CHAPTER I

INTRODUCTION AND STATEMENT OF THE PROBLEM

1.1 INTRODUCTION

During recent years, there has been a growing interest in the application of state-variable techniques for the study of dynamical systems and networks. This may be attributed to the fact that the state-variable approach is computationally more attractive, especially in terms of computer-aided design (CAD) [7]. Moreover, the approach is more general than the classical Laplace and Fourier transform theory and hence is applicable to many systems for which transform theory breaks down [15]. Since the approach is in time-domain, it is equally applicable to both non-linear and time-varying systems in addition to the time invariant linear systems [7]. Apart from providing a more general representation of a physical process, a very important advantage of this technique lies in its flexibility in generating "equivalent" canonical representations which are very useful in system analysis. Another important contribution of this approach is that it permits problems in networks and systems to be treated in an unified manner [10]. Besides, the technique is particularly useful in multiport network synthesis [10] and consequently new synthesis procedures using this approach are being developed [6], [8] - [11], [13], [15], [17], [57], [24], [12], [13], [147], [150], [15].
This thesis is concerned with the state-variable realization of dynamical systems and its application to the synthesis of finite, lumped, linear, time-invariant, passive and active multiport networks.

It is well-known that a state-variable characterization of finite, lumped, linear, time-invariant, passive p-port network is given by the dynamical equations or state equations

\[
\begin{align*}
\dot{X} &= AX + BU, \\
Y &= CX + DU,
\end{align*}
\]

... (1.1)

where \( X \) is n-vector, the state, having its components as capacitor voltages and inductor currents, \( U \) is p-vector, the input, and \( Y \) is q-vector, the output. The matrices \( A, B, C, \) and \( D \) are real constant matrices of dimensions \( n \times n, n \times p, q \times n \) and \( q \times p \), respectively.

In network synthesis, we are mainly concerned with the realization of a passive or active network that has a prescribed immittance or transfer function matrix \( G(s) \); whereas the system realization problem is to pass from an input-output description of a system in the form of an impulse response matrix \( G(t) \), or a transfer function matrix, \( G(s) \), to a state-space description of the type (1.1). Thus, system realization problem is intimately related to modern network synthesis.
Once the dynamical equation (1.1) is known for a system, the system can be easily simulated on an analog computer. Further, transfer function is an input-output description of a system, whereas a dynamical equation describes not only the input-output relation but also the internal structure of a system. Thus, the realization problem may also be considered as an "identification problem", a problem of identifying the internal structure of a system from the knowledge obtained through direct measurements at the input and output terminals. Because of its wide applications, the realization problem has been actively considered over the past decade by several investigators and consequently, a well developed theory of realization is now available in the technical literature. In the field of network synthesis, the first-step is to determine a minimal realization $[A, B, C, D]$ of a given input-output description. Since the realization is minimal, the number of dynamic or reactive elements and integrators needed to synthesize a network will be minimum, which is desirable for reasons of economy and sensitivity. If a given state-model $[A, B, C, D]$ satisfies Anderson's positive real lemma, a synthesis of the network using only passive elements is possible. Further, the realization set $[A, B, C, D]$ satisfying reciprocity criterion due to Yarlagadda will lead to reciprocal network realizations.

Modern system theory concepts have also been exploited
to give state-space interpretation of some of the well-known classical synthesis procedures, and properties of network functions. Recently, the state-variable technique has also found application in evolving novel active RC multiport network synthesis methods suitable for integrated circuit fabrication \[13, 57, 69, 71, 96, 98, 122, 123, 158\].

Thus, the problem of realization of dynamical systems assumes great significance because of its manifold applications in studying problems of various engineering disciplines such as optimal control, system theory and network theory.

Having introduced the problem of state-variable realization for linear, time-invariant dynamical systems and discussed its implications, the specific problems considered in the present thesis are stated in the next section.

1.2 STATEMENT OF THE PROBLEM

The work embodied in this thesis can be broadly classified in three sections:

1. State-variable realization of linear, time-invariant dynamical systems,

2. State-space interpretation of some classical synthesis procedures, and

3. Multiport active RC network synthesis with a minimum number of capacitors.
Specifically, the following problems are considered in this thesis:

(1) New algorithms are developed for obtaining minimal reciprocal realization from a given symmetric transfer-function matrix and symmetric impulse response matrix. The algorithm for symmetric transfer-function matrix uses Markov-parameters and gives a simpler procedure, whereas, moments of the impulse response are used for the reciprocal realization of impulse response matrix. The minimal reciprocal realization is useful for passive reciprocal network synthesis of symmetric positive real (SPR) immittance matrices.

(2) In order to establish a link between state-variable characterization and specifications in s domain, a state-space interpretation of Foster multiport LC network synthesis, Cauer driving point (dp) synthesis, and well-known coefficient matching technique of active RC filter design, is presented.

(3) An active RC multiport network synthesis procedure, suitable for integrated circuit fabrication, is evolved. Specifically, the proposed procedure is applied to the synthesis of short-circuit admittance matrix, open-circuit impedance matrix, and transfer-impedance matrix using operational amplifiers.

(4) Given a symmetric positive real (SPR) immittance matrix, a synthesis procedure, based on the preceding
results, is stated for passive-reciprocal multi-port realization using RCT (resistor, capacitor and Ideal Transformer) network.

It is worthwhile to mention that some aspects of these problems have been studied by many authors [39], [115], [116], [13], [99], [151], [156], [160] and some results are available. The work reported in [39] and [116] is concerned with the first problem where the minimal realization of a symmetric matrix is obtained by modifying the well-known Ho-Kalman algorithm [55]. The procedures proposed in the present thesis reduces the computations considerably by requiring Hankel matrices of lower order.

The second problem i.e. state-space interpretation has been considered in [5], [9], [20], [59], [74], [87], [115], [139] where the classical synthesis procedures and network properties have been revisited via state-space characterization with a view to bridge the gap between the synthesis procedures in s domain and state-space.

As regards the third problem, the procedures due to Bickart and Melvin [18], [98], Mann and Pike [96], and Huang [57] are available. But the upper bound on the number of active elements required in these methods is quite large. In some cases, the number of resistors required in the realization is also more.
The fourth problem i.e. passive reciprocal multiport synthesis has been investigated by Youla and Tissi, Vongpanitlerd and Anderson and Yarlagadda using RLCT network. The proposed procedure in this thesis is for passive reciprocal multiport RLCT network and requires a minimum number of capacitors.

1.3 ORGANISATION OF THE THESIS

Having stated the problem in the preceding section, the organization of the remaining part of the thesis is given below.

In Chapter II, the problem of state-variable realization of linear, time-invariant dynamical systems is introduced. Having given some system theory preliminaries, a historical review of some selective literature on minimal realization methods of linear dynamical systems, state-space interpretation of some well-known network properties and synthesis procedures, general state-space passive network synthesis based on reactance extraction technique, and multiport active RC network synthesis procedures, is presented. The well-known Ho-Kalman algorithm is also discussed because of its importance and use in the subsequent work in this thesis.

The minimal reciprocal realization algorithms, for symmetric transfer-function matrix, and symmetric impulse response matrix using moments, are developed in Chapter III. The use of moments of the impulse response
is advantageous in the presence of noise. The proposed procedures are simpler and computations are considerably reduced as they require Hankel matrices of lower dimensions. Superiority of the realization techniques evolved here in regard to simplicity and elegance is amply illustrated with the help of suitable examples.

Chapter IV is devoted to seeking state-space interpretation of Foster multiport LC network synthesis, Cauer driving point synthesis, and coefficient matching technique for active RC second order filter design.

In Chapter V, new active RC multiport network synthesis procedure, with a minimum number of capacitors and suitable for integrated circuit fabrication, is discussed. The proposed approach of active RC multiport network synthesis is first outlined. Subsequently, the synthesis of short-circuit admittance matrix, open-circuit impedance matrix, and transfer-impedance matrix using operational amplifiers is considered. The proposed method is illustrated with the help of suitable examples. Also, based on the above approach, a passive reciprocal multiport synthesis procedure using RCT network for SPR immittance matrices is briefly described. Examples are given to illustrate the procedure.

Chapter VI contains a summary of the results presented in this thesis. Some suggestions, for further investigations in this field which might lead to some interesting results, have been included at the end of this chapter.