CHAPTER 5

THERMAL RADIATION AND MHD EFFECTS ON
FLOW PAST AN INFINITE VERTICAL OSCILLATING
PLATE WITH CHEMICAL REACTION OF FIRST ORDER

5.1 INTRODUCTION

Magnetoconvection plays an important role in agriculture, petroleum industries, geophysics and in astrophysics. Important applications in the study of geological formations, in exploration and thermal recovery of oil and in the assessment of aquifers, geothermal reservoirs and underground nuclear waste storage sites. Radiative heat and mass transfer play an important role in manufacturing industries for the design of fins, steel rolling, nuclear power plants, gas turbines and various propulsion device for aircraft, missiles, satellites and space vehicles are examples of such engineering applications.

MHD effects on impulsively started vertical infinite plate with variable temperature in the presence of transverse magnetic field were studied by Soundalgekar et al. (1983). Mazumdar and Deka (2007) presented the MHD flow past an impulsively started infinite vertical plate in the presence of thermal radiation. Deka and Neog (2009) studied the unsteady MHD flow past a vertical oscillating plate with thermal radiation and variable mass diffusion. The dimensionless governing equations were solved using Laplace transform technique.

Das et al. (1994) have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Again, mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction studied by Das et al. (1999). The dimensionless governing equations were solved by the usual Laplace transform technique and the solutions are valid only at lower time level.
Raptis and Perdikis (1999) studied the effects of thermal radiation and free convection flow past a moving vertical plate. Radiation effect on mixed convection along an isothermal vertical plate were studied by Hossain and Takhar (1996). The governing equations were solved analytically. Das et al. (1999) have analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate.

However the combined study of MHD and thermal radiation effects on infinite oscillating isothermal vertical plate in the presence of chemical reaction of first order is not studied in the literature. It is proposed to study the chemical reaction effects on unsteady flow past infinite isothermal vertical oscillating plate, in the presence of magnetic field and thermal radiation. The dimensionless governing equations are tackled using the Laplace transform technique. The solutions are in terms of exponential and complementary error function.

5.2 BASIC EQUATIONS AND ANALYSIS

Here the unsteady flow of a viscous incompressible fluid which is initially at rest and surrounds an infinite vertical plate with temperature \( T_w \) and concentration \( C_w' \). Here, the \( x \)-axis is taken along the plate in the vertically upward direction and the \( y \)-axis is taken normal to the plate. Initially, it is assumed that the plate and the fluid are of the same temperature and concentration. At time \( t' > 0 \), the plate starts oscillating in its own plane with frequency \( \omega' \) and the temperature of the plate is raised to \( T_w \) and the concentration level near the plate is also raised to \( C_w' \). The plate is also subjected to a uniform magnetic field of strength \( B_0 \). The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. It is assumed that the effect of viscous dissipation is negligible in the energy equation and there is a first order chemical reaction between the diffusing species and the fluid. Then by usual Boussinesq's approximation, the unsteady flow is governed by the following:
Equation of momentum with MHD:

\[
\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta'(C' - C'_\infty) + \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u
\]  

(5.1)

Energy equation with radiation:

\[
\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y}
\]  

(5.2)

Mass diffusion equation with chemical reaction:

\[
\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y^2} - K \cdot C'
\]  

(5.3)

With the following initial and boundary conditions:

\begin{align*}
t' \leq 0: & \quad u = 0, \quad T = T_\infty, \quad C' = C'_\infty \quad \text{for all } y \\
t' > 0: & \quad u = u_0 \cos \omega t', \quad T = T_w, \quad C' = C'_w \quad \text{at } y = 0 \\
& \quad u = 0, \quad T \rightarrow T_\infty, \quad C' \rightarrow C'_\infty \quad \text{as } y \rightarrow \infty
\end{align*}

(5.4)

The local radiant for the case of an optically thin gray gas is expressed by

\[
\frac{\partial q_r}{\partial y} = -4a^* \sigma(T_\infty^4 - T^4)
\]  

(5.5)

It is assumed that the temperature differences within the flow are sufficiently small such that \(T^4\) may be expressed as a linear function of the temperature. This is accomplished by expanding \(T^4\) in a Taylor series about \(T_\infty\) and neglecting higher order terms, thus

\[
T^4 \approx 4T_\infty^3 T - 3T_\infty^4
\]  

(5.6)
By using equations (5.5) and (5.6), equation (5.2) reduces to

$$\rho C_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2} + 16a^* \sigma T_w^3 (T_w - T) \tag{5.7}$$

On introducing the following non dimensional quantities:

$$U = \frac{u}{u_0}, \quad t = \frac{t' u_0^2}{v}, \quad Y = \frac{v u_0}{v}, \quad \theta = \frac{T - T_x}{T_w - T_x},$$

$$Gr = \frac{g \beta v (T_w - T_x)}{u_0^3}, \quad C = \frac{C' - C_x}{C_w - C_x}, \quad Gc = \frac{v \beta^* (C' - C_x)}{u_0^3}, \tag{5.8}$$

$$\omega = \frac{\omega u_0'}{u_0}, R = \frac{16a^* v^2 \sigma T_w^3}{ku_0^2}, \quad Pr = \frac{\mu C_p}{k}, \quad Sc = \frac{v}{D}, \quad M = \frac{\sigma B_0^2 v}{\rho u_0^2}, \quad K = \frac{v K_l}{u_0^2}$$

in equations (5.1) to (5.4) lead to

$$\frac{\partial U}{\partial t} = Gr \theta + Gc C + \frac{\partial^2 U}{\partial Y^2} - MU \tag{5.9}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial Y^2} - \frac{R}{Pr} \theta \quad \tag{5.10}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Y^2} - KC \quad \tag{5.11}$$

The initial and boundary conditions in non dimensional form are

$$U = 0, \quad \theta = 0, \quad C = 0, \quad \text{for all } Y, t \leq 0$$

$$t > 0: \quad U = \cos \omega t, \quad \theta = 1, \quad C = 1, \quad \text{at } Y = 0 \tag{5.12}$$

$$U = 0, \quad \theta \to 0, \quad C \to 0, \quad \text{as } Y \to \infty$$
The solutions are obtained for hydromagnetic flow field in the presence of first order chemical reaction and thermal radiation.

5.3 SOLUTION PROCEDURE

The equations (5.9) to (5.11), subject to the boundary conditions (5.12), are solved by the usual Laplace transform technique and the solutions are derived as follows:

\[
U = \frac{\exp(i\omega t)}{4} \left[ \exp(2\eta \sqrt{(M + i\omega)t} \text{erfc}(\eta + \sqrt{(M + i\omega)t}) + \exp(-2\eta \sqrt{(M + i\omega)t} \text{erfc}(\eta - \sqrt{(M + i\omega)t}) \quad (5.13) \\
+ \exp(-2\eta \sqrt{(M - i\omega)t} \text{erfc}(\eta + \sqrt{(M - i\omega)t}) + \exp(-2\eta \sqrt{(M - i\omega)t} \text{erfc}(\eta - \sqrt{(M - i\omega)t}) \right] \\
+ (d + e) \left[ \exp(2\eta \sqrt{Mt} \text{erfc}(\eta + \sqrt{Mt}) + \exp(-2\eta \sqrt{Mt} \text{erfc}(\eta - \sqrt{Mt}) \\
- d \exp(bt) \left[ \exp(2\eta \sqrt{(M + b)t} \text{erfc}(\eta + \sqrt{(M + b)t}) + \exp(-2\eta \sqrt{(M + b)t} \text{erfc}(\eta - \sqrt{(M + b)t}) \right] \\
- e \exp(ct) \left[ \exp(2\eta \sqrt{(M + c)t} \text{erfc}(\eta + \sqrt{(M + c)t}) + \exp(-2\eta \sqrt{(M + c)t} \text{erfc}(\eta - \sqrt{(M + c)t}) \right] \\
- d \left[ \exp(2\eta \sqrt{Rt} \text{erfc}(\eta \sqrt{Pr} + \sqrt{at}) + \exp(-2\eta \sqrt{Rt} \text{erfc}(\eta \sqrt{Pr} + \sqrt{at}) \right] \\
+ d \exp(bt) \left[ \exp(2\eta \sqrt{Pr(a + b)t} \text{erfc}(\eta \sqrt{Pr} + \sqrt{(a + b)t}) + \exp(-2\eta \sqrt{Pr(a + b)t} \text{erfc}(\eta \sqrt{Pr} + \sqrt{(a + b)t}) \right] \\
- e \left[ \exp(2\eta \sqrt{KtSc} \text{erfc}(\eta \sqrt{Sc} + \sqrt{Kt}) + \exp(-2\eta \sqrt{KtSc} \text{erfc}(\eta \sqrt{Sc} - \sqrt{Kt}) \right] \\
+ e \exp(ct) \left[ \exp(2\eta \sqrt{Sc(K + c)t} \text{erfc}(\eta \sqrt{Sc} + \sqrt{(K + c)t}) + \exp(-2\eta \sqrt{Sc(K + c)t} \text{erfc}(\eta \sqrt{Sc} - \sqrt{(K + c)t}) \right]
\]
\[ \theta = \frac{1}{2} \left[ \exp(2\eta \sqrt{Rt}) \text{erfc}(\eta \sqrt{Pr} + \sqrt{at}) + \exp(-2\eta \sqrt{Rt}) \text{erfc}(\eta \sqrt{Pr} - \sqrt{at}) \right] \]  \hspace{1cm} (5.14)

\[ C = \frac{1}{2} \left[ \exp(2\eta \sqrt{KtSc}) \text{erfc}(\eta \sqrt{Sc} + \sqrt{Kt}) + \exp(-2\eta \sqrt{KtSc}) \text{erfc}(\eta \sqrt{Sc} - \sqrt{Kt}) \right] \]  \hspace{1cm} (5.15)

Where, \( a = \frac{R}{Pr}, \quad b = \frac{M - R}{Pr - 1}, \quad c = \frac{M - KSc}{Sc - 1}, \quad d = \frac{Gr}{2a(1 - Pr)}, \quad e = \frac{Gc}{2b(1 - Sc)} \) and \( \eta = \frac{Y}{2\sqrt{t}} \).

### 5.4 RESULTS AND DISCUSSION

The numerical values of the velocity, concentration and temperature are computed for different parameters like phase angle, magnetic field parameter, radiation parameter, chemical reaction parameter, Schmidt number, thermal Grashof number and mass Grashof number. The purpose of the calculations given here is to assess the effects of the parameters \( M, K, Gr, Gc \) and \( Sc \) upon the nature of the flow and transport (\( Sc = 0.6, Pr = 0.71 \)).

Figure 5.1 illustrates the effect of the concentration profiles for different values of the chemical reaction parameter \( (K = 0.2, 2, 5, 10) \) at \( t = 0.4 \). The effect of chemical reaction parameter is important in concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the concentration increases with decreasing chemical reaction parameter. The temperature profiles are calculated for different values of thermal radiation parameter \( (R = 0.2, 2, 5, 10) \) at time \( t = 0.4 \) and these are shown in figure 5.2. The effect of thermal radiation parameter is important in temperature profiles. It is observed that the temperature increases with decreasing radiation parameter.

The velocity profiles for different phase angles \( (\omega t = 0, \pi/6, \pi/3, \pi/2) \), \( R = 10, M = 0.2, K = 2, Gr = Gc = 2 \) and \( t = 0.6 \) are shown in figure 5.3. It is observed that the velocity increases with decreasing phase angle \( \omega t \). Figure 5.4 demonstrates the
effects of the magnetic field parameter on the velocity when \((M = 0.2, 2, 5),\) \(\omega \tau = \pi/6, Gr = Gc = 5, R = 10, K = 5, Pr = 0.71\) and \(t = 0.6\). It is observed that the velocity increases with decreasing magnetic field parameter.

The effect of velocity for different values of the radiation parameter \((R = 2, 5, 10),\) \(\omega \tau = \pi/6, K = 2, M = 0.2, Gr = Gc = 2\) and \(t = 0.6\) are shown in figure 5.5. The trend shows that the velocity decreases in the presence of high thermal radiation. Figure 5.6 illustrates the effect of the velocity for different values of the reaction parameter \((K = 0.5, 5, 15),\) \(\omega \tau = \pi/4, R = 10, M = 0.2, Gr = Gc = 2\) and \(t = 0.6\). The trend shows that the velocity increases with decreasing chemical reaction parameter.

The effect of velocity profiles for different time \((t = 0.2, 0.4, 0.6, 0.8),\) \(R = 10, M = 0.2, K = 2, Gr = Gc = 2\) and \(\omega \tau = \pi/6\) are shown in figure 5.7. In this case, the velocity increases gradually with respect to time \(t\). The velocity profiles for different thermal Grashof number \((Gr = 5,10),\) mass Grashof number \((Gc = 5,10),\) \(\omega \tau = \pi/6, K = 15, R = 5, M = 2\) and time \(t = 0.8\) are shown in figure 5.8. It is clear that the velocity increases with increasing thermal Grashof number or mass Grashof number.
Fig. 5.1: Concentration profiles for different values of $K$

Fig. 5.2: Temperature profiles for different values of $R$
Fig. 5.3: Velocity profiles for different values of $\omega t$

Fig. 5.4: Velocity profiles for different values of $M$
Fig. 5.5: Velocity profiles for different values of $R$

Fig. 5.6: Velocity profiles for different values of $K$
Fig. 5.7: Velocity profiles for different values of $t$

Fig. 5.8: Velocity profiles for different values of $Gr$, $Gc$
5.5 SUMMARY AND CONCLUSION

In this chapter MHD and thermal radiation effects on flow past an oscillating infinite isothermal vertical plate with uniform temperature and uniform mass diffusion, in the presence of chemical reaction of first order is studied. The dimensionless equations are solved using Laplace transform technique. The effects of velocity, temperature and concentration for different parameters like $\omega t, M, R, K, Gr, Gc, Sc$ and $t$ are analyzed.

The study concludes that

(i) The velocity increases with decreasing phase angle, magnetic field parameter, radiation parameter and chemical reaction parameter.

(ii) The velocity increases with increasing thermal Grashof number or mass Grashof number. But the trend is just reversed with respect to time.

(iii) The plate concentration increases with decreasing chemical reaction parameter.

(iv) The temperature increases with decreasing radiation parameter.