Abstract

The present thesis embodies the researches carried out at the Aligarh Muslim University, Aligarh. The thesis has been divided into seven chapters. Besides the introductory first chapter, second chapter deals with Jacobi type and Gegenbauer type generalization of certain polynomials, third and fourth chapters concern with $q$-analogues of the Jacobi type and Gegenbauer type generalized polynomials and certain classical polynomials respectively. The last three chapters i.e. fifth, sixth and seventh deal with the generalization of the exponential operators obtained by Dattoli and his collaborators.

Each chapter is divided into a number of sections. Definitions and equations have been numbered chapter wise. The section number is followed by number of equation e.g. (4.3.2) refers to equation number 2 of section 3 of chapter 4.

A brief review of some important special functions, the classical orthogonal polynomials, some generating functions, integral transformations, exponential shift operators, Hermite Kampé de Fériet polynomials, basic hypergeometric series, basic numbers, $q$-analogues of some polynomials, $q$-analogues of some identities and the definition and notations that commonly arise in practices are given in chapter-I.

Chapter-II considers Jacobi type and Gegenbauer type generalization of certain polynomials and their generating functions. Relationships among generalized polynomials have also been included.
Chapter-III refers to $q$-analogues of Jacobi type and Gegenbauer type generalization of certain polynomials studied in chapter-II. Moreover, $q$-analogues of their generating functions have also been established.

In chapter-IV, $q$-analogues of Bateman’s polynomials, Pasternack’s polynomials, Shively’s pseudo-Laguerre polynomials, Cesàro polynomials, Gottlieb polynomials, generalized Hypergeometric polynomial set obtained by S. D. Bajpai and M. S. Arora, have been studied. Further $q$-analogues of their generating functions have also been derived.

Chapter-V deals with the generalization of exponential operators used by Dattoli and Levi for translation and diffusive operator which were utilized to establish analytical solutions of difference and integral equations. The generalization of their technique is expected to cover wide range of such utilization.

The aim of Chapter-VI is to introduce and use the generalized exponential shift operators, operators on the base $a$ ($a > 0, \neq 1$), to deal with the families of Pseudo-Kampé de Fériet polynomials, which can be viewed as the natural complement for the theory of fractional derivatives and partial fractional differential equations of evolutive type. We show that these families allow the possibility of treating a large variety of exponential operators, operators on the base $a$ ($a > 0, \neq 1$), providing generalized fractional forms of shift operators.

The objective of the last chapter is to introduce the generalized exponential operators and to use it for dealing with the families of partial differential
equations of evolution type and to treat the problems involving fractional operators. Further, the properties of the families of special polynomials or special functions (like the Riemann $\zeta$ function) are naturally associated with the proposed formalism.

In the end an exhaustive and up to date list of writings and original papers of the subject matter of this thesis have been provided in the form of a bibliography.