CHAPTER- IV

INTUITIONISTIC L-FUZZY TRANSLATION

4.1 Introduction:

This chapter contains the intuitionistic L-fuzzy translation of intuitionistic L-fuzzy subgroups of a group. These concepts are used in the development of some important results and theorems.

4.1.1 Definition: Let A be an intuitionistic L-fuzzy subset of X and $\alpha$ and $\beta$ in $[0, 1- \text{Sup}\{ \mu_A(x) + \nu_A(x) : x \in X, 0 < \mu_A(x) + \nu_A(x) < 1 \}]$. Then $T = T_{(\alpha, \beta)}$ is called an intuitionistic L-fuzzy translation of A if

$\mu_T(x) = \mu_A(x) + \alpha$ , $\nu_T(x) = \nu_A(x) + \beta$,

$\alpha+\beta \leq 1-\text{Sup}\{\mu_A(x) + \nu_A(x) : x \in X, 0 < \mu_A(x) + \nu_A(x) < 1\}$, for all $x$ in $X$.

4.2 – PROPERTIES OF INTUITIONISTIC L-FUZZY TRANSLATION:

4.2.1 Theorem: If T is an intuitionistic L-fuzzy translation of an intuitionistic L-fuzzy subgroup A of a group G, then $\mu_T(x^{-1}) = \mu_T(x)$ and $\nu_T(x^{-1}) = \nu_T(x)$, $\mu_T(x) \leq \mu_T(e)$ and $\nu_T(x) \geq \nu_T(e)$, for all $x$ and $e$ in G.

Proof: Let $x$ and $e$ be elements of G.

Now, $\mu_T(x) = \mu_A(x) + \alpha$

$= \mu_A( (x^{-1})^{-1} ) + \alpha$

$\geq \mu_A(x^{-1}) + \alpha$
\[ \mu_T(x) = \mu_T(x^{-1}) \]
\[ = \mu_A(x^{-1}) + \alpha \]
\[ \geq \mu_A(x) + \alpha \]
\[ = \mu_T(x). \]

Therefore, \( \mu_T(x) = \mu_T(x^{-1}) \), for \( x \) in \( G \).

And \( \nu_T(x) = \nu_A(x) + \beta \)
\[ = \nu_A((x^{-1})^{-1}) + \beta \]
\[ \leq \nu_A(x^{-1}) + \beta \]
\[ = \nu_T(x^{-1}) \]
\[ = \nu_A(x^{-1}) + \beta \]
\[ \leq \nu_A(x) + \beta \]
\[ = \nu_T(x). \]

Therefore, \( \nu_T(x) = \nu_T(x^{-1}) \), for \( x \) in \( G \).

Now, \( \mu_T(e) = \mu_A(e) + \alpha \)
\[ = \mu_A(xx^{-1}) + \alpha \]
\[ \geq \{ \mu_A(x) \land \mu_A(x^{-1}) \} + \alpha \]
\[ = \mu_A(x) + \alpha \]
\[ = \mu_T(x). \]

Therefore, \( \mu_T(e) \geq \mu_T(x) \), for \( x \) in \( G \).

And \( \nu_T(e) = \nu_A(e) + \beta \)
\[ = \nu_A(xx^{-1}) + \beta \]
\[ \leq \{ \nu_A(x) \lor \nu_A(x^{-1}) \} + \beta \]
\[= \nu_A(x) + \beta\]

\[= \nu_T(x).\]

Therefore, \(\nu_T(e) \leq \nu_T(x)\), for \(x\) in \(G\).

**4.2.2 Theorem:** If \(T\) is an intuitionistic L-fuzzy translation of an intuitionistic L-fuzzy subgroup \(A\) of a group \(G\), then

(i) \(\mu_T(xy^{-1}) = \mu_T(e)\) implies \(\mu_T(x) = \mu_T(y)\),

(ii) \(\nu_T(xy^{-1}) = \nu_T(e)\) implies \(\nu_T(x) = \nu_T(y)\), for all \(x, y\) and \(e\) in \(G\).

**Proof:** Let \(x, y\) and \(e\) be elements of \(G\).

Now, \(\mu_T(x) = \mu_A(x) + \alpha\)

\[= \mu_A(xy^{-1}y) + \alpha\]

\[\geq \{ \mu_A(xy^{-1}) \land \mu_A(y) \} + \alpha\]

\[= (\mu_A(xy^{-1}) + \alpha) \land (\mu_A(y) + \alpha)\]

\[= \mu_T(xy^{-1}) \land \mu_T(y)\]

\[= \mu_T(e) \land \mu_T(y)\]

\[= \mu_T(y) = \mu_A(y) + \alpha\]

\[= \mu_A(yx^{-1}x) + \alpha\]

\[\geq \{ \mu_A(yx^{-1}) \land \mu_A(x) \} + \alpha\]

\[= (\mu_A(yx^{-1}) + \alpha) \land (\mu_A(x) + \alpha)\]

\[= \mu_T(yx^{-1}) \land \mu_T(x)\]

\[= \mu_T(e) \land \mu_T(x)\]

\[= \mu_T(x).\]

Therefore, \(\mu_T(x) = \mu_T(y)\), for all \(x\) and \(y\) in \(G\).
And \( v_T(x) = v_A(x) + \beta \)
\[
= v_A(xy^{-1}y) + \beta \\
\leq \{ v_A(xy^{-1}) \lor v_A(y) \} + \beta \\
= ( v_A(xy^{-1}) + \beta ) \lor ( v_A(y) + \beta ) \\
= v_T(xy^{-1}) \lor v_T(y) \\
= v_T(e) \lor v_T(y) \\
= v_T(y) = v_A(y) + \beta \\
= v_A(yx^{-1}x) + \beta \\
\leq \{ v_A(yx^{-1}) \lor ( v_A(x) \} + \beta \\
= ( v_A(yx^{-1}) + \beta ) \lor ( v_A(x) + \beta ) \\
= v_T(yx^{-1}) \lor v_T(x) \\
= v_T(e) \lor v_T(x) \\
= v_T(x).
\]

Therefore, \( v_T(x) = v_T(y) \), for all \( x \) and \( y \) in \( G \).

4.2.3 Theorem: If \( T \) is an intuitionistic L-fuzzy translation of an intuitionistic L-fuzzy subgroup \( A \) of a group \( G \), then \( T \) is an intuitionistic L-fuzzy subgroup of \( G \), for all \( x \) and \( y \) in \( G \).

Proof: Assume that \( T \) is an intuitionistic L-fuzzy translation of an intuitionistic L-fuzzy subgroup \( A \) of a group \( G \). Let \( x \) and \( y \) in \( G \).

We have, \( \mu_T(xy^{-1}) = \mu_A(xy^{-1}) + \alpha \)
\[
\geq \{ \mu_A(x) \land \mu_A(y^{-1}) \} + \alpha \\
= \{ \mu_A(x) \land \mu_A(y) \} + \alpha
\]
\[ \begin{aligned}
\mu_A(x) + \alpha \land \mu_A(y) + \alpha \\
= \mu_T(x) \land \mu_T(y).
\end{aligned} \]

Therefore, \( \mu_T(xy^{-1}) \geq \mu_T(x) \land \mu_T(y) \), for all \( x \) and \( y \) in \( G \).

And \( \nu_T(xy^{-1}) = \nu_A(xy^{-1}) + \beta \)

\[ \begin{aligned}
& \leq \{ \nu_A(x) \lor \nu_A(y^{-1}) \} + \beta \\
& = \{ \nu_A(x) \lor \nu_A(y) \} + \beta \\
& = (\nu_A(x) + \beta) \lor (\nu_A(y) + \beta) \\
& = \nu_T(x) \lor \nu_T(y).
\end{aligned} \]

Therefore, \( \nu_T(xy^{-1}) \leq \nu_T(x) \lor \nu_T(y) \), for all \( x \) and \( y \) in \( G \).

Hence \( T \) is an intuitionistic L-fuzzy subgroup of \( G \).

4.2.4 Theorem: If \( T \) is an intuitionistic L-fuzzy translation of an intuitionistic L-fuzzy subgroup \( A \) of a group \( G \), then \( H = \{ x \in G : \mu_T(x) = \mu_T(e) \land \nu_T(x) = \nu_T(e) \} \) is either empty or a subgroup of \( G \).

Proof: If no element satisfies this condition, then \( H \) is empty.

If \( x \) and \( y \) satisfies this condition, then

\[ \begin{aligned}
\mu_T(x^{-1}) = \mu_T(x) = \mu_T(e) \land \nu_T(x^{-1}) = \nu_T(x) = \nu_T(e).
\end{aligned} \]

Therefore, \( \mu_T(x^{-1}) = \mu_T(e) \land \nu_T(x^{-1}) = \nu_T(e) \).

Therefore, \( x^{-1} \in H \).

Now, \( \mu_T(xy^{-1}) \geq \mu_T(x) \land \mu_T(y) \)

\[ \begin{aligned}
& = \mu_T(e) \land \mu_T(e) \\
& = \mu_T(e),
\end{aligned} \]

and \( \mu_T(e) = \mu_T(xy^{-1}(xy^{-1})^{-1}) \)
\[ \geq \mu_T(xy^{-1}) \land \mu_T(xy^{-1}) \]
\[ = \mu_T(xy^{-1}). \]
Therefore, \( \mu_T(e) = \mu_T(xy^{-1}) \), for all \( x \) and \( y \) in \( G \).

Now, \( \nu_T(xy^{-1}) \leq \nu_T(x) \lor \nu_T(y) \)
\[ = \nu_T(e) \lor \nu_T(e) \]
\[ = \nu_T(e), \]
and \( \nu_T(e) = \nu_T((xy^{-1})(xy^{-1})^{-1}) \)
\[ \leq \nu_T(xy^{-1}) \lor \nu_T(xy^{-1}) \]
\[ = \nu_T(xy^{-1}). \]
Therefore, \( \nu_A(e) = \nu_A(xy^{-1}) \), for all \( x \) and \( y \) in \( G \).

Therefore, \( xy^{-1} \) in \( H \). Hence \( H \) is a subgroup of \( G \).

**4.2.5 Theorem:** If \( T \) is an intuitionistic L-fuzzy translation of an intuitionistic L-fuzzy subgroup \( A \) of a group \( G \), then \( H = \{ < x, \mu_T(x) > : \mu_T(x) = \mu_T(e) \land \nu_T(x) = \nu_T(e) \} \) is either empty or a L-fuzzy subgroup of \( G \).

**Proof:** If no element satisfies this condition, then \( H \) is empty.

If \( x \) and \( y \) satisfies this condition, then

by Theorem 4.2.4, \( xy^{-1} \) in \( H \).

Therefore, \( \mu_T(xy^{-1}) = \mu_T(e) \land \nu_T(xy^{-1}) = \nu_T(e) \), for all \( x \) and \( y \) in \( G \).

But, \( \mu_T(xy^{-1}) \geq \mu_T(x) \land \mu_T(y^{-1}) \)
\[ = \mu_T(x) \land \mu_T(y), \text{for all } x \text{ and } y \text{ in } G. \]
Hence \( H \) is a L-fuzzy subgroup of \( G \).
4.2.6 **Theorem:** If $T$ is an intuitionistic L-fuzzy translation of an intuitionistic L-fuzzy subgroup $A$ of a group $G$, then $H = \{ < x, \nu_T(x) > : \mu_T(x) = \mu_T(e) \text{ and } \nu_T(x) = \nu_T(e) \}$ is either empty or an antiL-fuzzy subgroup of $G$.

**Proof:** It is trivial.

4.2.7 **Theorem:** Let $T$ be an intuitionistic L-fuzzy translation of an intuitionistic L-fuzzy subgroup $A$ of a group $G$. If $\mu_T(xy^{-1}) = 1$, then $\mu_T(x) = \mu_T(y)$ and if $\nu_T(xy^{-1}) = 0$, then $\nu_T(x) = \nu_T(y)$.

**Proof:** Let $x$ and $y$ be elements of $G$.

Now, $\mu_T(x) = \mu_T(xy^{-1}y)$

$\geq \mu_T(xy^{-1}) \land \mu_T(y)$

$= 1 \land \mu_T(y)$

$= \mu_T(y) = \mu_T(y^{-1})$

$= \mu_T(x^{-1}xy^{-1})$

$\geq \mu_T(x^{-1}) \land \mu_T(xy^{-1})$

$= \mu_T(x) \land \mu_T(xy^{-1})$

$= \mu_T(x) \land 1 = \mu_T(x)$.

Therefore, $\mu_T(x) = \mu_T(y)$, for all $x$ and $y$ in $G$.

Now, $\nu_T(x) = \nu_T(xy^{-1}y)$

$\leq \nu_T(xy^{-1}) \lor \nu_T(y)$

$= 0 \lor \nu_T(y)$
\[ T(y) = T(x^{-1}xy^{-1}) \]
\[ \leq T(x^{-1}) \lor T(xy^{-1}) \]
\[ = T(x) \lor T(xy^{-1}) \]
\[ = T(x) \lor 0 = T(x). \]

Therefore, \( T(x) = T(y) \), for all \( x \) and \( y \) in \( G \).

**4.2.8 Theorem:** Let \( G \) be a group. If \( T \) is an intuitionistic L-fuzzy translation of an intuitionistic L-fuzzy subgroup \( A \) of \( G \), then \( \mu_T(xy) = \mu_T(x) \land \mu_T(y) \) and \( \nu_T(xy) = \nu_T(x) \lor \nu_T(y) \), for each \( x \) and \( y \) in \( G \) with \( \mu_T(x) \neq \mu_T(y) \) and \( \nu_T(x) \neq \nu_T(y) \).

**Proof:** Let \( x \) and \( y \) be elements of \( G \).

Assume that \( \mu_T(x) > \mu_T(y) \) and \( \nu_T(x) < \nu_T(y) \).

Then, \[ \mu_T(y) = \mu_T(x^{-1}xy) \]
\[ \geq \mu_T(x^{-1}) \land \mu_T(xy) \]
\[ = \mu_T(x) \land \mu_T(xy) \]
\[ = \mu_T(xy) \]
\[ \geq \mu_T(x) \land \mu_T(y) = \mu_T(y). \]

Therefore, \( \mu_T(xy) = \mu_T(y) = \mu_T(x) \land \mu_T(y) \), for all \( x \) and \( y \) in \( G \).

Then, \[ \nu_T(y) = \nu_T(x^{-1}xy) \]
\[ \leq \nu_T(x^{-1}) \lor \nu_T(xy) \]
\[ = \nu_T(x) \lor \nu_T(xy) \]
\[ = \nu_T(xy) \]
\[ \leq \nu_T(x) \vee \nu_T(y) = \nu_T(y). \]

Therefore, \( \nu_T(xy) = \nu_T(y) = \nu_T(x) \vee \nu_T(y) \), for all \( x \) and \( y \) in \( G \).

**4.2.9 Theorem:** Let \((G, \cdot)\) and \((G', \cdot)\) be any two groups. If \( f : G \to G' \) is a homomorphism, then the homomorphic image of an intuitionistic L-fuzzy translation of an intuitionistic L-fuzzy subgroup \( A \) of \( G \) is an intuitionistic L-fuzzy subgroup of \( G' \).

**Proof:** Let \((G, \cdot)\) and \((G', \cdot)\) be any two groups and \( f : G \to G' \) be a homomorphism. That is \( f(xy) = f(x)f(y) \), for all \( x \) and \( y \) in \( G \).

Let \( V = f(T^A_{(\alpha, \beta)}) \), where \( T^A_{(\alpha, \beta)} \) is an intuitionistic L-fuzzy translation of an intuitionistic L-fuzzy subgroup \( A \) of \( G \).

We have to prove that \( V \) is an intuitionistic L-fuzzy subgroup of \( G' \).

Now, for \( f(x) \) and \( f(y) \) in \( G' \), we have

\[
\mu_V[f(x) \left( f(y)^{-1} \right)] = \mu_V[f(x) f(y^{-1})]
\]

\[
= \mu_V[f(x^{-1})]
\]

\[
\geq \mu^A_\alpha(x^{-1})
\]

\[
= \mu_A(x) + \alpha
\]

\[
\geq \{ \mu_A(x) \land \mu_A(y^{-1}) \} + \alpha
\]

\[
\geq \{ \mu_A(x) \land \mu_A(y) \} + \alpha
\]

\[
= (\mu_A(x) + \alpha) \land (\mu_A(y) + \alpha)
\]

\[
= \mu^A_\alpha(x) \land \mu^A_\alpha(y)
\]

which implies that \( \mu_V[f(x) \left( f(y)^{-1} \right)] \geq \mu_V(f(x)) \land \mu_V(f(y)) \), for all \( f(x) \) and \( f(y) \) in \( G' \).
And, \( \nu_V[ f(x) ( f(y)^{-1} ) ] = \nu_V[ f(x) f(y)^{-1} ] \)

\[ = \nu_V[ f(x y^{-1}) ] \]

\[ \leq \nu_{\beta}^\delta (x y^{-1}) \]

\[ = \nu_A( x y^{-1}) + \beta \]

\[ \leq \{ \nu_A(x) \lor \nu_A( y^{-1}) \} + \beta \]

\[ \leq \{ \nu_A(x) \lor \nu_A( y) \} + \beta \]

\[ = ( \nu_A(x) + \beta ) \lor ( \nu_A(y) + \beta ) \]

\[ = \nu_{\beta}^\delta (x) \lor \nu_{\beta}^\delta (y) \]

which implies that \( \nu_V[ f(x) ( f(y)^{-1} ) ] \leq \nu_V( f(x) ) \lor \nu_V( f(y) ) \), for all \( f(x) \) and \( f(y) \) in \( G \).

Therefore, \( V \) is an intuitionistic L-fuzzy subgroup of \( G \).

Hence the homomorphic image of an intuitionistic L-fuzzy translation of \( A \) of \( G \) is an intuitionistic L-fuzzy subgroup of \( G \).

4.2.10 Theorem: Let \((G, \cdot)\) and \((G', \cdot')\) be any two groups. If \( f : G \rightarrow G' \) is a homomorphism, then the homomorphic pre-image of an intuitionistic L-fuzzy translation of an intuitionistic L-fuzzy subgroup \( V \) of \( G' \) is an intuitionistic L-fuzzy subgroup of \( G \).

Proof: Let \( (G, \cdot) \) and \( (G', \cdot') \) be any two groups and \( f : G \rightarrow G' \) be a homomorphism. That is \( f(xy) = f(x)f(y) \), for all \( x \) and \( y \) in \( G \).

Let \( T = T\alpha,\beta = f(A) \), where \( T\alpha,\beta \) is an intuitionistic L-fuzzy translation of intuitionistic L-fuzzy subgroup \( V \) of \( G' \).

We have to prove that \( A \) is an intuitionistic L-fuzzy subgroup of \( G \).
Let \( x \) and \( y \) in \( G \). Then,

\[
\mu_A(xy^{-1}) = \mu_T(f(xy^{-1}))
\]

\[
= \mu_T(f(x)f(y^{-1}))
\]

\[
= \mu_T[f(x)(f(y))^{-1}]
\]

\[
= \mu_V[f(x)(f(y))^{-1}] + \alpha
\]

\[
\geq \{ \mu_V(f(x)) \wedge \mu_V(f(y)) \} + \alpha
\]

\[
= (\mu_V(f(x)) + \alpha) \wedge (\mu_V(f(y)) + \alpha)
\]

\[
= \mu_T(f(x)) \wedge \mu_T(f(y))
\]

\[
= \mu_A(x) \wedge \mu_A(y)
\]

which implies that \( \mu_A(xy^{-1}) \geq \mu_A(x) \wedge \mu_A(y) \), for all \( x \) and \( y \) in \( G \).

And, \( \nu_A(xy^{-1}) = \nu_T(f(xy^{-1})) \)

\[
= \nu_T(f(x)f(y^{-1}))
\]

\[
= \nu_T[f(x)(f(y))^{-1}]
\]

\[
= \nu_V[f(x)(f(y))^{-1}] + \beta
\]

\[
\leq \{ \nu_V(f(x)) \lor \nu_V(f(y)) \} + \beta
\]

\[
= (\nu_V(f(x)) + \beta) \lor (\nu_V(f(y)) + \beta)
\]

\[
= \nu_T(f(x)) \lor \nu_T(f(y))
\]

\[
= \nu_A(x) \lor \nu_A(y)
\]

which implies that \( \nu_A(xy^{-1}) \leq \nu_A(x) \lor \nu_A(y) \), for all \( x \) and \( y \) in \( G \).

Therefore, \( A \) is an intuitionistic L-fuzzy subgroup of \( G \).
Hence the homomorphic pre-image of an intuitionistic L-fuzzy translation of an intuitionistic L-fuzzy subgroup \( V \) of \( G^l \) is an intuitionistic L-fuzzy subgroup of \( G \).

**4.2.11 Theorem:** Let \((G,\ast)\) and \((G^l, \ast)\) be any two groups. If \( f : G \rightarrow G^l \) is an anti-homomorphism, then the anti-homomorphic image of an intuitionistic L-fuzzy translation of an intuitionistic L-fuzzy subgroup \( A \) of \( G \) is an intuitionistic L-fuzzy subgroup of \( G^l \).

**Proof:** Let \((G,\ast)\) and \((G^l, \ast)\) be any two groups and \( f : G \rightarrow G^l \) be an anti-homomorphism. That is \( f(x \ast y) = f(y) f(x) \), for all \( x \) and \( y \) in \( G \).

Let \( V = f(T^{\alpha}_{(\alpha,\beta)}) \), where \( T^{\alpha}_{(\alpha,\beta)} \) is an intuitionistic L-fuzzy translation of an intuitionistic L-fuzzy subgroup \( A \) of \( G \).

We have to prove that \( V \) is an intuitionistic L-fuzzy subgroup of \( G^l \).

Now, for \( f(x) \) and \( f(y) \) in \( G^l \), we have

\[
\begin{align*}
\mu_V[ f(x) ( f(y)^{-1} ) ] &= \mu_V[ f(x) f(y^{-1}) ] \\
&= \mu_V[ f(y^{-1}x) ] \\
&\geq \mu^\times_{A^l}(y^{-1}x) \\
&= \mu_A(y^{-1}x) + \alpha \\
&\geq \{ \mu_A(x) \land \mu_A(y^{-1}) \} + \alpha \\
&\geq \{ \mu_A(x) \land \mu_A(y) \} + \alpha \\
&= (\mu_A(x) + \alpha) \land (\mu_A(y) + \alpha) \\
&= \mu^\times_{A^l}(x) \land \mu^\times_{A^l}(y)
\end{align*}
\]
which implies that \( \mu_V[ f(x) ( f(y)^{-1} ) ] \geq \mu_V( f(x) ) \land \mu_V( f(y) ) \), for all \( f(x) \) and \( f(y) \) in \( G^l \).

And, \( \nu_V[ f(x) ( f(y)^{-1} ) ] = \nu_V[ f(x) f(y^{-1}) ] \)

\[
= \nu_V[ f(y^{-1}x) ] \\
\leq \nu_V^{\beta}(y^{-1}x) \\
= \nu_A(y^{-1}x) + \beta \\
\leq \{ \nu_A(x) \lor \nu_A(y^{-1}) \} + \beta \\
\leq \{ \nu_A(x) \lor \nu_A(y) \} + \beta \\
= ( \nu_A(x) + \beta ) \lor ( \nu_A(y) + \beta ) \\
= \nu_V^{\beta}(x) \lor \nu_V^{\beta}(y),
\]

which implies that \( \nu_V[ f(x) ( f(y)^{-1} ) ] \leq \nu_V( f(x) ) \lor \nu_V( f(y) ) \), for all \( f(x) \) and \( f(y) \) in \( G^l \).

Therefore, \( V \) is an intuitionistic L-fuzzy subgroup of a group \( G^l \).

Hence the anti-homomorphic image of an intuitionistic L-fuzzy translation of \( A \) of \( G \) is an intuitionistic L-fuzzy subgroup of \( G^l \).

**4.2.12 Theorem:** Let \((G, \cdot)\) and \((G^l, \cdot)\) be any two groups. If \( f : G \to G^l \) is an anti-homomorphism, then the anti-homomorphic pre-image of an intuitionistic L-fuzzy translation of an intuitionistic L-fuzzy subgroup \( V \) of \( G^l \) is an intuitionistic L-fuzzy subgroup of \( G \).

**Proof:** Let \((G, \cdot)\) and \((G^l, \cdot)\) be any two groups and \( f : G \to G^l \) be an anti-homomorphism. That is \( f(xy) = f(y)f(x) \), for all \( x \) and \( y \) in \( G \).
Let $T = T^{\vee}_{(\alpha, \beta)} = \mathbb{I}(A)$, where $T^{\vee}_{(\alpha, \beta)}$ is an intuitionistic L-fuzzy translation of an intuitionistic L-fuzzy subgroup $V$ of $G^i$.

We have to prove that $A$ is an intuitionistic L-fuzzy subgroup of $G$.

Let $x$ and $y$ in $G$. Then,

$$
\mu_A(x^{-1}y) = \mu_T( f(xy^{-1}) )
$$

$$
= \mu_T( f(y^{-1}) f(x) )
$$

$$
= \mu_T[ ( f(y^{-1}) )^{-1} f(x) ]
$$

$$
= \mu_V[ f( f(y^{-1}) )^{-1} f(x) ] + \alpha
$$

$$
\geq \{ \mu_V( f(x) ) \land \mu_V( f(y) ) \} + \alpha
$$

$$
= ( \mu_V( f(x) ) + \alpha ) \land ( \mu_V( f(y) ) + \alpha )
$$

$$
= \mu_T( f(x) ) \land \mu_T( f(y) )
$$

$$
= \mu_A(x) \land \mu_A(y),
$$

which implies that $\mu_A(x^{-1}y) \geq \mu_A(x) \land \mu_A(y)$, for all $x$ and $y$ in $G$.

And, $\nu_A(x^{-1}y) = \nu_T( f(xy^{-1}) )$

$$
= \nu_T( f(y^{-1}) f(x) )
$$

$$
= \nu_T[ ( f(y^{-1}) )^{-1} f(x) ]
$$

$$
= \nu_V[ ( f( f(y^{-1}) )^{-1} f(x) ] + \beta
$$

$$
\leq ( \nu_V( f(x) ) \lor \nu_V( f(y) ) ) + \beta
$$

$$
= ( \nu_V( f(x) ) + \beta ) \lor ( \nu_V( f(y) ) + \beta )
$$

$$
= \nu_T( f(x) ) \lor \nu_T( f(y) )
$$

$$
= \nu_A(x) \lor \nu_A(y)
$$

which implies that $\nu_A(x^{-1}y) \leq \nu_A(x) \lor \nu_A(y)$, for all $x$ and $y$ in $G$. 

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Therefore, A is an intuitionistic L-fuzzy subgroup of G.

Hence the anti-homomorphic pre-image of an intuitionistic L-fuzzy translation of an intuitionistic L-fuzzy subgroup V of $G^l$ is an intuitionistic L-fuzzy subgroup of G.

4.2.13 Theorem: Let $(G, \cdot)$ and $(G^l, \cdot)$ be any two groups. If $f : G \rightarrow G^l$ is a homomorphism, then the homomorphic image of an intuitionistic L-fuzzy translation of an intuitionistic L-fuzzy normal subgroup A of G is an intuitionistic L-fuzzy normal subgroup of $G^l$.

**Proof:** Let $(G, \cdot)$ and $(G^l, \cdot)$ be any two groups and $f : G \rightarrow G^l$ be a homomorphism. That is $f(xy) = f(x)f(y)$, for all $x$ and $y$ in G.

Let $V = f(T^A_{(\alpha, \beta)})$, where $T = T^A_{(\alpha, \beta)}$ is an intuitionistic L-fuzzy translation of an intuitionistic L-fuzzy normal subgroup A of G.

We have to prove that V is an intuitionistic L-fuzzy normal subgroup of $G^l$.

Now, for $f(x)$ and $f(y)$ in $G^l$,

clearly V is an intuitionistic L-fuzzy subgroup of $G^l$.

We have $\mu_V( f(x) f(y) ) = \mu_V( f(xy) )$,

\[ \geq \mu_T(xy) \]

\[ = \mu_A(xy) + \alpha, \]

\[ = \mu_A(yx) + \alpha \]

\[ = \mu_T(yx) \]

\[ \leq \mu_V( f(yx) ) \]

\[ = \mu_V( f(y) f(x) ), \]
which implies that \( \mu_V(f(x)f(y)) = \mu_V(f(y)f(x)) \), for all \( f(x) \) and \( f(y) \) in \( G^l \).

And, \( \nu_V( f(x)f(y) ) = \nu_V( f(xy) ) \),

\[
\leq \nu_T(xy) \\
= \nu_A(xy) + \beta, \\
= \nu_A(yx) + \beta \\
= \nu_T(yx) \\
\geq \nu_V( f(yx) ) \\
= \nu_V( f(y)f(x) )
\]

which implies that \( \nu_V(f(x)f(y)) = \nu_V(f(y)f(x)) \), for all \( f(x) \) and \( f(y) \) in \( G^l \).

Therefore, \( V \) is an intuitionistic L-fuzzy normal subgroup of a group \( G^l \).

Hence the homomorphic image of an intuitionistic L-fuzzy translation of \( A \) of \( G \) is an intuitionistic L-fuzzy normal subgroup of \( G^l \).

4.2.14 Theorem: Let \((G,\cdot)\) and \((G^l,\cdot^l)\) be any two groups. If \( f : G \to G^l \) is a homomorphism, then the homomorphic pre-image of an intuitionistic L-fuzzy translation of an intuitionistic L-fuzzy normal subgroup \( V \) of \( G^l \) is an intuitionistic L-fuzzy normal subgroup of \( G \).

Proof: Let \((G,\cdot)\) and \((G^l,\cdot^l)\) be any two groups and \( f : G \to G^l \) be a homomorphism. That is \( f(xy) = f(x)f(y) \), for all \( x \) and \( y \) in \( G \).

Let \( T = T_{(\alpha,\beta)}^V = f(A) \), where \( T_{(\alpha,\beta)}^V \) is an intuitionistic L-fuzzy translation of intuitionistic L-fuzzy normal subgroup \( V \) of \( G^l \).

We have to prove that \( A \) is an intuitionistic L-fuzzy normal subgroup of \( G \).

Let \( x \) and \( y \) in \( G \). Then,
clearly $A$ is an intuitionistic L-fuzzy subgroup of $G$,

$$\mu_A(xy) = \mu_T( f(xy) )$$

$$= \mu_V( f(xy) ) + \alpha$$

$$= \mu_V( f(x)f(y) ) + \alpha$$

$$= \mu_V( f(y)f(x) ) + \alpha$$

$$= \mu_V( f(xy) ) + \alpha$$

$$= \mu_T( f(xy) )$$

$$= \mu_A(yx),$$

which implies that $\mu_A(xy) = \mu_A(yx)$, for all $x$ and $y$ in $G$.

And $\nu_A(xy) = \nu_T( f(xy) )$

$$= \nu_V( f(xy) ) + \beta$$

$$= \nu_V( f(x)f(y) ) + \beta$$

$$= \nu_V( f(y)f(x) ) + \beta$$

$$= \nu_V( f(xy) ) + \beta$$

$$= \nu_T( f(xy) )$$

$$= \nu_A(yx),$$

which implies that $\nu_A(xy) = \nu_A(yx)$, for all $x$ and $y$ in $G$.

Therefore, $A$ is an intuitionistic L-fuzzy normal subgroup of $G$.

Hence the homomorphic pre-image of an intuitionistic L-fuzzy translation of an intuitionistic L-fuzzy normal subgroup $V$ of $G^l$ is an intuitionistic L-fuzzy normal subgroup of $G$. 
4.2.15 Theorem: Let \((G, \cdot)\) and \((G^1, \cdot)\) be any two groups. If \(f : G \rightarrow G^1\) is an anti-homomorphism, then the anti-homomorphic image of an intuitionistic L-fuzzy translation of an intuitionistic L-fuzzy normal subgroup \(A\) of \(G\) is an intuitionistic L-fuzzy normal subgroup of \(G^1\).

Proof: Let \((G, \cdot)\) and \((G^1, \cdot)\) be any two groups and \(f : G \rightarrow G^1\) be an anti-homomorphism. That is \(f(xy) = f(y)f(x)\), for all \(x, y \in G\).

Let \(V = f(T^d_{(\alpha, \beta)})\), where \(T^d_{(\alpha, \beta)}\) is an intuitionistic L-fuzzy translation of an intuitionistic L-fuzzy normal subgroup \(A\) of \(G\).

We have to prove that \(V\) is an intuitionistic L-fuzzy normal subgroup of \(G^1\).

Now, for \(f(x)\) and \(f(y)\) in \(G^1\), clearly \(V\) is an intuitionistic L-fuzzy subgroup of \(G^1\). We have

\[
\mu_V(f(x)f(y)) = \mu_V(f(y)f(x)) \\
\geq \mu_T(yx) \\
= \mu_A(yx) + \alpha \\
= \mu_A(xy) + \alpha \\
= \mu_T(xy) \\
\leq \mu_V(f(xy)) \\
= \mu_V(f(y)f(x))
\]

which implies that \(\mu_V(f(x)f(y)) = \mu_V(f(y)f(x))\), for \(f(x)\) and \(f(y)\) in \(G^1\).
And \( \nu( f(x) f(y) ) = \nu( f(y) f(x) ) \)

\[ \leq \nu( f(y) x ) \]

\[ = \nu( y x ) + \beta \]

\[ = \nu( x y ) + \beta \]

\[ = \nu( x y ) \]

\[ \geq \nu( f(x) y ) \]

\[ = \nu( f(y) f(x) ) \]

which implies that \( \nu( f(x) f(y) ) = \nu( f(y) f(x) ) \), for \( f(x) \) and \( f(y) \) in \( G^I \).

Therefore, \( V \) is an intuitionistic L-fuzzy normal subgroup of a group \( G^I \).

Hence the anti-homomorphic image of an intuitionistic L-fuzzy translation of \( A \) of \( G \) is an intuitionistic L-fuzzy normal subgroup of \( G^I \).

**4.2.16 Theorem:** Let \( (G, \cdot) \) and \( (G^I, \cdot) \) be any two groups. If \( f : G \rightarrow G^I \) is an anti-homomorphism, then the anti-homomorphic pre-image of an intuitionistic L-fuzzy translation of an intuitionistic L-fuzzy normal subgroup \( V \) of \( G^I \) is an intuitionistic L-fuzzy normal subgroup of \( G \).

**Proof:** Let \( (G, \cdot) \) and \( (G^I, \cdot) \) be any two groups and \( f : G \rightarrow G^I \) is an anti-homomorphism. That is \( f(x y) = f(y) f(x) \), for all \( x \) and \( y \) in \( G \).

Let \( T = T^F_{(a, \beta)} = f(A) \), where \( T^F_{(a, \beta)} \) is an intuitionistic L-fuzzy translation of an intuitionistic L-fuzzy normal subgroup \( V \) of \( G^I \).

We have to prove that \( A \) is an intuitionistic L-fuzzy normal subgroup of \( G \).

Let \( x \) and \( y \) in \( G \).

Then, clearly \( A \) is an intuitionistic L-fuzzy subgroup of \( G \),

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\[ \mu_A(xy) = \mu_T(f(xy)) \]
\[ = \mu_V(f(xy)) + \alpha \]
\[ = \mu_V(f(y)f(x)) + \alpha \]
\[ = \mu_V(f(x)f(y)) + \alpha \]
\[ = \mu_V(f(yx)) + \alpha \]
\[ = \mu_T(f(yx)) \]
\[ = \mu_A(yx), \]

which implies that \( \mu_A(xy) = \mu_A(yx) \), for all \( x \) and \( y \) in \( G \).

And \( \nu_A(xy) = \nu_T(f(xy)) \)
\[ = \nu_V(f(xy)) + \beta \]
\[ = \nu_V(f(y)f(x)) + \beta \]
\[ = \nu_V(f(x)f(y)) + \beta \]
\[ = \nu_V(f(yx)) + \beta \]
\[ = \nu_T(f(yx)) \]
\[ = \nu_A(yx), \]

which implies that \( \nu_A(xy) = \nu_A(yx) \), for all \( x \) and \( y \) in \( G \).

Therefore, \( A \) is an intuitionistic L-fuzzy normal subgroup of \( G \).

Hence the anti-homomorphic pre-image of an intuitionistic L-fuzzy translation of an intuitionistic L-fuzzy normal subgroup \( V \) of \( G \) is an intuitionistic L-fuzzy normal subgroup of \( G \).