HYSTERESIS IN FERROMAGNETIC RANDOM FIELD ISING, XY AND HEISENBERG MODELS

(ABSTRACT)

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Abstract

Hysteresis is caused by the delay in responding to a changing force. The response of a system subjected to a cyclic force is different in increasing force than it is in decreasing force. This makes a hysteresis loop. The area of the hysteresis loop is a measure of the energy dissipated in each cycle of the driving field. It is a quantity of practical as well as theoretical interest. It depends on the period $\tau$ of the driving field as well as the nature of the material being driven. In simple situations it is often possible to characterize a material by a typical relaxation time $\tau_{\text{relax}}$ but it is fixed for a material and we do not have much control over it. We can choose $\tau$ in a hysteresis experiment. We are interested in the limit when the frequency of the driving field $\omega \to 0$, or equivalently $\tau \to \infty$. This is of course a theoretical limit. Realistically $\tau$ cannot exceed the maximum time available to the experimentalist. If $\tau \gg \tau_{\text{relax}}$ the system is able to keep up with the changing field and the area of the hysteresis loop is negligibly small. However there are materials that cannot be characterized by a single $\tau_{\text{relax}}$ but rather a spectrum of relaxation times that diverge on practical time scales. It is not uncommon to find such materials. Most materials with quenched disorder fall in this category. The quenched disorder creates a large number of metastable states in the system. These are separated from each other by large barriers [1, 2]. The relaxation time (escape from a metastable state) in these materials diverges exponentially with the ratio of the barrier height to the thermal energy of the system. The distribution of barrier heights translates into a distribution of relaxation times. A spectrum of diverging relaxation times means that these materials show hysteresis even in the limit $\omega \to 0$.

Sethna et al [3] introduced the non-equilibrium random-field Ising model to study hysteresis and other phenomena such as return point memory and Barkhausen noise.
in disordered ferromagnets at zero temperature and in the limit $\omega \to 0$. The disorder in their model is characterized by on-site random fields having a Gaussian distribution with average value zero and standard deviation $\sigma$. They employed the zero temperature Glauber dynamics of Ising spins i.e. a spin flips only if it is not aligned along the net field at its site. It is an iterative dynamics. If a spin flips it changes the net field on its neighbors and may cause them to flip as well. This may result in an avalanche of spin flips. The dynamics is applied till each spin in the system is stable. It is assumed that the applied field $h$ changes infinitely slowly as compared with the time (i.e. the number of iterations) that the system takes to reach a stable state. This is implemented in the model by holding $h$ constant during an avalanche. This model has been studied extensively using mean field theory, renormalization group, and numerical simulations.

Exact solutions of the model have also been obtained in one dimension [4, 5, 6] and on a Bethe lattice of coordination number $z$ [7, 8]. Numerical work suggests that in three and higher dimensions, the model exhibits a non-equilibrium critical point at $\sigma = \sigma_c$ and $h = h_c$. If $\sigma < \sigma_c$, there is a first order jump in the magnetization on each half of the hysteresis loop. As $\sigma$ increases, the jumps decrease in size and move towards $h = 0$. The size of the jump goes to zero continuously at the critical point and the hysteresis loop is smooth for $\sigma > \sigma_c$. The non-equilibrium critical point shows scaling and universality similar to the critical-point phenomena in equilibrium systems. Exact results on a Bethe lattice show the absence of a critical point if $z \leq 3$.

The work presented in this thesis is inspired and motivated by the phenomena and the model mentioned above. As in the earlier studies, we restrict to hysteresis at zero temperature and zero frequency of the driving field. However we work with Ising as well as $XY$ and Heisenberg models. We have obtained a few exact results for hysteresis in these models in the mean field limit. We have also performed numerical simulations
in order to check our analytic results and compare these with the behavior of the models on simple cubic lattices. The outline of the thesis is as follows.

Chapter 1 contains a brief introduction to hysteresis and places the present study in the broad context of earlier studies in the field of disorder driven hysteresis.

In Chapter 2, we study hysteresis in the case when a fraction of sites on the lattice are not occupied by Ising spins. We call this the dilute random field Ising model and obtain exact expressions for hysteresis loops on a Bethe lattice. This is an extension of the earlier work without dilution \[7, 8\]. The qualitative behavior of the dilute model is similar to that of the non-dilute model if the dilution is not too strong. In the limit of extreme dilution, the system breaks up into isolated clusters of spins. In this limit there is no possibility of avalanches that span across the system and consequently no possibility of macroscopic jumps in the magnetization, and no possibility of critical behavior. However an interesting effect is seen in this limit. The hysteresis loops acquire a wasp-waisted shape. This is essentially a result of the surface effects of the isolated clusters. We discuss the relevance of the dilute random field model in understanding some hysteresis experiments in geological rocks \[9, 10, 11\].

Chapter-3 is devoted to hysteresis in the random field XY and Heisenberg models \[12\]. We also perform numerical simulations to check our analytic results and to examine the behavior of the models on simple cubic lattices. The quenched random fields are taken to be randomly oriented vectors of a fixed length. Exact expressions for hysteresis loops are obtained in the mean-field limit. The results are somewhat unexpected and surprising at first sight. The reason is that this problem has been studied earlier by Silveira and Kardar \[13\] using the renormalization group approach. They used a slightly different variant of the random field distribution. One may think that
the form of quenched disorder may not alter the behavior of the model in any drastic fashion. However, we find that perhaps it does. Silveira and Kardar find critical-point hysteresis in $XY$ and Heisenberg models similar to the one in the Ising model. We find that the three models are strikingly different. The Ising model has a critical point. The $XY$ model has only a first order phase transition and no critical point. The $XY$ model also has unusual wasp-waisted hysteresis loops similar to those seen in the case of extremely dilute random field Ising model. These features are also born out qualitatively in the simulations on a simple cubic lattice. The Heisenberg model has a peculiar critical point quite unlike the one in the Ising case. Do these differences arise from different variants of the random field distribution used in the two studies? Or does the discrepancy originate somewhere else? Our mean field results are based on the limit of infinitely long-range but infinitely weak interactions between spins. We compare these results with the results of the renormalization group theory in six and higher dimensions when all the terms in the action other than the quadratic term are irrelevant. The renormalization group theory above the upper critical dimension (six for the non-equilibrium random field model) is also known as the mean field theory but there appears to be no reason why the two mean field theories should agree with each other.

Chapter 4 is devoted to resolving the discrepancy mentioned in Chapter 3. In chapter 3 we considered random fields in the form of randomly oriented vectors of a fixed length $a$. We now relax this restriction and consider randomly oriented vectors of variable lengths $\{|\vec{a}_i|\}$ [14]. We consider two distributions for $\{|\vec{a}_i|\}$: (i) each cartesian component of $\vec{a}_i$ has a Gaussian distribution with average value zero and standard deviation $\sigma$, and (ii) $|\vec{a}_i|$ has a uniform distribution in the interval 0 to $\Delta$. We obtain exact expressions for the hysteresis loop for both distributions and also examine critical
point hysteresis in each case. In the case of the Gaussian distribution we recover the results of Silveira and Kardar in six and higher dimensions. We find that the uniform bounded distribution in case (ii) also gives the same critical behavior as the Gaussian distribution. This is somewhat surprising in view of the fact that the two distributions produce different critical behavior for the hysteresis in random field Ising model [15]. At present we do not fully understand why a uniform bounded distribution and an unbounded Gaussian distribution should yield the same critical behavior in the case of $XY$ and Heisenberg spins. However some possible reasons for this are discussed.

Chapter 5 contains some concluding remarks.