in 14 into an arc

and this is the longitudinal difference.
DETERMINATION OF THE DISTANCE BETWEEN BUKHARA AND BALKH FROM THEIR LONGITUDES AND LATITUDES

1 Longitudinal difference between the two cities 2;49,48 degrees
2 Converting 1 into a chord 2;57,55
3 Multiplying 2 by cos lat. of Bukhara 137,34,29,44,0
4 Dividing the product in 3 by the total sine 2;17,34
5 Multiplying 2 by cos lat. of Balkh 142,37,15,50,56
6 Dividing the product in 5 by the total sine 2;22,37
7 Multiplying 4 by 6 5,26,59,17,58
8 Obtaining the latitudinal difference between the two cities 2,38,24 degrees
9 Converting 8 into chord 2,45,52
10 Finding square of 9 7,38,31,45,4
11 Adding 10 and 7 13,5,31,3,2
12 Its square root 3,36,56
13 Converting 12 into an arc 3,27,11 degrees
14 Multiplying 13 by 56, 40 miles which is a measure of 1 equatorial degree, the distance comes to 195,40,23 miles
15 Converting 14 into farsakhs by dividing the value by 3 miles 65,13,26 farsakhs

DETERMINATION OF THE LONGITUDE AND LATITUDE OF AMUYA FROM THE LONGITUDES AND LATITUDES OF BALKH AND JURJANIYA

1 The linear distance between Jurjaniya and Amuya 85 farsakhs
2 Dropping 5 farsakhs and converting into miles 240 miles
3 Converting 2 into equatorial degrees
by dividing 56, 40 miles = 4; 14, 17 degrees

4 Calculating the chord (sine) of 3

5 Multiplying 4 by sine

6 Dividing 5 by total sine

7 Finding the square of 6

8 Finding square of 4

9 Subtracting the two squares

10 Extracting square root of 9

11 Finding its arc

12 Adding 11 to the co-latitude of Jurjaniya, i.e., 47; 43 degrees

13 Converting 12 into chord

14 Subtracting 12 from 90 degrees

15 Dividing 5 by 13

16 Converting 15 into an arc

17 Adding 16 to the longitude of Jurjaniya
(84; 0.54 degrees already known)93

All above examples were given by Alberuni in Tahdid Nihayat
al-Amakin. However, realizing the significance of his work, he
took up the subject afresh in the second chapter of the sixty

treatise of al-Qanun al-Mas'udi in his discourse on the verification
of the longitudes of Ghasna and al-Islamdariya. It is certainly

93 Alberuni, Tahdid, p. 215.
a credit to Alberuni that he evolved dependable methods which would yield remarkable results.

DETERMINATION OF THE DIRECTION OF QIBLA

Alberuni was keen to determine the correct longitudes and latitudes of towns not only for the sake of scientific accuracy but also because it had other uses. One of the most significant uses, for example, which the Muslims can make, is for the determination of the exact direction of the Ka'ba from the place of worship, for they are required to face the Ka'ba while saying their prayers. Alberuni argues that such benefits do not accrue only to the Muslims but followers of other religions also need to know the exact azimuths of their holy shrines from the places where they live. For the Jews, the knowledge of the direction of Jerusalem is important. Similarly, the east-west line is for the Christians and the meridian line is for the Sabians.94

Furthermore, it may be pointed out that the determination of the exact time of the noon, the midafternoon, the setting of the twilight, and the break of the dawn as well as the computation of the time of the visibility of the new moon, all of which have religious significance, depend on the precise determination of the latitudes and longitudes.

Alberuni's four methods of determining the latitudes and longitudes of towns gave reasonably correct results and he proved theoretically also that if the latitude and longitude of Mecca in

94 ibid., pp. 258-9.
association with those of another town, say, Ghazna, be known, the
azimuth of the former can be computed from the latter. In all
his methods he aims to measure either the 'arc of azimuth' or the
'gauge' by means of which the direction of the Qibla may be fixed.
It may however be mentioned that the results of his four methods,
which all are directed towards Ghazna, do not exactly tally with
each other. For instance, in the first method the value of the arc
which gives the direction of the Qibla at Ghazna measured from its
true south point on the horizon circle is computed as 70; 48, 15
degrees; in the second method, it is 70; 47, 13; in the third
method the value of the arc of displacement is 70; 49, 16 degrees;
while in the fourth method it is 70; 46, 56 degrees. These minor
variations in the results may partly be attributed to the
inconsistent rounding off the figures in his calculations.

The four methods are based on very elaborate and intricate
trigonometrical methods. Alberuni himself remarks that they are
beyond the comprehension of layman, for whom he had devised a
simpler method for the determination of the direction of the
Qibla. For lack of space, we shall confine ourselves to only one
of his trigonometrical methods along with the simpler method.

**TRIGONOMETRICAL METHOD**

The pre-requisite of this method is the knowledge of the
latitudes and longitudes of Mecca and of the other town, say,
Ghazna. From one of these data the longitudinal difference should also be calculated. Now let the longitude of Ghazna is west 94; 22, 24 degrees and its latitude north 33; 35 degrees. The latitude and longitude of Mecca are 21; 50 degrees and 68; 0 degrees respectively. The longitudinal difference between the two towns is 26; 22, 24 degrees.

PROCEDURE OF THE METHOD

1. Calculating the cosine of the latitude of Ghazna
   49; 59, 5

2. Calculating sine of the longitudinal difference
   27; 35, 14

3. Multiplying 1 by 2
   137; 56, 22, 42, 10

4. Dividing 3 by the total sine
   22; 58, 56 (sine of the perpendicular 22; 31, 19 degrees)

5. Determining the cosine of 5, i.e., the sine of the complement of 5
   55; 25, 26

6. Finding sine of the latitude of Ghazna
   33; 11, 20

7. Multiplying 7 by the total sine
   199; 20, 0

8. Dividing 8 by 6
   35; 55, 44

9. Converting 9 into arc of degrees
   36; 56, 48 degrees

10. Difference between 10 and the latitude of Mecca (21; 50 degrees)
    15; 6, 46 degrees

11. Calculating the cosine of 11
    57; 55, 29

12. Multiplying 12 by 6
    321; 24, 48, 7, 34

13. Dividing 13 by the total sine
    53; 30, 25

14. Converting 14 into an arc
    63; 5, 54 degrees

15. Calculating the cosine of 15, i.e., the sine of its complement, 26; 54, 6 degrees
    27; 8, 51
17. Multiplying the cosine of the latitude of Mecca and the sine of the longitudinal difference in 2 1538;17,11,24,6
18. Dividing 17 by 16 56;39,50
19. Converting 16 into an arc 70;48,15 degrees

This is the arc which gives the direction of the qibla at Ghazna measured from its true south point on the horizon circle.99

SIMPLE METHOD

Alberuni does not give the basis of this method and, therefore, its authenticity is rather questionable. The method is as follows: Draw a circle of a suitable dimension on the surface of the ground with a diameter north to south and representing the meridian line. Divide this diameter into two equal halves by another diameter running east and west and passing through the centre of the circle. Now divide the radius which joins the centre with the south point into three equal parts. Draw a perpendicular on the first part counting from the centre westward until it meets the circumference. Join the point of intersection of the perpendicular with the circumference from the centre of the circle. This line gives the direction of the qibla.

If more precision is needed, the method can slightly be modified. Instead of erecting a perpendicular on the south going radius of the circle. Alberuni suggests that the radius which joins the centre to the west point of the cardinal may be divided into eighteen equal parts. On the first division from the west point draw a perpendicular and stretch it to meet the circumference.

southward. The intersection of this perpendicular with the
circumference would give the azimuth point. 100

COMPARATIVE STUDY OF ALBERUNI'S AND MODERN
LONGITUDES

The basic difference in Alberuni's longitudes and the modern
ones is that in the former the longitudes have been determined
from the western-most point in North Africa, i.e., Susul Aga
while in the latter they are given with reference to the meridian
which passes through Greenwich. It has been found, with a fair
degree of accuracy, that the longitudinal difference between these
two initial meridians amount to 14 degrees 28 minutes 101 and
consequently it would mean that Alberuni's longitudes would be
ahead of their modern counterparts by that amount. However, when
this value is deducted from the longitudes given by Alberuni then
in no case do the results come nearer to the modern values, as
would be seen from the Table I given below. In the table, which
follows, some thirteen well known places have been taken at random
and are arranged in the ascending order of their longitudes.

<table>
<thead>
<tr>
<th>Towns &amp; places</th>
<th>Alberuni's longitudes</th>
<th>Value obtained by subtracting 14d28m from (2)</th>
<th>Modern longitudes</th>
<th>Difference between (3) and (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
</tr>
<tr>
<td>1 Debal</td>
<td>92d 30m</td>
<td>78d 2m</td>
<td>68d 0m E + 11d 2m</td>
<td></td>
</tr>
<tr>
<td>2 Nirun</td>
<td>94d 30m</td>
<td>80d 2m</td>
<td>68d 30m E+ 11d 32m</td>
<td></td>
</tr>
<tr>
<td>3 Bamshana</td>
<td>95d 0m</td>
<td>80d 32m</td>
<td>68d 30m E+ 12d 2m</td>
<td></td>
</tr>
</tbody>
</table>

100 Ibid., pp. 255-6.
101 Bani, S.H., 'Al-Biruni and his Magnum Opus al-Qanun ul Mas'udi',
    al-Qanun al-Mas'udi, op.cit., p.xxxv.
<table>
<thead>
<tr>
<th>Place</th>
<th>Latitude</th>
<th>Longitude</th>
<th>Difference (E)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sanam</td>
<td>96° 10'</td>
<td>81° 42'</td>
<td>70° 17' + 11° 25'</td>
</tr>
<tr>
<td>Anhilwara</td>
<td>90° 20'</td>
<td>83° 52'</td>
<td>72° 5' + 11° 47'</td>
</tr>
<tr>
<td>Khambayat</td>
<td>99° 20'</td>
<td>84° 52'</td>
<td>72° 42' + 12° 10'</td>
</tr>
<tr>
<td>Dhar</td>
<td>100° 15'</td>
<td>85° 47'</td>
<td>75° 5' + 10° 42'</td>
</tr>
<tr>
<td>Bharoch</td>
<td>101° 5'</td>
<td>86° 37'</td>
<td>73° 10' + 13° 27'</td>
</tr>
<tr>
<td>Mahura</td>
<td>104° 0'</td>
<td>89° 32'</td>
<td>77° 48' + 11° 14'</td>
</tr>
<tr>
<td>Prayag</td>
<td>106° 20'</td>
<td>91° 52'</td>
<td>81° 50' + 10° 2'</td>
</tr>
<tr>
<td>Kankarh</td>
<td>107° 0'</td>
<td>92° 32'</td>
<td>81° 40' + 10° 52'</td>
</tr>
<tr>
<td>Banarasri</td>
<td>107° 20'</td>
<td>92° 52'</td>
<td>82° 37' + 10° 15'</td>
</tr>
<tr>
<td>Gangasayar</td>
<td>110° 40'</td>
<td>96° 12'</td>
<td>87° 50' + 6° 22'</td>
</tr>
</tbody>
</table>

From the above table it would be clear that the difference between the values of modern longitudes, shown in column (4) of the table, and the value obtained by subtracting 14 degrees 26 minutes from Alberuni's longitudes in column (2), is not the same everywhere nor does it follow any uniform pattern of variation.

It would be seen that the difference is always on the plus side, i.e., the value arrived at after subtracting 14 degrees 26 minutes from Alberuni's longitudes. There are, however, exceptions where it is on the negative side. Thus from this study it is now clear that 14 degrees 26 minutes is not the common difference between Alberuni's and the modern longitudes and that by deducting or adding this value to Alberuni's longitudes, the modern value cannot be found out.

As a matter of fact the difference between modern longitudes and Alberuni's, as is evident from Table II, is so great and inconsistent that this difference cannot be considered as a workable media in transferring one longitude into another.
Table II

<table>
<thead>
<tr>
<th>Towns &amp; Places</th>
<th>Alberuni's longitudes</th>
<th>Modern longitudes</th>
<th>Difference between (2) and (4)</th>
<th>Difference between to consecutive longitudes Alberuni (5a)</th>
<th>Modern (5b)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td></td>
</tr>
<tr>
<td>1 Kabul</td>
<td>95d 20m</td>
<td>69d 30m E</td>
<td>25d 50m</td>
<td>0d 55m</td>
<td>2d 0m</td>
</tr>
<tr>
<td>2 Multan</td>
<td>96d 15m</td>
<td>71d 30m E</td>
<td>24d 45m</td>
<td>0d 5m</td>
<td>2d 0m</td>
</tr>
<tr>
<td>3 Peshawar</td>
<td>97d 10m</td>
<td>71d 30m E</td>
<td>25d 40m</td>
<td>1d 10m</td>
<td>2d 53m</td>
</tr>
<tr>
<td>4 Lahore</td>
<td>98d 20m</td>
<td>74d 23m E</td>
<td>23d 57m</td>
<td>1d 20m</td>
<td>2d 0m</td>
</tr>
<tr>
<td>5 Ujjain</td>
<td>100d 50m</td>
<td>75d 45m E</td>
<td>35d 5m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 Meerut</td>
<td>102d 10m</td>
<td>77d 45m E</td>
<td>25d 25m</td>
<td>1d 20m</td>
<td>2d 0m</td>
</tr>
<tr>
<td>7 Prayag</td>
<td>106d 20m</td>
<td>81d 50m E</td>
<td>24d 30m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 Varanasi</td>
<td>107d 20m</td>
<td>82d 37m E</td>
<td>24d 43m</td>
<td>1d 0m</td>
<td>0d 47m</td>
</tr>
<tr>
<td>9 Gangasayar</td>
<td>110d 40m</td>
<td>87d 50m E</td>
<td>22d 50m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 Setabandhu (Adams Bridge)</td>
<td>119d 0 m</td>
<td>79d 30m E</td>
<td>39d 30m</td>
<td>8d 20m</td>
<td>8d 20m</td>
</tr>
</tbody>
</table>

In the above table it may be seen that the difference between the two sets of longitudes normally exceeds 23 degrees and invariably fluctuates between 24 degrees and 26 degrees. It may also be noted that this difference does not exceed or diminish at a constant rate, neither towards the east nor towards the west. From the study of this table a revealing fact emerges that though no common denominator is found between Alberuni's longitudes and the modern ones, yet one can be worked out which should range between 25 and 26 degrees. The need is to evolve a formula wherein this difference should have been properly calculated and used for converting Alberuni's longitudes into their modern equivalents.
THE FORMULA

The formula is based on the assumption that in order to eliminate the difference between Alberuni's and the Greenwich longitudes the first essential is to find out a place where Alberuni's longitudes and modern longitudes would differ with each other by that amount. The longitudes of a number of well-known medieval towns, for example, Damascus, Shiraz, Ghazni, Kabul and Baghdad and many others can be of use in this regard but the last one is found to be preferable to the rest for a number of reasons. First, this town in the medieval period had been an important centre of trade and commerce and therefore its exact location was well known. Secondly, the town still survives as a flourishing metropolis and is the capital of an important country of the Arab world and therefore its modern longitude is available. Thirdly, in the medieval period, at least for three centuries, it had been the seat of learning and it was here that a host of geographers engaged themselves to the task of determining the latitude and longitude of the place with greater precision than in the case of other towns. Fourthly, Baghdad heads over other towns in respect of its almost central location in the then known 'inhabitable world' which would minimize the errors of the longitudes caused by more eastward or westward displacement in comparison with those towns which are located on either extremes of the Oikumene. Fifthly, Baghdad seems to be one of those towns where the shifting in the site of the town has been slight and negligible.

In the formula such a standard town has been termed a 'base town'.

In the proposed formula two quantities are involved. One is a variable quantity and the other is a constant one. The variable quantity (called L) in the formula is the substituted value of the longitude of the town whose modern equivalent is to be determined while the constant quantity is the difference between Alberuni's and modern longitudes of the 'base town'. The subtraction of this constant quantity, which in the formula is 25d 30m, from the known Alberuni's longitude of the to-be-determined town, that is X in the formula, gives its modern equivalent with reference to Greenwich.

The algebraic form of the formula may be developed as below:

\[ X = \text{modern longitude of the 'base town'} - \text{Alberuni's known longitude of the 'base town'}. \]

Now substituting the corresponding values of Baghdad for the 'base town' in the formula, the formula becomes:

\[ X - 44d 30m = \text{Alberuni's longitude of } X - 70d \]

taking 44d 30m to the other side, the formula:

\[ X = \text{Alberuni's known longitude of } X - 70d + 44d 30m \]

which after simplification becomes:

\[ X = \text{Alberuni's known longitude of } X - 25d 30m \]

Therefore, on the final shape the formula comes out thus:

\[ X = L - 25d 30m \]

(where X is the unknown modern longitude to be determined and L is the longitude of the same town as given by Alberuni).

APPLICATION OF THE FORMULA

Example 1: Kabul, Alberuni's longitude 95d 20m (mod. long. 6.9d 30m E).
Inserting the value of the given longitude of Alberuni in the formula:

\[ x = 95d 20m - 25d 30m = 69d 50m \]

**Example 2:** Ghazni, Alberuni's longitude 94d 20m (mod. long. 68d 25m E).

\[ x = 94d 20m - 25d 30m = 68d 50m \]

**Example 3:** Antakya (Antioch), Alberuni's longitude 61d 35m (mod. long. 36d 15m E).

\[ x = 61d 35m - 25d 30m = 36d 5m \]

**Example 4:** Hali, Alberuni's longitude 66d 20m (mod. long. 41d 51m E).

\[ x = 66d 20m - 25d 30m = 41d 50m \]

**Example 5:** Mecca, Alberuni's longitude 67d (mod. long. 40d 14m E).

\[ x = 67d - 25d 30m = 41d 30m \]

In this connection some other results which may be of interest are given table III.

<table>
<thead>
<tr>
<th>Alberuni's towns</th>
<th>Modern names</th>
<th>Alberuni's longitudes</th>
<th>Converted longitudes</th>
<th>Modern longitudes</th>
<th>Difference between (4) and (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Mastank</td>
<td>Mastung</td>
<td>95d 0m</td>
<td>69d 30m</td>
<td>66d 42m E</td>
<td>2d 42m</td>
</tr>
<tr>
<td>2 Sanam</td>
<td>Somnath</td>
<td>96d 10m</td>
<td>70d 40m</td>
<td>70d 17m E</td>
<td>0d 23m</td>
</tr>
<tr>
<td>3 Mullistan</td>
<td>Multan</td>
<td>96d 15m</td>
<td>70d 45m</td>
<td>71d 30m E</td>
<td>0d 45m</td>
</tr>
<tr>
<td>4 Parsavar</td>
<td>Peshawar</td>
<td>97d 10m</td>
<td>71d 40m</td>
<td>71d 30m E</td>
<td>0d 10m</td>
</tr>
<tr>
<td>5 Waibling</td>
<td>Atteck</td>
<td>97d 50m</td>
<td>72d 20m</td>
<td>72d 32m E</td>
<td>0d 12m</td>
</tr>
<tr>
<td>6 Jailam</td>
<td>Jhelum</td>
<td>98d 20m</td>
<td>72d 50m</td>
<td>73d 45m E</td>
<td>0d 55m (Contd.)</td>
</tr>
</tbody>
</table>

Table III
A close scrutiny of the results worked out above on the basis of this formula will show that the margin of difference between the computed value and the modern value is very small and when the difference happens to be great, it may be attributed to a number of other reasons, such as:

1) Faulty determination of the longitudes by Alberuni himself.
2) Wrong reporting and confusion in Alberuni's statements.
3) Faulty transcription of the text.

MERITS AND DEMERITS

Generally the formula gives the results which are exceedingly dependable. Of the sixteen examples cited at random in Table III above the results obtained in all the instances are within 3 degrees of their right value; only in 3 cases does the value crosses 2 degrees. Of the 16 instances, 13 results are well within 2 degrees and 8 results differ from their right value by only a margin of less than a degree.
And in those cases where the results differ from their right value by more than 1 degree the difference just exceeds 1 degree. Three are the cases where the results are within a range of 2 to 3 degrees from their right value. These examples clearly establish that the formula is quite useful in converting Alberuni's longitudes into their modern counterparts.

Another very interesting study the formula helps to undertake is the identification of medieval towns. By the conversion of Alberuni's longitudes into Greenwich longitudes and with the help of the latitudes given by Alberuni the site can be located on the map and can be compared with modern sites. In this connection an interesting case is that of Bari. In Qanun al-Mas'udi, Alberuni besides giving its latitude and longitude reports that it was then the capital of those kings whose reigns extended east of the Ganges. We learn from other medieval sources that the Rai kings of central India had then abandoned their old capital Kanoj and had chosen Bari instead. This town according to Alberuni's notes lies near the confluence of the three rivers, the Rahab, the Kuhi or Kawini, and the Saryu or Sarwa or Saraini uniting with the Ganges, at a distance of about 10 farsakhs or 3 or 4 days journey east of Kanoj. All these statements, as will be noticed, are confusing and in the wake of such a confusion an accurate knowledge of the latitude and longitude of Bari alone can be of help. Its geometrical position has been given as 26 degrees 30 minutes north and 105 degrees 50 minutes east. This

101 Alberuni, Kitab al-Hind, Vol.I, p.261. See also The History of India as Told by its own Historians, Elliot and Dowson, Allahabad, Vol.I, pp.49-50, for discussion and identification of the three rivers, namely, the Kuhi, the Rahab and the Saryu.
position suggests, at least tentatively that its site should be that of modern Rae Bareli whose latitude and longitude are 26 degrees 20 minutes north and 81 degrees 5 minutes east. Moreover, the name of Rae Bareli seems to be a corruption of Rai Bari which is situated near the confluence of two rivers the Sai and the Ganges, the former being a tributary of the latter. The town still shows the signs of being a capital of an old kingdom which speak of its medieval glory. The description of other rivers, the Kuhà and the Rahab joining the Ganges at some place in the east seems to be a misconception based on hearsay. And if the identification of these rivers, the Rahab and the Kuhà may be taken for Behta or Rahrai and Gomti respectively, as suggested by General Cunningham, then it would be difficult to find a place east of the Ganges where all of them would unite. Any attempt made in identifying Bari on the aforementioned confusing knowledge would be misleading. Thus it would be seen that this identification of Bari has been facilitated by the application of this formula. This is just one of the examples cited, but there may be a number of cases where it will be found to lead to correct identification of medieval towns mentioned in Alberuni's accounts.

The demerits of the formula, however, is that it cannot be used successfully in the case of those places whose longitudes are less than 14 degrees 28 minutes, i.e., those places whose

103 The name of the river Sai seems to be the Sarwa or Sarain or Sarayu.
modern longitudes are west of Greenwich. In that case the
formula has to be slightly modified. The formula then would be:

\[ X = L - 14d 26m - 25d 30m \]

which would come to

\[ X = L - 11d 2m \]

A couple of examples are given below:

Example 1: Saragossa. Alberuni’s longitude 12 degrees, modern
longitude 0 degree 51 minutes west.

Applying the formula: \( X = 12d - 11d 2m = 0d 50m \)

Example 2: Murcia, Alberuni’s longitude 12d 50 m, modern
longitude 1 degree 45 minutes west.

Applying the formula: \( X = 12d 50m - 11d 2m = 1d 48 m \).

Alberuni has mentioned about 19 places, which would now lie
west of Greenwich, in his table in *al-Qanun al-Mas‘udi*. Out of
these 19 towns four are in the 'Third Climate' and the rest in
the 'Fourth Climate'. Most of these towns still await proper
identification and as such their modern longitudes are not
available, therefore, they cannot be tested for the accuracy of
the formula.

ALBERUNI’S SINE TABLE

Alberuni in the third book of *al-Qanun al-Mas‘udi* (Canon of
Masud) concentrates mainly on the problems and applications of
trigonometry, plane as well as spherical, and it is here that he

gives an exhaustive table of sines of angles. The table is, perhaps, the only complete extant record of its type which has reached us from the Medieval Arabs.

The Table comprises the whole of the sixth chapter of the third treatise of al-Qanun al-Mas'udi. It was disposed in Arabic alphabet (Huruf) which according to the general practice in those times expressed numerical values. Another feature of the table is its composition in sexagesimal fractions (the scale of sixty) rather than the more convenient decimal fractions.

106 The Arabs in the Middle Ages generally employed Arabic letters for notations in their writings, though Arabic-Hindu numerals were also not uncommon. The practice seems to be a legacy from the Greeks who likewise used their own alphabet for numerical expression. In this system each letter is assigned a specific and fixed value, as for instance in Arabic alphabetic numeral system (Huruf al-jimal) alif is 1, be 2, jim 3, dal 4... mim 40, nun 50, sin 60... shin 300 and so on. This is difficult to see why the medieval Arabs remained attached to this cumbersome and outmoded system when already Huruf al-Gobar (Dust Numerals) and other forms were there. One reason for frequent use of alphabetic numerals was definitely not the ease of handling them but the fact that the medieval Arab world was extended over a large region where Arabic-Hindu numerals, though coming from the same source differed considerably in form and structure. For example, numerals used in Spain were at variance in shape and form from those in the east, say Khorasan or Kabul. Probably for the sake of homogeneity and universality alphabetic representation was preferred and retained. At any rate this numeral system has its own drawbacks which often recur in subsequent copying and transmission of the original text. An omission or misplacing of a dot brings into the text inscalable error, for example, just for a dot, a guad (90) may change into a ghued (800) or an 'ain (70) into a chain (1000) and vice versa.

107 The practice in the entire Muslim medieval period had unsparingly been to use the scale of sixty to express the fractions. This system which is now commonly referred to as 'sexagesimal system' was not a traditional follow up of the Greeks, specially Ptolemy (the Greeks probably got it from...
The language and the method of its presentation were probably the factors which prevented European scholars from examining it. Had this been done, Alberuni would certainly have earned his due place in the history of mathematical sciences.

107 (Contd.)

the Babylonians), but had some definite advantages over the rival developing decimal system. In sexagesimal system the fractions are cut much lower than those in the decimal system, compare for instance $1/10$ and $1/60$; the former in decimal fractions supplies a value 0.1 and the latter 0.0166 (recurring). Above that, the number of factors are large in the case of 60 than in 10; in the former are 2, 3, 4, 5, 6, 10, 12, 15, 20 and 30, while in the latter only 2 and 5. This proposition helps the ready use of halves, thirds, fourths, fifths, sixths, tenths, twelfths, fifteenths and so on.

As far as the origin of the system is concerned the claim that the Babylonians started it since they divided the circle into 360 parts and a circle into six equal parts, equal to six radii of the circle, found sixty as a convenient radix, is only a history. The fact is that the number 60 did play an important part in the number system of that country but never as a one-sixth part of a circle of 360 degrees. The Babylonians divided their circle into 8, 12, 120, 240 and 460 equal parts. In fact, the Greeks of the later period invented and developed the sexagesimal system though a suggestion of 60 might have come from the Babylonians. Interestingly the division of a circle into 360 parts itself seems an outgrowth rather than the cause of origin of the system.

The sexagesimal system still persists in our writings of measures of time, angles and arcs. Ordinarily in such degrees, minutes and seconds are shown. In classical Greek times as well the fractional values were taken to this order only. The Arabs, however, took the calculations to generally, fifth and sixth order which are notated as $1 \ 45 \ 30 \ 40 \ 20 \ 40 \ 42$ meaning thereby

\[
1 + \frac{45}{60} + \frac{30}{60^2} + \frac{40}{60^3} + \frac{20}{60^4} + \frac{40}{60^5} + \frac{42}{60^6}
\]

or

\[
1 + \frac{45}{60} + \frac{30}{3600} + \frac{40}{21600} + \frac{20}{1296000} + \frac{40}{77760000} + \frac{42}{46656000000}
\]

It may be noted that symbols (° , ′ , ″) are comparatively modern; the Arabs, instead, wrote darj/ayman/aiga, dawia, thawani, thawalah, rayab'a, khawands and sawads, etc. in converting the sexagesimal values into the decimal ones, the procedure here has been to start from the lowest denomination and moving upward.
The contents of the Table in Alberuni's text have been arranged into four columns. The first column gives arc measurements, the second sines, the third 'equalisations' and the fourth 'differences'. The Arabic heads are \textit{satr added }al-qusi, al-juvab, al-ta'adil and al-fudul\textit{ respectively}. The particularity of the Table is that the different values in it have been computed at an interval of fifteen minutes and this enhances the utility of the Table manifold.

The values inserted under the heads of \textit{al-ta'adil} and \textit{al-fudul} (Col.4) are very important as they help in checking the correctness of sines. All the figures disposed in the last column are actually one-fourth of the corresponding figures in the third column. In order to find the sine of the next angle one has simply to add the corresponding difference in the previous sine value. For instance, if we add the difference \(15^{I} 38^{I} 43^{IV}\) noted against 5 degrees to its sine \(5^{I} 12^{II} 45^{III} 38^{IV}\) the result will give the sine value \(5^{I} 29^{II} 24^{III} 21^{IV}\) of 5 degrees 15 minutes. On this basis the entire Table can be checked and any indiscrepancy creeping in as a result of copying and transmission of the MS can be corrected. It would be seen from the Table that the difference is not uniform but diminishes proportionately with the increase of the sine as the angles advance till it becomes zero with the sinus totus.

In Appendix I, Alberuni's sines as given in his Table, have been converted into decimal fractions up to the sixth place and compared with the corresponding natural sines. It would be
observed that Alberuni in his appropriations of sines has been very accurate, so much so that his computed values tally closely with the modern values even up to the sixth place of decimal. This is, undoubtedly, no small achievement. There are, however, instances where slight deviations, from the right value do occur but such cases are rare.

In the computation of his Sine Table, Alberuni excelled not only his predecessors but had few peers in the following two centuries. It is really unfortunate that Europe did not begin to focus its attention on Alberuni until 1866. Had a proper study been done earlier it would have saved the labours of European geometers of the fifteenth and sixteenth centuries -- for example, Peurbach (c. 1460), Regiomontanus (c. 1464), Copernicus (c. 1520) -- who attempted to recast the trigonometrical tables afresh.

108 Amongst the Arab scholars who enriched the science of trigonometry mention may be made of Al-Khwarizmi (c. 825), Al-Battani (c. 920), Abul Wafa (c. 980), Ibn al-Zarqala (c. 1050), Jabir Ibn Aflah (c. 1145), Nasiruddin Tusi (c. 1250) and Ulugh Beg (c. 1435).