FIRST THESIS.

on the \(3\)-operator associated with Lie group of transformations.

INTRODUCTION.

The deformation of a space \(V_n\), whether metric or of affine connexion, is realized by remarking that a change of variables

\[ \bar{x} = f(x), \]

admits two different geometrical interpretations. Kosambi has pointed out in his paper "Continuous groups and two theorems of Euler, Math. Student, II, 1934, pp.94-100" that the change of variables can be regarded either (a) as a mapping of the \(x\)-space upon the \(\bar{x}\)-space, i.e. as a straightforward transformation of the \(x\)-space into the \(\bar{x}\)-space, or (b) as a mapping of the \(x\)-space upon itself, i.e. the point of coordinates \(\bar{x}\) is regarded as the point of coordinates \(x\) in the \(x\)-space. The change of variables, in the second interpretation, will be called a displacement, i.e. the point \(x\) is displaced in the original space \(V\) to \(\bar{x}\). Thus
in the displaced $V_n$ vectors and tensors at $\bar{\mathbf{x}}$ are just the vectors and tensors of $V_n$ at $\mathbf{x}$, and the metric and connexion parameters are $g_{ij}(\bar{x})$ and $L^i_{jk}(\bar{x})$. This displaced space will also be referred to as the deformation of $V_n$. Introducing, then, the notations $\bar{g}_{ij}(\bar{x})$ and $\bar{L}^i_{jk}(\bar{x})$ for the metric and connexion parameters of the transformed space $V_n$ at $\bar{x}$, we consider $\bar{g}_{ij}(\bar{x})$ and $\bar{L}^i_{jk}(\bar{x})$ as the representatives of $V_n$ at the point $\bar{x}$. The measure of deformation of $V_n$ can be obtained by comparing the metric and connexion parameters of the new geometry of the deformed $V_n$ at $\bar{x}$, with the representatives of $V_n$ at $\bar{x}$.

In my paper "one parameter continuous group of deformations" I have shown that by subjecting a general space of affine connexion to general finite one parameter groups of transformations $G_i$, defined by

$$\bar{x}^i = x^i + tDx^i + \ldots + t^n D^i x + \ldots$$

where $t$ is a parameter, and

$$D^i = u^r \frac{\partial}{\partial x^i} , u^r$$

the vector field defining the infinitesimal transformation that generates the group, the deformations of the vectors, tensors and the connexion parameters ($L^i_{jk}$) of the space are determined by a single operator, which I call $S$-operator
associated with the given one-parameter group of Lie.

It is to be noticed that the exposition of this problem though given in a different manner in my paper cited above as well as in Chapter II of this work, is the same at the foundation as that given above.

Chapter I of this work has its object to give a brief exposition of the method of 'Repère mobile' as initiated by Darboux and developed, in all its generalities, by E. Cartan to study the invariants of a variety and the generalizations of Riemann spaces. The method of 'Repère mobile' was later on generalized by E. Cartan to establish the structure of the finite continuous groups and to make the connection of his theory with the classical theory of S. Lie, because the elements of the theory of E. Cartan are found in the notions introduced by S. Lie.

In Chapter II, I give a short proof of my results about the $S$-operator which I have already defined in my paper to be published shortly in Ind. Jour. Math., by the methods of tensor calculus. The present method of defining the $S$-operator is based on the theory of 'Repère mobile' and was very kindly indicated to me by Father C. Racine.
4.

I give here, once for all, some formulae of 'Covariant bilinear' introduced by Frobenius and 'Exterior multiplication' defined by H. Grassmann as they will be constantly applied in the present work.

'Covariant bilinear'. - If
\[ \omega(x, dx) = \sum_K a_K(x) dx_K \]
be a form of Pfaff,

the expression
\[ d\omega(x, dx) - \delta \omega(x, dx) = \sum_K \left( da_K \delta x_K - \delta a_K dx_K \right) \]
\[ = \sum_{(iK)} \left( \frac{\partial a_K}{\partial x_i} - \frac{\partial a_i}{\partial x_K} \right) \left( dx_i \delta x_K - \delta x_i dx_K \right) \]
\[ (1 \leq i \leq K \leq n) \]

will be denoted by an abridged symbol

| \omega'(x, dx, \delta x) = d\omega(x, dx) - \delta \omega(x, dx) |

'Exterior multiplication' - The 'exterior' product of p differentials \( dx_1, dx_2, \ldots, dx_p \) is denoted by

\[ [dx_1 dx_2 \ldots dx_p] \]

and stands for the determinant
The 'exterior' product of the forms

$$\omega_1 = \sum a_i \, dx_i , \quad \omega_2 = \sum b_i \, dx_i , \quad \omega_3 = \sum c_i \, dx_i$$

will be by definition

$$[\omega_1, \omega_2, \omega_3] = \sum_{i,j,k} a_i b_j c_k \ [dx_i \, dx_j \, dx_k]$$