CHAPTER 2

RADIATION AND HYDROMAGNETIC EFFECTS ON VERTICAL OSCILLATING PLATE WITH VARIABLE TEMPERATURE AND CHEMICAL REACTION

2.1 INTRODUCTION

Thermal radiation effect on heat and mass transfer plays an important role in manufacturing industries for the design of reliable equipment. Nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering applications. Magnetohydrodynamic (MHD) plays an important role in agriculture, petroleum industries, geophysics and in astrophysics. It has also important applications in the study of geological formations, in exploration and thermal recovery of oil and in the assessment of aquifers, geothermal reservoirs and underground nuclear waste storage sites.

The effect of chemical reaction depend whether the reaction is homogeneous or heterogeneous. This depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, the reaction is heterogeneous if it takes place at an interface and homogeneous if it takes place in solution. Chambre and Young (1958) have analyzed a first order chemical reaction in the neighbourhood of a horizontal plate. The mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction was studied by Das et al. (1994).

England and Emery (1969) have studied the thermal radiation effects of an optically thin gray gas bounded by a stationary vertical plate. The flow of a viscous, incompressible fluid past an infinite isothermal vertical plate, oscillating in its own plane was solved by Soundalgekar (1979b). Radiation effects on the oscillatory flow past
vertical plate in the presence of uniform temperature analyzed by Mansour (1990). The radiation effects on flow past an impulsively started infinite isothermal vertical plate was analyzed by Das et al. (1996a). Raptis and Perdikis (2003a) analyzed the thermal radiation of an optically thin gray gas.

The effects of transversely applied magnetic field, on the flow of an electrically conducting fluid past an impulsively started infinite isothermal vertical plate was studied by Soundalgekar et al. (1979). Also the effect on the flow past a vertical oscillating plate due to a combination of concentration and temperature differences was studied by Soundalgekar and Akolkar (1983).

It is proposed to study the thermal radiation effects and the chemical reaction effects on unsteady flow of a viscous incompressible fluid past an infinite vertical oscillating plate with variable temperature and uniform mass diffusion, in the presence of external magnetic field. In this chapter, section 2.2 studies the effects of radiation and chemical reaction and the section 2.3 analyzes the effects of MHD and chemical reaction. The dimensionless governing equations are tackled using the Laplace transform technique. The solutions are in terms of exponential and complementary error function.

2.2 EFFECTS OF RADIATION AND CHEMICAL REACTION

2.2.1 BASIC EQUATIONS AND ANALYSIS

Here the unsteady flow of a viscous incompressible fluid which is initially at rest and surrounds an infinite vertical plate with temperature $T_\infty$ and concentration $C_\infty'$. Here, the $x$-axis is taken along the plate in the vertically upward direction and the $y$-axis is taken normal to the plate. Initially, it is assumed that the plate and the fluid are of the same temperature and concentration. At time $t' > 0$, the plate starts oscillating in its own plane with frequency $\omega'$ and the temperature of the plate is raised linearly with respect to time and the concentration level near the plate is raised to $C_\infty'$. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. It is also assumed
that there exists a homogeneous first order chemical reaction between the fluid and species concentration. Then by usual Boussinesq’s approximation, the unsteady flow is governed by the following equations:

**Equation of momentum:**

\[
\frac{\partial u}{\partial t'} = g \beta (T - T_\infty) + g \beta' (C' - C'_\infty) + \nu \frac{\partial^2 u}{\partial y^2}
\]  

(2.2.1)

**Energy equation with radiation:**

\[
\rho C_v \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y}
\]

(2.2.2)

**Mass diffusion equation with chemical reaction:**

\[
\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y^2} - K_I C'
\]

(2.2.3)

In most cases of chemical reactions, the reaction rate depends on the concentration of the species itself. A reaction is said to be of the order \(n\), if the reaction rate is proportional to the \(n^{th}\) power of the concentration. In particular, a reaction is said to be first order, if the rate of reaction is directly proportional to concentration itself.

With the following initial and boundary conditions:

\[ t' \leq 0 : \quad u = 0, \quad T = T_\infty, \quad C' = C'_\infty \quad \text{for all } y \]

\[ t' > 0 : \quad u = u_0 \cos \omega t', \quad T = T_\infty + (T_w - T_\infty) A t', \quad C' = C'_w \quad \text{at } y = 0 \]

\[ u = 0, \quad T \to T_w, \quad C' = C'_w \quad \text{as } y \to \infty \]

(2.2.4)

The local radiant for the case of an optically thin gray gas is expressed by

\[
\frac{\partial q_r}{\partial y} = -4a^* \sigma (T_\infty^4 - T^4)
\]

(2.2.5)
It is assumed that the temperature differences within the flow are sufficiently small such that $T^4$ may be expressed as a linear function of the temperature. This is accomplished by expanding $T^4$ in a Taylor series about $T_\infty'$ and neglecting higher order terms, thus

$$ T^4 \approx 4 T_\infty^3 T - 3 T_\infty^4 $$

(2.2.6)

By using equations (2.2.5) and (2.2.6), equation (2.2.2) reduces to

$$ \rho \ C_p \frac{\partial T}{\partial t'} = k \ \frac{\partial^2 T}{\partial y^2} + 16 \ a^* \ \sigma T_\infty^3 (T_\infty - T) $$

(2.2.7)

The dimensionless quantities are defined as

$$ U = \frac{u}{u_0}, \quad i = \frac{t' u_0^2}{v}, \quad Y = \frac{y u_0}{v}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, $$

(2.2.8)

$$ Gr = \frac{g \ \beta v (T_w - T_\infty)}{u_0^3}, \quad C = \frac{C' - C_\infty}{C_w - C_\infty}, \quad Gc = \frac{v g \ \beta^* (C_w' - C_\infty')}{u_0^3}, \quad Pr = \frac{\mu C_p}{k}, \quad Sc = \frac{v}{D}, \quad R = \frac{16 a^* v^2 \ \sigma T_\infty^3}{k u_0^2}, \quad K = \frac{v K}{u_0^2}, \quad \omega = \frac{\omega'}{u_0^2} $$

Equations (2.2.1) to (2.2.4) reduces to

$$ \frac{\partial U}{\partial t} = Gr \ \theta + Gc \ C + \frac{\partial^2 U}{\partial Y^2} $$

(2.2.9)

$$ \frac{\partial \theta}{\partial t'} = \frac{1}{Pr} \ \frac{\partial^2 \theta}{\partial Y^2} - \frac{R}{Pr} \ \theta $$

(2.2.10)

$$ \frac{\partial C}{\partial t} = \frac{1}{Sc} \ \frac{\partial^2 C}{\partial Y^2} - KC $$

(2.2.11)
The initial and boundary conditions in non dimensional form are

\[ U = 0, \quad \theta = 0, \quad C = 0, \quad \text{for all } Y,t \leq 0 \]

\[ t > 0: \quad U = \cos \omega t, \quad \theta = t, \quad C = 1, \quad \text{at } Y = 0 \]

\[ U = 0, \quad \theta \to 0, \quad C \to 0, \quad \text{as } Y \to \infty \]  

(2.2.12)

All the physical variables are defined in the nomenclature. The solutions are obtained for hydrodynamic flow field in the presence of thermal radiation and chemical reaction.

### 2.2.2 SOLUTION PROCEDURE

The equations (2.2.9) to (2.2.11), subject to the boundary conditions (2.2.12), are solved by the usual Laplace transform technique and the solutions are derived as follows:

\[
\theta = \frac{t}{2} \left[ \exp(2\eta \sqrt{R} t) \ \text{erfc}(\eta \sqrt{Pr} + \sqrt{at}) + \exp(-2\eta \sqrt{R} t) \ \text{erfc}(\eta \sqrt{Pr} - \sqrt{at}) \right] 
\]

\[
- \frac{\eta \sqrt{Pr} \sqrt{t}}{2\sqrt{R}} \left[ \exp(-2\eta \sqrt{R} t) \ \text{erfc}(\eta \sqrt{Pr} - \sqrt{at}) - \exp(2\eta \sqrt{R} t) \ \text{erfc}(\eta \sqrt{Pr} + \sqrt{at}) \right]
\]

(2.2.13)

\[
C = \frac{1}{2} \left[ \exp(2\eta \sqrt{KtSc} \ \text{erfc}(\eta \sqrt{Sc} + \sqrt{Kt}) + \exp(-2\eta \sqrt{KtSc}) \ \text{erfc}(\eta \sqrt{Sc} - \sqrt{Kt}) \right]
\]

(2.2.14)
\begin{equation}
U = \frac{\exp(i\omega t)}{4} \left[ \exp(2\eta \sqrt{i\omega t}) \erfc(\eta + \sqrt{i\omega t}) + \exp(-2\eta \sqrt{i\omega t}) \erfc(\eta - \sqrt{i\omega t}) \right]

+ \frac{\exp(-i\omega t)}{4} \left[ \exp(2\eta \sqrt{-i\omega t}) \erfc(\eta + \sqrt{-i\omega t}) + \exp(-2\eta \sqrt{-i\omega t}) \erfc(\eta - \sqrt{-i\omega t}) \right]

+ 2(d + e) \erfc(\eta) + 2tb \left[ (1 + 2\eta^2) \erfc(\eta) - \frac{2\eta}{\sqrt{\pi}} \exp(-\eta^2) \right]

- d \exp(bt) \left[ \exp(2\eta \sqrt{bt}) \erfc(\eta + \sqrt{bt}) + \exp(-2\eta \sqrt{bt}) \erfc(\eta - \sqrt{bt}) \right]

- e \exp(ct) \left[ \exp(2\eta \sqrt{ct}) \erfc(\eta + \sqrt{ct}) + \exp(-2\eta \sqrt{ct}) \erfc(\eta - \sqrt{ct}) \right]

- d(1 + bt) \left[ \exp(2\eta \sqrt{Rt}) \erfc(\eta \sqrt{\Pr} + \sqrt{at}) + \exp(-2\eta \sqrt{Rt}) \erfc(\eta \sqrt{\Pr} - \sqrt{at}) \right]

+ \frac{bd\eta\Pr\sqrt{t}}{\sqrt{R}} \left[ \exp(-2\eta \sqrt{Rt}) \erfc(\eta \sqrt{\Pr} - \sqrt{at}) - \exp(2\eta \sqrt{Rt}) \erfc(\eta \sqrt{\Pr} + \sqrt{at}) \right]

+ d \exp(bt) \left[ \exp(2\eta \sqrt{Pr(a + b)t}) \erfc(\eta \sqrt{Pr} + \sqrt{(a + b)t}) + \exp(-2\eta \sqrt{Pr(a + b)t}) \erfc(\eta \sqrt{Pr} - \sqrt{(a + b)t}) \right]

- e \left[ \exp(2\eta \sqrt{KtSc}) \erfc(\eta \sqrt{Sc} + \sqrt{Kt}) + \exp(-2\eta \sqrt{KtSc}) \erfc(\eta \sqrt{Sc} - \sqrt{Kt}) \right]

+ e \exp(ct) \left[ \exp(2\eta \sqrt{Sc(K + c)t}) \erfc(\eta \sqrt{Sc} + \sqrt{(K + c)t}) + \exp(-2\eta \sqrt{Sc(K + c)t}) \erfc(\eta \sqrt{Sc} - \sqrt{(K + c)t}) \right]

(2.2.15)

Where, \( a = \frac{R}{\Pr}, b = \frac{R}{1 - \Pr}, c = \frac{KSc}{1 - Sc}, d = \frac{Gr}{2b^2(1 - \Pr)}, e = \frac{Gc}{2c(1 - Sc)}, \eta = \frac{Y}{2\sqrt{t}} \) and \( \erfc \) is called as complementary error function.
In order to get the physical insight into the problem, the numerical values of \( U \) have been computed from equation (2.2.15). While evaluating this expression, it is observed that the argument of the error function is complex and hence, we have separated it into real and imaginary parts by using the following formula:

\[
\text{erf}(a + ib) = \text{erf}(a) + \frac{\exp(-a^2)}{2a\pi} \left[ 1 - \cos(2ab) + i\sin(2ab) \right] \\
+ \frac{\exp(-a^2)}{2a\pi} \sum_{n=1}^{\infty} \frac{\exp(-n^2/4)}{n^2 + 4} \left[ f_n(a,b) + i\ g_n(a,b) \right] + \varepsilon(a,b)
\]

Where, \( f_n(a,b) = 2a - 2a \cosh(nb) \cos(2ab) + n \sinh(nb) \sin(2ab) \)
\( g_n(a,b) = 2a \cosh(nb) \sin(2ab) + n \sinh(nb) \cos(2ab) \)

\( |\varepsilon(a,b)| \approx 10^{-6} |\text{erf}(a + ib)| \)

2.2.3 RESULTS AND DISCUSSION

The numerical values of the velocity, temperature and concentration are calculated for different values of the phase angle, radiation parameter, Schmidt number, thermal Grashof number, mass Grashof number and time. The purpose of the calculations given here is to assess the effect of different \( \omega t, K, R, Sc \) and \( t \) upon the nature of the flow and transport in the presence of air \( (Pr = 0.71) \) and water vapor \( (Sc = 0.6) \). The Laplace transform solutions are in terms of exponential and complementary error function.

The velocity profiles for different phase angles \( (\omega t = 0, \pi/6, \pi/4, \pi/2) \), \( R=5, K=2, Gr=Gr_c=2 \) and \( t=0.2 \) are shown in figure 2.2.1. It is observed that the velocity increases with decreasing phase angle \( \omega t \). It is interesting to note that at \( \omega t = 0 \), the plate is considered to be vertical and the velocity profile developed from \( U = 1 \). At \( \omega t = \pi/2 \), the plate is considered to be horizontal and the velocity profiles developed from the origin.
The effect of velocity for different values of the radiation parameter \((R = 0.2, 5, 20)\), \(\omega t = \pi / 6\), \(K = 0.2\), \(Gr = 10\), \(Gc = 2\) and \(t = 0.4\) are shown in figure 2.2.2. The trend shows that the velocity increases with decreasing radiation parameter. It is observed that the velocity decreases in the presence of high thermal radiation.

Figure 2.2.3 illustrates the effect of the velocity for different values of the reaction parameter \((K = 0.2, 5, 15)\), \(\omega t = \pi / 6\), \(R = 5\), \(Gr = Gc = 2\) and \(t = 0.4\). The trend shows that the velocity increases with decreasing chemical reaction parameter. The effect of velocity profiles for different time \((t = 0.2, 0.3, 0.4)\), \(Pr = 0.71\), \(Sc = 0.6\), \(\omega t = \pi / 6\), \(R = 5\), \(K = 0.2\), \(Gr = Gc = 2\), are shown in figure 2.2.4. In this case, the velocity increases gradually with respect to time \(t\). The velocity profiles for different thermal Grashof number \((Gr = 2, 10)\), mass Grashof number \((Gc = 2, 5)\), \(\omega t = \pi / 6\), \(K = 0.2\), \(R = 5\) and time \(t = 0.3\) are shown in figure 2.2.5. It is clear that the velocity increases with increasing thermal Grashof number or mass Grashof number.

Figure 2.2.6 represents the effect of concentration profiles at time \(t = 0.2\) for different Schmidt numbers \((Sc = 0.16, 0.3, 0.6, 2.01)\) and \(K = 0.2\). The effect of concentration is important in concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the wall concentration increases with decreasing values of the Schmidt number. Figure 2.2.7 demonstrates the effect of the concentration profiles for different values of the chemical reaction parameter \((K = 0.2, 2, 5, 10)\), \(Sc = 0.6\) and time \(t = 0.2\). It is observed that the concentration increases with decreasing chemical reaction parameter.

The temperature profiles are calculated for different values of thermal radiation parameter \((R = 0.2, 2, 5, 10)\) from equation (2.2.13) and these are shown in figure 2.2.8 for air at time \(t = 1\). The effect of thermal radiation parameter is important in temperature profiles. It is observed that the temperature increases with decreasing radiation parameter.
The temperature profiles are calculated for different values of time 
\((t = 0.2, 0.4, 0.6, 1)\) are shown in figure 2.2.9 at \(R=0.2\). It is observed that the 
temperature increases with increasing time \(t\).
Fig. 2.2.2: Velocity profiles for different values of R

Fig. 2.2.3: Velocity profiles for different values of K
Fig. 2.2.4: Velocity profiles for different values of $t$

Fig. 2.2.5: Velocity profiles for different values of $Gr, Gc$
Fig. 2.2.6: Concentration profiles for different values of $\text{Sc}$

Fig. 2.2.7: Concentration profiles for different values of $\text{K}$
Fig. 2.2.8: Temperature profiles for different values of $R$

Fig. 2.2.9: Temperature profiles for different values of $t$
2.3 EFFECTS OF MHD AND CHEMICAL REACTION

2.3.1 MATHEMATICAL FORMULATION

First order chemical reaction effects on infinite isothermal vertical oscillating plate with variable temperature and uniform mass diffusion in the presence of magnetic field is studied. Initially, it is assumed that the plate and the fluid are of the same temperature and concentration. At time $t' > 0$, the plate starts oscillating in its own plane with frequency $\omega'$ and the temperature of the plate is raised linearly with respect to time and the concentration level near the plate is raised uniformly. The plate is also subjected to a uniform magnetic field of strength $B_0$. It is assumed that the effect of viscous dissipation is negligible in the energy equation and there is a first order chemical reaction between the diffusing species and the fluid. Then by usual Boussinesq’s approximation, the unsteady flow is governed by the following:

Equation of momentum with MHD:

$$\frac{\partial u}{\partial t'} = g \beta (T - T_\infty) + g \beta' (C' - C'_\infty) + \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u$$  \hspace{1cm} (2.3.1)

Energy equation:

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2}$$  \hspace{1cm} (2.3.2)

Mass diffusion equation with chemical reaction:

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y^2} - K_i C'$$  \hspace{1cm} (2.3.3)

With the following initial and boundary conditions:

$$t' \leq 0 : \quad u = 0, \quad T = T_\infty, \quad C' = C'_\infty \quad \text{for all } y$$

$$t' > 0 : \quad u = u_0 \cos \omega t', \quad T = T_\infty + (T_w - T_\infty) A t', \quad C' = C'_w \quad \text{at } y = 0$$  \hspace{1cm} (2.3.4)

$$u = 0, \quad T \rightarrow T_w, \quad C' \rightarrow C'_w \quad \text{as } y \rightarrow \infty$$
Where, \( A = \frac{u_0^2}{v} \).

The following non dimensional quantities are introduced in equations (2.3.1) to (2.3.4):

\[
U = \frac{u}{u_0}, \quad t = \frac{t' u_0^2}{v}, \quad Y = \frac{y u_0}{v}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty},
\]

\[
Gr = \frac{g \beta \nu (T_w - T_\infty)}{u_0^3}, \quad C = \frac{C' - C'_\infty}{C_w - C'_\infty}, \quad Gc = \frac{\nu g \beta' (C_w - C'_\infty)}{u_0^3}, \quad (2.3.5)
\]

\[
P_r = \frac{\mu C_p}{k}, \quad S_c = \frac{v}{D}, \quad M = \frac{\sigma B_0^2 v}{\rho u_0^2}, \quad K = \frac{v K_t}{u_0^2}, \quad \omega = \frac{\omega' v}{u_0^2}
\]

Equations (2.3.1) to (2.3.4) lead to

\[
\frac{\partial U}{\partial t} = Gr \theta + Gc C + \frac{\partial^2 U}{\partial Y^2} - M U \quad (2.3.6)
\]

\[
\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial Y^2} \quad (2.3.7)
\]

\[
\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Y^2} - K C \quad (2.3.8)
\]

The initial and boundary conditions in non dimensional form are

\[
U = 0, \quad \theta = 0, \quad C = 0, \quad \text{for all } Y, t \leq 0
\]

\[
t > 0: \quad U = \cos \omega t, \quad \theta = t, \quad C = 1, \quad \text{at } Y = 0
\]

\[
U = 0, \quad \theta \to 0, \quad C \to 0, \quad \text{as } Y \to \infty
\]

The solutions are obtained for hydromagnetic flow field in the presence of first order chemical reaction.
2.3.2 METHOD OF SOLUTION

The equations (2.3.6) to (2.3.8), subject to the boundary conditions (2.3.9), are solved by the usual Laplace transform technique and the solutions are derived as follows:

\[ U = \frac{\exp(i\omega t)}{4} \left[ \exp(2\eta \sqrt{(M + i\omega) t}) \text{erfc}(\eta + \sqrt{(M + i\omega) t}) 
+ \exp(-2\eta \sqrt{(M + i\omega) t}) \text{erfc}(\eta - \sqrt{(M + i\omega) t}) \right] 
+ \frac{\exp(-i\omega t)}{4} \left[ \exp(2\eta \sqrt{(M - i\omega) t}) \text{erfc}(\eta + \sqrt{(M - i\omega) t}) 
+ \exp(-2\eta \sqrt{(M - i\omega) t}) \text{erfc}(\eta - \sqrt{(M - i\omega) t}) \right] 
+ (c(1 + at) + d) \left[ \exp(2\sqrt{MT}) \text{erfc}(\eta + \sqrt{MT}) + \exp(-2\eta \sqrt{MT}) \text{erfc}(\eta - \sqrt{MT}) \right] 
- 2c \text{erfc}(\eta \sqrt{Pr}) - c \exp(at) \left[ \exp(2\eta \sqrt{(M + a) t}) \text{erfc}(\eta + \sqrt{(M + a) t}) 
+ \exp(-2\eta \sqrt{(M + a) t}) \text{erfc}(\eta - \sqrt{(M + a) t}) \right] 
- \frac{ac \eta \sqrt{t}}{\sqrt{M}} \left[ \exp(-2\eta \sqrt{MT}) \text{erfc}(\eta - \sqrt{MT}) - \exp(2\eta \sqrt{MT}) \text{erfc}(\eta + \sqrt{MT}) \right] 
- d \exp(bt) \left[ \exp(2\eta \sqrt{(M + b) t}) \text{erfc}(\eta + \sqrt{(M + b) t}) 
+ \exp(-2\eta \sqrt{(M + b) t}) \text{erfc}(\eta - \sqrt{(M + b) t}) \right] 
- 2act \left[ (1 + 2\eta^2 Pr) \text{erfc}(\eta \sqrt{Pr}) - \frac{2\eta \sqrt{Pr}}{\sqrt{\pi}} \exp(-\eta^2 Pr) \right] 
+ c \exp(at) \left[ \exp(2\eta \sqrt{a Pr t}) \text{erfc}(\eta \sqrt{Pr} + \sqrt{at}) + \exp(-2\eta \sqrt{a Pr t}) \text{erfc}(\eta \sqrt{Pr} - \sqrt{at}) \right] 
- d \left[ \exp(2\eta \sqrt{KtSc}) \text{erfc}(\eta \sqrt{Sc} + \sqrt{Kt}) + \exp(-2\eta \sqrt{KtSc}) \text{erfc}(\eta \sqrt{Sc} - \sqrt{Kt}) \right] 
+ d \exp(bt) \left[ \exp(2\eta \sqrt{Sc(K + b) t}) \text{erfc}(\eta \sqrt{Sc} + \sqrt{(K + b) t}) 
+ \exp(-2\eta \sqrt{Sc(K + b) t}) \text{erfc}(\eta \sqrt{Sc} - \sqrt{(K + b) t}) \right] \]

(2.3.10)
\[
\theta = t \left[ (1 + 2\eta^2 \, \Pr) \text{erfc}(\eta \sqrt{\Pr}) - \frac{2}{\sqrt{\pi}} \eta \sqrt{\Pr} \exp(-\eta^2 \Pr) \right]
\]  
\[\text{(2.3.11)}\]

\[
C = \frac{1}{2} \left[ \exp(2\eta \, \sqrt{\Kt \Sc}) \text{erfc}(\eta \sqrt{\Sc} + \sqrt{\Kt}) + \exp(-2\eta \, \sqrt{\Kt \Sc}) \text{erfc}(\eta \sqrt{\Sc} - \sqrt{\Kt}) \right]
\]
\[\text{(2.3.12)}\]

Where, \(a = \frac{M}{\Pr - 1}\), \(b = \frac{M - K \, \Sc}{\Sc - 1}\), \(c = \frac{Gr}{2a^2(1 - \Pr)}\), \(d = \frac{Gc}{2b(1 - \Sc)}\), and \(\eta = \frac{Y}{2\sqrt{t}}\).

### 2.3.3 DISCUSSION OF RESULTS

The numerical values of the velocity, concentration and temperature are computed for different parameters like magnetic field parameter, chemical reaction parameter, Schmidt number, thermal Grashof number and mass Grashof number. The purpose of the calculations given here is to assess the effects of the parameters \(M, K, Gr, Gc\) and \(Sc\) upon the nature of the flow and transport. The solutions are in terms of exponential and complementary error function.

The temperature profiles are calculated for different values of time \((t = 0.2, 0.4, 0.6, 1)\) and \(\Pr = 7.0\) (water) are shown in figure 2.3.1. It is observed that the temperature increases with increasing time.

The velocity profiles for different phase angles \((\omega t = 0, \pi / 4, \pi / 3, \pi / 2)\), \(M = 0.2, K = 8, Gr = Gc = 5, Sc = 0.6, \Pr = 7.0\) and \(t = 0.4\) are shown in figure 2.3.2. It is observed that the velocity increases with decreasing phase angle. Figure 2.3.3 illustrates the effect of the velocity for different values of the reaction parameter \((K = 2, 8, 15)\), \(\omega t = \pi / 4\), \(M = 0.2, Gr = Gc = 5, Sc = 0.6, \Pr = 7.0\) and \(t = 0.6\). The trend shows that the velocity increases with decreasing chemical reaction parameter.

Figure 2.3.4 demonstrates the effects of the magnetic field parameter on the velocity when \((M = 0.2, 2, 5)\), \(\omega t = \pi / 4\), \(K = 8, Gr = Gc = 5, Sc = 0.6, \Pr = 7.0\) and \(t = 0.4\). It is observed that the velocity increases with decreasing magnetic field.
parameter. The effect of velocity profiles for different time \((t = 0.1, 0.2, 0.3)\), \(M = 0.2\), \(K = 8\), \(\omega t = \pi / 4\), \(Gr = Gc = 5\), \(Sc = 0.6\), \(Pr = 7.0\) are shown in figure 2.3.5. In this case, the velocity increases gradually with respect to time.

The velocity profiles for different thermal Grashof number \((Gr = 2, 10)\), mass Grashof number \((Gc = 2, 5)\), \(\omega t = \pi / 4\), \(K = 8\), \(M = 0.2\), \(Sc = 0.6\), \(Pr = 7.0\) and time \(t = 0.6\) are shown in figure 2.3.6. It is clear that the velocity increases with increasing thermal Grashof number or mass Grashof number.

Figure 2.3.7 demonstrates the effect of the concentration profiles for different values of the chemical reaction parameter \((K = 0.2, 2, 5, 10)\), \(Sc = 0.6\) and \(t = 0.2\). It is observed that the concentration increases with decreasing chemical reaction parameter. Figure 2.3.8 represents the effect of concentration profiles at time \(t = 0.2\) for different Schmidt numbers \((Sc = 0.16, 0.3, 0.6, 2.01)\) and \(K = 0.2\). It is observed that the wall concentration increases with decreasing values of the Schmidt number.
Fig. 2.3.1: Temperature profiles for different values of $t$

Fig. 2.3.2: Velocity profiles for different values of $\omega t$
Fig. 2.3.3: Velocity profiles for different values of $K$

Fig. 2.3.4: Velocity profiles for different values of $M$
Fig. 2.3.5: Velocity profiles for different values of $t$

Fig. 2.3.6: Velocity profiles for different values of $Gr$, $Gc$
Fig. 2.3.7: Concentration profiles for different values of K

Fig. 2.3.8: Concentration profiles for different values of Sc
2.4 CONCLUSION

Thermal radiation effects on unsteady flow past an infinite vertical oscillating plate, in the presence of variable temperature and uniform wall concentration are studied in section 2.2 and the section 2.3 deals with the MHD flow past an oscillating infinite vertical plate with variable temperature and uniform mass diffusion, in the presence of chemical reaction of first order. The effects of velocity, temperature and concentration for different parameters like $\omega t, R, K, Gr, Gc, Sc$ and $t$ are discussed. Conclusions of the study are as follows:

(i) The velocity increases with decreasing phase angle, radiation parameter, chemical reaction parameter and magnetic field parameter.

(ii) The trend is just reversed with respect to time.

(iii) It is observed that the velocity increases with increasing thermal Grashof number or mass Grashof number.

(iv) The wall concentration increases with decreasing Schmidt number and chemical reaction parameter.

(v) The plate temperature decreases due to high thermal radiation and temperature increases with increasing time.

(vi) It is also observed that the leading edge effect is not affected by the oscillation of the plate.