INTRODUCTION

Fuzzy subsets:

Fuzzy subsets were introduced by Zadeh in 1965 to represent / manipulate data and information possessing non-statistical uncertainties. It was specifically designed to mathematically represent uncertainty and vagueness and to provide formalized tools for dealing with the imprecision intrinsic to many problems.

The first publication in fuzzy subset theory by Zadeh (1965) and then by Goguen (1967, 1969) show the intention of the authors to generalize the classical set. In classical set theory, a subset A of a set X can be defined by its characteristic function \( \chi_A : X \rightarrow \{0, 1\} \) is defined by \( \chi_A( x ) = 0 \), if \( x \notin A \) and \( \chi_A( x ) = 1 \), if \( x \in A \).

The mapping may be represented as a set of ordered pairs \( \{ ( x, \chi_A( x ) ) \} \) with exactly one ordered pair present for each element of X. The first element of the ordered pair is an element of the set X and the second is its value in \( \{ 0, 1 \} \). The value ‘0’ is used to represent non-membership and the value ‘1’ is used to represent membership of the element A. The truth or falsity of the statement “x is in A” is determined by the ordered pair. The statement is true, if the second element of the ordered pair is ‘1’, and the statement is false, if it is ‘0’
Similarly, a fuzzy subset $A$ of a set $X$ can be defined as a set of ordered pairs $\{ (x, \chi_A(x)) : x \in X \}$, each with the first element from $X$ and the second element from the interval $[0, 1]$ with exactly one ordered pair present for each element of $X$. This defines a mapping, $\mu_A$ between elements of the set $X$ and values in the interval $[0, 1]$. That is, $\mu_A : X \rightarrow [0, 1]$.

The value ‘0’ is used to represent complete non-membership, the value ‘1’ is used to represent complete membership and values in between are used to represent intermediate degrees of membership.

The set $X$ is referred to as the Universe of discourse for the fuzzy subset $A$. Frequently, the mapping $\mu_A$ is described as a function, the membership function of $A$, the degree to which the statement ‘$x$ is in $A$’ is true, is determined by finding the ordered pair $(x, \mu_A(x))$. The degree of truth of the statement is the second element of the ordered pair.

**Intuitionistic fuzzy subsets:**

Prof. K.T. Atanassov, a Bulgarian Engineer, introduced a new component which determines the degree of non-membership also in defining Intuitionistic Fuzzy Subset (IFS) theory. In 1983, he came across A.Kauffmann’s book “Introduction to the theory of Fuzzy subsets” Academic Press, New York, 1975, then he tried to introduce intuitionistic fuzzy subsets to study the properties of the new objects so defined. He defined ordinary operations as $\cap$, $\cup$, $+$ and $-$ over the new
sets, then defined operators similar to the operators ‘necessity’ and ‘possibility’.

George Gargov named new sets as the ‘‘Intuitionistic Fuzzy Subsets’’, as their fuzzification denies the law of the excluded middle, $A \cup A^c = X$. This has encouraged Prof. K.T. Atanassov to continue his work on intuitionistic fuzzy subsets.

An Intuitionistic Fuzzy Subset $A$ of a set $X$ is defined as an object of the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$, where $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$ define the degree of membership and the degree of non-membership of the element $x$ in $X$ respectively and for every $x$ in $X$ satisfying $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Let $(L, \leq)$ be a complete lattice with an involutive order reversing operation $N : L \to L$. An intuitionistic $L$-fuzzy subset (ILFS) $A$ in $X$ is defined as an object of the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$, where $\mu_A : X \to L$ and $\nu_A : X \to L$ define the degree of membership and the degree of non-membership of the element $x$ in $X$ respectively and for every $x$ in $X$ satisfying $\mu_A(x) \leq N(\nu_A(x))$. 