CHAPTER II

THE DUCTILE APPROXIMATION

(II.1) Differential Cross Section

The differential cross section in the deuteron rest frame is given by\(^52\)

\[
\frac{d\sigma}{dq^2} = \frac{1}{96\pi^4} \frac{m^2}{E^2} \int \frac{M^2}{E_1 E_2} \sum_{\text{spins}} |M|^2 \, d^3p_1
\]

with

\[q^2 = (\vec{q} + \vec{\gamma})^2 = m^2 \quad \text{and} \quad q_2^2 = (\vec{q} - \vec{p}_1)^2 = m^2\]  \hspace{1cm} (II.1b)

where the square of the transition matrix element which is summed over initial and final spins is needed to calculate the differential cross section. These matrix elements are defined in equation (I.13) and equation (I.30) for helicity conserving and helicity flipping neutral current theories.

The matrix element \(M\) can also be written as

\[
M = \frac{G_F}{\sqrt{2}} l_\lambda J_\lambda
\]

(II.2)

with \(l_\lambda\) of the matrix element of the leptonic part due to the leptonic neutral current and \(J_\lambda\) is the matrix element of the hadronic neutral current defined between the initial deuteron and final dimuon states and written as

\[
J_\lambda = \langle mp | J_{\lambda}^{\text{NC}} | d \rangle
\]

(II.3)
The square of the matrix element with summed over all the initial and final spins is calculated as

$$
\sum_{\text{spins}} |M|^2 = \sum_{\text{spins}} G_2^2 \left| J^1 \right|^2 = \frac{G_2^2}{2} \sum_{\text{spins}} \mathcal{J}^\eta \mathcal{J}^\eta
$$

(77.4)

where \( \mathcal{J}^\eta \) is the leptonic tensor which is obtained by summing the leptonic matrix elements of the leptonic neutral current, and \( \mathcal{J}^\eta \) is the hadronic tensor obtained by summing the hadronic neutral current matrix element as described by equation (77.4).

(77.1) Helicity-conserving theories

In order to calculate the differential cross section in the helicity conserving, \( V-A \) theories of weak neutral currents we evaluate the product of the leptonic tensor and the hadronic tensor as described by equation (77.4).

The leptonic tensor \( \mathcal{J}^\eta \) in the \( V-A \) theories is given by

$$
\mathcal{J}^\eta = \mathcal{J}_\eta^\dagger = \left( \overline{U}(K) X_\eta \left( U(K) \right)^\dagger \right) \left( \overline{U}(K') X_\eta \left( U(K') \right)^\dagger \right)
$$

(77.5)

which is calculated by using simple algebra and is given by the following relation

$$
\mathcal{J}^\eta = \left( \nu \nu_\alpha \right) \left( k_i' \eta \cdot k_i \eta \right) \left( k_i' \eta \cdot s_\eta \right) \left( k_i' \eta \cdot p_i \right) \delta_{\alpha \eta} \delta_{\alpha \eta}
$$

(77.6)

The tensor \( \mathcal{J}^\eta \) is defined by

$$
\mathcal{J}^\eta = \sum_{\text{spins}} \mathcal{J}^\eta \mathcal{J}^\eta
$$

(77.7)
\[ \mathcal{I}_{\eta} \] is evaluated using equation (7.7) and equation (7.18) and the different components of the hadronic tensor \[ \mathcal{I}_{\eta} \] are calculated by performing the spin sum using the trace rules derived in Appendix B.

In order to calculate the differential cross section in equation (7.18), we perform the \( \pi^0 \) integration in the closure approximation. Following the standard procedure, we assume that (a) for a given \( n^2 \), \( \pi^0 \) coupling in the expression \( \int \mathcal{L}^{\eta} \mathcal{I}_{\eta} \, d^3 \pi^0 \) varies slowly over the range of integration and can be replaced by a suitably chosen average \( \langle \mathcal{L}^{\eta} \rangle \). (b) We have chosen \( \langle \mathcal{L}^{\eta} \rangle \) to be equal to the \( \mathcal{L}^{\eta} \) corresponding to the one-particle elastic case i.e., \( \mathcal{L}^{\eta} = -2 \beta^2 / m \). The corrections to this approximation will involve the internal motion of the nucleons inside the deuteron and can be important at low \( \gamma^2 \) and \( n^2 \).

(b) In the integration \( \int \mathcal{L}^{\eta} \mathcal{I}_{\eta} \, d^3 \pi^0 \), the hadronic tensor is replaced by \( \langle \mathcal{L}^{\eta} \rangle \) and taken out of the integration since we can write

\[ \int \mathcal{L}^{\eta} \mathcal{I}_{\eta} \, d^3 \pi^0 = \langle \mathcal{L}^{\eta} \rangle \int \mathcal{I}_{\eta} \, d^3 \pi^0 \]  \hspace{1cm} (7.3)

The \( d^3 \pi^0 \) integration is performed over the hadronic tensor alone using the closure approximation for final state wave functions

\[ \int \phi_i^*(\vec{r}^0) \phi_j(\vec{r}^0) \, d^3 \vec{p}^0 = (2\pi)^3 \delta(\vec{r}^0 - \vec{r}^0) \]  \hspace{1cm} (7.0)
The $J^\pi$ interaction extends from zero to certain $n'_{\text{max}}$ allowed by the energy-momentum conservation. This $n'_{\text{max}}$ is extended to infinity by assuming that the contribution from $n'_{\text{max}}$ to infinity which is otherwise not allowed, is very small.

Using these assumptions we have evaluated the terms entering in the tensor $\Gamma_{\alpha\beta}$ and the other terms entering in $(\dot{\mathcal{L}}_{\alpha\beta})$ in appendix C. The different components of $\mathcal{L}^\alpha \cdot \mathcal{L}^\beta$ and $\mathcal{L}^\alpha \times \mathcal{L}^\beta$ are given by

$$
\mathcal{L}^\alpha = \ell_{\alpha \beta} \Gamma^\beta = \left( \frac{2}{E_0 E_0'} \right) \left[ n'_{\text{max}} \cdot \mathbf{v}' \mathbf{v}' \right]
$$

$$
\mathcal{L}^\alpha = \ell_{\alpha \beta} \Gamma^\beta = \left( \frac{2}{E_0 E_0'} \right) \left[ n'_{\text{max}} \cdot \mathbf{v}' + n'_{\text{max}} \mathbf{k} \cdot \mathbf{v} \right]
$$

$$
\mathcal{L}^\alpha = \ell_{\alpha \beta} \Gamma^\beta = \left( \frac{2}{E_0 E_0'} \right) \left[ \mathbf{v}' \mathbf{v}' + n'_{\text{max}} \mathbf{k} \cdot \mathbf{v} \right] + \frac{2}{E_0 E_0'} \mathbf{v} \cdot \mathbf{v}'
$$

and the different components of $\mathcal{J}_{\alpha\beta}$ are given by

$$
\mathcal{J}_{00} = \int \mathcal{J}_{\alpha\beta} \Gamma^\beta = 2 \mu_0 \left[ \left( \mathbf{F}_c \mathbf{v}' \right) + \mathbf{F}_e \mathbf{v}' \right] \left( 1 + \frac{\mathbf{v}'^2}{4M^2} - \frac{K}{M^2} \right)
$$

$$
\mathcal{J}_{00} = \int \mathcal{J}_{\alpha\beta} \Gamma^\beta = \left( \mathbf{F}_c \mathbf{v}' \right) + \mathbf{F}_e \mathbf{v}' \mathbf{v}^2 \left( \mathbf{v}' \mathbf{v}' - \frac{K}{M^2} \right)
$$

$$
\mathcal{J}_{00} = \int \mathcal{J}_{\alpha\beta} \Gamma^\beta = 2 \mu_0 \left[ \mathbf{F}_c \mathbf{v}' \right] + \mathbf{F}_e \mathbf{v}' \mathbf{v}^2 \left( \mathbf{v}' \mathbf{v}' - \frac{K}{M^2} \right)
$$

$$
\mathcal{J}_{00} = \int \mathcal{J}_{\alpha\beta} \Gamma^\beta = 2 \mu_0 \left[ \mathbf{F}_c \mathbf{v}' \right] + \mathbf{F}_e \mathbf{v}' \mathbf{v}^2 \left( \mathbf{v}' \mathbf{v}' - \frac{K}{M^2} \right)
$$

$$
\mathcal{J}_{00} = \int \mathcal{J}_{\alpha\beta} \Gamma^\beta = 2 \mu_0 \left[ \mathbf{F}_c \mathbf{v}' \right] + \mathbf{F}_e \mathbf{v}' \mathbf{v}^2 \left( \mathbf{v}' \mathbf{v}' - \frac{K}{M^2} \right)
$$

$$
\mathcal{J}_{00} = \int \mathcal{J}_{\alpha\beta} \Gamma^\beta = 2 \mu_0 \left[ \mathbf{F}_c \mathbf{v}' \right] + \mathbf{F}_e \mathbf{v}' \mathbf{v}^2 \left( \mathbf{v}' \mathbf{v}' - \frac{K}{M^2} \right)
$$
\[ T_{ij} = \sum_{\text{Sym}} \left[ (f_i^b(y_i) + f_i^b(y_i) + 2 F_E^b(y_i) F_{E^b}(y_i) I(y_i)) \frac{\alpha_i}{4 M^2} \right. \]

\[ - (f_i^b(y_i)^2 + f_i^b(y_i) \frac{K}{3 M^2} \delta_{ij} - 2 F_E^b(y_i) F_{E^b}(y_i) \frac{H(y_i)}{3 M^2} \delta_{ij} \]

\[ + \left( F_M^b(y_i) + F_M^b(y_i) + 2 F_M^b(y_i) F_M^b(y_i) I(y_i) \right) \left( \frac{y_i - \frac{q^2}{4 M^2}}{4 M^2} \right) + \left( F_M^b(y_i) g_A^b(y_i) \right) \]

\[ + F_M^b(y_i) g^n_A(y_i) + \left( F_M^b(y_i) g^n_A(y_i) + F_M^b(y_i) g^n_A(y_i) \right) \frac{I(y_i)}{3} \left( \frac{y_i + \frac{q^2}{4 M^2}}{M} \delta_{ij} - \left( g^n_A(y_i) \right) \right) \]

\[ + g^n_A(y_i) \frac{K}{3 M^2} \delta_{ij} - 2 g^n_A(y_i) g^n_A(y_i) \frac{H(y_i)}{g M^2} \left( \frac{y_i}{4 M^2} \right) \]

\[ (7.19a) \]

where \( f_i^b(n^2) \), \( f_i^b(n^2) \), \( f_i^b(n^2) \), \( f_i^b(n^2) \) are related with vector form factor by equation (7.19) and \( p_A^n(n^2) \), \( p_A^n(n^2) \) are the axial vector form factors for proton and neutron respectively and

\[ T(n) = \int \hat{q}_i \cdot \hat{v} \, d^3v \, \hat{r} \frac{d^3r}{\hat{v}} \]

\[ T(n) = \int \hat{p}_i \cdot \hat{v} \, (\nabla^2 \hat{P}_i(\hat{v})) \, \hat{r} \frac{d^3r}{\hat{v}} \]

\[ F = T(n) \]

\[ T(n) = 1 \]

with the help of equations (7.19a) and (7.19b), we get the differential cross section as

\[ \frac{d\sigma}{dQ^2} = \left( \frac{2}{4\pi} \frac{F_E}{E} \right) \left( 1 - \frac{Q^2}{4M^2} \right) \left( \left( F_E^b(y_i) + F_E^b(y_i) + 2 F_E^b(y_i) F_{E^b}(y_i) I(y_i) \right) \right) \]

\[ (7.19b) \]
\[
x^\prime \left\{ 1 + \frac{(E_2^2 + E_2^2 - 2 E_2 E_2 \cos \theta)}{4 M^2} \right. - \left( \frac{E_2 - E_2'}{M} \right) + \left( \frac{(E_2 - E_2')^2}{M^2} \right) \left( 1 + \cos \theta \right) \\
+ \left( \frac{g_{A}^b (q^2) + g_{A}^h (q^2) + 2 g_{A}^b (q^2) g_{A}^h (q^2) \frac{I(q)}{3}}{M} \right) \left( 3 - \cos \theta \right) + \left( \frac{E_2^2 + E_2^2 - 2 E_2 E_2 \cos \theta}{M^2} \right) \\
- \left( \frac{E_2 - E_2'}{M} \right) \left( 1 + \cos \theta \right) \right\} - \left( F_{M}^b (q^2) + F_{M}^h (q^2) + 2 F_{M}^b (q^2) F_{M}^h (q^2) \frac{I(q)}{3} \right) \\
\times \left( \frac{E_2 + E_2' + E_2 E_2 (1 - \cos \theta) (1 - \cos \theta)}{2 M^2} \right) \pm 2 \left( F_{M}^b (q^2) g_{A}^b (q^2) + F_{M}^h (q^2) g_{A}^h (q^2) \right) \\
+ \left( F_{M}^b (q^2) g_{A}^h (q^2) + F_{M}^h (q^2) g_{A}^b (q^2) \frac{I(q)}{3} \right) \left( \frac{E_2 + E_2'}{M} \right) \left( 1 - \cos \theta \right) - 2 \left( 1 - \cos \theta \right) \\
\times \left( \left( F_{E}^b (q^2) + F_{E}^h (q^2) + g_{A}^b (q^2) + g_{A}^h (q^2) \right) \frac{K}{M^3} + \left( 2 F_{E}^b (q^2) F_{E}^h (q^2) \right) \right. \\
+ \left. \frac{2}{3} \left( g_{A}^b (q^2) g_{A}^h (q^2) \frac{H(q)}{M^2} \right) \right) \\
\] (17.11)

Consider: in lowest order of \( \langle 1/F \rangle \), the expression (11.13) reduces as

\[
\frac{d\sigma}{dq^2} = \frac{G^2}{4\pi} \left( \frac{E_2'}{E_2} \right) \left[ \left( F_{E}^b (q^2) + F_{E}^h (q^2) + 2 F_{E}^b (q^2) F_{E}^h (q^2) \frac{I(q)}{3} \right) \\
\times \left\{ 1 - \left( \frac{E_2 - E_2'}{M} \right) \left( 1 + \cos \theta \right) + \left( \frac{g_{A}^b (q^2) + g_{A}^h (q^2) + 2 g_{A}^b (q^2) g_{A}^h (q^2) \frac{I(q)}{3}}{M} \right) \right\} \\
\times \left( \frac{3 - \cos \theta}{3} - \left( \frac{E_2 - E_2'}{M} \right) \left( 1 + \cos \theta \right) \right) \right] \pm 2 \left( F_{M}^b (q^2) g_{A}^b (q^2) + F_{M}^h (q^2) g_{A}^h (q^2) \frac{I(q)}{3} \right) \\
\times \left( \frac{E_2 + E_2'}{M} \right) \left( 1 - \cos \theta \right) \\
\] (17.14)

where + sign used for neutrino transition and - sign used for antineutrino transition.
The total cross section $\sigma$ can be obtained by integrating the equation (II.13) over $q^2$. The deuteron effects thus enter in equations (II.13) and (II.14) through the functions $V(q)$, $\Pi(q)$ and $R$ as defined in equation (II.12).

(II-1.2) Helicity-flipping theories

The differential cross section in helicity flipping ("p\n\n\n") theories can be evaluated in a similar manner as described for $\gamma$, a theory in previous section (II-1.1). The differential cross section $d\sigma/dq^2$ is obtained by substituting the value of $|M|^2$ in equation (II.1) which is formally written as

$$\frac{d\sigma}{dq^2} = \frac{G^2}{192\pi^4} \frac{M_0^2}{E_2^2} \int \frac{M^2}{E_1'E_2'} \mathbf{L} \cdot \mathbf{F} d^3p$$

(II.15)

where $\mathbf{L}^\gamma\nu$ and $\mathbf{T}^\gamma\nu$ are the leptonic tensor and the hadronic tensor occurring in $|M|^2$. [Note that the tensorial structure of $\mathbf{L}^\gamma\nu$ and $\mathbf{T}^\gamma\nu$ will be different for $S$, $P$, and $T$ interactions.]

The various components of the leptonic tensor $\mathbf{L}^\gamma\nu$ and the hadronic tensor integrated over $d^3p$ (i.e., $\int \mathbf{L}^\gamma\nu d^3p$), are evaluated considering the matrix element in scalar, vector, pseudo scalar and tensor couplings. The various components for the leptonic tensor i.e., $L_{\ell\nu}$, $L_{\ell\nu}$, $L_{\ell\nu}$, etc., have been evaluated in appendix B and quoted in equations (7.11) - (8.16).

In order to calculate the differential cross section in equation (II.15), we perform the $p^2$ integration in closure approximation assuming that for a given $q^2$, "$\nu$" occurring in the
expression $\mathcal{L} \cdot \mathcal{J} \, d^3p$ varies slowly over the range of integration and can be replaced by $\langle E_0' \rangle$. We have chosen the average $\langle E_0' \rangle$ to be equal to $E_0' = E_0 - q^2/2M$ corresponding to quasi-elastic peak. The correction to this approximation involves the internal motion of the nucleons inside deuteron and will be small at intermediate energies considered here. In the integration $\int \mathcal{L} \cdot \mathcal{J} \, d^3p$, the leptonic tensor $\mathcal{L}^{\lambda \eta \phi \sigma}$ is replaced by $\langle \mathcal{L}^{\lambda \eta \phi \sigma} \rangle$ and taken out of the integration which is performed over $p'$, the limit of integration extends from zero to $p_{\text{max}}'$ allowed by energy momentum conservation. The $p_{\text{max}}'$ is then extended to infinity considering that the contribution from $p_{\text{max}}'$ to infinity is very small.

In this approximation the momentum integration in equation (II.13) is performed over the hadronic tensor alone using the completeness property of the final dimuon state wave function as described in previous section (II-1.1).

The differential cross section $d\sigma/dq^2$ defined in equation (II.15) can therefore be written as

$$\frac{d\sigma}{dq^2} = \frac{G^2}{192\pi^4} \frac{m^2}{E_0^2} \left(1 - \frac{q^2}{4M^2}\right) \langle \mathcal{L}^{\lambda \eta \phi \sigma} \rangle \int \mathcal{J}^{\lambda \eta \phi \sigma} \, d^3p' \quad (II.16)$$

The hadronic tensor $\mathcal{J}^{\lambda \eta \phi \sigma}$ is defined as

$$\mathcal{J}^{\lambda \eta \phi \sigma} = \int \mathcal{L}^{\lambda \eta \phi \sigma} \, d^3p' \quad (II.17)$$

The various components of hadronic tensor $\mathcal{J}^{\lambda \eta \phi \sigma}$ are evaluated in helicity flipping theories and the results
are quoted in Appendix B (see equations (A-20) - (A-23)).

By considering the equations for leptonic tensors and hadronic tensors and substituting them in equation (II.17), we can evaluate the differential cross section \(d\sigma/dq^2\). The results for \(d\sigma/dq^2\) in helicity flipping theories containing terms up to order 0 \((1/M)^2\) are given by

\[
\frac{d\sigma}{dq^2} = \frac{G^2}{4\pi} \left( \frac{E'_0}{E_0} \right) \left( 1 - \frac{q^2}{4M^2} \right) \left[ (F^l_{\pm}(q^2) + F^l_{\mp}(q^2)) + 2 F^l_{\pm}(q^2) F^l_{\mp}(q^2) \right] \frac{I(q)}{3}
\]

\[
x \left( 1 + \frac{q^2}{4M^2} \right) (1 - \cos \theta) + \left( F^l_{\pm}(q^2) + F^l_{\mp}(q^2) + 2 F^l_{\pm}(q^2) F^l_{\mp}(q^2) \right) \frac{I(q)}{3}
\]

\[
x \frac{q^2}{4M^2} (1 - \cos \theta) + \left( T^l_{\pm}(q^2) + T^l_{\mp}(q^2) + 2 T^l_{\pm}(q^2) T^l_{\mp}(q^2) \right) \frac{I(q)}{3}
\]

\[
\left\{ 4 \left( 1 + \frac{q^2}{4M^2} \right) (3 + \cos \theta) - 4 \left( \frac{3E_0' - E_0 (2 + \cos \theta)}{M^2} \right) \right\}
\]

\[
x \left( \frac{q^2}{4} + (E'_0 - E_0)^2 \right) \frac{I(q)}{3} + \left( (T^l_{\pm}(q^2) + T^l_{\mp}(q^2)) K + 2 T^l_{\pm}(q^2) T^l_{\mp}(q^2) H(q) \right)
\]

\[
x \frac{8(3 + \cos \theta)}{3M^2} + \left( F^l_{\pm}(q^2) T^l_{\pm}(q^2) + F^l_{\mp}(q^2) T^l_{\mp}(q^2) + (F^l_{\pm}(q^2) T^l_{\mp}(q^2) \right)
\]

\[
+ F^l_{\pm}(q^2) T^l_{\mp}(q^2) \right) \frac{I(q)}{3} \left( E_0 + E_0' \right) \frac{(1 - \cos \theta)}{M} \pm \left( F^l_{\pm}(q^2) T^l_{\mp}(q^2) \right)
\]

\[
+ F^l_{\mp}(q^2) T^l_{\pm}(q^2) + \left( F^l_{\pm}(q^2) T^l_{\mp}(q^2) + F^l_{\mp}(q^2) T^l_{\pm}(q^2) \right) \frac{I(q)}{3}
\]

\[
x \left( \frac{2(3 - \cos \theta)}{M} \right) - \left( T^l_{\pm}(q^2) + T^l_{\mp}(q^2) + 2 T^l_{\pm}(q^2) T^l_{\mp}(q^2) \right)
\]

\[
\left( T^l_{\pm}(q^2) T^l_{\pm}(q^2) \right) \frac{I(q)}{3} \frac{1}{M^2} \left\{ 2(3 - \cos \theta) \frac{q^2}{3} + (E_0 - E_0')^2 \right\} \left( 1 + \cos \theta \right)
\]
\[+ \left( T_1^p(q^2) T_1^b(q^2) + T_1^b(q^2) T_1^p(q^2) + T_1^p(q^2) T_2^b(q^2) + T_1^b(q^2) T_2^p(q^2) \right) \times \left\{ \frac{8 \left( \frac{(E_2' - E_2)}{M^2} \right) \cos^2 \theta - \frac{E_2 E_2'}{M^2} (1 - \cos^2 \theta)}{\lambda^2} + \frac{4 \left( 3 \frac{E_2'}{E_2} - 2 + \cos \theta \right)}{M^2} \right\} \]

(II.18)

Considering the terms in lowest order of \(1/M\), we get

\[
\frac{d\sigma}{dq^2} = \frac{G_1^2}{4 \pi} \left( \frac{E_2'}{E_2} \right) \left[ (F_S^{b2}(q^2) + F_S^{h2}(q^2) + 2 F_S^{L}(q^2) F_S^{n}(q^2) I(q)) (1 - \cos \theta) \right.
\]

\[+ \left( T_1^b(q^2) + T_1^h(q^2) + 2 T_1^b(q^2) T_1^h(q^2) \right) I(q) \left( \frac{E_2 + E_2'}{(1 - \cos \theta)} \right) \]

\[- \frac{4 \left( 3 \frac{E_2'}{E_2} - 2 + \cos \theta \right)}{M} \] + \left( F_S^{b}(q^2) T_1^b(q^2) + F_S^{h}(q^2) T_1^h(q^2) \right) I(q) \left( \frac{E_2 + E_2'}{(1 - \cos \theta)} \right) \]

\[+ \left( F_S^{L}(q^2) T_1^L(q^2) + F_S^{n}(q^2) T_1^n(q^2) \right) I(q) \left( \frac{E_2 + E_2'}{(1 - \cos \theta)} \right) \]

\[\pm \left( F_p^{b}(q^2) T_1^b(q^2) + F_p^{h}(q^2) T_1^h(q^2) + F_p^{L}(q^2) T_1^L(q^2) \right) \left( \frac{2 (1 - \cos \theta)}{M} (E_2 + E_2') \right) \]

(II.19)

where + sign is used for neutrino transition - sign is used for antineutrino transition.

The functions \(I(q), \mu(q)\) and \(K\) occurring in equations (II.18) and (II.19) are same as given in equation (II.12).

The total cross section \(\sigma\) is obtained by integrating equation (II.18) over \(q^2\).
(II.2) Weak form factors

For helicity conserving theories the vector and axial vector neutral current form factors \( F_1(x^2) \) and \( F_A(x^2) \) are used in analogy with the weak form factors of charged currents. The \( x^2 \) dependence of these form factors used in the present calculation are given by

\[
F_1(x^2) = \frac{F_1(x)}{(1 + x^2/\Lambda^2)^2}
\]

\[
F_2(x^2) = \frac{F_2(x)}{(1 + x^2/\Lambda^2)^2}
\]

\[
F_A(x^2) = \frac{F_A(x)}{(1 + x^2/\Lambda^2)^2}
\]

with \( \Lambda = 340 \text{ MeV} \).

where the form factors \( F_1(0), F_2(0) \) and \( F_A(0) \) are the various form factors for nucleons i.e. proton and neutron, at \( x^2 = 0 \). The numerical values of these form factors at very small \( x^2 \) (i.e. \( x^2 \approx 0 \)) are given for various neutral current models in V, A theories, in Table II.

In helicity fliping theories the various neutral current form factors for scalar, pseudo scalar and tensor couplings are not at all established even phenomenologically. In calculating the total cross sections in these theories, we have taken the values of \( F_3(0), F_5(0) \) and \( T_1(0) \) as calculated by Adler \( ^{19,36} \) at very small \( x^2 \) (i.e. \( x^2 \approx 0 \)). The numerical values of various nucleon form factors at
\( q^2 = 0 \) in quark model and MIT model as calculated by Adler \(^{36} \) have been reported in Table IV. The \( q^2 \) dependence on these form factors \(^{77} \) is taken to be

\[
F_S(q^2) = \frac{F^S(0)}{(1 + q^2/M^2)} \]

\[
F_P(q^2) = \frac{F^P(0)}{(1 + q^2/M^2)} \tag{II.21}
\]

\[
T_{1,2}^N(q^2) = \frac{T_{1,2}^N(0)}{(1 + q^2/M^2)^2}
\]

with \( M = 840 \text{ MeV} \), and \( M_p = 910 \text{ MeV} \).

(II-3) Results and Discussion

The neutrino (antineutrino) disintegration cross section \( \sigma^- \) is calculated from equations (II,13) and (II,18) in helicity conserving theories and in helicity flipping theories using the form factors given in equation (II,20) and (II,21) respectively and the initial deuteron has been described by the Dalitz wave function neglecting the \( P \) state. We write the radial part of the wave function as \(^{76}\)

\[
f_1(\lambda) = \frac{\sqrt{6}}{2\pi(a+b)} \frac{(a+b)}{(b-a)} \left( \frac{e^{-\lambda R} - e^{-bR}}{R} \right) \tag{II.22}
\]

with \( a = 450 \text{ MeV} \) and \( b = 257 \text{ MeV} \).
The various integrals $I(q)$, $H(q)$ and $K$ defined in equation (II,12) are solved using the above wave function $\psi_1(x)$ for deuteron. These functions are evaluated as

$$I(q) = \frac{2\alpha_q(\alpha+\beta)}{(\beta-\alpha)^2} \left( \frac{1}{q} \right) \left[ \tan^{-1} \frac{q}{2\alpha} + \tan^{-1} \frac{q}{2\beta} - 2\tan^{-1} \frac{q}{\alpha+\beta} \right]$$

$$H(q) = \frac{2\alpha_q(\alpha+\beta)}{(\beta-\alpha)^2} \left( \frac{1}{q} \right) \left[ -\alpha^2 \tan^{-1} \frac{q}{2\alpha} - \beta^2 \tan^{-1} \frac{q}{2\beta} + (\alpha^2+\beta^2) \tan^{-1} \frac{q}{\alpha+\beta} \right]$$

$$K = H(0) = \sigma_0$$  \hspace{1cm} (II,23)

We plot the value of function $I(q)$ as a function of $q^2$ in Figure 1. It can be seen from Figure 1 that the function $I(q)$ which depends upon the deuteron wave function play an important role at small $q^2$. At a larger $q^2$ value the $I(q)$ approaches to zero. Thus the effect of entering $q^2$ in the function $I(q)$ is deuteron effect. The other functions $H(q)$ and $K$ also contain the effect of deuteron wave function.

The neutrino disintegration cross section $\sigma^-$ is calculated in helicity conserving theories and helicity flipping theories from equations (II,13) and (II,18) respectively and the results are plotted as a function of incident neutrino energy, $E$, in Figures 2 and 3 for two distinct theories. Similar calculations have been done for the antineutrino disintegration process and the results for $\sigma^-$ vs antineutrino energy $E'_{\bar{\nu}}$ are presented in Figures 4 and 5 for helicity conserving and helicity flipping theories.
respectively.

The results presented in figures 2 and 3 for all the neutral current models considered show that the disintegration cross section increases with energy but the rate of increase is smaller as the energy becomes higher. This is because the increase in the cross section due to increase in energy is partially compensated by the damping due to \( q^2 \) dependence of various form factors. In helicity conserving vector, axial vector theories, the various nucleon form factors \( F_1^N(q^2) \), \( F_2^N(q^2) \) and \( F_A^N(q^2) \) are used in analogy with weak interaction form factors of charged current. The \( q^2 \) dependence of these form factors used in the calculation is given in equation (II.20) where the form factors for proton and neutron at \( q^2 = 0 \) are given in various SU(2) \& U(1) models of neutral current in Table II. The conventional Weinberg-Salam model seems to give the larger effect for total cross section in helicity conserving theories.

In helicity flipping theories the various form factors for scalar, pseudo scalar and tensor couplings are not at all established even phenomenologically. In calculating the cross section we have taken these form factors as \( F_0^N(q^2) \), \( F_P^N(q^2) \) and \( F_{1,2}^N(q^2) \). The \( q^2 \) dependence of these form factors is given in equation (II.21). The form factors \( F_0^N(0) \), \( F_P^N(0) \) and \( F_{1,2}^N(0) \) at \( q^2 = 0 \) were numerically calculated by Adler in phenomenological quark model and "IT model. The values of these form factors in two models are reported in
Table IV. There are indications that the phenomenological values of some of the form factors (especially $T_q^2(q^2)$ for proton and neutron respectively) may be smaller than the values calculated by Adler. The results for the anti-neutrino disintegration cross sections presented in helicity conserving as well as in helicity flipping theories in figures 4 and 5 are similar but smaller than the cross sections calculated in figures 2 and 3 for the neutrino disintegration.

The disintegration process has been earlier analyzed at intermediate energies by Ali and Dominguez and Osipov and Tyutin. They have calculated the production of $L = 0$ (S waves) and $L = 1$ (P waves) waves at intermediate energies and have not considered the contribution of higher waves. Our calculation includes the contribution of all the partial waves through the closure approximation over the final dinucleon state. We do not make significant error in extending the limit of $p^*$ integration to infinity because the contribution to double differential cross section from $p_{max}$ to infinity, which is not allowed, is very small. The major uncertainty in our calculation lies in replacing the value of $E_{j'}$ by the average value $<E_{j'}>$ i.e., $<E_{j'}> = E_{j'} - q^2/2M - x/M$. This approximation is quite good except at very low energies. This is because kinematically $<E_{j'}> = E_{j'} - q^2/2M - <p^2>/M$, where $p$ is the internal momentum of the nucleons inside deuteron. The quantitative estimate of $<p^2>/M$ for various wave function of the deuteron vary from 2.5 MeV to 10 MeV.
Therefore the closure approximation should be good for energies $E_D \gg 10\text{MeV}$ where the uncertainties in determining the values of $\langle r_D^2 \rangle$ are small. The factor which relates the internal motion of nucleons inside deuteron become important at low energies. This is the reason that the closure approximation results do not give reliable results for the low energy experiments of Passierb et al.\textsuperscript{56}

At intermediate energies where $E_D \gg 10\text{MeV}$, present results calculated in closure approximation are quite reliable and should be used in analyzing these experiments as has been done in the case of $\nu + d \rightarrow \mu^- + p + p$ (ref. 62).