CHAPTER 4

MHD AND CHEMICAL REACTION EFFECTS ON OSCILLATING ISOTHERMAL VERTICAL PLATE WITH VARIABLE MASS DIFFUSION

4.1 INTRODUCTION

MHD flow has application in metrology, solar physics and in motion of earth’s core. Also it has applications in the field of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics. MHD finds applications in electromagnetic pumps, controlled fusion research, crystal growing, MHD couples and bearings, plasma jets, chemical synthesis and MHD power generators, etc. Examples include magnetic control of molten iron flow in the steel industry, liquid metal cooling in nuclear reactors and magnetic suppression of molten semi-conducting materials.

MHD effects on impulsively started vertical infinite plate with variable temperature in the presence of transverse magnetic field were studied by Soundalgekar et al. (1981). The dimensionless governing equations were solved using Laplace transform technique. Mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction studied by Das et al. (1999). The dimensionless governing equations were solved by the usual Laplace transform technique and the solutions are valid only at lower time level. Vajravelu and Rivera (2003) presented the hydromagnetic flow at an oscillating plate.

However the simultaneous heat and mass transfer effects on infinite oscillating isothermal vertical plate in the presence of chemical reaction and MHD is not studied. Hence it is proposed to study MHD and chemical reaction effects on oscillating isothermal vertical plate with uniform temperature and variable mass diffusion.
The dimensionless governing equations are tackled using the Laplace transform technique. The solutions are in terms of exponential and complementary error function.

### 4.2 ANALYSIS

First order chemical reaction effects on infinite isothermal vertical oscillating plate with variable mass diffusion in the presence of magnetic field is studied. Here the unsteady flow of a viscous incompressible fluid which is initially at rest and surrounds an infinite vertical plate with temperature $T_\infty$ and concentration $C'_\infty$. Here, the x-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. Initially, it is assumed that the plate and the fluid are of the same temperature and concentration. At time $t' > 0$, the plate starts oscillating in its own plane with frequency $\omega'$ and the temperature of the plate is raised to $T_w'$ and the concentration level near the plate is raised linearly with respect to time. The plate is also subjected to a uniform magnetic field of strength $B_0$. It is assumed that the effect of viscous dissipation is negligible in the energy equation and there is a first order chemical reaction between the diffusing species and the fluid. Then by usual Boussinesq’s approximation, the unsteady flow is governed by the following:

**Equation of Momentum with MHD**

$$\frac{\partial u}{\partial t'} = g \beta (T - T_\infty) + g \beta' (C' - C'_\infty) + v \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u$$  \hspace{1cm} (4.1)

**Energy Equation**

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2}$$  \hspace{1cm} (4.2)

**Equation of Mass diffusion with chemical reaction**

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y^2} - K_i C'$$  \hspace{1cm} (4.3)
With the following initial and boundary conditions:

\[
\begin{align*}
t' & \leq 0 : \quad u = 0, \quad T = T_w, \quad C' = C'_w, \quad \text{for all } y \\
t' > 0 : \quad u = u_0 \cos \omega t', \quad T = T_w, \quad C' = C'_w + (C'_w - C'_\infty)A t' \quad \text{at } y = 0 \\
& \quad u = 0, \quad T \to T_w, \quad C' \to C'_\infty \quad \text{as } y \to \infty 
\end{align*}
\]

(4.4)

Where, \( A = \frac{u_0^2}{\nu} \).

The following non-dimensional quantities are introduced in equations (4.1) to (4.4)

\[
\begin{align*}
U &= \frac{u}{u_0}, \quad t = \frac{t' u_0^2}{\nu}, \quad Y = \frac{y u_0}{\nu}, \quad \theta = \frac{T - T_w}{T_w - T_\infty}, \\
Gr &= \frac{g \beta \nu (T_w - T_\infty)}{u_0^3}, \quad C = \frac{C'_w - C'_\infty}{C'_w - C'_\infty}, \quad Gc = \frac{\nu g \beta^2 (C'_w - C'_\infty)}{u_0^3}, \\
Pr &= \frac{\mu C_p}{k}, \quad Sc = \frac{\nu}{D}, \quad M = \frac{\sigma B_0^2 v}{\rho u_0^2}, \quad K = \frac{\nu K_1}{u_0^2}.
\end{align*}
\]

(4.5)

Equations (4.1) to (4.4) lead to

\[
\frac{\partial U}{\partial t} = Gr \theta + Gc C + \frac{\partial^2 U}{\partial Y^2} - M U
\]

(4.6)

\[
\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial Y^2}
\]

(4.7)

\[
\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Y^2} - K C
\]

(4.8)
The initial and boundary conditions in non dimensional form are

\[ U = 0, \quad \theta = 0, \quad C = 0, \quad \text{for all } Y, t \leq 0 \]

\[ t > 0: \quad U = \cos \omega t, \quad \theta = 1, \quad C = t, \quad \text{at } Y = 0 \]

\[ U = 0, \quad \theta \to 0, \quad C \to 0, \quad \text{as } Y \to \infty \] \hfill (4.9)

All the physical variables are defined in the nomenclature. The solutions are obtained for hydromagnetic flow field in the presence of first order chemical reaction with mass diffusion.

### 4.3. METHOD OF SOLUTION

The equations (4.6) to (4.8) subject to the boundary conditions (4.9), are solved by the usual Laplace transform technique and the solutions are derived as follows:

\[
U = \frac{\exp(i\omega t)}{4} \left[ \exp(2\eta \sqrt{(M + i\omega)t}) \text{erfc}(\eta + \sqrt{(M + i\omega)t}) + \exp(-2\eta \sqrt{(M + i\omega)t}) \right]
\]

\[ + \frac{\exp(-i\omega t)}{4} \left[ \exp(2\eta \sqrt{(M - i\omega)t}) \text{erfc}(\eta + \sqrt{(M - i\omega)t}) + \exp(-2\eta \sqrt{(M - i\omega)t}) \right]
\]

\[ + (c + (1 + bt)d) \left[ \exp(2\eta \sqrt{Mt}) \text{erfc}(\eta + \sqrt{Mt}) + \exp(-2\eta \sqrt{Mt}) \right]
\]

\[ - 2c \text{erfc}(\eta \sqrt{Pr}) - c \exp(at) \left[ \exp(2\eta \sqrt{(M + a)t}) \text{erfc}(\eta + \sqrt{(M + a)t}) + \exp(-2\eta \sqrt{(M + a)t}) \right]
\]

\[ - \frac{bd \eta \sqrt{t}}{\sqrt{M}} \left[ \exp(-2\eta \sqrt{Mt}) \text{erfc}(\eta - \sqrt{Mt}) - \exp(2\eta \sqrt{Mt}) \text{erfc}(\eta + \sqrt{Mt}) \right]
\]

\[ - d \exp(bt) \left[ \exp(2\eta \sqrt{(M + b)t}) \text{erfc}(\eta + \sqrt{(M + b)t}) + \exp(-2\eta \sqrt{(M + b)t}) \right] \]
\[ + c \exp(at) \left[ \exp(2\eta \sqrt{a \Pr t}) \operatorname{erfc}(\eta \sqrt{Pr + \sqrt{at}}) + \exp(-2\eta \sqrt{a \Pr t}) \operatorname{erfc}(\eta \sqrt{Pr} - \sqrt{at}) \right] \\
- d(1+bt) \left[ \exp(2\eta \sqrt{KtSc}) \operatorname{erfc}(\eta \sqrt{Sc + \sqrt{Kt}}) + \exp(-2\eta \sqrt{KtSc}) \operatorname{erfc}(\eta \sqrt{Sc} - \sqrt{Kt}) \right] \\
+ \frac{bd \eta \sqrt{Sc t}}{\sqrt{K}} \left[ \exp(-2\eta \sqrt{KtSc}) \operatorname{erfc}(\eta \sqrt{Sc} + \sqrt{(K+b)t}) + \exp(-2\eta \sqrt{Sc(K+b)t}) \operatorname{erfc}(\eta \sqrt{Sc} - \sqrt{(K+b)t}) \right] 
\]

\[
\theta = \operatorname{erfc}(\eta \sqrt{Pr}) \tag{4.11}
\]

\[
C = \frac{t}{2} \left[ \exp(2\eta \sqrt{KtSc}) \operatorname{erfc}(\eta \sqrt{Sc} + \sqrt{Kt}) + \exp(-2\eta \sqrt{KtSc}) \operatorname{erfc}(\eta \sqrt{Sc} - \sqrt{Kt}) \right] \\
- \frac{\eta \sqrt{Sc t}}{2\sqrt{K}} \left[ \exp(-2\eta \sqrt{KtSc}) \operatorname{erfc}(\eta \sqrt{Sc} + \sqrt{(K+b)t}) - \exp(2\eta \sqrt{KtSc}) \operatorname{erfc}(\eta \sqrt{Sc} - \sqrt{(K+b)t}) \right] \tag{4.12}
\]

Where, \( a = \frac{M}{Pr-1} \), \( b = \frac{M - K \cdot Sc}{Sc - 1} \), \( c = \frac{Gr}{2a(1-Pr)} \), \( d = \frac{Gc}{2b^2(1-Sc)} \) and \( \eta = \frac{Y}{2\sqrt{t}} \).

**4.4 DISCUSSION OF RESULTS**

The numerical values of the velocity and concentration are computed for different parameters like magnetic field parameter, chemical reaction parameter, Schmidt number, thermal Grashof number and mass Grashof number. The purpose of the calculations given here is to assess the effects of the parameters \( \omega t, M, K, Gr, Gc \) and \( Sc \) upon the nature of the flow and transport.

Figure 4.1 demonstrates the effect of the concentration profiles for different values of the chemical reaction parameter \( (K = 2, 5, 10) \), \( Sc = 0.6 \) and time \( t = 0.4 \). It is observed that the concentration increases with decreasing chemical reaction parameter.
Figure 4.2 represents the effect of concentration profiles at time $t = 0.4$ for different Schmidt number ($Sc = 0.16, 0.3, 0.6, 2.01$) and $K = 2$. It is observed that the wall concentration increases with decreasing values of the Schmidt number. The effect of concentration profiles for different values of time ($t = 0.2, 0.4, 0.6, 1$), $K = 2$ and $Sc = 0.6$ are presented in figure 4.3. The trend shows that the wall concentration increases with increasing time $t$.

The velocity profiles for different phase angles ($\omega t = 0, \pi / 6, \pi / 3, \pi / 2$), $M = 0.2$, $Gr = Gc = 5$, $K = 8$, $Sc = 0.6$, $Pr = 7.0$ and $t = 0.4$ are shown in figure 4.4. It is observed that the velocity increases with decreasing phase angle. Figure 4.5 demonstrates the effects of the magnetic field parameter on the velocity when $(M = 0.2, 2.5)$, $\omega t = \pi / 4$, $K = 8$, $Gr = Gc = 5$, $Sc = 0.6$, $Pr = 7.0$ and $t = 0.6$. It is observed that the velocity increases with decreasing magnetic field parameter. Figure 4.6 illustrates the effect of the velocity for different values of the reaction parameter $(K = 0.5, 5, 15)$, $\omega t = \pi / 4$, $M = 0.2$, $Gr = Gc = 5$, $Sc = 0.6$, $Pr = 7.0$ and $t = 0.6$. The trend shows that the velocity increases with decreasing chemical reaction parameter.

The velocity profiles for different thermal Grashof number ($Gr = 5, 10$), mass Grashof number ($Gc = 5, 10$), $\omega t = \pi / 4$, $K = 15$, $M = 0.2$, $Sc = 0.6$, $Pr = 7.0$ and $t = 0.6$ are shown in figure 4.7. It is clear that the velocity increases with increasing thermal Grashof number or mass Grashof number. The effect of velocity profiles for different time ($t = 0.2, 0.4, 0.6$), $M = 0.2$, $K = 8$, $\omega t = \pi / 4$, $Gr = Gc = 5$, $Pr = 7.0$ and $Sc = 0.6$ are shown in figure 4.8. In this case, the velocity increases gradually with respect to time $t$. 

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Fig. 4.1: Concentration profiles for different values of $K$

Fig. 4.2: Concentration profiles for different values of $Sc$
Fig. 4.3: Concentration profiles for different values of $t$

Fig. 4.4: Velocity profiles for different values of $\omega t$
Fig. 4.5: Velocity profiles for different values of $M$

Fig. 4.6: Velocity profiles for different values of $K$
Fig. 4.7: Velocity profiles for different values of Gr, Gc

Fig. 4.8: Velocity profiles for different values of t
4.5 CONCLUSION

The problem of MHD flow past an oscillating infinite isothermal vertical plate with uniform temperature and variable mass diffusion, in the presence of chemical reaction of first order is studied. The effects of velocity and concentration for different parameters like $\omega$, $M$, $K$, $Sc$, $Gr$, $Gc$ and $t$ are studied.

The study concludes that the velocity increases with decreasing phase angle, magnetic field parameter and chemical reaction parameter. The trend is just reversed with respect to time $t$ and also the velocity increases with increasing thermal Grashof number or mass Grashof number. The plate concentration increases with decreasing chemical reaction parameter and Schmidt number. Also the concentration increases with increasing time $t$. 