Derivation of equation for extent of reduction of the partially reduced product in the direct oxide electrochemical reduction of UO$_2$

The term ‘extent of reduction ($E_R$)’ should essentially imply the percentage of original UO$_2$ that is reduced to metallic U after subjecting it to electro-reduction. It means the extent of reduction will be expressed in terms of UO$_2$ only.

$$E_R = \left( \frac{W_R}{W_T} \right) \times 100\%$$

$W_R$ = Wt. of UO$_2$ corresponding to wt. of electro-generated U metal ($W_U$ g) and $W_T$ = Wt. of the reduced product taken ($W_P$) expressed in terms of wt. of UO$_2$ = wt. of unreduced UO$_2$ in the electro-reduced product + wt. of UO$_2$ corresponding to $W_U$ g of U metal. The above equation shows the percentage of the total UO$_2$ of the sample that was converted to U metal.

Wt. of UO$_2$ in the electrolysed product = $W_P - W_U$

Where, $W_P$ is the weight of the electrolysed product taken and $W_U$, the wt. of U metal estimated in the electrolysed product by MEHS method.

Now, 270 g of UO$_2$ contains 238 g of U metal

$W_R$ = Wt. of UO$_2$ corresponding to $W_U$ g of U metal = $W_U \times (270/238) = 1.1344 \times W_U$

$$E_R = \left( \frac{W_R}{W_T} \right) \times 100\%$$

$$= \left\{ \frac{\text{wt. of UO}_2 \text{ corresponding to } W_U \text{ g of U metal}}{\text{wt. of unreduced } \text{UO}_2 \text{ in the electro-reduced product + wt. of UO}_2 \text{ corresponding to } W_U \text{ g of U metal}}} \right\} \times 100\%$$

$$= \left\{ \frac{1.1344W_U}{[W_P - 1.1344W_U]} \right\} \times 100\%$$

Dividing the numerator and denominator by $W_U$, we get

$$E_R = \left\{ \frac{1.1344}{[0.1344 + (W_P / W_U)]} \right\} \times 100\%$$

Dividing the numerator and denominator by 1.1344, we get

$$E_R = \left\{ 1 / [0.1185 + 0.8815 (W_P / W_U)] \right\} \times 100\%$$
Therefore, the extent of reduction of UO$_2$ after electrolysis ($E_R$) is given as

$$E_R = \{1 / [\alpha + \beta (W_p / W_U)]\} \times 100\%$$

Where $\alpha$ and $\beta$ are constants with values as 0.1185 and 0.8815 respectively.