Chapter 6

Magnetization reversal through soliton in an anisotropic ferromagnet with octupole-dipole interaction

The increasing interest in both of the aspects of magnetic spin chain with different boundary conditions has been recently by means of physical finite systems, such as non-equal edge spin chain, and unconventional magnetic interactions, particularly the interplay with the spin-wave aspects, such as high-energy spin-Wigner excitations. New ideas are expected from the anisotropy of the magnetic moment of the magnetizing particles, which can be represented by a model containing a magnetic spin chain extended over a large number of layers. It is anti-parallel to the direction of magnetization, which is achieved by successively reversing parts of the spin chain with octupole-dipole interactions in a linear fashion. For this purpose, the magnetization reversal process by flipping of the spins is shown to be a robust process. During this process, linear spin interactions have been accounted for, and the results are compared with numerical data obtained from the semi-classical equations. The semi-classical equations are solved using the finite difference method, which is in excellent agreement with the numerical data obtained from the semi-classical equations. The solutions are in excellent agreement with the numerical data obtained from the semi-classical equations.

The exact one-dimensional magneto-elastic motion in the form of the total spin vector $S(t) = S_x(t) + i S_y(t)$ is numerically calculated, which we have proposed for topic.
6.1 Introduction

The increasing interest in the nonlinear spin excitations in ferromagnetic spin chain with different magnetic interactions have been driven by demands for their practical applications as well as by their scientific importance in the last decades. Remarkable progress has been made on their technological applications, particularly for magnetic and spintronic devices such as high-density data storage and magnetic field sensors. The future of magnetic recording is a matter of considerable debate because it appears to be approaching various physical limits imposed by the dynamics of the recording process. In conventional magnetic recording, for example, information is recorded on a thin film containing a magnetic layer that is divided into small domains, each with a well defined magnetization. In data storage, the reversal of magnetization is used to write bits of information onto magnetic materials. Information is written by reversing the direction of magnetization using an external magnetic field that is anti-parallel to this direction [328]. In this connection, we study the nonlinear spin excitations in the form of soliton in an anisotropic ferromagnetic spin chain with octupole-dipole interaction and we establish the magnetization reversal process by flipping of soliton in the ferromagnetic media.

For the few decades, considerable effort have been devoted for understanding the nonlinear spin excitations in terms of soliton in the Heisenberg ferromagnetic spin chain with different magnetic interactions in the classical and semi-classical limits [21,22,290,299,300,295,382-384]. The dynamics associated with different magnetic interactions are governed by nonlinear Schrödinger family of equations which admit localized nonlinear spin excitations. The soliton spin excitations have been identified in the ferromagnetic spin chain under semi-classical approximation in the long wave length and low temperature limit in which the spin operators are treated as bosonic operators through Holstein-Primakoff approximation in connection with Glauber's coherent state representation.

The existence of a third order spin-spin exchange interaction of the form $(\vec{s}_{i} \cdot \vec{s}_{i+1})(\vec{s}_{i+1} \cdot \hat{k})^2$, where $\hat{k}$ is a constant vector, was proposed by Toru Moriya
[84,89,385]. It is pointed out that the dynamics of a Heisenberg spin chain with site-dependent bilinear and biquadratic exchange interaction in the classical continuum limit is governed by an inhomogeneous higher order nonlinear Schrödinger equation and exhibits an interesting phenomenon of soliton flipping which leads to magnetization reversal in the ferromagnetic medium due to the presence of inhomogeneity [290]. In the different context, it is found that the magnetization reversal through soliton flip has been investigated in a site-dependent weak ferromagnet with linear inhomogeneity by solving the evolution equations associated with soliton parameters like amplitude and velocity by using the fourth order Runge-Kutta method numerically [386]. Conceived by the above in mind, in this chapter we investigate the effect of next higher order octupole-dipole interaction on the nonlinear spin excitations.

## 6.2 Model and spin dynamics

The Heisenberg Hamiltonian for an anisotropic ferromagnetic spin chain with octupole-dipole interaction can be written as

\[
\tilde{H} = -\sum_i \left[ J_1 [S_i^z S_{i+1}^x + S_i^y S_{i+1}^y] + J_2 [S_i^x S_{i+1}^x + J_3 (\vec{S}_i \cdot \vec{S}_{i+1})(\vec{S}_{i+1} \cdot \hat{k})^2 \\
- B_1 (S_i^z)^2 - B_2 (S_i^z)^4 \right],
\]

(6.1)

where \(\hat{k} = (0, 0, 1)\) and \(\vec{S}_i = (S_i^x, S_i^y, S_i^z)\) represents a three component spin vector. In Eq. (6.1), \(J_1\) and \(J_2\) represent the bilinear exchange interaction parameters due to spin-spin coupling in the \(S^x - S^y\) plane and along the \(z\)-direction respectively and the term proportional to \(J_3\) corresponds to the octupole-dipole interaction. The terms proportional to \(B_1\) and \(B_2\) represent the single-ion uniaxial anisotropic energy due to crystal field effect and easy axis of magnetization is chosen along the \(z\)-direction. The dimensionless form of the Hamiltonian (6.1) can be written as

\[
H = -\frac{1}{2} \sum_i \left[ \frac{J_1}{S_2^2} [\dot{S}_i^+ \dot{S}_{i+1}^- + \dot{S}_i^- \dot{S}_{i+1}^+] + \frac{2J_2}{S_2^4} [\dot{S}_i^z \dot{S}_{i+1}^z] \\
+ \frac{J_3}{S_4^2} [\dot{S}_i^+ \dot{S}_{i+1}^- + \dot{S}_i^- \dot{S}_{i+1}^+ + 2\dot{S}_i^z \dot{S}_{i+1}^z] \right] (\dot{S}_{i+1}^z)^2 - \frac{2B_1}{S_2^2} (\dot{S}_i^z)^2 - \frac{2B_2}{S_4^2} (\dot{S}_i^z)^4,
\]

(6.2)
where $\hat{S}_i^+ = \hat{S}_i^x \pm i \hat{S}_i^y$ and $\hat{S}_i = \frac{\hat{S}_i^x}{S}$. While writing Eq. (6.2), we define $H = \hbar H / \gamma^2 S^2$, $J_3 = \hbar^2 S^2 J_3$ and $B_2' = \hbar^2 S^2 B_2$. The dimensionless spin operator $\hat{S}_i$ satisfies the commutation relations $[\hat{S}_n^+, \hat{S}_m^-] = 2\delta_{mn} \hat{S}_n^z$ and $[\hat{S}_n^x, \hat{S}_m^y] = +\delta_{mn} \hat{S}_n^z$ with $\hat{S}_n \cdot \hat{S}_n = S(S + 1)$. Now, we introduce the Holstein-Primakoff transformation [137, 145] for the spin operators in terms of Boson operators $\hat{S}_n^+ = \sqrt{2S} \left[ 1 - \frac{a_n a_n^\dagger}{2S} \right]^{1/2} a_n$, $\hat{S}_n^- = \sqrt{2S} \left[ 1 - \frac{a_n a_n^\dagger}{2S} \right]^{1/2}$ and $\hat{S}_n^z = [S - a_n a_n^\dagger]$. The bosonic operators $a_n^\dagger$ and $a_n$ satisfy the Bose commutation relations $[a_n, a_m^\dagger] = \delta_{mn}$ and $[a_n, a_m] = [a_n, a_m^\dagger] = 0$.

For treating the problem semi-classically at sufficiently low temperature, we use a truncated semiclassical expansion for $\hat{S}_n^+$ and $\hat{S}_n^-$ in the following form as

$$
\frac{\hat{S}_n^+}{S} = \sqrt{2} \left[ 1 - \frac{\epsilon^2}{4} a_n a_n^\dagger - \frac{\epsilon^4}{32} a_n^\dagger a_n^\dagger a_n a_n \right] + \mathcal{O}(\epsilon^6 a_n^\dagger a_n a_n^\dagger a_n),
$$

and

$$
\frac{\hat{S}_n^-}{S} = \sqrt{2} \epsilon a_n^\dagger a_n \left[ 1 - \frac{\epsilon^2}{4} a_n a_n^\dagger - \frac{\epsilon^4}{32} a_n^\dagger a_n^\dagger a_n a_n \right] + \mathcal{O}(\epsilon^6 a_n^\dagger a_n a_n^\dagger a_n),
$$

where $\epsilon = 1/\sqrt{S}$ is a small dimensionless parameter and while writing the above expansions it is assumed that $a_n^\dagger a_n \ll 2S$. Using the expansions for $\hat{S}_n^+$ and $\hat{S}_n^-$, the Hamiltonian (6.2) can be written in terms of the Bosonic operators as a power series in $\epsilon$ as

$$
H = - \sum_i \left\{ \epsilon^2 \left\{ J(a_n^\dagger a_{n+1}^\dagger + a_n^\dagger a_{n+1}) - J'(a_{n+1}^\dagger a_{n+2}^\dagger + a_{n+1}^\dagger a_n) - 2J'' a_{n+1}^\dagger a_{n+1} \right. \\
+ 2B_1 a_n^\dagger a_n + 4B_2 a_n^\dagger a_n a_n + \frac{\epsilon^4}{4} \left\{ J \left[ a_n^\dagger a_n a_n^\dagger a_{n+1} + a_n a_n^\dagger a_{n+1} a_n^\dagger a_{n+1} \right] - 4J_2 a_n^\dagger a_n a_n^\dagger a_{n+1} a_n^\dagger a_{n+1} a_n \right. \\
- 2J'' \left[ 6a_n^\dagger a_{n+1}^\dagger a_{n+1} a_n + 6a_n a_n^\dagger a_{n+1}^\dagger a_{n+1} a_n - 4a_n^\dagger a_n^\dagger a_{n+1} a_n^\dagger a_{n+1} a_n \right] \\
- 4a_n^\dagger a_n^\dagger a_{n+1}^\dagger a_{n+1} a_n + \left. 2B_1 a_n^\dagger a_n a_n^\dagger a_n^\dagger - 12B_2 a_n^\dagger a_n a_n^\dagger a_n^\dagger a_n \right\} \\
+ \frac{\epsilon^6}{32} \left\{ J \left[ 2a_n^\dagger a_n a_n^\dagger a_{n+1} a_n + 2a_n a_n^\dagger a_n^\dagger a_{n+1} a_{n+1} a_{n+1} \right] - a_n^\dagger a_n a_n^\dagger a_n a_n^\dagger a_n a_n^\dagger a_n a_n^\dagger + a_n^\dagger a_n a_n^\dagger a_n a_n^\dagger a_n a_n^\dagger a_n a_n^\dagger + \\
- a_n^\dagger a_n^\dagger a_n^\dagger a_n a_n + 6a_n^\dagger a_n^\dagger a_n a_n^\dagger a_n a_n^\dagger a_n a_n^\dagger a_n a_n^\dagger + \\
+ 16J'' \left[ 3a_n^\dagger a_{n+1}^\dagger a_{n+1} a_n a_n^\dagger a_n^\dagger a_n + 6a_n^\dagger a_n a_n^\dagger a_n a_n^\dagger a_n^\dagger a_n a_n^\dagger a_n a_n^\dagger + \\
- 2a_n^\dagger a_n a_n^\dagger a_n a_n^\dagger a_n^\dagger a_n a_n^\dagger a_n a_n^\dagger a_n a_n^\dagger + \\
+ a_n^\dagger a_n^\dagger a_n^\dagger a_n^\dagger a_n^\dagger a_n a_n^\dagger a_n a_n^\dagger a_n a_n^\dagger a_n a_n^\dagger + \\
+ a_n^\dagger a_n^\dagger a_n^\dagger a_n^\dagger a_n a_n^\dagger a_n a_n^\dagger a_n a_n^\dagger a_n a_n^\dagger + \\
+ 2(J_2 + J'_3 - B_1 - B_2) \right]. \\
\right.
$$

(6.3)
where \( J = J_1 + J'_2 \), \( J' = J_2 + J'_3 \) and \( J'' = J'_3 \). The equation of motion associated with Heisenberg ferromagnetic spin chain can be expressed in terms of Bose operators by substituting the Hamiltonian (6.3) in the following equation of motion

\[
\hbar \frac{\partial a_n}{\partial t} = [a_n, H] = F(a_n^\dagger, a_n, a_{n+1}^\dagger, a_{n+1}).
\]

We are concerned with the nonlinear spin excitations induced by nonlinearity in the magnon system, in which a cluster of spins may undergo a large excursion as compared to the rest of the spins. Quantum state of such large amplitude collective modes may be represented by coherent states. Hence, we introduce Glauber's coherent-state representation [149] for the Bose operators \( a_n^\dagger |u\rangle = u_n^* |u\rangle \), \( a_n |u\rangle = u_n |u\rangle \), \( |u\rangle = \Pi_n |u_n\rangle \) with \( \langle u|u\rangle = 1 \), where \( |u(n)\rangle \) is the coherent-state eigenvector for the operator \( a_n \) and \( u_n \) is the coherent amplitude for the system in the state \( |u\rangle \), we write down the discrete equation of motion \( < u|a_j |u > \) corresponding to the new Hamiltonian (6.3) as given by

\[
\frac{d u_n}{dt} = -\sum \left\{ \frac{\epsilon^2}{4} \left[ J (u_{n-1} + u_{n+1}) - (2J' + 2J'' - 2B_1 - 4B_2') u_n \right] \\
- \frac{\epsilon^4}{4} \left[ J \left( u_{n+1}^* u_{n-1} + u_{n-1}^* u_{n+1} \right) + 2u_n^2 (u_{n-1} + u_{n+1}) + \left( u_{n-1}^2 u_{n-1}^* \right) \right] + 4J_2 (u_{n+1}^2 + |u_{n-1}|^2) u_n - 2J'' \left[ 6(|u_{n+1}|^2 \\
+ |u_{n-1}|^2) u_n + 12 |u_n|^2 u_{n-1} - 8 |u_n|^2 u_{n+1} - 4 (u_{n+1}^2 u_{n-1}^* + u_{n+1}^* u_{n-1}^2) \right] \\
+ 8B_1 |u_n|^2 u_n + 48B_2' |u_n|^2 u_n \right\} + \frac{\epsilon^6}{32} \left[ J \left( 2u_n^2 (|u_{n+1}|^2 u_{n+1}^* + |u_{n-1}|^2 u_{n-1}^*) \right) \\
+ |u_{n+1}|^2 u_{n+1}^* + 4 |u_n|^2 (|u_{n+1}|^2 u_{n+1}^* + |u_{n-1}|^2 u_{n-1}^*) \right] \\
+ 2 |u_n|^2 (u_{n+1}^* u_{n+1}^* + u_{n-1}^* u_{n-1}^*) - 3 |u_n|^4 (u_{n+1} + u_{n-1}) - (|u_{n+1}|^4 u_{n+1} \\
+ |u_{n-1}|^4 u_{n-1}) + 16J'' \left[ 9 |u_n|^4 u_{n+1} - 6 |u_{n+1}|^4 u_n - 12 |u_{n-1}|^2 |u_n|^2 u_{n+1} \\
- 6 |u_n|^4 u_{n+1} + 3 |u_{n+1}|^4 u_{n+1} + 6 |u_n|^2 u_{n}^2 u_{n-1}^* + u_{n}^2 (|u_{n+1}|^2 u_{n+1}^* \right) \\
+ |u_{n-1}|^2 u_{n-1}^* + 2 |u_n|^2 (|u_{n+1}|^2 u_{n+1} + |u_{n-1}|^2 u_{n-1}) + 384B_2' |u_n|^4 u_n \right].
\] (6.4)

The presence of nonlinearity and discreteness in Eq. (6.4) is very difficult to solve in the present form. Hence, we make continuum approximation which is valid in the low temperature and long wave length limit by assuming that the lattice constant is very small compared to the length of the lattice. For this,
we assume that the spins vary slowly over the distance of the lattice parameter \( \gamma \) and replace \( u(t) \) as \( u_{i\pm 1} = u(x, t) \pm \gamma u_x + \frac{\gamma^2}{2!} u_{xx} \pm \frac{\gamma^3}{3!} u_{xxx} + \frac{\gamma^4}{4!} u_{xxxx} + O(\gamma^5) \), where \( x = n\gamma \) and suffix \( x \) represents the partial derivative with respect to \( x \).

**Case (i)**

Using the above expansion for \( u \) in Eq. (6.4) and retaining the terms proportional to \( \gamma^m e^n \) up to order \( O(m + n = 4) \), the resultant equation can be written as

\[
\begin{align*}
    i u_t + J \gamma^2 u_{xx} + \frac{J^2 \gamma^4}{12} u_{xxxx} - \frac{\epsilon^2}{4} \left( (8J - 8J_2 - 16J'' + 8B_1 + 48B_2) |u|^2 u \\
    + \gamma^2 \left((4J - 4J_2 - 4J'')|u|^2 u_{xx} + (2J - 4J_2 - 4J'') u^2 u^*_{xx}
    + (4J - 4J_2 - 4J'') u_x^2 u + (2J - 8J''|u_x^2 u^* \right)\right) + 12\epsilon^4 B_2 |u|^4 u &= 0. \quad (6.5)
\end{align*}
\]

After redefinition of the coefficients of Eq. (6.5), we get the following equation

\[
\begin{align*}
    i u_t + u_{xx} + 2l_1 |u|^2 u + l_2 (|u|^2 u_{xx} + |u_x|^2 u) + l_3 u^2 u^*_{xx} + l_4 u^2 u^* + l_5 |u|^4 u + l_6 u_{xxxx} &= 0, \quad (6.6)
\end{align*}
\]

where \( l_1 = -\frac{\epsilon^2}{J^2} (J - J_2 - 2J'' + B_1 + 6B_2) \), \( l_2 = -\frac{\epsilon^2}{J} (J - J_2 - J'') \), \( l_3 = -\frac{\epsilon^2}{2J} (J - 2J_2 - 2J'') \), \( l_4 = -\frac{\epsilon^2}{2J} (J - 2J'') \), \( l_5 = \frac{12\epsilon^4 B_2}{J^2 \gamma^2} \) and \( l_6 = \frac{\gamma^2}{12} \). Eq. (6.6) describes the dynamics of a one-dimensional anisotropic Heisenberg ferromagnetic spin system with octupole-dipole interaction in the semiclassical limit which is governed by a generalized nonlinear Schrödinger (GNLS) equation. It is found that the structure of Eq. (6.6) resembles that of the GNLS equation of bi-quadratic Heisenberg ferromagnetic spin chain in the classical continuum limit obtained through space curve mapping [387].

**Case (ii)**

Moreover, making use of \( u \) in Eq. (6.4) and retaining the terms proportional to \( \gamma^m e^n \) up to order \( O(m + n = 5) \), we get the following higher order NLS equation

\[
\begin{align*}
    i u_t + J \gamma^2 u_{xx} + \frac{J^2 \gamma^4}{12} u_{xxxx} - \frac{\epsilon^2}{4} \left( (8J - 8J_2 - 16J'' + 8B_1 + 48B_2) |u|^2 u \\
    + \gamma^2 \left((4J - 4J_2 - 4J'')|u|^2 u_{xx} + (2J - 4J_2 - 4J'') u^2 u^*_{xx}
    + (4J - 4J_2 - 4J'') u_x^2 u + (2J - 8J''|u_x^2 u^* \right)\right) + 384B_2 |u|^4 u + 96J'' |u|^2 u^2 u^* &= 0. \quad (6.7)
\end{align*}
\]
After redefinition of the coefficients of Eq. (6.7), we obtain the following higher order nonlinear Schrödinger equation of the form

\[ iu_t + u_{xx} + 2k_1|u|^2u + k_2(|u|^2u_{xx} + |u_x|^2u) + k_3|u|^4u + k_4u_{xx}^2u_x^* + k_5(uu_xu_{xx}^*) + uu_x^*u_{xx} + u^*u_xu_{xx} + |u_x|^4u_x + k_6|u|^4u + k_7|u|^2u_x^2u_x^* + k_8u_{xxxx} = 0, \]  

(6.8)

where \( k_1 = -\frac{J^2}{J^2}(J - J_2 - 2J'' + B_1 + 6B_2^2) \), \( k_2 = -\frac{J^2}{J} (J - J_2 - J'') \), \( k_3 = -\frac{J^2}{J} (J - 2J_2 - 2J'') \), \( k_4 = -\frac{J^2}{J^2} (J - 4J'') \), \( k_5 = -\frac{J^2}{J^2} \), \( k_6 = \frac{12B_2^2}{J^2} \), \( k_7 = \frac{3J'^2}{J^2} \) and \( k_8 = \frac{J^2}{12} \).

### 6.3 Perturbation of soliton due to discreteness effect

By choosing the coefficients of Eq. (6.6) as \( J_1 = -2J'_3 \), \( J_2 = -\frac{1}{3}J'_3 \), \( B_1 = -\frac{21}{4}J_2 \), \( B_2 = -\frac{5}{24}J_2 \) and \( \gamma^2 = 3\epsilon^2 \) and making use of it in Eq. (6.6), we get

\[ iu_t + u_{xx} + 2|u|^2u + \lambda \left[ \frac{1}{4}u_{xxxx} - \frac{2}{3}|u|^4u - \frac{14}{5}(|u|^2u_{xx} + |u_x|^2u) - \frac{7}{10}u_{xx}^2u_x^* - \frac{5}{2}u_x^2u_x^* \right] = 0, \]  

(6.9)

where \( \lambda = \epsilon^2 \). In order to understand the nonlinear spin excitations of a Heisenberg spin chain under the influence of octupole-dipole interaction perturbatively by treating the terms proportional to \( \lambda \) as weak perturbation using the perturbation method as laid down by Kodama and Ablowitz [301]. When \( \lambda = 0 \), Eq. (6.9) reduces to the completely integrable cubic NLS equation which admit N-soliton solutions [7]. The envelope one soliton solution for cubic NLS can be written as

\[ u = \eta \operatorname{sech}\eta(\theta - \theta_0) \exp[i\xi(\theta - \theta_0) + i(\sigma - \sigma_0)], \]  

(6.10)

where \( \theta_t = -2\xi, \ \theta_x = 1, \ \sigma_t = \eta^2 + \xi^2 \) and \( \sigma_x = 0 \). The parameter \( \eta \) and \( \xi \) are related to the scattering parameter of the inverse scattering transform analysis. Now, we write \( \eta, \ \xi, \ \theta, \ \theta_0 \) and \( \sigma_0 \) as functions of a new time scale \( T = \lambda t \) and hence \( u = \hat{u}(\theta, T; \lambda) \exp[i\xi(\theta - \theta_0) + i(\sigma - \sigma_0)] \). Then under the assumption of quasi-stationarity, we expand \( \hat{u} \) in terms of \( \lambda \) using Poincare type expansion as

\[ \hat{u}(\theta, T; \lambda) = \hat{u}_0(\theta, T) + \lambda \hat{u}_1(\theta, T) + ..., \]  

(6.11)
where $\hat{u}_0 = \eta \sech \eta (\theta - \theta_0)$. Making use of $u$ in Eq. (6.9) and then collecting the coefficient of different powers of $\lambda$, we obtain at $O(\lambda)$

$$-\eta^2 \ddot{u}_1 + \dot{u}_{1\theta \theta} + 2u_0^2 \ddot{u}_1 + 4u_0^2 \dot{u}_1 = F_1(\hat{u}_0).$$

(6.12)

After substituting $\hat{u}_1 = \hat{\phi}_1 + i \hat{\psi}_1$ in Eq. (6.12), we obtain

$$L_1 \ddot{\hat{\phi}_1} = -\eta^2 \ddot{\hat{\phi}_1} + \dot{\phi}_{1\theta \theta} + 6u_0^2 \ddot{\hat{\phi}_1} = \Re F_1(\hat{u}_0),$$

(6.13)

$$L_2 \ddot{\hat{\psi}_1} = -\eta^2 \ddot{\hat{\psi}_1} + \dot{\psi}_{1\theta \theta} + 2u_0^2 \ddot{\hat{\psi}_1} = \Im F_1(\hat{u}_0),$$

(6.14)

where $L_1, L_2$ are self-adjoint operators and $\hat{\phi}_1, \hat{\psi}_1$ are real functions. The real $\Re F_1$ and imaginary $\Im F_1$ part of $F_1$ is given by

$$\Re F_1(\hat{u}_0) = \xi_T (\theta - \theta_0) \dot{u}_0 + \frac{7}{2} \ddot{u}_0 \dot{u}_{\theta \theta} + \frac{53}{10} \dddot{u}_0 \dot{u}_{\theta \theta} - \frac{16}{5} \xi^2 \dddot{u}_0 - \frac{2}{3} \xi^5 \dot{u}_0,$$

$$- \frac{1}{4} (\dddot{u}_{\theta \theta \theta \theta} - 6 \xi^2 \dddot{u}_{\theta \theta} + \xi^4 \dddot{u}_0),$$

(6.15)

$$\Im F_1(\hat{u}_0) = -\dddot{u}_0 + 12 \xi \dddot{u}_0 \dot{u}_{\theta \theta} - \xi \dddot{u}_{\theta \theta \theta \theta} + \xi^3 \dddot{u}_0.$$

(6.16)

It is pointed out that $\ddot{u}_{\theta \theta}$ and $\dot{\hat{u}}_0$ are the solutions of the homogeneous parts of Eqs. (6.13) and (6.14) for $\hat{\phi}_1$ and $\hat{\psi}_1$ respectively and the secularity conditions yield

$$\int_{-\infty}^{\infty} \dot{u}_{\theta \theta} \Re F_1 d\theta = 0 \quad \text{and} \quad \int_{-\infty}^{\infty} \dot{u}_0 \Im F_1 d\theta = 0.$$  

(6.17)

By substituting the values of $\ddot{u}_{\theta \theta}, \dot{\hat{u}}_0, \Re F_1$ and $\Im F_1$ in Eqs. (6.17) and after evaluating the integrals, we obtained $\xi_T = \eta_T = 0$. This implies that the discreteness effect does not alter the velocity and amplitude of the soliton during propagation in an anisotropic ferromagnetic spin chain with octupole-dipole interaction at lowest order of expansion. Now, we consider Eq. (6.8) and by choosing the coefficients of $J_1 = -2J_3, J_2 = -\frac{4}{5}J_3, B_1 = -\frac{21}{4}J_2, B'_2 = -\frac{5}{21}J_2$ and $\gamma^2 = 3\epsilon^2$, finally we arrive at

$$iu_t + u_{xx} + 2|u|^2 u + \lambda \left[ \frac{1}{4} u_{xxxx} - \frac{2}{3} |u|^4 u - \frac{14}{5} (|u|^2 u_{xx} + |u|^2 u_x^2) ight]$$

$$- \frac{7}{10} u^2 u_{xx} - \frac{5}{2} u_x^2 u_x^2 + 2 \delta_1 (uu_x u_{xx} + uu_x^2 u_{xx}) + u_x u_{xx}$$

$$+ |u_x|^2 u_x - 3 \delta_2 |u|^2 u_x^2 u_x^2 = 0,$$

(6.18)
where \( \lambda = \epsilon^2 \). In Eq. (6.18), the term proportional to \( \lambda \) are responsible for the discreteness effect and hence we treat the terms as weak perturbation by taking \( \lambda \) as the perturbation parameter. Similarly, we perform the same procedure as that of the previous case (c) and then after substituting \( \hat{u}_1 = \phi_1 + i\hat{\psi}_1 \) in the resultant equation, we obtain a system of two equations for \( \phi_1 \) and \( \hat{\psi}_1 \). The real and imaginary parts of \( F_2 \) is given by

\[
\Re F_2(\hat{u}_0) = \xi_T(\theta - \theta_0)\hat{u}_0 + \frac{7}{2}\hat{u}_0^2\hat{u}_{000} + \frac{53}{10}\hat{u}_0\hat{u}_{00}^2 - \frac{16}{5}\xi^2\hat{u}_0^3 - 2\delta_1(3\hat{u}_0\hat{u}_{00}\hat{u}_{000} + \hat{u}_{00}^3) + 2\xi\hat{u}_0^5 + 3\delta_2\hat{u}_0^4\hat{u}_{00} - \frac{1}{4}(\hat{u}_{000000} - 6\xi^2\hat{u}_{000} + \xi^4\hat{u}_0),
\]

\[\text{(6.19)}\]

\[
\Im F_2(\hat{u}_0) = -\hat{u}_0\xi - \xi\hat{u}_{000} + \xi^3\hat{u}_0 + \frac{46}{5}\xi\hat{u}_0^2\hat{u}_{00} - 2\delta_1(\hat{u}_0\hat{u}_{00}^2 + \hat{u}_{00}^2\hat{u}_{000}) - 3\delta_2\xi\hat{u}_0^5.
\]

\[\text{(6.20)}\]

By substituting the solutions \( \hat{u}_{00} \), \( \hat{u}_0 \), \( \Re F_2 \) and \( \Im F_2 \) in the associated secularity conditions

\[
\int_{-\infty}^{\infty} \hat{u}_{00}\Re F_2 d\theta = 0 \quad \text{and} \quad \int_{-\infty}^{\infty} \hat{u}_0\Im F_2 d\theta = 0,
\]

\[\text{(6.21)}\]

and after evaluating the integrals, we get the following evolution equations for velocity and amplitude of the soliton

\[
\xi_T = (12\delta_1 + \frac{106}{35}\delta_2)\eta^6,
\]

\[\text{(6.22)}\]

\[
\eta_T = (\frac{16}{15}\delta_1 - \frac{16}{5}\delta_2)\xi^5.
\]

\[\text{(6.23)}\]

As Eqs. (6.22-6.23) are highly nonlinear in nature, it is very difficult to solve. Then, we rescale \( \eta \) suitably as \( \eta_1 \rightarrow \eta^2 \), we get

\[
\xi_{1T} = \beta_1\eta_1^3,
\]

\[\text{(6.24)}\]

\[
\eta_{1T} = 2\beta_2\xi_1\eta_1^3,
\]

\[\text{(6.25)}\]

where \( \beta_1 = (12\delta_1 + \frac{106}{35}\delta_2) \) and \( \beta_2 = (\frac{16}{15}\delta_1 - \frac{16}{5}\delta_2) \). By solving Eq. (6.24-6.25) through modified extended tanh-function method [300,324,325,388], we get

\[
\xi_1(T) = a_0 + i\sqrt{\frac{3\beta_2}{20\beta_1}} b_0 \tanh\left(\frac{20}{3}\beta_1\beta_2 b_0^2 T\right) - \frac{\beta_2 b_0}{10a_0\beta_1} \tanh^2\left(\frac{20}{3}\beta_1\beta_2 b_0^2 T\right),
\]

\[\text{(6.26)}\]
Figure 6.1: Evolution of amplitude (a,b) $\eta(T)$ and velocity (c,d) $\xi(T)$ of the soliton for (a,c) $b_0 = 0.8$ and (b,d) $b_0 = 1.0$ with $a_0 = 1.0$, $\delta_1 = -0.45$ and $\delta_2 = -0.133$.

\begin{equation}
\eta_1(T) = b_0 - i \sqrt{\frac{3b_0\beta_2}{5\beta_1\alpha_0^2}} \tanh \left( \sqrt{\frac{20}{3}} \beta_1 \beta_2 b_0^2 T \right).
\end{equation}

(6.27)

From the results after careful analysis, it is found that the discreteness effect alter the velocity and the amplitude of the soliton during propagation which leads to magnetization reversal behaviour through flipping of soliton. Now, we demonstrate the magnetization reversal through flipping of soliton under the influence of octupole-dipole interaction in an anisotropic ferromagnetic spin chain by plotting Eqs. (6.26-6.27) for the velocity and amplitude of the soliton by choosing specific values for the parameters. From the Fig. (6.1a), it is inferred that the amplitude of the soliton increases as time passes and reaches maximum value when $T = 1.9$ ns, then the soliton suddenly flipped into negative direction and it reaches negative maximum leading to magnetization reversal.
6.3 Perturbation of soliton due to discreteness effect

process. On careful analysis, it is found that for $b_0 = 0.8$ the soliton takes $4 \text{ ns}$ to complete one cycle. More interestingly, it is also found that when we increase the value of $b_0$, the switching time is reduced to $2 \text{ ns}$ as shown in Fig. (6.1b). Thus, this type of lossless switching of solitons in the ferromagnetic media may find immense applications in the magnetic data-storage industry.

6.3.1 Perturbed solitons

The perturbed soliton solutions can be constructed by solving Eq. (6.13) for $\dot{\phi}_1$ and Eq. (6.14) for $\dot{\psi}_1$ using $\xi_T$ and $\eta_T$ for the case (ii). The homogeneous part of Eq. (6.13) admits two particular solutions $\phi_{11}$ and $\phi_{12}$ which are of the form

$$\phi_{11} = \text{sech}\eta(\theta - \theta_0) \tanh \eta(\theta - \theta_0),$$

$$\phi_{12} = \frac{1}{2} \frac{[\text{sech}\eta(\theta - \theta_0) - \frac{3}{2} \eta(\theta - \theta_0) \text{sech}\eta(\theta - \theta_0) \tanh \eta(\theta - \theta_0)] - \frac{1}{2} \tanh \eta(\theta - \theta_0) \sinh \eta(\theta - \theta_0)]}{\eta}.$$  

The general solution for $\dot{\phi}_1$ can then be written using the formula

$$\dot{\phi}_1 = \alpha_1 \phi_{11} + \alpha_2 \phi_{12} - \phi_{11} \int_{-\infty}^{\theta} \phi_{12} \Re F_2 d\theta' + \phi_{12} \int_{-\infty}^{\theta} \phi_{11} \Re F_2 d\theta',$$

with $\alpha_1$ and $\alpha_2$ being arbitrary constants. After evaluating the integrals in Eq. (6.30), we obtain

$$\phi_{11} \int_{-\infty}^{\theta} \phi_{12} \Re F_2 d\theta = \left[ \frac{2203}{720} \eta^4 + \frac{17}{40} \xi^2 \eta + \frac{3}{16} \xi^4 - \frac{12}{35} \delta_2 \eta^4 - 3 \frac{\xi_T}{\eta} \right] \text{sech}(\eta \theta)$$
$$+ \left[ \frac{2839}{360} \eta^3 + \frac{19}{10} \xi^2 \eta - \frac{3}{16} \xi^4 - 4 \delta_1 \eta^4 + \frac{6}{35} \delta_2 \eta^4 + 3 \frac{\xi_T}{\eta} \right] \text{sech}^3(\eta \theta)$$
$$+ \left[ \frac{304}{45} \eta^3 - \frac{123}{40} \xi^2 \eta + \frac{33}{35} \delta_2 \eta^4 + \frac{38}{5} \delta_1 \eta^4 - \left( \frac{152}{5} \delta_1 + \frac{18}{5} \delta_2 \right) \eta^5 \Theta \right] \text{sech}^5(\eta \theta)$$
$$+ \left[ \frac{569}{120} \eta^3 - \frac{9}{35} (14 \delta_1 + 3 \delta_2) \eta^4 + \frac{99}{70} (14 \delta_1 + 3 \delta_2) \eta^5 \Theta \right] \text{sech}^7(\eta \theta)$$
$$- \frac{3}{14} (14 \delta_1 + 3 \delta_2) \eta^5 \Theta \text{sech}(\eta \theta) + \left[ \frac{1}{4} \xi_T \Theta^2 + \frac{1}{8} (8 \xi^2 \eta^2 - \xi^4 - \eta^4) \right] \tanh(\eta \theta)$$
$$\times \text{sech}(\eta \theta) + \left[ \frac{3}{4} \xi_T \Theta^2 - \frac{3}{16} (6 \xi^2 \eta^2 - \xi^4 - \eta^4) \right] \tanh(\eta \theta) \text{sech}^3(\eta \theta).$$
\[ \begin{align*}
&+ \left[ \frac{39}{16} \xi^2 \eta^2 - \frac{207}{40} \eta^4 \right] \Theta + \frac{3}{10} \delta_1 \eta^4 + \frac{3}{20} \delta_2 \eta^4 \right] \tanh(\eta \Theta) \sech^5(\eta \Theta) \\
&+ \left[ \frac{569}{120} \eta^4 \Theta - \frac{3}{14} (14 \delta_1 + 3 \delta_2) \eta^4 \right] \tanh(\eta \Theta) \sech^7(\eta \Theta) \\
&- 8 \delta_1 \eta^4 \ln \cosh(\eta \Theta) \tanh(\eta \Theta) \sech(\eta \Theta),
\end{align*} \]

and

\[ \begin{align*}
&\phi_{12} \int_{-\infty}^{\Theta} \phi_{11} R F_2 d\theta = \left[ \frac{1}{2} \xi \eta \Theta - \frac{12}{35} \delta_2 \eta^5 \Theta - \frac{1}{16} (6 \xi^2 \eta - \frac{3}{4} \xi^4 - \eta^3) \right] \\
&\times \sech(\eta \Theta) + \left[ \frac{6}{35} \delta_2 \eta^5 (\Theta - \theta_0) - \frac{3}{16} \xi^4 + \frac{19}{10} \xi^2 \eta - \frac{153}{80} \eta^3 \right] \sech^3(\eta \Theta) \\
&+ \left[ \frac{1097}{168} \eta^3 - \frac{93}{40} \xi^2 \eta + \left( \frac{3}{5} \delta_1 + \frac{3}{70} \delta_2 \right) \eta^5 (\Theta - \theta_0) \right] \sech^5(\eta \Theta) \\
&+ \left[ \frac{569}{140} \eta^3 + \left( 6 \delta_1 + \frac{27}{35} \delta_2 \right) \eta^5 (\Theta - \theta_0) \right] \sech^7(\eta \Theta) \\
&- \frac{3}{14} (14 \delta_1 + 3 \delta_2) \eta^5 (\Theta - \theta_0) \sech^9(\eta \Theta) \\
&- \left[ \frac{3}{4} \xi T \eta - \frac{1}{2} \delta_2 \eta^4 \right] \tanh(\eta (\Theta - \theta_0)) \sech(\eta \Theta) \\
&- \frac{3}{4} \xi T \Theta^2 + \frac{3}{16} (6 \xi^2 \eta - \xi^4 - \eta^4) \Theta + \delta_1 \eta^4 - \frac{9}{70} \delta_2 \eta^4 \right] \\
&\times \tanh(\eta \Theta) \sech^3(\eta \Theta) + \left[ \frac{93}{10} \xi^2 \eta^2 - \frac{207}{40} \eta^4 \right] \Theta \\
&+ 4 \delta_1 \eta^4 + \frac{12}{35} \delta_2 \eta^4 \right] \tanh(\eta (\Theta - \theta_0)) \sech^5(\eta \Theta) \\
&+ \left[ \frac{569}{140} \eta^4 (\Theta - \theta_0) - \frac{3}{14} (14 \delta_1 + 3 \delta_2) \eta^4 \right] \tanh(\eta \Theta) \sech^7(\eta \Theta) \\
&+ \left[ \frac{1}{4 \eta^2} \xi T - \frac{4}{35} \delta_2 \eta^4 \right] \sinh(\eta \Theta),
\end{align*} \]

where \( \Theta = (\theta - \theta_0) \). Substituting Eqs. (6.31) and (6.32) in the formula for the general solution for \( \phi_1 \), we obtain

\[ \phi_1 = \left[ \frac{7}{2} \xi \eta \Theta - \frac{12}{35} \delta_2 \eta^5 \Theta + \frac{12}{35} \delta_2 \eta^4 - \frac{1}{8} \eta \xi^4 - \frac{4}{5} \xi^2 \eta - \frac{517}{180} \eta^3 \right] \sech(\eta \Theta) \\
- \left[ (4 \delta_1 - \frac{6}{35} \delta_2) \eta^5 \Theta + \frac{3}{5} \xi T \Theta - (4 \delta_1 - \frac{6}{35} \delta_2) \eta^5 - \frac{569}{180} \eta^3 \right] \sech^3(\eta \Theta) \\
+ \left[ (3 \delta_1 + \frac{51}{14} \delta_2) \eta^5 \Theta - \frac{569}{180} \eta^3 - (\frac{38}{5} \delta_1 + \frac{33}{35} \delta_2) \eta^4 \right] \sech^5(\eta \Theta) \\
- \left[ \frac{69}{5} \delta_1 + \frac{243}{70} \delta_2 \right] \eta^5 \Theta - \frac{569}{840} \eta^3 - \frac{9}{35} (14 \delta_1 + 3 \delta_2) \eta^4 \right] \sech^7(\eta \Theta) \]
\[ \text{However, the solution } \hat{\phi}_1 \text{ contains the secular terms which makes the solution unbounded is removed by choosing the arbitrary constant } \alpha_2 = 0. \text{ Further using the boundary conditions } \hat{\phi}_1(0)|_{\theta_0=0} = 0; \hat{\phi}_{1\theta}(0)|_{\theta_0=0} = 0, \text{ we obtain } \alpha_1 = -146\delta_1 \eta^4 - \frac{107}{28} \delta_2 \eta^4. \text{ Using the above secular and boundary conditions, the explicit form of } \hat{\phi}_1 \text{ is constructed as} \]

\[
\hat{\phi}_1 = \left[\frac{7}{2} \frac{\xi_r}{\eta} - \frac{12}{35} \delta_2 \eta^5 + \frac{12}{35} \delta_2 \eta^4 - \frac{1}{8 \eta} \xi_r^4 - \frac{4}{5} \xi_r^2 \eta - \frac{517}{180} \eta^3 \right] \text{sech}(\eta \Theta) \\
- \left[\frac{1}{4 \delta_1} + \frac{6}{35} \delta_2 \eta^5 \Theta - \frac{569}{180} \eta^3 + \frac{3}{5} \delta_1 - \frac{33}{35} \delta_2 \eta^4 \right] \text{sech}^3(\eta \Theta) \\
+ \left[\frac{1}{4} \frac{\xi_r}{\eta} - \frac{1}{8 \eta} \xi_r^4 - \frac{3}{4 \eta^2} \xi_r^2 + \frac{146}{35} \eta^4 - \frac{95}{28} \delta_2 \eta^4 \right] \text{sech}^7(\eta \Theta) \\
\times \text{tanh}(\eta \Theta) \text{sech}(\eta \Theta) - (\delta_1 - \frac{9}{70} \delta_2) \eta^4 \text{tanh}(\eta \Theta) \text{sech}^3(\eta \Theta) \\
+ \frac{9}{140} (14 \delta_1 + 3 \delta_2) \eta^4 \text{tanh}(\eta \Theta) \text{sech}^5(\eta \Theta) + \frac{6259}{1680} \eta^4 \Theta \text{tanh}(\eta \Theta) \times \text{sech}^7(\eta \Theta) + 8 \delta_1 \eta^4 \ln \cosh(\eta \Theta) \text{tanh}(\eta \Theta) \text{sech}(\eta \Theta). \tag{6.34} \]

In the similar fashion, the solution for \( \hat{\psi}_1 \) can be evaluated by solving Eq. (6.14). The homogeneous part of Eq. (6.14) admits two particular solutions \( \psi_{11} \) and \( \psi_{12} \)

\[
\psi_{11} = \text{sech}\eta(\theta - \theta_0), \tag{6.35} \\
\psi_{12} = \frac{1}{2 \eta} [\eta(\theta - \theta_0) \text{sech}\eta(\theta - \theta_0) + \sinh\eta(\theta - \theta_0)]. \tag{6.36} 
\]

Knowing the two particular solutions, the general solution for \( \hat{\psi}_1 \) can be obtained using the formula...
\[ \dot{\psi}_1 = \alpha_3 \psi_{11} + \alpha_4 \psi_{12} - \psi_{11} \int_{-\infty}^{\theta} \psi_{12} \mathcal{S} F_2 d\theta' + \psi_{12} \int_{-\infty}^{\theta} \psi_{11} \mathcal{S} F_2 d\theta', \] (6.37)

in which \( \alpha_3 \) and \( \alpha_4 \) are the arbitrary constants. Using the two particular solutions in \( \dot{\psi}_1 \) and after evaluating the integrals in Eq. (6.37), we obtain

\[
\begin{align*}
\psi_{11} \int_{-\infty}^{\theta} \psi_{12} \mathcal{S} F_2 d\theta' &= \left[ \frac{1}{4} \eta_\tau \Theta^2 + \frac{1}{2} (\eta \Theta_T + \xi \eta^3 - \xi^3 \eta) \Theta \right] \text{sech}(\eta \Theta) \\
&\quad - \left[ \frac{1}{4} \eta_\tau \Theta^2 + \frac{1}{4} (\eta \Theta_T + \xi \eta^3 - \xi^3 \eta) \Theta - \frac{1}{15} \delta_1 \xi \eta^3 + \frac{1}{5} \delta_2 \xi^3 \right] \text{sech}^3(\eta \Theta) \\
&\quad + \left[ \frac{19}{10} \xi \eta^3 \Theta - \frac{1}{10} (6 \delta_1 - 3 \delta_2) \xi \eta^3 \right] \text{sech}^5(\eta \Theta) \\
&\quad - \left[ \frac{1}{2} \eta_\tau \Theta + \frac{2}{10} \delta_1 + \frac{2}{5} \delta_2 \right] \xi \eta^4 \Theta + \frac{1}{4} (\Theta_T + \xi \eta^2 - \xi^3) \\
&\quad + \frac{2}{3} \delta_1 \xi \eta^3 (19 \xi) \eta^2 \Theta \text{tanh}(\eta \Theta) \text{sech}(\eta \Theta) \\
&\quad + \left[ \frac{19}{10} \xi \eta^2 + \frac{2}{3} \delta_1 \xi \eta^3 + \left( \frac{2}{15} \delta_1 - \frac{6}{15} \delta_2 \right) \xi \eta^4 \Theta \right] \text{tanh}(\eta \Theta) \text{sech}^3(\eta \Theta) \\
&\quad + \frac{1}{10} (6 \delta_1 - 3 \delta_2) \xi \eta^4 \Theta (\eta \Theta) \text{sech}^5(\eta \Theta) \\
&\quad - \frac{2}{5} (2 \delta_1 - 6 \delta_2) \xi \eta^3 \Theta \text{sech} (\eta \Theta) \Theta (\theta_0) \ln \cosh (\eta \Theta),
\end{align*}
\] (6.38)

and

\[
\begin{align*}
\psi_{12} \int_{-\infty}^{\theta} \psi_{11} \mathcal{S} F_2 d\theta' &= \left[ \frac{4}{15} (6 \delta_1 - 3 \delta_2) \xi \eta^4 \Theta - \frac{4}{3} \delta_1 \xi \eta^4 \Theta + \frac{1}{4} (\Theta_T + \xi \eta^2 - \xi^3) \right] \\
&\quad \times \text{tanh}(\eta \Theta) \text{sech}(\eta \Theta) + \left[ \frac{19}{10} \xi \eta^2 + \left( \frac{2}{15} \delta_1 - \frac{2}{5} \delta_2 \right) \xi \eta^4 \Theta \right] \\
&\quad \times \text{tanh}(\eta \Theta) \text{sech}(\eta \Theta) + \left( \frac{2}{15} \delta_1 - \frac{2}{5} \delta_2 \right) \xi \eta^4 \Theta \\
&\quad \times \text{tanh}(\eta \Theta) \text{sech}^3(\eta \Theta) + \frac{1}{10} (6 \delta_1 - 3 \delta_2) \xi \eta^4 \Theta \\
&\quad \times \text{tanh}(\eta \Theta) \text{sech}^5(\eta \Theta) \\
&\quad - \left[ \frac{1}{4} \eta_\tau \Theta^2 + \frac{7}{4} (\Theta_T + \xi \eta^2 - \xi^3) \Theta - \left( \frac{14}{30} \delta_1 + \frac{1}{10} \delta_2 \right) \xi \eta^3 \right] \\
&\quad \times \text{sech}^3(\eta \Theta) + \left[ \frac{19}{10} \xi \eta^3 \Theta - \frac{1}{10} (6 \delta_1 - 3 \delta_2) \xi \eta^3 \right] \text{sech}^5(\eta \Theta) \\
&\quad + \left[ \frac{4}{15} (\delta_1 - \frac{4}{3} \delta_2) \xi \eta^3 - \frac{1}{4 \eta^2} \xi \eta T \right] \text{tanh}(\eta \Theta) \Theta. \quad (6.39)
\end{align*}
\]

Substituting Eqs. (6.38) and (6.39) in the formula for the general solution for \( \dot{\psi}_1 \), we obtain
\[ \hat{\psi}_1 = \frac{1}{2} \left[ \frac{\eta r}{\eta} \Theta + \frac{43}{5} \xi \eta^2 + (\Theta_T - \xi^3) \right] \tanh(\eta \Theta) \sech(\eta \Theta) \\
+ \frac{2}{3} \delta_1 \xi \eta^3 \tanh(\eta (\theta - \theta_0)) \sech^3(\eta \Theta) + \left[ \alpha_3 - \frac{1}{4} \eta r \Theta^2 \right] \\
+ \frac{\alpha_4}{2} \Theta - \frac{1}{2} (\Theta_T + \xi \eta^2 - \xi^3) \eta \Theta + \left( \frac{2}{15} \delta_1 - \frac{2}{5} \delta_2 \right) \xi \eta^3 \right] \sech(\eta \Theta) \\
\times \sech(\eta \Theta) + \left[ \left( \frac{2}{5} \delta_1 + \frac{3}{10} \delta_2 \right) \xi \eta^3 \right] \sech^3(\eta \Theta) \\
+ \left( \frac{\alpha_4}{2} + \left( \frac{4}{15} \delta_1 + \frac{4}{5} \delta_2 \right) \xi \eta^3 - \frac{1}{4 \eta^2 \eta r} \right) \tanh(\eta \Theta) \sinh(\eta \Theta) \\
+ \frac{2}{15} (2 \delta_1 - 6 \delta_2) \xi \eta^3 \sech(\eta \Theta) \sinh(\eta \Theta). \right] \]

(6.40)

In order to remove the secularity terms which makes the solution unbounded, we choose \( \alpha_4 = 0 \). On using the boundary conditions \( \hat{\psi}_1(0)|_{\theta_0=0} = 0; \hat{\psi}_{10}(0)|_{\theta_0=0} = 0 \), we get \( \alpha_3 = -\frac{1}{30} (16 \delta_1 - 3 \delta_2) \xi \eta^3 \). Using the above secular and boundary conditions, we write the final form of solution for \( \hat{\psi}_1 \) as

\[ \hat{\psi}_1 = -\frac{1}{4} \eta r \Theta^2 + \frac{1}{2} (\Theta_T + \xi \eta^2 - \xi^3) \eta \Theta + \left( \frac{2}{5} \delta_1 + \frac{3}{10} \delta_2 \right) \xi \eta^3 \right] \sech(\eta \Theta) \\
+ \left[ \frac{2}{15} \delta_1 + \frac{3}{10} \delta_2 \right] \xi \eta^3 \sech^3(\eta \Theta) + \frac{2}{15} \left[ 2 \delta_1 - 6 \delta_2 \right] \xi \eta^3 \sech(\eta \Theta) \\
\times \ln \cosh(\eta \Theta) + \frac{1}{2} \left[ \frac{\eta r}{\eta} \Theta + \frac{38}{5} \xi \eta^2 + (\Theta_T + \xi \eta^2 - \xi^3) \right] \\
\times \tanh(\eta \Theta) \sech(\eta \Theta) + \frac{2}{3} \delta_1 \xi \eta^3 \tanh(\eta \Theta) \sech^3(\eta \Theta). \right] \]

(6.41)

Having obtained the explicit form of \( \hat{\psi}_1 \) and \( \hat{\psi}_1 \), the first-order perturbed soliton solution \( \hat{\psi}_1 \) can be constructed through the relation \( \hat{\psi}_1 = \hat{\phi}_1 + i \hat{\psi}_1 \) as

\[ \hat{\psi}_1 = \left[ \frac{7 \xi T}{2 \eta} \Theta - \frac{12}{35} \xi \eta^5 \Theta + \frac{12}{35} \delta_2 \eta^4 - \frac{1}{8 \eta} \xi^4 - \frac{4}{5} \xi^2 \eta - \frac{517}{180} \eta^3 \right] \sech(\eta \Theta) \\
- \left[ (4 \delta_1 - \frac{6}{35} \delta_2) \eta^5 \Theta + \frac{3 \xi T}{\eta} \Theta - (4 \delta_1 - \frac{6}{35} \delta_2) \eta^4 - \frac{569}{180} \eta^3 \right] \sech^3(\eta \Theta) \\
+ \left[ (31 \delta_1 + \frac{51}{14} \delta_2) \eta^5 \Theta - \frac{569}{180} \eta^3 + \left( \frac{38}{5} \delta_1 - \frac{33}{35} \delta_2 \right) \eta^4 \right] \sech(\eta \Theta) \\
- \left[ \frac{69}{70} \delta_1 + \frac{243}{70} \delta_2 \eta^5 \Theta - \frac{569}{840} \eta^3 - \frac{9}{35} (14 \delta_1 + 3 \delta_2) \eta^4 \right] \sech(\eta \Theta) \\
- \left[ \frac{1}{4} \xi T \Theta^2 + \frac{1}{8} (6 \xi^2 \eta^2 - \xi^4 - \eta^4) \Theta + \frac{3}{4 \eta^2 \xi T + 146 \delta_1 \eta^4} \right] \right]. \]
Figure 6.2: Observation of soliton switching in the real part of perturbed solution with $\delta_1 = 3\delta_2$, $\delta_2 = 0.15$, $b_0 = -0.6$, $\theta_0 = 0.5$ (i) $a_0 = 0.06$ and (ii) $a_0 = 0.5$.

\[ \times \tanh(\eta \Theta) \text{sech}(\eta \Theta) - (\delta_1 - \frac{9}{70} \delta_2) \eta^4 \tanh(\eta \Theta) \text{sech}^3(\eta \Theta) \]
\[ + \frac{9}{140} (14\delta_1 + 3\delta_2) \eta^4 \tanh(\eta \Theta) \text{sech}^5(\eta \Theta) + \frac{6259}{1680} \eta^4 \Theta \tanh \Theta \]
\[ \times \text{sech}^7(\eta \Theta) + 8\delta_1 \eta^4 \ln \cosh(\eta \Theta) \tanh(\eta \Theta) \text{sech}(\eta \Theta) \]
\[ + i \left[ \frac{1}{2} \left( \frac{\eta \Theta}{\eta} + \frac{38}{5} \xi \eta^2 + (\Theta_T + \xi \eta^2 - \xi^3) \right) \tanh(\eta \Theta) \text{sech}(\eta \Theta) \right. \]
\[ - \frac{1}{4} \eta \Theta^2 + \frac{1}{2} (\Theta_T + \xi \eta^2 - \xi^3) \eta \Theta + \left( \frac{2}{5} \delta_1 + \frac{3}{10} \delta_2 \right) \eta \xi^3 \text{sech}^2(\eta \Theta) \]
\[ + \frac{2}{5} \delta_1 + \frac{3}{10} \delta_2 \eta \xi^3 \text{sech}^3(\eta \Theta) + \frac{2}{3} \delta_1 \xi \eta^3 \tanh(\eta \Theta) \text{sech}^3(\eta \Theta) \]
\[ + \frac{2}{15} \left[ 2\delta_1 - 6\delta_2 \right] \xi \eta^3 \text{sech}(\eta \Theta) \ln \cosh(\eta \Theta) \right] \]

(6.42)

We have observed the magnetization switching through soliton while plotting the real part of the perturbed solution with the choices of parameters $\delta_1 = 3\delta_2$, $\delta_2 = 0.15$, $b_0 = -0.6$ and $\theta_0 = 0.5$ as depicted in Figs. (6.2). From the Fig. (6.2a), it is inferred that when the soliton moves along the ferromagnetic medium with octupole-dipole interaction which triggers switching through flipping of solitons. But, when the value of the parameter $a_0$ is increased, the amplitude of the soliton is decreased appreciably (Fig. 6.2b). Moreover, this type of soliton
Figure 6.3: Observation of soliton switching in the imaginary part of perturbed solution with \( \delta_1 = 3\delta_2, \delta_2 = 0.5, b_0 = 0.8, \theta_0 = 0.5 \) (i) \( a_0 = 0.4 \) and (ii) \( a_0 = 0.5 \).

Switching is also observed in the imaginary part of perturbed soliton solution as shown in Figs. (6.3). Therefore the magnetization reversal through flipping of soliton can be achieved under the influence of octupole-dipole interaction in a ferromagnetic medium.

6.4 Conclusions

We have studied the nonlinear spin excitations in a one-dimensional anisotropic Heisenberg ferromagnetic spin chain with octupole-dipole interaction through semi-classical approach using Glauber’s coherent state method combined with the Holstein-Primakoff bosonic representation for spin operator in the continuum limit. The associated dynamics is governed by generalized higher order nonlinear Schrödinger equations. The effect of octupole-dipole interaction on the governing nonlinear dynamical equations was analyzed at different orders of the lattice parameter through the multiple scale perturbation analysis. The results show that the velocity and amplitude of the soliton unaltered by the influence of octupole-dipole interaction during propagation in an anisotropic ferromagnetic medium at lowest order of expansion which means
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that the soliton parameters travel with constant velocity and amplitude along the ferromagnetic medium. But, it was noted that the soliton undergoes dramatic changes during the propagation under the influence of octupole-dipole interaction at the higher orders. This higher order influence of octupole-dipole interaction induces magnetization reversal.