CHAPTER 3

MHD EFFECTS ON FLOW PAST AN EXPONENTIALLY ACCELERATED INFINITE VERTICAL PLATE

3.1 INTRODUCTION

The influence of magnetic field on viscous impressionlible flow of an electrically conducting fluid has its importance in many industrial applications such as extrusion of plastics in the manufacture of Rayon and Nylon, purification of crude oil, pulp, paper industry, textile industry, in different geophysical cases, magnetic control of molten iron flow in the steel industry, liquid metal cooling in nuclear reactors and magnetic suppression of molten semi conducting materials, such as sustained plasma confinement for controlled thermonuclear fusion ion propulsion, electromagnetic pumps, MHD power generators, controlled fusion research plasma jets and chemical synthesized electromagnetic casting of metals. In many process industries, the cooling of threads or sheets of some polymer materials is an important in the production line. The rate of cooling can be controlled effectively to achieve the final products of desired characteristics by drawing threads etc in the presence of an electrically conducting fluid subject to a magnetic field.

Gupta [22] analyzed the flow of a uniformly accelerated vertical plate in the presence of uniform magnetic field. Raptis et al [68] studied the effects of a uniform transverse magnetic field on the free-convection flow of a viscous
incompressible and electrically conducting fluid past an accelerated vertical infinite plate with variable suction or injection and heat flux. The effect of a uniform transverse magnetic field on the free convection flow of an electrically conducting fluid past an infinite vertical plate for both the classes of impulsive as well as uniformly accelerated motion of the plate was discussed by Raptis and Singh [63]. Singh [77] investigated the flow of an electrically conducting incompressible viscous fluid due to the time-varying motion of an infinite vertical plate in the presence of a transverse magnetic field. The free convection flow of an incompressible and viscous fluid past an exponentially accelerated infinite vertical plate was analyzed by Singh and Naveen kumar [79].

In this chapter, the first section 3.2 deals with the study of magnetohydrodynamics effects on flow past an exponentially accelerated vertical plate with variable temperature and the second section 3.3 deals with the study of heat and mass transfer effects on exponentially accelerated isothermal vertical plate with variable mass diffusion in the presence of magnetic field. The dimensionless governing equations are solved by using the Laplace transform technique.

3.2 VARIABLE TEMPERATURE IN THE PRESENCE OF MAGNETOHYDRODYNAMICS

3.2.1 Analysis

An unsteady MHD flow of a viscous incompressible fluid past an infinite vertical plate with variable temperature is studied. The $x$-axis is taken along the plate in the vertically upward direction and the $y$-axis is taken normal to the plate. A transverse magnetic field of uniform strength $B_0$ is assumed to
be applied normal to the plate. At time $t' \leq 0$, the plate and fluid are at the same temperature $T_\infty$. At time $t' > 0$, the plate is exponentially accelerated with a velocity $u = u_0 \exp(a't')$ in its own plane and the plate temperature is raised linearly with time $t$. It is assumed that the induced magnetic field and of viscous dissipation is negligible. Then by usual Boussinesq’s and boundary layer approximation, the unsteady flow is governed by the following equations:

$$\frac{\partial u}{\partial t'} = \frac{g\beta}{\nu} (T - T_\infty) + \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u$$  \hspace{1cm} (3.1)$$

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2}$$  \hspace{1cm} (3.2)$$

with the following initial and boundary conditions:

$$u = 0, \quad T = T_\infty, \quad \text{for all } y, t' \leq 0$$

$$t' > 0: \quad u = u_0 \exp(a't'), \quad T = T_\infty + (T_w - T_\infty) A t', \quad \text{at } y = 0$$

$$u \to 0 \quad T \to T_\infty, \quad \text{as } y \to \infty$$  \hspace{1cm} (3.3)$$

where $A = \frac{u_0^2}{\nu}$

On introducing the following non-dimensional quantities:

$$U = \frac{u}{u_0}, \quad t = \frac{t'u_0^2}{\nu}, \quad Y = \frac{yu_0}{\nu}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty},$$

$$Gr = \frac{g\beta \nu (T_w - T_\infty)}{u_0^3}, \quad Pr = \frac{\mu C_p}{k}, \quad a = \frac{a\nu}{u_0^2}, \quad M = \frac{\sigma B_0^2 \nu}{\rho u_0^2}$$  \hspace{1cm} (3.4)$$

in equations (3.1) to (3.4) leads to

$$\frac{\partial U}{\partial t} = Gr \theta + \frac{\partial^2 U}{\partial Y^2} - M U$$  \hspace{1cm} (3.5)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial Y^2}$$  \hspace{1cm} (3.6)$$
The initial and boundary conditions in non-dimensional quantities are

\[ U = 0, \quad \theta = 0, \quad \text{for all} \quad Y, t \leq 0 \]

\[ t > 0: \quad U = \exp(at), \quad \theta = t \quad \text{at} \quad Y = 0 \quad (3.7) \]

\[ U \to 0, \quad \theta \to 0 \quad \text{as} \quad Y \to \infty \]

The dimensionless governing equations (3.5) and (3.6), subject to the boundary conditions (3.7) are solved by the usual Laplace transform technique and the solutions are derived as follows:

\[ \theta = t \left[ (1 + 2 \eta^2 Pr) \text{erfc}(\eta \sqrt{Pr}) - \frac{2}{\sqrt{\pi}} \eta \sqrt{Pr} \exp(-\eta^2 Pr) \right] \quad (3.8) \]

\[ U = \frac{\exp(at)}{2} \left[ \exp(2\eta \sqrt{(M+a)t}) \text{erfc}(\eta + \sqrt{(M+a)t}) \\
+ \exp(-2\eta \sqrt{(M+a)t}) \text{erfc}(\eta - \sqrt{(M+a)t}) \right] \]

\[ + c(1 + bt) \left[ \exp(2\eta \sqrt{Mt}) \text{erfc}(\eta + \sqrt{Mt}) + \exp(-2\eta \sqrt{Mt}) \text{erfc}(\eta - \sqrt{Mt}) \right] \]

\[ - \frac{bc \eta \sqrt{t}}{\sqrt{M}} \left[ \exp(-2\eta \sqrt{Mt}) \text{erfc}(\eta - \sqrt{Mt}) - \exp(2\eta \sqrt{Mt}) \text{erfc}(\eta + \sqrt{Mt}) \right] \]

\[ - c \exp(bt) \left[ \exp(2\eta \sqrt{(M+b)t}) \text{erfc}(\eta + \sqrt{(M+b)t}) \\
+ \exp(-2\eta \sqrt{(M+b)t}) \text{erfc}(\eta - \sqrt{(M+b)t}) \right] \]

\[ -2c \text{erfc}(\eta \sqrt{Pr}t) - 2bc t \left[ (1 + 2\eta^2 Pr) \text{erfc}(\eta \sqrt{Pr}) - \frac{2\eta \sqrt{Pr}}{\sqrt{\pi}} \exp(-\eta^2 Pr) \right] \]

\[ + c \exp(bt) \left[ \exp(2\eta \sqrt{Pr \sqrt{bt}t}) \text{erfc}(\eta \sqrt{Pr} + \sqrt{bt}) \\
+ \exp(-2\eta \sqrt{Pr bt}) \text{erfc}(\eta \sqrt{Pr} - \sqrt{bt}) \right] \quad (3.9) \]

where \( b = \frac{M}{Pr - 1} \), \( c = \frac{Gr}{2b^2(1 - Pr)} \) and \( \eta = \frac{Y}{2\sqrt{t}} \).
3.2.2 Results and Discussion

The purpose of the calculations given here is to assess the effects of the parameters ‘$a$’, $M$ and $Gr$ upon the nature of the flow and transport. The numerical values of the velocity and temperature are computed for different parameters like magnetic field parameter, Prandtl number, ‘$a$’, thermal Grashof number and time.

The velocity profiles for different values of ($a = 0.2, 0.5, 0.8$), $Gr = 5$ and $Pr = 7$ at $t = 0.2$ are studied and presented in Figure 3.1. It is observed that the velocity increases with increasing values of ‘$a$’.

Figure 3.2 demonstrates the effects of the magnetic field parameter on the velocity when ($M = 2, 5, 8, 10$), $a = 0.5, Pr = 7$ and $t = 0.2$. It is observed that the velocity increases with decreasing values of the magnetic field parameter. This shows that the increase in the magnetic field parameter leads to a fall in the velocity. This agrees with the expectations, since the magnetic field exerts a retarding force on the free convective flow.

The temperature profiles calculated for different values of time are shown in Figure 3.3 for water ($Pr = 7.0$) and air ($Pr = 0.71$). The effect of Prandtl number is important in temperature profiles. It is observed that the temperature increases with increasing time $t$. The effect of heat transfer is more in the presence of air than in water.
Figure 3.1 Velocity profiles for different $a$
Figure 3.2  Velocity profiles for different $M$
Figure 3.3 Temperature profiles for different Pr and t
3.3 MHD EFFECTS ON ISOTHERMAL VERTICAL PLATE WITH VARIABLE MASS DIFFUSION

3.3.1 Governing Equations

Here the flow of a viscous incompressible fluid past an infinite isothermal vertical plate with variable mass diffusion in the presence of magnetic field is studied. At time \( t' \leq 0 \), the plate and fluid are at the same temperature \( T_\infty \) and concentration \( C'_\infty \). At time \( t' > 0 \), the plate is exponentially accelerated with a velocity \( u = u_0 \exp(a't') \) in its own plane and the temperature from the plate is raised to \( T_w \) and the mass is diffused from the plate to the fluid is raised linearly with time. Then under usual Boussinesq’s approximation the unsteady flow is governed by the following equations:

\[
\begin{align*}
\frac{\partial u}{\partial t'} &= g\beta(T - T_\infty) + g\beta^* (C' - C'_\infty) + \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u \\
\rho \ C_P \frac{\partial T}{\partial t'} &= k \frac{\partial^2 T}{\partial y^2} \\
\frac{\partial C'}{\partial t'} &= D \frac{\partial^2 C'}{\partial y^2}
\end{align*}
\]  

\( (3.10) \) \( (3.11) \) \( (3.12) \)

with the following initial and boundary conditions:

\( u = 0, \quad T = T_\infty, \quad C' = C'_\infty \) for all \( y, t' \leq 0 \)

\( t' > 0 : \quad u = u_0 \exp(a't'), \quad T = T_w, \quad C' = C'_\infty + (C'_w - C'_\infty) A t' \) at \( y = 0 \)

\( u \to 0 \quad T \to T_\infty, \quad C' \to C'_\infty \) as \( y \to \infty \)

\( (3.13) \)

where \( A = \frac{u_0^2}{\nu} \).
On introducing the following non-dimensional quantities:

\[ U = \frac{u}{u_0}, \quad t = \frac{t' u_0^2}{\nu}, \quad Y = \frac{y u_0}{\nu}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \]

\[ Gr = \frac{g \beta \nu (T_w - T_\infty)}{u_0^3}, \quad C = \frac{C' - C'_\infty}{C_w - C'_\infty}, \quad Gc = \frac{\nu g \beta^* (C_w - C'_w)}{u_0^3}, \]

\[ Pr = \frac{\mu C_p}{k}, \quad a = \frac{\alpha' \nu}{u_0^2}, \quad Sc = \frac{\nu}{D}, \quad M = \frac{\sigma B_0^3 \nu}{\rho u_0^2} \]

in equations (3.10) to (3.14), leads to

\[
\begin{align*}
\frac{\partial U}{\partial t} &= Gr \theta + Gc \ C + \frac{\partial^2 U}{\partial Y^2} - M \ U \\
\frac{\partial \theta}{\partial t} &= \frac{1}{Pr} \frac{\partial^2 \theta}{\partial Y^2} \\
\frac{\partial C}{\partial t} &= \frac{1}{Sc} \frac{\partial^2 C}{\partial Y^2}
\end{align*}
\]

The initial and boundary conditions in non-dimensional quantities are

\[
\begin{align*}
U &= 0, \quad \theta = 0, \quad C = 0 \quad \text{for all} \quad Y, t \leq 0 \\
t > 0: \quad U = \exp(at), \quad \theta = 1, \quad C = t \quad \text{at} \quad Y = 0 \\
U \rightarrow 0, \quad \theta \rightarrow 0, \quad C \rightarrow 0 \quad \text{as} \quad Y \rightarrow \infty
\end{align*}
\]

### 3.3.2 Solution Procedure

The dimensionless governing equations (3.15) to (3.17), subject to the boundary conditions (3.18), are solved by the usual Laplace transform technique and the solutions are derived as follows:

\[ \theta = \text{erfc}(\eta \sqrt{Pr}) \]

\[ C = t \left[ (1 + 2 \ \eta^2 \ Sc) \ \text{erfc}(\eta \sqrt{Sc}) - \frac{2\eta \ \sqrt{Sc}}{\sqrt{\pi}} \ \exp(-\eta^2 Sc) \right] \]
\[ U = \frac{\exp(at)}{2} \left[ \exp(2\eta \sqrt{(M + a)t}) \, \text{erfc}(\eta + \sqrt{(M + a)t}) \\
+ \exp(-2\eta \sqrt{(M + a)t}) \, \text{erfc}(\eta - \sqrt{(M + a)t}) \right] \\
+ [d + e(1 + ct)] \left[ \exp(2\eta \sqrt{Mt}) \, \text{erfc}(\eta + \sqrt{Mt}) \\
+ \exp(-2\eta \sqrt{Mt}) \, \text{erfc}(\eta - \sqrt{Mt}) \right] \\
- \frac{ecn\sqrt{t}}{\sqrt{M}} \left[ \exp(-2\eta \sqrt{Mt}) \, \text{erfc}(\eta - \sqrt{Mt}) - \exp(2\eta \sqrt{Mt}) \, \text{erfc}(\eta + \sqrt{Mt}) \right] \\
-d \exp(bt) \left[ \exp(2\eta \sqrt{(M + b)t}) \, \text{erfc}(\eta + \sqrt{(M + b)t}) \\
+ \exp(-2\eta \sqrt{(M + b)t}) \, \text{erfc}(\eta - \sqrt{(M + b)t}) \right] \\
-2d \, \text{erfc}(\eta \sqrt{Pr}) - 2e \, \text{erfc}(\eta \sqrt{Sc}) \\
-2ec \, t \left[ (1 + 2\eta^2 Sc) \, \text{erfc}(\eta \sqrt{Sc}) - \frac{2\eta \sqrt{Sc}}{\sqrt{\pi}} \exp(-\eta^2 Sc) \right] \\
-e \exp(ct) \left[ \exp(2\eta \sqrt{(M + c)t}) \, \text{erfc}(\eta + \sqrt{(M + c)t}) \\
+ \exp(-2\eta \sqrt{(M + c)t}) \, \text{erfc}(\eta - \sqrt{(M + c)t}) \right] \\
+d \exp(bt) \left[ \exp(2\eta \sqrt{Pr \, bt}) \, \text{erfc}(\eta \sqrt{Pr} + \sqrt{bt}) \\
+ \exp(-2\eta \sqrt{Pr \, bt}) \, \text{erfc}(\eta \sqrt{Pr} - \sqrt{bt}) \right] \\
+e \exp(ct) \left[ \exp(2\eta \sqrt{Sc \, ct}) \, \text{erfc}(\eta \sqrt{Sc} + \sqrt{ct}) \\
+ \exp(-2\eta \sqrt{Sc \, ct}) \, \text{erfc}(\eta \sqrt{Sc} - \sqrt{ct}) \right] \] (3.21)

where \( b = \frac{M}{Pr - 1}, c = \frac{M}{Sc - 1}, d = \frac{Gr}{2b(1 - Pr)}, e = \frac{Gc}{2c^2(1 - Sc)} \) and \( \eta = \frac{Y}{2\sqrt{t}} \).
3.3.3 Results and Discussion

For physical understanding of the problem numerical computations are carried out for different physical parameters like $M, a, Gr, Gc, Sc$ and $t$ upon the nature of the flow and transport. The value of Prandtl number $Pr$ is chosen such that they represent water ($Pr = 7.0$). The numerical values of the velocity and concentration are computed for different physical parameters like magnetic filed parameter, $a$, Prandtl number, thermal Grashof number, mass Grashof number, Schmidt number and time.

The velocity profiles for different $(a = 0.2, 0.5, 0.9), Gr = Gc = 5, M = 0.2, Sc = 2.01$ at $t = 0.2$ are studied and presented in Figure 3.4. It is observed that the velocity increases with increasing values of ‘$a$’. Figure 3.5 illustrates the effects of the magnetic field parameter on the velocity when $(M = 2, 5, 10), a = 0.5, Gr = Gc = 5, Sc = 2.01$ and $t = 0.2$. It is observed that the velocity increases with decreasing values of the magnetic field parameter. This shows that the increase in the magnetic field parameter leads to a fall in the velocity.

The velocity profiles for different values of time $(t = 0.2, 0.4, 0.6), M = 0.2, a = 0.5, Gr = Gc = 5$ and $Sc = 2.01$ are shown in Figure 3.6. It is clear that the velocity increases gradually with increasing values of time. Figure 3.7 demonstrates the effects of different thermal Grashof number $(Gr = 2, 10)$ and mass Grashof number $(Gc = 5, 15)$ on the velocity when $M = 0.2, a = 0.5, Sc = 2.01$ and $t = 0.2$. It is observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number.
Figure 3.4 Velocity profiles for different $a$
Figure 3.5 Velocity profiles for different $M$
Figure 3.6 Velocity profiles for different $t$
Figure 3.7 Velocity profiles for different Gr and Gc
Figure 3.8 Concentration profiles for different Sc
The concentration profiles for different values of Schmidt number 
\((Sc = 0.3, 0.6, 0.78, 2.01)\) and time \(t = 0.2\) are shown in Figure 3.8. It is observed 
that the concentration increases with decreasing values of the Schmidt number.

3.4 CONCLUDING REMARKS

A detailed theoretical study has been carried out for the flow past an 
exponentially accelerated infinite vertical plate in the presence of magnetic 
field for the following boundary conditions (i) variable plate temperature 
(ii) isothermal plate with variable mass diffusion. The plate temperature is raised 
linearly with time and concentration level near the plate is raised uniformly. 
The linearized dimensionless governing equations are solved using the Laplace 
transform technique. The fields of velocity, temperature and concentration are 
analyzed for different parameters like ‘\(a\)’, thermal Grashof number, mass Grashof 
number, magnetic field, Schmidt number and ‘\(t\)’. The conclusions of the study 
are as follows:

(i) The velocity decreases with increasing magnetic field parameter.

(ii) The velocity increases with increasing values of ‘\(a\)’ and time in the presence 
of magnetic field parameter.

(iii) The concentration level near the plate decreases with increasing values of 
Schmidt number.

(iv) As time increases, there is a rise in temperature.