CHAPTER I

INTRODUCTION AND REVIEW OF THE PROBLEM
1.1 **INTRODUCTION**

The knowledge of the features of the transmission and absorption of low and intermediate energy electrons and positrons in elemental materials is of great importance for the experimental methods in nuclear, atomic and solid state physics. It is also useful in understanding the various interactions of these particles with matter. In the field of nuclear spectroscopy, for work with internal-conversion electrons, β-rays, Auger electrons and photo electrons etc., one needs to know the exact fraction of the incident electrons transmitted through an absorber. The knowledge of the ranges of these particles in matter has useful applications for the study of biological effects, radiation damage, dosage-rates and energy dissipation at various depths of an absorber. It has also useful applications in the design of detection systems, radiation technology, semi-conductor detectors, shielding and choosing the proper thickness of the targets.

In spite of these important applications, the concept of ranges of electrons and positrons in matter is still ambiguous. The penetration of positrons through matter and their corresponding differences from electrons has been a subject of very little investigation both theoretically as well as experimentally. The ambiguous concept of the ranges, both from the experimental as well as theoretical point of view, is because of the fact that different experimentalists\(^1\) have used different definitions
The transmission curves of electrons and positrons have a long straight portion down to fairly low transmission and then a considerable tail going into the background. At low transmissions, when the electrons have penetrated a certain fraction of their range, the beam becomes diffused and the phenomenon of diffusion sets in, with the result that transmission curves have nearly the same shape at the end. Several definitions have been employed to obtain ranges from the transmission curves. The various range definitions will be discussed in Chapter - III.

With the development of linear accelerators, some workers like Hereford and Swan\textsuperscript{7)}, measured the absorption of monoenergetic electrons of few MeV. In their review article Katz and Penfold\textsuperscript{12)} gave a comprehensive summary of the earlier work of experimentally determined ranges of monoenergetic electrons and continuous beta-ray energies. They have pointed out that there is no distinct difference in range between monoenergetic electrons of energy \(E\) and beta-rays having the same end point energy.

1.2 SUMMARY OF THE EXPERIMENTAL WORK WITH ELECTRONS AND POSITRONS

The earlier experimental work was reviewed by Katz and Penfold\textsuperscript{12)} in 1952. The measurements were mostly in aluminium
absorber using electrons. The experimental geometries were given no consideration and the final percentage transmissions through absorbers in many cases were quite high and also different for different workers. The experimental work simply depicted the qualitative behaviour of electron transmission. In the early work on transmission experiments no attempt was made for the study of positron transmission. This was probably due to the non-availability of positron sources on the one hand and difficulty in their detection on account of background problems involved on the other hand. Seliger and Gubernator did some work on the transmission of positrons in the energy range of 180 KeV to 960 KeV, and 50 KeV to 160 KeV respectively. These measurements were made in limited absorbers and no considerations were given to the geometry of the detecting system. Nathu Ram et al. have studied the mass absorption coefficients of electrons and positrons of 0.324 MeV and 0.544 MeV in Be, Al, Cu, Ag and Pb. However, they did not correlate their measurements with any theory.

The experimental work with electrons and positrons is represented in table 1.1, indicating the energies at which the measurements were made; the absorbing materials used and the geometry of the experimental set up used, where it is known.

1.3 THEORETICAL APPROACHES

(a) C.s.d.a. ranges: The abbreviation C.s.d.a. stands for continuous-slowing-down-approximation. Berger and Seltzer
<table>
<thead>
<tr>
<th>Authors with reference</th>
<th>Electron energy</th>
<th>Positron energy</th>
<th>Absorbing Materials used</th>
<th>Geometry of the Experimental Set up used and other details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Widdoson and Champion</td>
<td>1.80 to 2.56 MeV</td>
<td>-</td>
<td>Al only</td>
<td>Geometry not mentioned.</td>
</tr>
<tr>
<td>Moore</td>
<td>1.24 to 2.60 MeV</td>
<td>-</td>
<td>Al only</td>
<td>Geometry not mentioned.</td>
</tr>
<tr>
<td>Glendenin</td>
<td>0.53 to 3.07 MeV</td>
<td>-</td>
<td>Al only</td>
<td>Geometry not mentioned.</td>
</tr>
<tr>
<td>Hereford and Swan</td>
<td>3 to 12 MeV</td>
<td>-</td>
<td>Al and Cu</td>
<td>Magnetically analysed β-ray spectrum was used and absorption curves obtained with triple coincidence counter train. No fixed Geometry used.</td>
</tr>
<tr>
<td>H.H. Seliger</td>
<td>180 to 960 KeV</td>
<td>180 to 960 KeV</td>
<td>Al, Ag, Sn, Au and Pb</td>
<td>The angle of deflection of the spectrometer is θ = 60°, the radius of curvature 4 cm, the solid angle used is 0.12 of 4π. The source is reflected on the counting tube in the approximate scale 1:1. The distance between the absorbing foils and the G.M. counter window is nearly 5 mm. Nothing has been mentioned about the suitability of this Geometry. The percentage transmission was very high in some cases.</td>
</tr>
<tr>
<td>Authors with reference</td>
<td>Electron energy</td>
<td>Positron energy</td>
<td>Absorbing Materials used</td>
<td>Geometry of the Experimental Set up used and other details</td>
</tr>
<tr>
<td>------------------------</td>
<td>-----------------</td>
<td>-----------------</td>
<td>--------------------------</td>
<td>---------------------------------------------------------</td>
</tr>
<tr>
<td>Trump et al (^{28})</td>
<td>2.00 MeV</td>
<td>-</td>
<td>Al only</td>
<td>No fixed Geometry used.</td>
</tr>
<tr>
<td>Katz and Penfold (^{12,24,25})</td>
<td>0.55 to 2.92 MeV</td>
<td>-</td>
<td>Al only</td>
<td>Geometry not defined. They have reviewed the measurements made upto 1952.</td>
</tr>
<tr>
<td>K. Gubernator (^{10,11})</td>
<td>50 to 167 KeV</td>
<td>50 to 167 KeV</td>
<td>Al, Ag, Au and Pb</td>
<td>The source at a considerable distance from the counter and absorber almost against the mica window. The results of any variation of Geometry are not discussed.</td>
</tr>
<tr>
<td>B.N.C. Agu et al (^{4})</td>
<td>0.25 to 0.75 MeV</td>
<td>-</td>
<td>Al, Cu, Ag and Au</td>
<td>No fixed Geometry used.</td>
</tr>
<tr>
<td>P.J. Ebert et al (^{5})</td>
<td>4 to 12 MeV</td>
<td>-</td>
<td>C, Al, Cu, Ag</td>
<td>No fixed Geometry used.</td>
</tr>
<tr>
<td>Patrick and Rupaal (^{33,34})</td>
<td>0.312 MeV</td>
<td>0.324 MeV</td>
<td>Al, Cu, Sn, Pb</td>
<td>Geometry not defined. Comparison of (\mu^+) and (\mu^-) at unequal energies.</td>
</tr>
<tr>
<td>P.S. Takhar (^{35})</td>
<td>1.77 MeV</td>
<td>1.8 MeV</td>
<td>C, Al, Cu, Sn, Pb</td>
<td>Geometry is identical for positron and electron transmissions. (\mu(e^+)) and (\mu(e^-)) compared at unequal energies.</td>
</tr>
</tbody>
</table>
have tabulated these C.s.d.a. ranges taking into consideration
the energy loss due to ionization and excitation\textsuperscript{15}) and also
the bremsstrahlung process\textsuperscript{16}). The C.s.d.a. range is the path
length which a particle would travel in the course of slowing
down, in an un-bounded homogeneous medium, from initial kinetic
energy $T$ to zero energy, if its rate of energy loss along the
entire track were always equal to the mean rate of energy loss.
C.s.d.a. range was calculated by integrating the reciprocal of
the total stopping power:–

$$R_{\text{C.s.d.a.}}^{\pm}(T) = \int_{T_1}^{T} \left[ - \frac{1}{\rho} (\frac{dE}{dx})_{\text{Total}}^{\pm} \right]^{-1} dT + R_{\text{Total}}^{\pm}(T_1) \ldots (1.1)$$

where $T_1$ is some lower limit of energy below which the calcula-
tions can not be performed because of the poor knowledge of the
stopping power and also $(\frac{dE}{dx})_{\text{Total}}^{\pm}$ becomes infinite as the energy
becomes zero. $T_1$ is normally taken as 1 KeV. For intermediate
and high energy electrons the contribution of $R_{\text{Total}}^{\pm}(T_1)$ is
negligible. Also

$$- \frac{1}{\rho} (\frac{dE}{dx})_{\text{Total}}^{\pm} = - \frac{1}{\rho} (\frac{dE}{dx})_{\text{Coll.}}^{\pm} - \frac{1}{\rho} (\frac{dE}{dx})_{\text{Rad.}}^{\pm} \ldots (1.2)$$

where the positive sign corresponds to positron and the negative
sign corresponds to electron. $\rho$ is the density of the material.

Berger and Seltzer\textsuperscript{17}) have tables for the energy loss due
to excitation and ionization as well as due to radiation in
different materials.
The C.s.d.a. range pertains to a particle track that is 'typical' but not experimentally realizable. If we compare the C.s.d.a. ranges of electrons and positrons in any material with the corresponding experimental values, it is observed that C.s.d.a. ranges are always greater than the measured values. It is also observed that although the C.s.d.a. range is an increasing function of the incident energy of the particle, and this is consistence with the experimental observation, nevertheless if these C.s.d.a. ranges for different elements are plotted as a function of energy, then at a fixed energy the values of these ranges are higher in a high Z material and smaller in low Z material. This is in contradiction with the experimental observations where the values of range at a fixed energy is less in high Z materials as compared with low Z materials. This fact is clear from Fig. 1.1 and 1.2. Hence the C.s.d.a. ranges fail to explain the observed values both qualitatively as well as quantitatively.

(b) Multiple scattering studies:— C.s.d.a. range was based on purely inelastic considerations. Using multiple scattering\(^{32}\), Rohrlich and Carlson\(^{15}\) calculated \(Z_d^+/Z_d^-\), where \(Z_d^+\) and \(Z_d^-\) are called the average penetration depth for positrons and electrons respectively, which is defined as the amount of absorber thickness to be placed in the path of the beam of positrons or electrons, such that the particles lose completely the memory of their initial orientation.
Fig. 1.1 Ranges of electrons as a function of energy.

- C.S. d.a. Ranges
- Experimental Ranges

ENERGY IN MeV

RANGE IN mg cm$^{-2}$
Fig. 1.2 Present experimental range in Al, Cu, Nd and Pb.

R_{sp} as a function of energy.
The average energy \( E_d = \gamma_d mc^2 \) at which the particle loses its memory of initial direction is defined by the condition\(^\text{31)}\)

\[ \langle \cos \Theta \rangle_{\text{average}} = \frac{1}{e} \quad \cdots \quad (1.3) \]

where \( \Theta \) is the angle of multiple scattering. They estimated approximately the values of \( \gamma_d^\pm \) which correspond to the instant when the average cosine of the multiple scattering angle drops to \( \frac{1}{e} \), for any value of \( \gamma_0 \). The average penetration depth corresponding to \( \gamma_d^\pm \) is given by:

\[ Z_d^\pm = \int_{\gamma_d^\pm}^{\gamma_0} k_l^\pm (\gamma_0, \gamma) \left| \frac{d\gamma}{ds} \right|^\pm \frac{1}{2} d\gamma \quad \cdots \quad (1.4) \]

where

\[ k_l^\pm (\gamma_0, \gamma) = \langle \cos \Theta \rangle_{\text{AV}}^\pm = \left[ \frac{G(\gamma_0)}{G(\gamma)} \right]^\pm \quad \cdots \quad (1.5) \]

and

\[ G^\pm (\gamma) = (\gamma + 1)^{\alpha^\pm Z} e^{\beta^\pm Z/\beta} \quad \cdots \quad (1.6) \]

The constants \( a^\pm \) and \( b^\pm \) were assumed to be approximately of the same order of magnitude for small and large values of atomic numbers.

Rohrlich and Carlson\(^\text{15)}\) calculated the values of penetration depths in aluminium and lead for the energy range from 0.1 MeV to 2.04 MeV taking into account the energy loss due to
excitation and ionization only. Their values of $Z_d^+/Z_d^-$ in aluminium increase with increasing energy up to about 1 MeV and after that this ratio decreases. In case of Pb, $Z_d^+/Z_d^-$ first increases with energy up to 1 MeV and then becomes constant. No explanation has been given for this behaviour.

Further limitations of these calculations are as follows:

1. The average penetration depth $Z_d^+$ or $Z_d^-$, defined by the condition $<\cos \theta>_\text{av} = 1/e$, has no physical significance and is not experimentally realizable quantity, hence it cannot be compared with range. The comparison of $Z_d^+/Z_d^-$ with measured range ratio $R^+/R^-$ does not seem to be justified. $Z_d^+/Z_d^-$ at the most provides only the qualitative nature of the multiple scattering differences between electrons and positrons.

2. Rohrlich and Carlson\textsuperscript{15} completely ignored the energy loss due to radiations and used only the energy loss due to collisions in their expressions for $<\cos \theta>_{\text{average}}$ and $Z_d^\pm$. The contribution of radiation losses at 2 MeV for the case of Pb was reported\textsuperscript{17} to be 21% of the total energy loss, hence their calculations are not at all reliable at these energies.

3. Due to the approximations used for the evaluation of the integrals in these calculations some percentage error is bound to appear in the final results, but no mention of this error has been made.

4. The values of $Z_d^+/Z_d^-$ has been reported only for the case of
Al and Pb, and these calculations are not possible for other materials. This is because the constants like $a^+$ and $b^+$ have been tabulated only for lead and aluminium.

(c) **Theoretical projected ranges:** Rohrlich and Carlson$^{15}$ calculated $Z_d^+$ taking into account only the elastic scattering of electrons and positrons. Also the C.s.d.a. ranges$^{14}$ were based on purely inelastic considerations, and both of these approaches$^{14, 15}$ do not interpret the measured ranges. While penetrating an absorber the electrons and positrons undergo both elastic as well as inelastic interactions with the atoms of the absorber. Batra and Sehgal$^{18-21}$ have taken into account both these processes while calculating the projected ranges $R_p^+$ for positrons and electrons. The mean projected range is defined as the mean projection of the path of these particles on the direction of incidence in the absorber. They assume that the inelastic scattering is statistically independent of energy loss fluctuations. Near the end of the range of electrons and positrons the energy of the particle becomes small and therefore multiple scattering is large. When the multiple scattering angle becomes very large, the electrons thereafter diffuse randomly and this contributes to straggling. For simplification they assume that the electrons first undergo a straight motion and their interaction with matter is only through inelastic process. The effect of multiple scattering is incorporated afterwards.

In order to apply the multiple scattering correction, the
mean square angle of multiple scattering $\langle \theta^2 \rangle_x^\pm$ (when electrons traverse a small thickness $x$ of the absorber) is required. The total stopping power $- \frac{1}{F} \left( \frac{dE}{dx} \right)_{\text{Total}}$, (where $F$ is the density of the material), is required as input parameter for calculating $\langle \theta^2 \rangle_x^\pm$. Batra and Sehgal\textsuperscript{18-21} found simple empirical relations of total stopping power, which are easily integrable. The situations of the random motion of electrons and positrons by multiple scattering have been visualized by them using the definition of transport mean free path $\lambda_{\text{tr}}$, i.e. the average distance a particle traverses before being scattered through an angle $\geq \pi/2$. This condition has been proved\textsuperscript{18-21} to be $\langle \theta^2 \rangle_T^\pm = \frac{2}{\pi}$. 

If $T_T^\pm$ and $\langle \theta^2 \rangle_T^\pm$ are the energy and the mean square projected angle respectively, corresponding to the instant when the motion of the particle becomes random, then the projected $R_p^\pm(T)$ is given as:

$$R_p^\pm (T) = R_{\text{C.s.d.a.}}^\pm (T) - R_{\text{C.s.d.a.}}^\pm (T_T^\pm) \quad \ldots \quad (1.7)$$

The values of the projected ranges\textsuperscript{18-21} are comparable with the present experimental values. The agreement for $6 < Z < 13$ materials is good. For intermediate and heavy elements these calculations\textsuperscript{18-21} give lower values of range as compared to the experimental values. The difference is small for intermediate $Z$ values but goes on increasing with increasing value of $Z$. At $Z = 82$ the theoretical values are off by 25% for particle energies of one MeV. The difference is more for still lower
energies. This difference in the theoretical and experimental values is probably because of two reasons.

(1) Batra and Sehgal\(^{18-21}\) have used Mott's\(^ {22}\) expression for elastic scattering cross section, which is in the form of a power series in \(aZ\), where \(a = 1/137\). They have used only the first term of this series, leaving the higher order terms. For small values of \(Z\) the higher order terms like \((aZ)^2\) and \((aZ)^3\) give negligible contribution, but for high values of \(Z\), the contribution of these higher order terms may be appreciable.

(2) In these calculations\(^{18-21}\) they have ignored the contribution to the range coming from the diffusion part. They considered diffusion to contribute to straggling only. From their reference - 21, Figs. 5 - 6, one observes that as the incident kinetic energy of electrons and positrons decreases and also with the increase of the atomic number \(Z\) of the absorber, the fraction of energy left with these particle increases. The contribution of the diffusion part of the ranges in the procedure based mainly on their\(^{18-21}\) calculations, if taken into account makes these calculated values of straggling free practical ranges agree very well with the experimental data.

1.4 PRESENT INVESTIGATIONS

(i) The penetration of electrons of energy \(E_{\max} = 0.25\) MeV, 0.77 MeV, 1.53 MeV and 1.71 MeV through a large number of elemental materials, \(6 \leq Z \leq 82\), has been studied experimentally.
Experimental measurements in the rare earth metals have been made for the first time. Simple empirical relations have been found for the straggling free range. Penetration of 1.88 MeV positrons in a number of materials including the rare earth metals has been experimentally investigated, and the comparison is made with the electron transmission.

(ii) An attempt has been made to build-up the transmission curves theoretically so as to get information about the practical ranges and absorption co-efficients of electrons and positrons for $6 < Z < 82$. The procedure used is to divide absorbing foils into a large number of thin slices. The scattering of the electrons and positrons in the backward direction in each slice has been taken into account to find the transmission through that particular slice. The energy loss in each slice has also been taken into account. This method is suitable for finding the absorption coefficients of different absorbers at different energies theoretically. The values of the absorption coefficients obtained from these theoretical transmission curves have been compared with the experimental values.

(iii) A simple method has been developed for calculating the straggling free practical ranges, $R_{sfp}^\pm$, of positrons and electrons of kinetic energy $< 5.0$ MeV in materials of any Z, taking into account the straggling part of the range and also accounting for higher order terms in the Mott's expression\textsuperscript{22} for the elastic scattering cross section. The procedure is
mainly based on existing theoretical calculations\textsuperscript{18-21} with the modification that the ranges of these particles are calculated by taking into account the contributions of both the parts of the range i.e. before and after the diffusion process starts. The ranges thus calculated would be of great use for better understanding of the present experimental data, and a meaningful comparison of the theoretical values with the experimental value:
REFERENCES

3. R.W. Varder, Phil. Mag. 29, 725 (1915).


