CHAPTER 4

Linearly Extensible Cube Network

The rationale for using multiprocessor is to create powerful computers by simply connecting multiple processors (nodes). The demand for higher computation speed and the signs of saturation in integrated circuit technology has given a flip to the development in multiprocessor systems. The multiprocessor approach to parallelism is the most generalized and flexible one, but to great extent its success depends on the interconnection topology. In this approach, multiple nodes are used to work in parallel for a given program and reduce the total execution time. Multiple processors in such a system are attached to interconnection network. There are a large number of choices for interconnection networks, such as crossbar, butterfly, mesh, ring, multistage ring, Torus Ring, tree, hypercube, hypernet, completely connected, de Bruijn, LET and so on [Ganeshan and Pradhan, 1993], [Hamid and Hall, 1994], [Abdel and Khaled, 1998], [Rafiq et al., 1999], [Yoo et al., 2000], [Kwai and Parhami, 2004]. There has been a lot of research for designing the appropriate topology of interconnection networks for massively parallel computer systems. However, there is no consensus on the best network organization [Kim and Veidenbaum, 1999]. Many large-scale multiprocessor systems have been developed with their own topologies [Kwak and Jhon, 2007]. Nevertheless, the basic goal is that a particular topology should have excellent properties such as regularity, scalability, small diameter, high connectivity, high fault tolerance,
low degree and small link complexity. Therefore, the choice of the topology of the interconnection network is critical in the design of a traditional parallel system and it may affect the overall system performance.

Study of parallel computer interconnection topology has emphasized wide use of cube based topologies. A variety of cube based multiprocessor networks have been reported, which exhibit the excellent properties of such multiprocessor systems such as regularity, symmetry, small diameter, low degree and good scalability [Esfahanian and Sagan, 1993], [Ghose and Desai, 1995], [Zhang, 2002]. Many interconnection networks such as trees and multidimensional meshes can be embedded in the cube. Motivated from the above discussion, a new multiprocessor network named as Linearly Extensible Cube (LEC) network has been proposed which exhibits the desirable properties of similar topological networks.

In this chapter, an analysis of the said network, its various properties and a brief comparison with other existing network such as hypercube, de Bruijn and Linearly Extensible Tree (LET) has been given in tabular form.

4.1 Multiprocessor Interconnection Networks

A suitable interconnection network is an integral part of any massively parallel system. The network is often modeled as undirected or directed graphs. The nodes (vertices) of such a graph represent the processing elements and the edges (arcs) denote the bidirectional communication channels/links. The length of a path between two nodes is the number of edges encountered in the path. The diameter of a network is the largest distance between two nodes. The degree of a network is the largest degree of all nodes in the network. Extensibility is the property which facilitates constructing large-sized systems out of small-sized systems with minimum changes in the configuration of a node of the system. Interconnection topologies are evaluated in terms of small diameter, low degree, simple extensibility and high fault tolerance. The
bisection width is another parameter to assess the performance of the network. It is the minimum number of edges required to be remove when a given network is cut into two halves. A high bisection width is desirable in the interconnection network. Many of these parameters make contradictory demands and therefore, a compromise is there in the design of the network. As the proposed network is a modification of the hypercube network hence a brief description of the hypercube network is given first in the next section.

4.2 Hypercube Network

The hypercube represents a class of message-passing architectures using cube (or exchange) interconnection topology. Hypercube networks are some of the first and most successful commercial multiprocessors. Each node in this network is connected through bidirectional, asynchronous point-to-point communication links to other nodes.

An n-dimensional hypercube multiprocessor consists of \( N = 2^n \) processors. Each processor is labeled by a different n-bit binary number \((b_{n-1} b_{n-2} \ldots b_1 b_0)\). Two processors are connected directly by a link if, and only if, their binary labels, differ exactly one bit position. The connection scheme places the processors at the vertices of an n-dimensional cube. Hypercube interconnection networks for different dimensions from 1 to 4 are shown in Figure 4.1. The hypercube has the property that it can be defined inductively. A hypercube of order 0 is a single node, and the hypercube of the order \( n+1 \) is constructed by taking two hypercubes of order \( n \) and connected their respective nodes. Figure 4.1 shows the hypercube architectures for \( n = 1, 2, 3, \) and 4. A one-cube architecture has \( n = 1 \) and \( 2^n = 2 \) nodes interconnected by a single path. A two-cube architecture has \( n = 2 \) and \( 2^n = 4 \) nodes interconnected as a square. A three-cube architecture has eight nodes interconnected as a cube and so on.
Figure 4.1: Hypercube interconnections
The reason for the popularity of the hypercube network can be attributed to its topological properties. Some important properties of this interconnection used in parallel computers are given below [Saad and Shultz, 1988], [Ganeshan and Pradhan, 1993], [Mano, 2003], [Hesham and Mustfa, 2005]:

1. The diameter of hypercube with \(2^n\) nodes is \(n\) i.e. the logarithmic of the number of nodes in the network. The node degree (number of edges per node) of hypercube is also equal to \(n\). The diameter and the node degree increase as the number of nodes in the hypercube network increase. The hypercube has a high bisection width \(b = 2^{n-1}\) and has good capability of fault tolerance.

2. A hypercube is a super set of other interconnection networks such as rings, multistage cube networks, meshes, trees etc. because these can be embedded into a hypercube by ignoring some hypercube connections.

3. Hypercubes have simple routing schemes. A message-routing policy may send a message to the neighbor whose binary tag agrees with the tag of the final destination in the next bit position, with the bits scanned in some order. The path length for sending a message between any two nodes is exactly the number of bits in which their tag bits differ. Numerous possible paths connecting any two nodes exist in the network, which produce a large communication bandwidth. For example, referring to Figure 4.1, in a three cube structure (\(n=3\)), node 000 can communicate directly with 001. It must cross at least two links to communicate with 011 (from 000 to 001 to 011 or from 000 to 010 to 011). Similarly, it is necessary to go through at least three links to communicate from node 000 to node 111 [Hwang and Briggs, 1985], [Bhuyan and Agrawal, 1984], [Mano, 2003].

4. The hypercube has poor scalability and it is difficult to package higher-dimensional hypercubes. In other words, when adding some few nodes
to the network, the network size must be duplicated to reach to the next specified network size [Ghose and Desai, 1995].

4.3 Linearly Extensible Cube (LEC) Multiprocessor Network

4.3.1 Design and Analysis

The LEC network grows linearly in a cube like shape. The network itself is recursively connected and is defined through connection functions in a manner similar to that of cube connection. Let Q be a set of N identical processors, represented as

\[ Q = \{P_0, P_1, P_2, \ldots, P_N\} \]

The number of processors N in the network is given by

\[ N = 2^n \]  \hspace{1cm} (4.1)

Where, \( n \) is the level or depth of the network \((n \in \mathbb{Z} \text{ and } n > 0)\). For different levels, network is having even numbers of processors. For \( n = 1 \), an LEC architecture of two processors interconnected by a single path can be obtained. For second level \((n = 2)\), the network is having \( 2^2 = 4 \) interconnected processors. Similarly, for \( n = 3 \), the LEC network has \( 2^3 = 8 \) interconnected processors.

In order to define the link functions we denote each processor in the set Q as \( P_n \), \( n \) being the level in server where the processor \( P_i \) resides. As per extension policy, only two processors exist at level \( n \). Thus at level 1, \( P_0 \) and \( P_1 \) exist and at level 2, \( P_2 \) and \( P_3 \) exist and so on. The arrangement is shown in Figure 4.2.
Figure 4.2: Arrangement of processors in LEC network

\[
\begin{array}{cc}
P_{01} & P_{11} \\
P_{22} & P_{32} \\
P_{43} & P_{53} \\
\end{array}
\]

Let \( Q' \) be the set of designated processors of \( Q \), thus

\[ Q' = \{P_i\}, \quad 0 \leq i \leq N-1 \]

The link function \( L_1 \) and \( L_2 \) define the mapping from \( Q' \) to \( Q \) as

\[
L_1 (P_i) = P_{(i+1) \mod N} \\
L_2 (P_i) = P_{(i+2) \mod N}, \text{ for all } P_i \text{ in } Q'
\] …… (4.2)

The two functions \( L_1 \) and \( L_2 \) in Equation (4.2) indicate the links between various processors in the network. These link functions can also be demonstrated by the adjacency matrix of order \( N \times N \), where \( N \) is the number of processors. Figure 4.3 (a) and Figure 4.3 (b) show the proposed network for six processors and its adjacency matrix respectively, where ‘1’ indicates a connection and ‘0’ indicates no connection between nodes.
Figure 4.3 (a): The LEC architecture with six processors

Figure 4.3 (b): Adjacency matrix for Figure 4.3 (a)
4.3.2 Properties of the LEC Network

Here some properties of the LEC network have been compared with hypercube, de Bruijn [Samathan and Pradhan, 1989], [Quinn, 2002] and LET [Rafiq et al., 1999] networks. These properties help to understand the effectiveness of a particular organization. The various properties are:

- **Number of Nodes (N):** The number of nodes in a multiprocessor network plays an important role to evaluate the performance of a multiprocessor system. Lesser the number of nodes, lesser is the system complexity and it is more economical. Therefore, number of nodes should be optimal. The number of nodes in LEC network is \( N = 2^n \) for \( n > 0 \), whereas, the number of nodes in the hypercube and de Bruijn network is \( 2^n \). In LET network, \( N = \sum_{k=1}^{n} k \) where, \( n \) is the depth or level of the network. Due to lesser number of processors in the LEC network, it may be considered more economical than other networks.

- **Diameter (D):** The diameter of a network is the measure of the maximum inter-node distance in the network. This property is important in determining the distance involved in communication and hence the performance of multiprocessor systems. In simple words diameter of a network is the maximum shortest path between source and destination node. The diameter of a network is bound to increase as the size grows unless there is no limit on the number of links.

In the case of de Bruijn, hypercube and LET the diameter increases by one as the number of processors is doubled. Ignoring the fold-back connections the diameter of the LEC network also increases by one on each extension. Table 4.1 shows, the diameter of various multiprocessor networks for different levels. These results have been obtained using shortest path algorithm. In case of LEC network, it is observed that the diameter does not always increase with
the addition of a layer of processors. It may be highlighted that the diameter of LEC shows a maximum value of 10 for 40 processors, which is lesser in comparison to other networks as given in Table 4.1.

The diameter in hypercube and in de Bruijn networks increases linearly by increasing the number of processors. In case of LEC and LET, the increment in diameter is not linear, however LEC has smaller diameter as compared to LET. This trend is depicted in Figure 4.4.

Figure 4.4: Comparison of Diameter of different multiprocessor networks
Table 4.1: Diameter of various sized multiprocessor network

<table>
<thead>
<tr>
<th>Number of processors</th>
<th>Level</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>(In LEC)</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
<td>14</td>
<td>16</td>
<td>18</td>
<td>20</td>
<td>22</td>
<td>30</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>(In LET)</td>
<td>1</td>
<td>3</td>
<td>6</td>
<td>10</td>
<td>15</td>
<td>21</td>
<td>28</td>
<td>36</td>
<td>45</td>
<td>55</td>
<td>66</td>
<td>78</td>
<td>136</td>
<td>231</td>
<td></td>
</tr>
<tr>
<td>(In Hypercube and in de Bruijn)</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>16</td>
<td>32</td>
<td>64</td>
<td>128</td>
<td>256</td>
<td>512</td>
<td>1024</td>
<td>2048</td>
<td>32768</td>
<td>1048576</td>
<td></td>
</tr>
<tr>
<td>Diameter</td>
<td>(In LEC)</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>6</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>(In LET)</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>8</td>
<td>9</td>
<td>12</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>(In Hypercube and in de Bruijn)</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>15</td>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>
• **Degree (d):** The degree or connectivity of a node in a network is defined as the number of connections required at each node. The connectivity of the nodes determines the complexity of the network. The higher the connectivity, the higher the hardware complexity and hence the cost of the network. Therefore, the node degree should be kept as low, as possible in order to reduce cost. It is best if the number of edges per node is a constant independent of the network size, because in that case the processor organization scales more easily to systems with large number of nodes. The degree of node in the proposed network is always 4. The connectivity of LET and of de Bruijn networks is also 4 or less, whereas, the connectivity of hypercube increases with the size.

• **Extensibility:** It is the property which facilitates large sized system out of small ones with minimum changes in the configuration of the nodes. It is the smallest increment by which the system can be expanded in a useful way. In the proposed network, the number of processors increases in a constant manner because each extension requires single layer of 2 nodes and no additional node is required at any extension. If we compare the extensibility of LET, the extension complexity increases linearly because each extension requires adding a single layer of \((n+1)\) nodes. Therefore, at higher levels, the number of nodes becomes large and complexity may increase. Similarly, the hypercube and de Bruijn networks though are extensible but the complexity increases exponentially by the power of 2. The constant growth of the proposed network makes the extension less costly. Besides, the LEC network can be extended in two directions, vertically upward and vertically downward and a chain of the network could be formed which is not available in the case of other networks. Figure 4.5 shows the extensibility of LEC network in both the directions.
Figure 4.5: Extensibility of LEC network
• **Bisection Width (b):** The bisection width of a network is the minimum number of edges that must be removed in order to divide the network into two halves (within one). High bisection width is better, because in algorithms requiring large amounts of data movements, the size of the data set divided by the bisection width puts a lower bound on the complexity of the parallel algorithm. The hypercube and de Bruijn networks have a high bisection width equal to $2^{n-1}$ and $2^n/n$ respectively. The LET network has a bisection width equal to $2\log_2(n-2)$, whereas, the LEC has a bisection width equal to $N$.

• **Fault Tolerance:** As more and more processors are incorporated into parallel machines, the size and the complexity of the network increases. If a fault in such a complex system occurs, it is required that all the active processors should remain as a connected component and be able to undertake part in significant parallel computation. In the proposed network, the bisection width is directly proportional to the number of processors available in the network. Therefore, the bisection width also increases with the increase in the network size. Thus, the proposed LEC network has a better capability of fault tolerance.

Table 4.2 summarizes the comparison of the above characteristics of various processor interconnection networks, which shows the superiority of the proposed network.
Table 4.2: Summary of parameters for various multiprocessor networks

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Hypercube</th>
<th>de Bruijn</th>
<th>LET</th>
<th>LEC</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of processor</td>
<td>$N=2^n$</td>
<td>$N=2^n$</td>
<td>$N=\sum_{k=1}^{n} k$</td>
<td>$N=2*n$</td>
</tr>
<tr>
<td>Degree</td>
<td>$n$</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Extensibility</td>
<td>$2^n$</td>
<td>$2^n$</td>
<td>$n+1$</td>
<td>2</td>
</tr>
<tr>
<td>Diameter</td>
<td>$O(\log_2 n)$</td>
<td>$O(\log_2 n)$</td>
<td>$O(\sqrt{N})$</td>
<td>$\left\lfloor N \right\rfloor$</td>
</tr>
<tr>
<td>Bisection Width</td>
<td>$2^{n-1}$</td>
<td>$2^n/n$</td>
<td>$2\log_2(n-2)$</td>
<td>$N$</td>
</tr>
</tbody>
</table>

In conclusion, it may be said that a new network topology (LEC) for multiprocessor systems has been proposed as an attempt to combine some desirable features of linearly extensible structures and compact hypercube or de Bruijn structures. The proposed architecture exhibits better connectivity, lesser number of nodes, lesser diameter and a constant extension of two nodes at each level over hypercube and de Bruijn networks. Therefore, the proposed LEC architecture may be considered as a low cost multiprocessor architecture.

In the next chapter, a dynamic scheduling scheme is described which takes into account the adjacency matrix of network interconnections for migration of load on the various processors of the network. To evaluate the performance, the proposed scheme is implemented on LEC by simulating different types of load. The performance of LEC network is compared by implementing the proposed scheme on other multiprocessor architectures discussed above.