CHAPTER II

NEMATIC DEFECTS IN MAGNETIC FIELD

2.1 Introduction

In this chapter we look at the effects of a magnetic field on the structure and properties of some defects in nematic liquid crystals. The diamagnetic anisotropy $\chi_a$ of a nematic can be positive or negative. Usually nematics with rod like molecules have positive $\chi_a$ and those with disc like molecules have negative $\chi_a$. We consider both the cases. When a sample of nematic with $\chi_a > 0$ is placed in a uniform magnetic field the director $n$ aligns parallel to the magnetic field. While in the case of $\chi_a < 0$ nematics the director aligns in a plane perpendicular to the magnetic field. In both the cases since $n$ is apolar solutions with $n$ and $-n$ have the same energy. Two such solutions get connected by a domain wall inside which the director turns through an angle $\pi$. These walls were first discussed by Helfrich [1] and are called Helfrich walls or planar solitons. We can have bend rich, splay rich or pure twist walls. Volovik and Mineev showed that these walls can end in disclination lines of half integral strength [2,3]. Two Helfrich walls can get interconnected through a disclination. Volovik and Mineev also showed the possibility of having cylindrical domains ending in point singularities. These has been named as linear solitons. Ranganath [1] predicted that in uniform magnetic field point singularities can result in discs inside which the director turns through $\pi$. In addition the symmetry of nematic liquid crystals allows one to consider cylindrical shell structures -Bubble domains- connecting the inside and outside uniform regions by twist or bend cylindrical shell.

It may be remarked in passing that such structures can exist in biaxial nematics as well. The simplest of biaxial nematics have orthorhombic symmetry and have three directors $a$, $b$ and $c$. The possible structures of various types of solitons in these systems was discussed by Ranganath [5].

In this chapter we have considered two types of magnetic fields. (1)The uniform magnetic field $H_z$ acting in the $z$ direction and (2)that of a circular magnetic field
Fig 2.1: A pair of unlike disclinations of strength 1/2 in a magnetic field acting normal to the disclination lines (a) Field perpendicular to the line joining the disclinations, (b) Field along the line joining disclinations.
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$H_a$ generated by a linear current element [1]. The effects of elastic anisotropy, i.e., $k_{11} \neq k_{22} \neq k_{33}$ have also been worked out.

2.2 Interaction between disclinations

Let us consider the case of $\pm 1/2$ disclination line in a nematic with $\chi_a$ positive. It was said earlier that this disclination line gets transformed to a domain wall terminating in a singular line under the action of a magnetic field $H$. The wall thickness is of the order of the magnetic coherence length $\xi = (k/\chi_a H)^{1/2}$ where $k = k_{11} = k_{22} = k_{33}$ is the elastic constant. We consider the interaction between such disclinations.

Consider now the case of a magnetic field $H$ acting perpendicular to the line connecting two unlike 1/2 wedge disclinations. The field induced structure is shown in Fig 2.1a. The two singularities are connected by a splay wall. The energy per unit area of these walls is given by $E = 2H(k\chi_a)^{1/2}$ [1]. It is clear from the figure that by moving the the two disclinations towards or away from each other the director pattern inside the dashed circles of radius $\xi$ or at far off distances is not much affected. But the size of the connecting wall is altered by moving the two disclinations towards each other. We assume the distance of separation to be much larger than $\xi$. Then for a change of distance $\Delta d$ the energy of the wall configuration is decreased (to a good approximation) by $\Delta E = \text{energy per unit area of the wall} \times \Delta d$ i.e., $\Delta E = 2H(k\chi_a)^{1/2}\Delta d$. Hence the change in total energy is proportional to the change in the distance of separation $\Delta d$ between two disclinations. Therefore two unlike disclinations attract with a force that is independent of distance separating them.

We now consider the case when the magnetic field is acting parallel to the line joining two unlike line disclinations. In this case we get a planar bend soliton extending on either side to infinity and away from the disclinations. This is shown in Fig 2.1b. The energy per unit area of this soliton is $E = \chi_a H^2/2$. Most of the region between the two defects is free from director distortion. To a good approximation we see that small displacements of the two defects do not change the director field either inside the dashed circles or at infinity. Hence by moving the two disclination away from each other by $\Delta d$ the change in energy is given by:
Fig 2.2: Pairs of like disclinations of strength $1/2$ in a magnetic field acting along the line connecting them. The force of interaction is independent of the distance of separation and can be repulsive (a) or attractive (b).
Fig 2.3: Pairs of like disclinations of strength 1/2 in a magnetic field perpendicular to the line connecting them. The interaction between them being repulsive (a) and attractive (b)
Fig 2.4: Poincaré defects of different strengths with line singularities (dashed line) ending at a point inside the material.

Fig 2.5: Generation of Poincaré defects in a nematic with negative diamagnetic anisotropy (a) before the Freedericksz threshold. (b) At much higher fields.
\[ \Delta E = \frac{-\gamma_s H^2}{2} \Delta d \]

Thus in this geometry we find that the field favors repulsion between two unlike disclinations. The force of repulsion being independent of distance. This is an unusual interaction, since in general two unlike defects always attract each other.

In the same way we see that two disclinations of opposite strength can experience an attractive or repulsive force in the presence of a magnetic field. The strengths of the attractive and repulsive forces are, however, different.

In the case of two like defects we have four walls being generated by a magnetic field. This is shown in Fig 2.2 and Fig 2.3 for magnetic field acting parallel and perpendicular to the line joining the defects. However, the interaction between the disclinations can be repulsive or attractive depending upon the geometry. Here again the force of interaction is independent of distance of separation.

In the above analysis we have ignored the formal elastic interaction. A calculation of the net interaction taking both contributions into account is not easy. However we can make some approximate estimates. We know that the elastic free energy density varies as \( k(\Delta \theta)^2 \) while the magnetic energy varies as \( \chi_s H^2 \sin^2 \theta \). Thus over distances \( d \) less than \( \xi \), to a good approximation the magnetic energy can be neglected compared to the elastic energy which dominates. Thus the distance independent interaction law will be valid only when \( d \gg \xi \).

Very similar arguments can be applied to nematics in electric field where the dielectric anisotropy \( \varepsilon_0 \) aligns the director to the field

### 2.3 Poincarè structures

In all the experimental situations known to date a line singularity is found to end either on itself forming a loop or on the surface of the sample. However it is known that a line singularity can also be terminated in a pair of half point disclinations [7,6]. While working out the defects in nematics, by Poincarè's technique Nabarro found that structures of the type shown in Fig 2.4 are also allowed. The first two structures have cylindrical symmetry with \( S = \pm 1/2 \) director pattern in the meridional plane. We can also have \( = +1/2 \) structures in one plane and \( -1/2 \) in the orthogonal plane. Poincarè structures are very unique. Firstly they are singular for
z < 0 and are strictly non-singular for z > 0, i.e., a singular line ends in the body of the material. Secondly the line singularity has a strength of ±1. Such topological defects have not been experimentally seen so far. In this section we suggest a method of generating such defects.

If a magnetic field is applied parallel to the director of a homeotropically aligned nematic with \( \chi_a < 0 \), it will undergo a Fredericks transition at a critical field given by \( H_c = (\pi^2 k/\chi_a d^2)^{1/2} \). Just above this threshold we get cylindrical nonsingular structure shown in Fig 2.5a. It should be noticed that in the central region the director is still opposing the magnetic torque. Hence at fields much higher than the critical field this structure can break down to a singular structure shown in Fig 2.5b. Here a \( S = +1 \) line singularity is shown to end in a pair of unlike Poincaré half point singularities.

This phenomena can be expected in nematic discotics since they usually have negative diamagnetic anisotropy. In rod-like nematics it is easier to get systems with negative dielectric anisotropy than negative diamagnetic anisotropy. Here the above arguments are valid *mutatis mutandis* in the presence of an electric field, provided the system is free of ions.

### 2.4 Bubble domains

As mentioned earlier Bubble domains are cylindrical shell structures connecting the inside and outside regions through twist, splay or bend deformation. We have investigated such structures in the presence of an all circular magnetic field of \( H_a = A/r \) (generated by a linear current element \( A \))in a nematic with \( \chi_a < 0 \).

In cylindrical polar coordinates \((r, \theta, \phi)\) the director \( n \) is given by

\[
n = [\sin \theta \cos(\phi - \alpha), \sin \theta \sin(\phi - \alpha), \cos \theta]\n\]

The distortion free energy density is:

\[
F = \frac{k}{2} \left[ (\nabla \theta)^2 + \sin^2 \theta (\nabla \phi)^2 \right] + \frac{\chi_a A^2}{2r^2} \sin^2 \theta \sin^2(\phi - \alpha)
\]

Minimization of the total energy \( \int F \, dv \) energy with respect to \( \theta \) results in

\[
k[\nabla^2 \theta - \sin \theta \cos \theta (\nabla \phi)^2] - \frac{\chi_a A^2}{r^2} \sin \theta \cos \theta \sin^2(\phi - \alpha) = 0
\]  
\[ (2.1) \]
Fig 2.6: A twist bubble domain in a nematic with $\chi_a < 0$, in an all circular field. The dashed lines are the boundary of the domain.

Fig 2.7: The director tilt $\theta$ in a twist bubble domain as a function of the distance from the center. (a) with $\eta = 1$. (b) with $\eta = 4$ and (c) $\eta = 6$
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similarly minimizing with respect to $\phi$ gives

$$k(\nabla^2\phi) - \frac{X_a A^2}{r^2} \sin(\phi - \alpha) \cos(\phi - \alpha) = 0$$

(2.2)

The solution satisfying both the equations (2.1) and (2.2) with the boundary conditions

$$\theta = 0 \text{ at } r = 0 \text{ and } \theta = \pi \text{ at } r = \infty$$

$$\theta = 2 \tan^{-1}(r/r_o)$$

and

$$\phi = \alpha + \pi/2$$

where

$$\eta = [1 + X_a A^2/k]^{1/2}$$

Here $r_o$ is the point at which $\theta$ becomes $\pi/2$. This solution represents a Bloch bubble domain which has been depicted in Fig 2.6. The variation of $\theta$ with respect to $r/r_o$ is shown in Fig 2.7 for different values of $\eta$. We see that the width of the cylindrical domain wall decreases as the field increases. Thus in such a field the bubble domain is a natural soliton solution. The total energy of this structure per unit height is found to be $4\pi k\eta$. Interestingly the energy is independent of its radius $r$. At high fields $\eta \rightarrow [X_a A^2/k]^{1/2}$ and the energy become $4\pi A(kX_a)^{1/2}$, which is $2\pi r$ times the energy/area of the planar soliton obtained in uniform fields.

Bubble domains can also exist in diamagnetically positive materials. Here for all circular fields $A < (k/X_a)^{1/2}$ we find a collapsed $+1$ all circular disclination [9]. This becomes a planar all circular singular structure at a critical value of $A$ given by $(k/X_a)^{1/2}$. On this structure we can now impose a twist-bubble or an inplanar bend-bubble domains. We consider here the case of twist bubble domain where the boundary conditions are:

$$\theta = -\pi/2 \text{ at } r = 0 \text{ & } \theta = \pi/2 \text{ at } r = \infty$$

the equations of equilibrium

$$k[\nabla^2 \theta + \sin \theta \cos \theta (\nabla \phi)^2] - X_a A^2/r^2 \sin \theta \cos \theta \sin^2(\phi - \alpha) = 0$$

and

$$k(\nabla^2 \phi) + X_a A^2/r^2 \sin(\phi - \alpha) \cos(\phi - \alpha) = 0$$
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yield the following solutions.

\[ \phi = \alpha + \pi/2 \quad \& \quad \theta = 2 \tan^{-1}[r/r_\theta]^{\eta} - \pi/2 \]

where \( \eta = [\chi_\alpha A^2/k - 1]^{1/2} \)

The energy of the bubble domains in this case is \( 4\pi k \eta + E_c \), where \( E_c \) is the energy of the singular core.

It should be mentioned in passing that although it is also possible to have such bubble domains in a uniform \( H_z \) field, an energy analysis is not possible in a linear theory of elasticity.

To conclude, we notice that effect of an external field on defects are non trivial. And in some cases the field can induce some defects. In either case we find not only interesting but in some cases even unexpected results.
References


