6.1 INTRODUCTION

The problem of realization of a rational transfer-function matrix into an irreducible (controllable and observable) dynamical equation is one of the fundamental problems in linear system theory. In this thesis, the problem of minimal reciprocal realization from a given symmetric transfer-function matrix and symmetric impulse response matrix has been discussed. New methods for the design of multiport active RC, and passive reciprocal networks using state-variable techniques are evolved. Some endeavours are also made to re-examine some of the well-known classical synthesis procedures via state-space characterization. The present chapter gives the summary of the various results obtained in this thesis, along with some suggestions for new research problems to be pursued for further investigations in this area.

6.2 SUMMARY OF THE RESULTS

A mathematic description of linear time-invariant dynamical systems and networks in the input-output form and state-variable vector differential equation form is reviewed first. Having stated some system theory preliminaries, the problem of state-variable realization of linear, time-invariant dynamical systems and networks is discussed with a view to
have a clear understanding of the subsequent results obtained in this thesis.

The problem of minimal reciprocal realization of linear time-invariant dynamical systems is considered. Two new and simplified algorithms have been evolved for obtaining minimal reciprocal realization from a given symmetric transfer-function matrix and symmetric impulse response matrix, one using the Markov-parameters and the other requiring moments of the impulse response matrix. Both the methods exploit the symmetry of the given transfer-function matrix or the impulse response matrix. In both the algorithms, the order of the Hankel matrices required in the procedure is much less than the existing methods and consequently, the computations and memory storage required are considerably reduced. The methods are essentially a modification of the Chen and Mital algorithm [24]. The realizations obtained by the proposed algorithms result in reciprocal networks. Further, a method based on the computation of the moments of the impulse response matrix is preferable when a realization is to be constructed from an empirically obtained data of \( G(t) \) which may be contaminated with noise.

An attempt has been made to establish yet another link between state-space and frequency domain methods. A state-space interpretation of the classical Foster multiport synthesis method for LC network has been presented. The proposed method is essentially an extension of the one given by Puri and Takeda [115] for 1-port Foster LC network realization.
State-variable techniques are also exploited to re-examine the well-known Cauer driving point synthesis of RC and LC networks, and active RC filter design using coefficient matching technique. A non-singular observability matrix has been employed as a canonical transformation to convert the state-model representation of the Cauer network or the active RC filter section into a canonical state-model.

A new and systematic synthesis procedure, based on a state-variable approach and the reactance extraction principle, has been developed whereby any qxp matrix \( T(s) \), of real rational functions of the complex frequency variable \( s \), can be realized as an active RC multiport network with the ports grounded. Specifically, the proposed procedure is applied to the active synthesis of a pxp short-circuit admittance matrix \( Y(s) \) when \( Y(\infty) \) is the sum of a strictly hyperdominant matrix plus a non-negative matrix, a pxp open-circuit impedance matrix, and a qxp transfer-impedance matrix with operational amplifiers. The realized network contains a minimum number of \( n \) grounded capacitors with unity capacitance spread, \( n \) being the McMillan's degree of \( T(s) \), and at most \( (p+2n) \) inverting, grounded voltage amplifiers. Of course, in the case of qxp transfer-impedance matrix synthesis \( q \)-operational amplifiers are also required. The facts that all the minimum number of capacitors have the same value, and that all the active elements, capacitors and ports are grounded, are very much desirable if the network is to be fabricated as an integrated circuit.

The proposed method is essentially a modification over the one given by Bicksart and Melvin [18]. The modification
reduces the upper bound on the number of active elements from \((2p + 2n)\) to only \((p + 2n)\). Also, it will usually require fewer resistors because the sub-matrix \(g_{22}\), as discussed in the procedure and illustrated in the examples, can always be chosen to be hyperdominant. Moreover, in the case of transfer-impedance matrix synthesis, the proposed method uses commercially available operational amplifiers instead of voltage amplifiers as used in \([18]\). The other advantages of \([18]\) are retained. Further, it is conjectured that the realization of \(T(s)\) will be relatively insensitive to capacitance variations because of their minimum number used in the network.

Based on the approach of multiport active RC network synthesis, considered here, and the results of minimal reciprocal realization from a given symmetric rational matrix, a new method for passive reciprocal multiport synthesis of a SPR immittance matrix using RCT network with a minimum number of capacitors, has been evolved. Since the given immittance matrix is symmetric positive real, the minimal realization set \(\{A, B, C, D\}\), obtained with the help of the algorithm discussed earlier, will satisfy both reciprocity and passivity constraints, a necessary and sufficient condition for \(\{A, B, C, D\}\) to be realizable with passive and reciprocal network elements \([10]\).

6.3 SUGGESTIONS FOR FURTHER INVESTIGATIONS

The state-variable approach to linear systems realization, and passive and active network synthesis has been reviewed and applied to minimal reciprocal realization of
linear time-invariant dynamical systems, classical synthesis methods, and modern active as well as passive multiport network synthesis procedures. Based on the research contribution of the thesis, some suggestions are given for further investigations in the following paragraphs:

1. Algorithms for obtaining minimal state-models \([A, B, C, D]\), satisfying reciprocity constraint, from a given symmetric rational transfer-function matrix and a symmetric impulse response matrix have been given in Chapter III. These state-models in general do not result in any canonical structure. It would be worthwhile to develop a method by which the minimal reciprocal state-model is in some standard canonical form, such that they can be realized further by standard techniques. In this connection, the references [19], [25], [28], [80], [100] and [161] will be useful.

2. In network problems, usually the given transfer-function matrix or impulse response matrix are symmetric. By exploiting the symmetry of the positive real immittance matrices, a passive reciprocal synthesis procedure using multiport RCT network with a minimum number of capacitors has been presented in Section (5.3). A passive reciprocal synthesis method using a minimum number of resistors was proposed in [151]. Investigations leading to a passive reciprocal synthesis procedure, from a symmetric positive real matrix, resulting in minimum number of reactive as well as resistive elements will be quite useful.

3. From a given symmetric impulse matrix, a method for constructing a minimal reciprocal realization using moments of
the impulse response has been given in Section (3.3). It would be interesting to extend this technique for time-varying impulse response matrices.

4. Because of some interest in the problem of sub-optimal approximation of a linear system of large dimension by one of the smaller dimension, a method has been recently given in [3] for obtaining a sub-optimum reduced model from the given input-output data in the form of Markov-parameters. It would be worthwhile to further reduce the order of Hankel-matrices used in [3] by exploiting the technique given in Chapter III in order to reduce the computation time and memory storage required.

5. The existing state-space techniques for the synthesis of positive real functions result either in RLC networks with transformers [10] or transformerless active networks. The equivalence of even simple transformerless procedures such as Bott-Duffin method [15] etc. in state-space has not been done so far [10]. It will be worthwhile to give state-space interpretation to such simple classical techniques possibly resulting in transformerless RLC synthesis.

6. A multiport active RC network synthesis procedure with a minimum number of capacitors for the realization of immittance matrices has been given in Section (5.2). The procedure reduces the upper bound on the number of active elements from \((2p + 2n)\) required in [18], to only \((p+2n)\). Investigations leading to further reduction of upper bound on the number of active elements, of course, with a minimum number of capacitors.
will be quite useful.

7. The network structure proposed in Section (5.2) is for the active RC realization of immittance functions with a minimum number of capacitors. Since the number of capacitors is minimum, it is conjectured that the realization of immittance matrices will be relatively insensitive to capacitance variations. However, further investigations are required in order to provide a quantitative assessment of the sensitivity of the selected network attributes and validity of the conjecture.

8. An approach of multiport active RC network synthesis presented in Section (5.2) is applied to the realization of a short-circuit admittance matrix, open-circuit impedance matrix, and transfer-impedance matrix. It may be extended to the active synthesis of other multiport network functions such as current gain matrix, transfer-admittance matrix, etc.

9. A passive reciprocal multiport RCT network synthesis of SPR immittance matrices is given in Section (5.3). It is worthwhile to extend this technique to the realization of other SPR multiport network functions such as voltage gain matrix, current gain matrix, transfer-impedance matrix etc. which are often available as given specifications in network synthesis.

10. Synthesis procedures described in this thesis are limited to linear networks and systems. Hardly any work has been done in the synthesis of non-linear networks. Recently, state-variable formulation of Lagrangian and Hamiltonian
equations for nonlinear networks has been proposed by Chua and McPherson[27]. It is hoped that a break through in the synthesis of nonlinear networks would be possible in the light of the procedure suggested in [27].

In conclusion, the theory of state-variables has opened new vistas in the realization of dynamical systems and lumped networks. It is hoped that the applications of the new techniques suggested in this thesis will help in solving more fascinating practical problems of nonlinear and distributed networks and systems.